A

(21119)

Roll No. ....

Total Questions : 13] [Printed Pages : 4

NP–3573

B.Sc. (Computer Science) Ist Semester Examination, Nov., 2019

APPLIED MATHEMATICS–I

(BCS–102)

Time : 3 Hrs.] *[M.M. : 75*

Note :– Attempt questions from all Sections as per instructions.

Section–A 3×5=15

Note :– Attempt all *five* questions.

1. Find the rank of the matrix :

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 6 \\ 2 & 3 & 4 \end{bmatrix}$$

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2. Verify Euler's theorem for :

$$u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$$

3. Find Jacobian $J\left(\frac{u, v}{x, y}\right)$ for $u = x^2$, $v = y^2$.

4. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

5. Show that the vector $\vec{V} = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$ is solenoidal.

Section–B 7½×2=15

Note :– Attempt any *two* questions.

6. Find the eigen values and corresponding eigen vectors of the matrix :

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

7. If $y = a\cos(\log x) + b\sin(\log x)$, then show that :

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

8. A fluid motion is given by

$$\vec{V} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}.$$

Show that the motion is irrotational and hence find the velocity potential.

Section–C 15×3=45

Note :- Attempt any *three* questions.

9. Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

and hence find A^{-1}.

10. (a) Expand $\sin(xy)$ in powers of $(x - 1)$ and $\left(y - \frac{\pi}{2}\right)$ as far a terms of second degree.

(b) Change the independent variable x to z in the equation :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2)$$

11. (a) Show that the minimum value of $u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$ is $3a^2$.

(b) Find the point upon the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a minimum value and find this minimum f.

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12. Prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma (m+n)}$ also, show that :

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta \, d\theta = \frac{\Gamma\frac{p+2}{2} \times \Gamma\frac{q+1}{2}}{2\Gamma\frac{p+q+2}{2}}$$

13. (a) Using Green's theorem, evaluate $\int_C (x^2 y\, dx + x^2 dy)$, where C is the boundary described counter clockwise of the triangle with vertices (0, 0), (1, 0), (1, 1).

(b) Evaluate $\oint_C \vec{F} \cdot \vec{dr}$ by Stoke's theorem, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$, and C is the boundary of the rectangle $x = \pm a$, $y = 0$ and $y = b$.

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