

**NP-3602 (CV-III)****B.Sc. (Computer Science)****Examination, Dec.-2021****DISCRETE STRUCTURE****(BCS-301)***Time : 1 1/2 Hours ]**[Maximum Marks : 75*

**Note :** Attempt **all** the sections as per instructions.

**Section-A**

**Note :** Attempt any **two** questions. Each question carries **7.5** marks.

$$2 \times 7.5 = 15$$

1. Consider the function  $f: \mathbb{N} \rightarrow \mathbb{N}$ , recursively given by  $f(0)=1$  and  $f(n+1)=2-f(n)$ . Find  $f(10)$ .
2. How many Lattice paths start as  $(3, 3)$  and end at  $(10, 10)$ ?

**P.T.O.**

9. (a) Prove that the maximum number of nodes in a binary tree of depth  $\alpha$  is  $2^{\alpha}-1$ , where  $\alpha \geq 1$ .
- (b) Show that the set  $M$  of all elements  $\{0, 1, 2, 3, 4, 5\}$  with addition modulo 6 and multiplication modulo 6 as a composition is a ring with zero divisors.

**OR**

Show that the set  $M$  of all matrices of the form  $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$  where  $n \in \mathbb{Z}$  is a semigroup under multiplication and it is isomorphic to  $(\mathbb{Z}, +)$ .

10. (a) Prove that product of two lattices is a lattices.
- (b) State and prove DeMorgan's law of Boolean Algebra.

**NP-3602(CV-III)/3****P.T.O.**

3. Find the order of the elements of  $(\mathbb{Z}_8, +_8)$
4. Define multi graph with example.
5. Use a membership table to show that:  
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

### Section-B

**Note :** Attempt any **one** question out of the following three questions. Each question carries 15 marks.  $1 \times 15 = 15$

6. Show that there are only two non isomorphic groups of order 4.
7. Show that in a Boolean algebra, the idempotent laws  $X \vee X = X$  and  $X \wedge X = X$  holds for every element X.
8. Give an example of a graph which is Hamiltonian but not Eulerian and vice-versa.

### Section-C

**Note :** Attempt any **two** questions out of the following five questions. Each question carries 22.5 marks.

$$2 \times 22.5 = 45$$

11. (a) Define K-regular graph. Give example of 2 regular, 3-regular, 4-regular graphs.

(b) Show that the function  $f(x, y) = x + y$  is primitive recursive.

(c) Use mathematical induction to show that

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

12. (a) State and prove pigeonhole principle.

(b) Define minimum spanning tree.

(c) Negate the statement: Every city in Canada is clean.

13. (a) Prove that a simple graph has a spanning tree iff it is connected.

(b) If a group has four elements, show that it must be abelian.