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(20321) Roll No. ......

B.Sc.(Com.Sc.)-I Sem.

NP-3573

B.Sc. (Computer Science) Examination, Dec.-2020

Applied Mathematics-I

(BCS-102)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from **all** the sections as per instructions.

Section-A

Note : Attempt all the **five** questions. Each question carries 3 marks. Very short answer is required not exceeding 75 words. 3×5=15

1. Find the rank of the following matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

P.T.O.

2. Change the independent variable from x to z in the equation

$$\frac{x^4 d^2y}{dx^2} + a^2y = 0 \text{ where } x = \frac{1}{z}$$

3. Write Taylor's expansion for log (1+x)

4. Evaluate $\int_0^{\infty} \frac{x^4(1+x^5)}{(1+x)^{15}} dx$

5. If $f(x,y,z) = 3x^2y - y^3z^2$. Find grad f at the point (1,−2,−1).

Section-B

Note : The section contains three questions. Attempt any **two** questions. Each question carries 7½ marks.

7½×2=15

6. Examine if the system of equations: $x+y+4z=6$, $3x+2y-2z=9$, $5x+y+2z=13$ is consistent.

7. If $u = xyz$, $v = x^2+y^2+z^2$, $w = x+y+z$ find the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

8. Evaluate $\int_{-1}^{1}\int_{0}^{1-x} x^{\frac{1}{3}} y^{\frac{-1}{2}} (1-x-y)^{\frac{1}{2}}\, dy\,dx$

Section-C

Note : This section contains **five** questions. Attempt any **three** questions. Each question carries 15 marks. $15\times3=45$

9. Determine the eigen values and the corresponding eigenvectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

10. If $y=(\sin^{-1}x)^2$, prove that

(i) $(1-x)^2y_2-xy_1-2=0$

(ii) $(1-x^2)y_{n+2}-(2n+1)xy_{n-1}-n^2y_n=0$

11. Find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2+y^2+z^2=1$

12. Prove that

(i) $\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta = \int_0^{\pi/2}\sqrt{\cot\theta}\,d\theta = \frac{\pi}{\sqrt{2}}$

(ii) $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta = \pi$

13. (i) If $F = 2z\hat{i}-x\hat{j}+y\hat{k}$ evaluate $\iiint_v F\,dv$ where v is the region bounded by the surfaces x=0, y=0, x=2, y=4, $z=x^2$, $z=2$

(ii) Evaluate $\iint_s F.n\,ds$ over the entire surface of the region above the xy-plane bounded by the cone $z^2=x^2+y^2$ and the plane $z=4$, if $F = 4xz\hat{i}+xyz^2\hat{j}+3z\hat{k}$