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(21218) **Roll No. ....**

B. Sc. (Com. Sc.)-I Sem.

NP-3573

B. Sc. (Computer Science) Examination, Dec. 2018

Applied Mathematics-I

(BCS-102)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from all Sections as per instructions.

Section-A

(Very Short Answer Questions)

Attempt all the *five* questions. Each question carries 3 marks. Very short answer is required. 3×5=15

1. Find the eigenvalues and eigenvectors of the matrix : 3

$$A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}.$$

2. If $z = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, prove that : 3

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}.$$

3. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, show that : 3

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta.$$

4. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. 3

5. Find the mass of an octant of the ellipsoid :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

the density at any point being $P = kxyz$. 3

Section-B

(Short Answer Questions)

Attempt any *two* questions out of the following three questions. Each question carries 7½ marks. Short answer is required. 7½×2=15

6. (a) If $y^{1/m} + y^{-1/m} = 2x$, prove that : 5

$$(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0.$$

(b) Find the nth derivative of $x^3 e^{ax}$. 2½

7. (a) For what values of λ and μ the following system of equation :

$$2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$$

will have (i) unique solution (ii) no solution. 5

(b) If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, find A^{-1}. 2½

8. (a) Apply Green's theorem to evaluate

$$\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy],$$

where C is the boundary of the area enclosed by the x-axis and the upper half of circle $x^2 + y^2 = a^2$. 5

(b) Find the work done by a force $y_i + x_j$ which displaces from origin to a point $(i + j)$. 2½

Section-C

(Detailed Answer Questions)

Attempt any *three* questions out of the following five questions. Each question carries 15 marks. Answer is required in detail. 15×3=45

9. (a) Find the rank of the following matrix by reducing it to normal form : 7½

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}.$$

(b) Find the characteristics equation of the symmetric matrix :

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix},$$

and hence find A^{-1} by Cayley-Hamilton theorem. 7½

10. (a) If $y = \tan^{-1} x$ prove that :

$$(1 + x^2)y_{n+1} + (2n + 1)xy_{n+1} + n(n+1)y_n = 0$$

hence, determine the values of all derivatives of y w. r. t. x when $x = 0$. 10

(b) If $u = e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$. 5

11. (a) Evaluate $\iint_S (yz\hat{i} + zx\hat{j} + xy\hat{k}).\overline{ds}$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. 7½

(b) If $\overline{F} = 2zi - xj + yk$, evaluate $\iiint_V \overline{F} dV$, where V is the region bounded by the surfaces $x = 0, y = 0,$ $x = 2, y = 4, z = x^2, z = 2.$ 7½

12. (a) If $P^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$, prove that : 10

$$P + \frac{d^2 P}{d\theta^2} = \frac{a^2 b^2}{P^3}.$$

(b) If $x = a(\cos\theta + \theta \sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, find $\frac{d^2 y}{dx^2}$. 5

13. (a) Evaluate $\oint_C \overline{F}.d\overline{r}$ Stokes' theorem, where:

$$\overline{F} = y^2 i + x^2 j - (x + z)k$$

and C is the boundary of triangle with vertices at $(0, 0, 0), (1, 0, 0)$ and $(1, 1, 0)$. 7½

(b) Find $\iint \overline{F}.\hat{n} dS$, where :

$$\overline{F} = (2x + 3z)i - (xz + y)j + (y^2 + 2z)k$$

and S is the surface of the sphere having centre $(3, -1, 2)$ and radius 3. 7½