Mathematics

(Chapter – 6) (The Triangle and its Properties)
(Class – VII)

Exercise 6.1

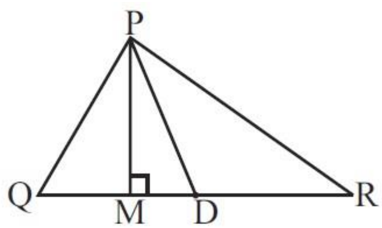
Question 1:

In ΔPQR, D is the mid-point of $\overline{QR}$.

$\overline{PM}$ is ____________

PD is ____________

Is QM = MR?



Answer 1:

Given: QD = DR

$\therefore$ $\overline{PM}$ is altitude.

PD is median.

No, QM $\neq$ MR as D is the mid-point of QR.

Question 2:

Draw rough sketches for the following:

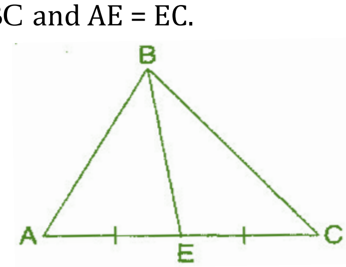
(a) In ΔABC, BE is a median.

(b) In ΔPQR, PQ and PR are altitudes of the triangle.

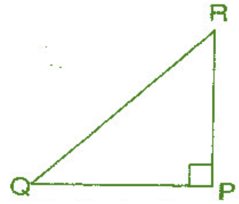
(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.

Answer 2:

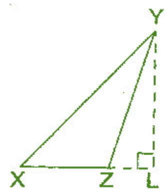
(a) Here, BE is a median in ΔABC and AE = EC.



(b) Here, PQ and PR are the altitudes of the ΔPQR and $RP \perp QP$.



(c) YL is an altitude in the exterior of ΔXYZ.



Question 3:

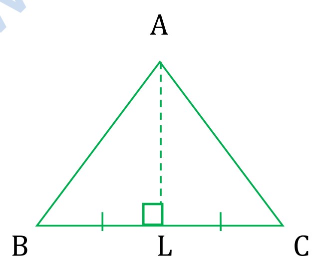
Verify by drawing a diagram if the median and altitude of a isosceles triangle can be same.

Answer 3:

Isosceles triangle means any two sides are same.

Take ΔABC and draw the median when AB = AC.

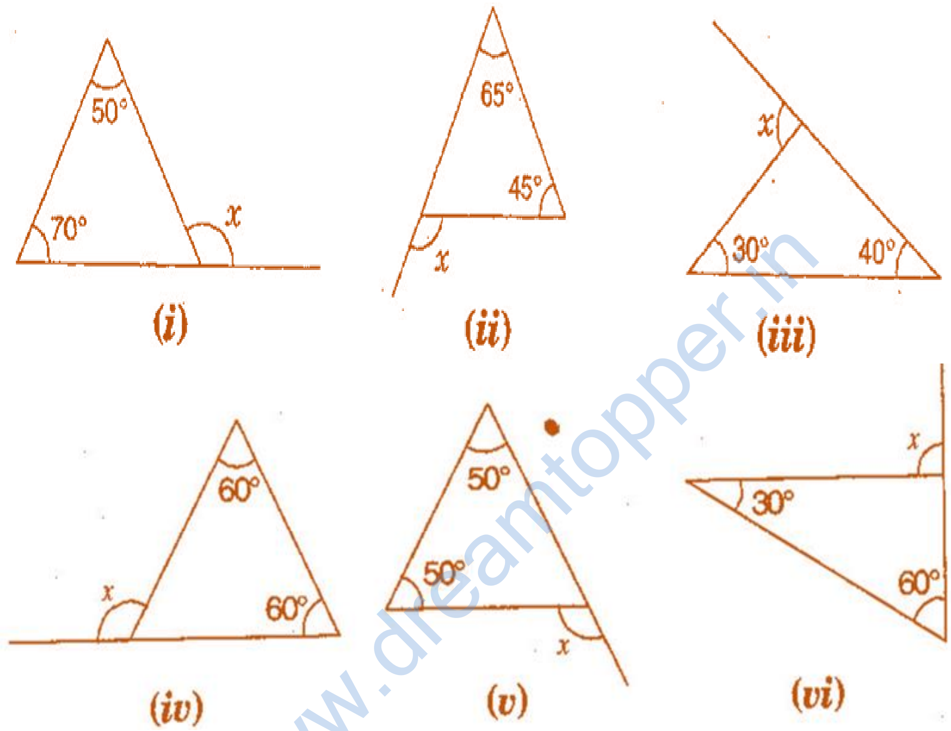
AL is the median and altitude of the given triangle.



Exercise 6.2

Question 1:

Find the value of the unknown exterior angle x in the following diagrams:



Answer 1:

Since, Exterior angle = Sum of interior opposite angles, therefore

(i) $x = 50° + 70° = 120°$

(ii) $x = 65° + 45° = 110°$

(iii) $x = 30° + 40° = 70°$

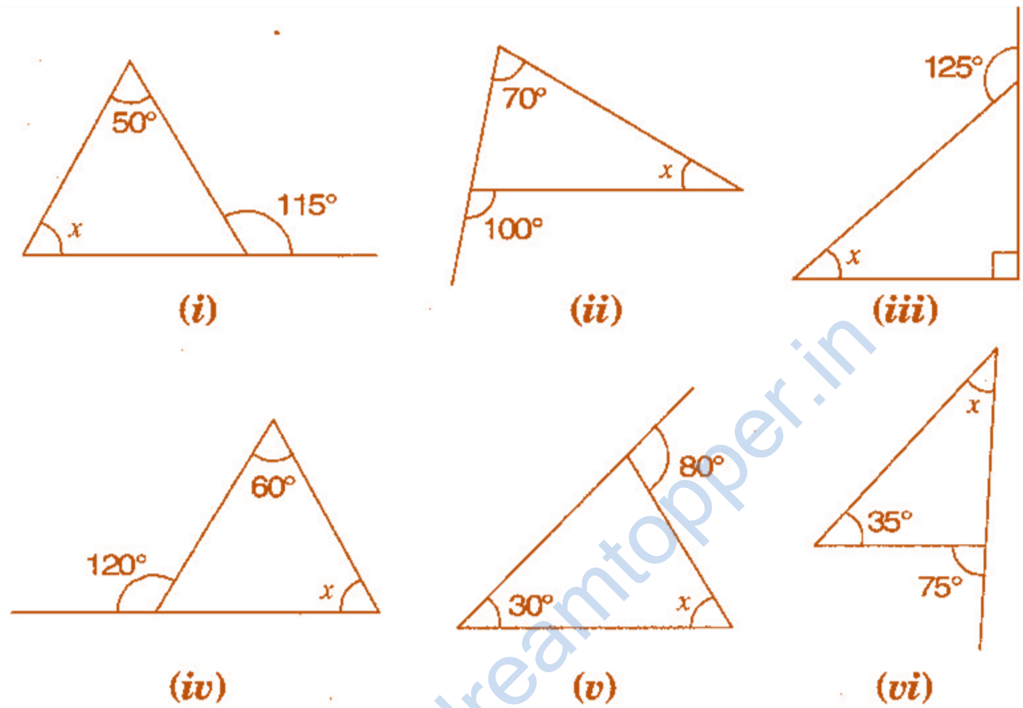
(iv) $x = 60° + 60° = 120°$

(v) $x = 50° + 50° = 100°$

(vi) $x = 60^{\circ} + 30^{\circ} = 90^{\circ}$

Question 2:

Find the value of the unknown interior angle x in the following figures:



Answer 2:

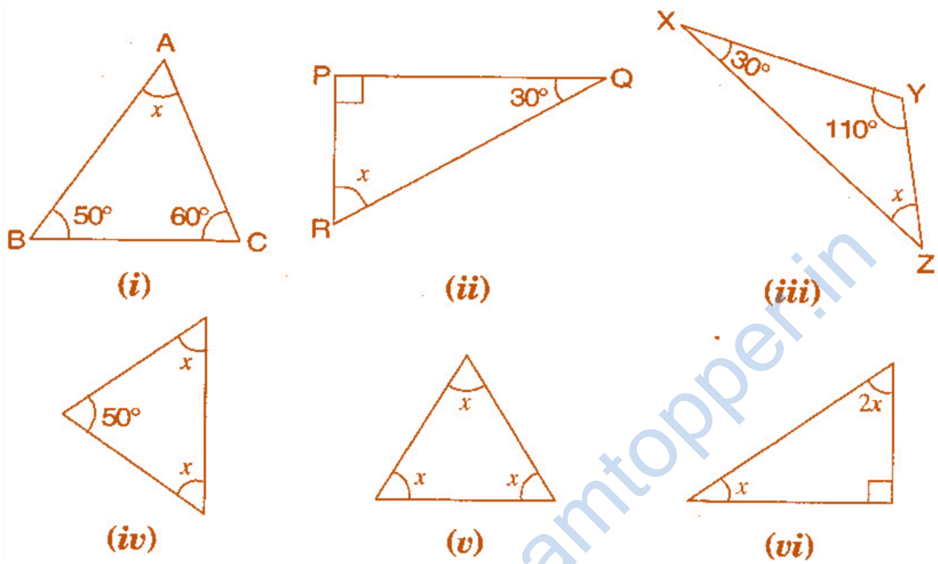
Since, Exterior angle = Sum of interior opposite angles, therefore

(i)	$x+50°=115°$	$\Rightarrow$	$x=115°-50°=65°$
(ii)	$70°+x=100°$	$\Rightarrow$	$x=100°-70°=30°$
(iii)	$x+90°=125°$	$\Rightarrow$	$x=120°-90°=35°$
(iv)	$60°+x=120°$	$\Rightarrow$	$x=120°-60°=60°$
(v)	$30°+x=80°$	$\Rightarrow$	$x=80°-30°=50°$
(vi)	$x+35°=75°$	$\Rightarrow$	$x=75°-35°=40°$

Exercise 6.3

Question 1:

Find the value of unknown x in the following diagrams:



Answer 1:

(i) In ΔABC,

$\angle BAC + \angle ACB + \angle ABC = 180°$ [By angle sum property of a triangle]

$\Rightarrow x + 50° + 60° = 180°$

$\Rightarrow x + 110° = 180°$

$\Rightarrow x = 180° - 110° = 70°$

(ii) In ΔPQR,

$\angle RPQ + \angle PQR + \angle RPQ = 180°$ [By angle sum property of a triangle]

$\Rightarrow 90° + 30° + x = 180°$

$\Rightarrow x + 120° = 180°$

$\Rightarrow x = 180° - 120° = 60°$

(iii) In ΔXYZ,

$\angle ZXY + \angle XYZ + \angle YZX = 180°$ [By angle sum property of a triangle]

$\Rightarrow 30° + 110° + x = 180°$

$\Rightarrow x + 140° = 180°$

$\Rightarrow x = 180° - 140° = 40°$

(iv) In the given isosceles triangle,

$x+x+50^\circ=180^\circ$ [By angle sum property of a triangle]

$\Rightarrow \quad 2x+50^\circ=180^\circ$

$\Rightarrow \quad 2x=180^\circ-50^\circ$

$\Rightarrow \quad 2x=130^\circ$

$\Rightarrow \quad x=\dfrac{130^\circ}{2}=65^\circ$

(v) In the given equilateral triangle,

$x+x+x=180^\circ$ [By angle sum property of a triangle]

$\Rightarrow \quad 3x=180^\circ$

$\Rightarrow \quad x=\dfrac{180^\circ}{3}=60^\circ$

(vi) In the given right angled triangle,

$x+2x+90^\circ=180^\circ$ [By angle sum property of a triangle]

$\Rightarrow \quad 3x+90^\circ=180^\circ$

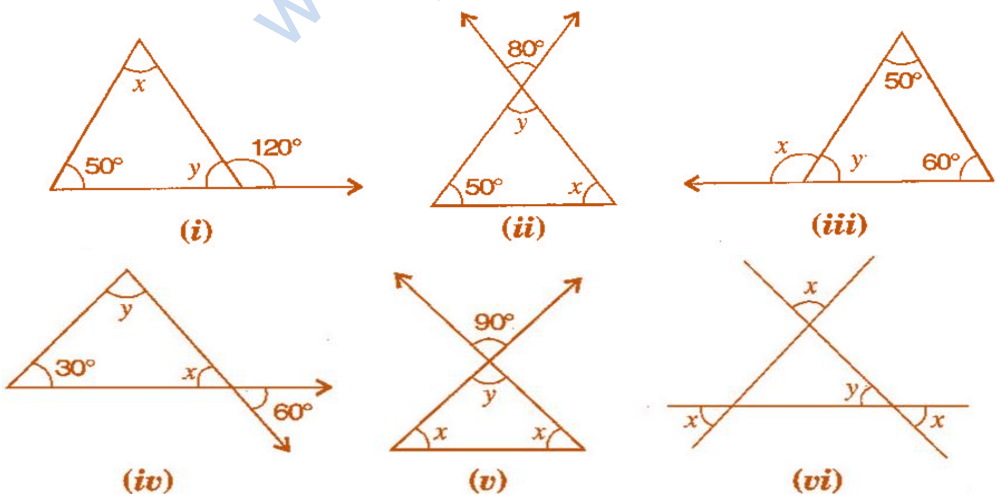
$\Rightarrow \quad 3x=180^\circ-90^\circ$

$\Rightarrow \quad 3x=90^\circ$

$\Rightarrow \quad x=\dfrac{90^\circ}{3}=30^\circ$

Question 2:

Find the values of the unknowns x and y in the following diagrams:



Answer 2:

(i) $50^\circ + x = 120^\circ$ [Exterior angle property of a Δ]

$\Rightarrow \quad x = 120^\circ - 50^\circ = 70^\circ$

Now, $50^\circ + x + y = 180^\circ$ [Angle sum property of a Δ]

$\Rightarrow \quad 50^\circ + 70^\circ + y = 180^\circ$

$\Rightarrow \quad 120^\circ + y = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 120^\circ = 60^\circ$

(ii) $y = 80^\circ$ ....(i) [Vertically opposite angle]

Now, $50^\circ + x + y = 180^\circ$ [Angle sum property of a Δ]

$\Rightarrow \quad 50^\circ + 80^\circ + y = 180^\circ$ [From equation (i)]

$\Rightarrow \quad 130^\circ + y = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 130^\circ = 50^\circ$

(iii) $50^\circ + 60^\circ = x$ [Exterior angle property of a Δ]

$\Rightarrow \quad x = 110^\circ$

Now $50^\circ + 60^\circ + y = 180^\circ$ [Angle sum property of a Δ]

$\Rightarrow \quad 110^\circ + y = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 110^\circ$

$\Rightarrow \quad y = 70^\circ$

(iv) $x = 60^\circ$ ....(i) [Vertically opposite angle]

Now, $30^\circ + x + y = 180^\circ$ [Angle sum property of a Δ]

$\Rightarrow \quad 50^\circ + 60^\circ + y = 180^\circ$ [From equation (i)]

$\Rightarrow \quad 90^\circ + y = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 90^\circ = 90^\circ$

(v) $y = 90^\circ$ ....(i) [Vertically opposite angle]

Now, $y + x + x = 180^\circ$ [Angle sum property of a Δ]

$\Rightarrow \quad 90^\circ + 2x = 180^\circ$ [From equation (i)]

$\Rightarrow \quad 2x = 180^\circ - 90^\circ$

$\Rightarrow \quad 2x = 90^\circ$

$\Rightarrow \quad x = \dfrac{90^\circ}{2} = 45^\circ$

(vi) $x = y$ ....(i) [Vertically opposite angle]

Now, $x + x + y = 180°$ [Angle sum property of a Δ]

$\Rightarrow$ $2x + x = 180°$ [From equation (i)]

$\Rightarrow$ $3x = 180°$

$\Rightarrow$ $x = \frac{180°}{3} = 60°$

Exercise 6.4

Question 1:

Is it possible to have a triangle with the following sides?

(i) 2 cm, 3 cm, 5 cm

(ii) 3 cm, 6 cm, 7 cm

(iii) 6 cm, 3 cm, 2 cm

Answer 1:

Since, a triangle is possible whose sum of the lengths of any two sides would be greater than the length of third side.

(i) 2 cm, 3 cm, 5 cm

$2 + 3 > 5$ No

$2 + 5 > 3$ Yes

$3 + 5 > 2$ Yes

This triangle is not possible.

(ii) 3 cm, 6 cm, 7 cm

$3 + 6 > 7$ Yes

$6 + 7 > 3$ Yes

$3 + 7 > 6$ Yes

This triangle is possible.

(iii) 6 cm, 3 cm, 2 cm

$6 + 3 > 2$ Yes

$6 + 2 > 3$ Yes

$2 + 3 > 6$ No

This triangle is not possible.

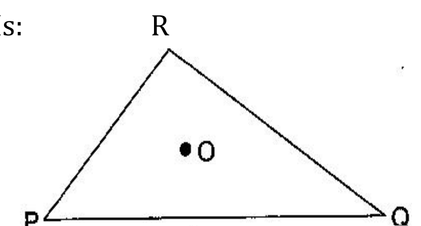
Question 2:

Take any point O in the interior of a triangle PQR. Is:

(i) OP + OQ > PQ ?

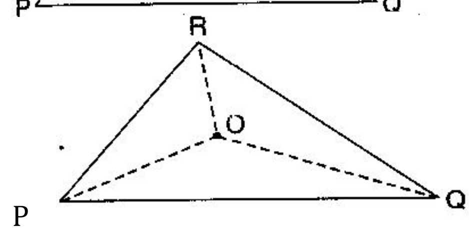
(ii) OQ + OR > QR ?

(iii) OR + OP > RP ?



Answer 2:

Join OR, OQ and OP.



(i) Is OP + OQ > PQ ?

Yes, POQ form a triangle.

(ii) Is OQ + OR > QR ?

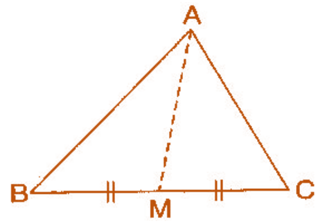
Yes, RQO form a triangle.

(iii) Is OR + OP > RP ?

Yes, ROP form a triangle.

Question 3:

AM is a median of a triangle ABC. Is AB + BC + CA > 2AM? (Consider the sides of triangles ΔABM and ΔAMC.)



Answer 3:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.

Therefore, In ΔABM, $AB + BM > AM$ ... (i)

In ΔAMC, $AC + MC > AM$ ... (ii)

Adding eq. (i) and (ii),

$AB + BM + AC + MC > AM + AM$

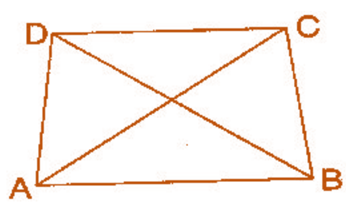
$\Rightarrow AB + AC + (BM + MC) > 2AM$

$\Rightarrow AB + AC + BC > 2AM$

Hence, it is true.

Question 4:

ABCD is a quadrilateral. Is AB + BC + CD + DA > AC + BD?



Answer 4:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.

Therefore, In ΔABC, $AB + BC > AC$ ....(i)

In ΔADC, $AD + DC > AC$ ....(ii)

In ΔDCB, $DC + CB > DB$ ....(iii)

In ΔADB, $AD + AB > DB$ ....(iv)

Adding equations (i), (ii), (iii) and (iv), we get

$$AB + BC + AD + DC + DC + CB + AD + AB > AC + AC + DB + DB$$
$\Rightarrow \quad (AB + AB) + (BC + BC) + (AD + AD) + (DC + DC) > 2AC + 2DB$
$\Rightarrow \quad 2AB + 2BC + 2AD + 2DC > 2(AC + DB)$
$\Rightarrow \quad 2(AB + BC + AD + DC) > 2(AC + DB)$
$\Rightarrow \quad AB + BC + AD + DC > AC + DB$
$\Rightarrow \quad AB + BC + CD + DA > AC + DB$

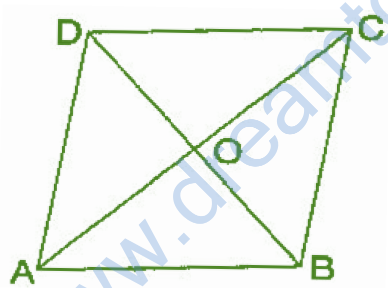
Hence, it is true.

Question 5:

ABCD is quadrilateral. Is AB + BC + CD + DA < 2 (AC + BD)?

Answer 5:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.



Therefore, In ΔAOB, $AB < OA + OB$ ....(i)
In ΔBOC, $BC < OB + OC$ ....(ii)
In ΔCOD, $CD < OC + OD$ ....(iii)
In ΔAOD, $DA < OD + OA$ ....(iv)
Adding equations (i), (ii), (iii) and (iv), we get
$AB + BC + CD + DA < OA + OB + OB + OC + OC + OD + OD + OA$
$\Rightarrow \quad AB + BC + CD + DA < 2OA + 2OB + 2OC + 2OD$
$\Rightarrow \quad AB + BC + CD + DA < 2[(AO + OC) + (DO + OB)]$
$\Rightarrow \quad AB + BC + CD + DA < 2(AC + BD)$

Hence, it is proved.

Question 6:

The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Answer 6:

Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side.

It is given that two sides of triangle are 12 cm and 15 cm.

Therefore, the third side should be less than 12 + 15 = 27 cm.

And also the third side cannot be less than the difference of the two sides.

Therefore, the third side has to be more than 15 – 12 = 3 cm.

Hence, the third side could be the length more than 3 cm and less than 27 cm.

Exercise 6.5

Question 1:

PQR is a triangle, right angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Answer 1:

Given: PQ = 10 cm, PR = 24 cm

Let QR be x cm.

In right angled triangle QPR,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$ [By Pythagoras theorem]

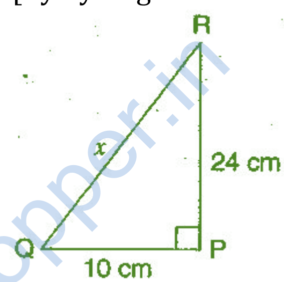
$\Rightarrow$ $(QR)^2 = (PQ)^2 + (PR)^2$

$\Rightarrow$ $x^2 = (10)^2 + (24)^2$

$\Rightarrow$ $x^2 = 100 + 576 = 676$

$\Rightarrow$ $x = \sqrt{676} = 26$ cm

Thus, the length of QR is 26 cm.



Question 2:

ABC is a triangle, right angled at C. If AB = 25 cm and AC = 7 cm, find BC.

Answer 2:

Given: AB = 25 cm, AC = 7 cm

Let BC be x cm.

In right angled triangle ACB,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$ [By Pythagoras theorem]

$\Rightarrow$ $(AB)^2 = (AC)^2 + (BC)^2$

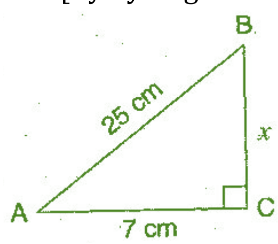
$\Rightarrow$ $(25)^2 = (7)^2 + x^2$

$\Rightarrow$ $625 = 49 + x^2$

$\Rightarrow$ $x^2 = 625 - 49 = 576$

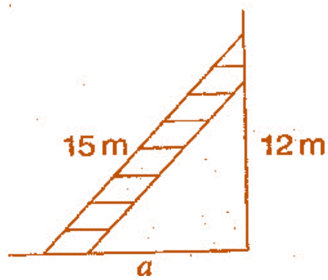
$\Rightarrow$ $x = \sqrt{576} = 24$ cm

Thus, the length of BC is 24 cm.



Question 3:

A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.



Answer 3:

Let AC be the ladder and A be the window.

Given: AC = 15 m, AB = 12 m, CB = a m

In right angled triangle ACB,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$ [By Pythagoras theorem]

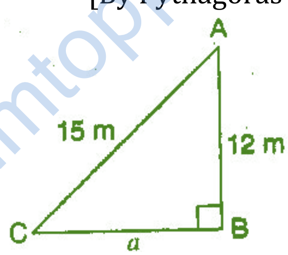
$\Rightarrow (AC)^2 = (CB)^2 + (AB)^2$

$\Rightarrow (15)^2 = (a)^2 + (12)^2$

$\Rightarrow 225 = a^2 + 144$

$\Rightarrow a^2 = 225 - 144 = 81$

$\Rightarrow a = \sqrt{81} = 9$ cm



Thus, the distance of the foot of the ladder from the wall is 9 m.

Question 4:

Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm

(ii) 2 cm, 2 cm, 5 cm

(iii) 1.5 cm, 2 cm, 2.5 cm

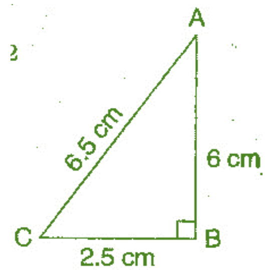
In the case of right angled triangles, identify the right angles.

Answer 4:

Let us consider, the larger side be the hypotenuse and also using Pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

(i) 2.5 cm, 6.5 cm, 6 cm



In ΔABC, $(AC)^2 = (AB)^2 + (BC)^2$

L.H.S. = $(6.5)^2$ = 42.25 cm

R.H.S. = $(6)^2 + (2.5)^2$ = 36 + 6.25 = 42.25 cm

Since, L.H.S. = R.H.S.

Therefore, the given sides are of the right angled triangle.

Right angle lies on the opposite to the greater side 6.5 cm, i.e., at B.

(ii) 2 cm, 2 cm, 5 cm

In the given triangle, $(5)^2 = (2)^2 + (2)^2$

L.H.S. = $(5)^2$ = 25

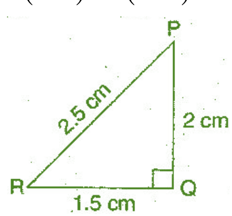
R.H.S. = $(2)^2 + (2)^2$ = 4 + 4 = 8

Since, L.H.S. $\neq$ R.H.S.

Therefore, the given sides are not of the right angled triangle.

(iii) 1.5 cm, 2 cm, 2.5 cm

In ΔPQR, $(PR)^2 = (PQ)^2 + (RQ)^2$



L.H.S. = $(2.5)^2$ = 6.25 cm

R.H.S. = $(1.5)^2 + (2)^2$ = 2.25 + 4 = 6.25 cm

Since, L.H.S. = R.H.S.

Therefore, the given sides are of the right angled triangle.

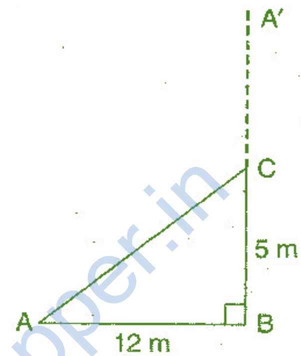
Right angle lies on the opposite to the greater side 2.5 cm, i.e., at Q.

Question 5:

A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Answer 5:

Let A'CB represents the tree before it broken at the point C and let the top A' touches the ground at A after it broke. Then ΔABC is a right angled triangle, right angled at B.



AB = 12 m and BC = 5 m

Using Pythagoras theorem, In ΔABC

$$(AC)^2 = (AB)^2 + (BC)^2$$

$\Rightarrow \quad (AC)^2 = (12)^2 + (5)^2$

$\Rightarrow \quad (AC)^2 = 144 + 25$

$\Rightarrow \quad (AC)^2 = 169$

$\Rightarrow \quad$ AC = 13 m

Hence, the total height of the tree = AC + CB = 13 + 5 = 18 m.

Question 6:

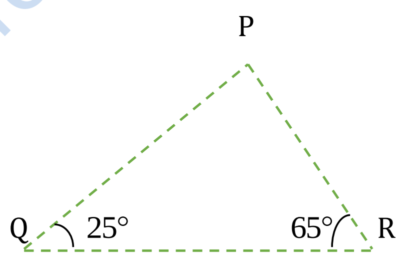
Angles Q and R of a ΔPQR are $25°$ and $65°$.

Write which of the following is true:

(i) $PQ^2 + QR^2 = RP^2$

(ii) $PQ^2 + RP^2 = QR^2$

(iii) $RP^2 + QR^2 = PQ^2$



Answer 6:

In ΔPQR,

$\angle PQR + \angle QRP + \angle RPQ = 180°$ [By Angle sum property of a Δ]

$\Rightarrow \quad 25° + 65° + \angle RPQ = 180°$

$\Rightarrow \quad 90° + \angle RPQ = 180°$

$\Rightarrow \quad \angle RPQ = 180° - 90° = 90°$

Thus, ΔPQR is a right angled triangle, right angled at P.

$\therefore \quad (\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$ [By Pythagoras theorem]

$\Rightarrow \quad (QR)^2 = (PR)^2 + (QP)^2$

Hence, Option (ii) is correct.

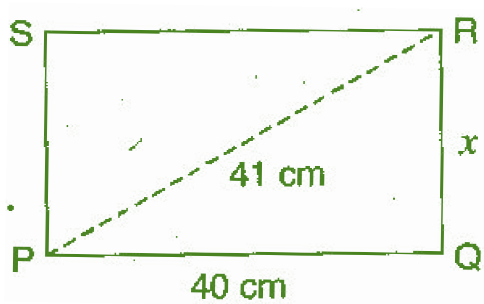
Question 7:

Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.

Answer 7:

Given diagonal (PR) = 41 cm, length (PQ) = 40 cm

Let breadth (QR) be x cm.



Now, in right angled triangle PQR,

$(PR)^2 = (RQ)^2 + (PQ)^2$ [By Pythagoras theorem]

$\Rightarrow \quad (41)^2 = x^2 + (40)^2$

$\Rightarrow \quad 1681 = x^2 + 1600$

$\Rightarrow \quad x^2 = 1681 - 1600$

$\Rightarrow \quad x^2 = 81$

$\Rightarrow \quad x = \sqrt{81} = 9$ cm

Therefore the breadth of the rectangle is 9 cm.

Perimeter of rectangle = 2(length + breadth)

= 2 (9 + 49)

= 2 x 49 = 98 cm

Hence, the perimeter of the rectangle is 98 cm.

Question 8:

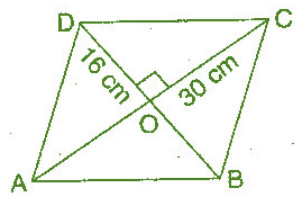
The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Answer 8:

Given: Diagonals AC = 30 cm and DB = 16 cm.

Since the diagonals of the rhombus bisect at right angle to each other.

Therefore, $OD = \frac{DB}{2} = \frac{16}{2}$ = 8 cm



And $OC = \frac{AC}{2} = \frac{30}{2}$ = 15 cm

Now, In right angle triangle DOC,

$(DC)^2 = (OD)^2 + (OC)^2$ [By Pythagoras theorem]

$\Rightarrow \quad (DC)^2 = (8)^2 + (15)^2$

$\Rightarrow \quad (DC)^2$ = 64 + 225 = 289

$\Rightarrow \quad DC = \sqrt{289}$ = 17 cm

Perimeter of rhombus = 4 x side = 4 x 17 = 68 cm

Thus, the perimeter of rhombus is 68 cm.