Chapter 4

Simple equation

Introduction to Linear Equation

Two students Rajan and Faizaan were being taught mathematics by their teacher for class-7^{th}

The teacher asked Rajan to think of a number. Then, asked him to multiply that number by 3 and add 4 to the product. Then she asked Rajan to share the number he got. He says the number is 19. The teacher instantly announced the number Rajan had thought of is 5. Rajan says yes. Faizaan and Rajan were surprised.

Now, everyone wants to know how they got to know the number thought by Rajan. This process is a game known as the mind reader. In this chapter, we will understand and learn how this game works.

When the teacher asked Rajan to think a number, the teacher did not know the number. It could be any number like 1, 2, 3,4,........7,.......100,........1000........ etc.

Suppose, we denote this number by a letter say, x. We can also use letters like y or s or some other letter instead of x. When the teacher multiplies x by 3, she gets 3x. She then adds 4 to the product, which gives $3x + 4$.

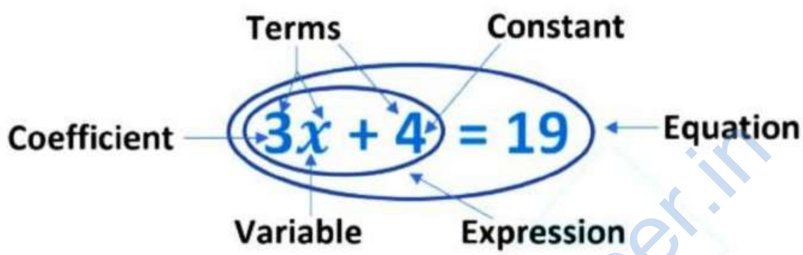
Then, the teacher asks Rajan to share his final number. When Rajan gave the final number as 19, we get an equation i.e. $3x + 4 = 19$.

What is an Equation?

When an expression is equal to a quantity which may be a constant or other expression is known as the equation. For example, $3x + 4 = 19$ is an equation. Here, expression $3x + 4$ is equal to a constant 19.

The value of $(3x + 4)$ depends upon the value of x. If $x = 1$, then $3x + 4 = 3 \times 1 + 4 = 7$. Similarly, when Rajan thought of 5, then for $x = 5$, $3x + 4 = 3 \times 5 + 4 = 19$.

Components of an Equation



Variable: A symbol for an unknown number is known as the variable. They are usually denoted by small alphabets x, y, z, u, v, w, s, t, a, b, etc. For example, x is a variable in $3x + 4$.

Coefficient: The product of the variable is called a coefficient. For example, 3 is a coefficient of x in $3x + 4$.

Constant: It is a fixed number. For example: In $3x + 4$, 4 is a constant.

Expression: Numbers, symbols, and operators (such as $+$, $-$, $\times$, and $\div$) grouped together that make an expression. For example, $3x + 4$ is an expression.

Terms: In expression, the terms are separated by + or – or ÷ signs.For example, 3x and 4 are the terms of expression $3x + 4$.

Root or Solution: The value of a variable that satisfies the equation is called the root or solution of the equation. For example: $x = 5$ is the root of the equation $3x + 4 = 19$.

Conversion of Statements to Equations & Vice-versa

To express a given statement in the form of an algebraic equation is called the conversion of statements to an algebraic equation.

Steps of conversion of statements to equation

Step 1: Read the statement carefully.

Step 2: The value that is not known or to be found, is assigned as the variable like x or y or a or b, etc.

Step 4: The expressions are formed as per the conditions given in the questions. Make this L.H.S (the left-hand side).

Step 5: The expressions are equated with values given in the questions. Make this R.H.S (the right-hand side).

Example 1: 5 times a number added to 6 is 32. Form an algebraic equation.

Solution: Suppose, the number be x.

5 times the number = 5x.

According to question, $5x + 6 = 32$.

Example 2: A number decreased by 10 is 25. Form an algebraic equation.

Solution: Suppose that the number be y.

Number decreased by 10 = y – 10.

According to question, $y - 10 = 25$.

Example 3: A number increased by 15 is equal to 25. Form an algebraic equation.

Solution: Let the number be ***v***.

Number increased by 15 = $v + 15$.

According to question, $v + 15 = 25$.

Example 4: Sum of a number and 5 times the number is 72. Form an algebraic equation.

Solution: Let the number be a.

5 times the number = 5a.

According to question, a + 5a = 72.

Example 5: Half of a number is 20 less than twice of itself. Form an algebraic equation.

Solution: Let the number be w.

$\frac{1}{2}$ of the number = $\frac{1}{2}w$

Twice the number = 2w.

According to question, $\frac{1}{2}w = 2w - 20$.

Example 6: One-fourth of a number plus 6 is 8. Form an algebraic equation.

Solution: Let the number be x.

$\frac{1}{4}$ of x = $\frac{1}{4}$ x.

One-fourth of a number plus 6 is $\frac{1}{4}x + 4$.

According to question, $\frac{1}{4}x + 6 = 8$.

Conversion of Equations to Statement

To express an algebraic equation in the form of a given statement is called the conversion of equations to statement.

Let us consider some examples:

Example 7:Convert the following equations in statement form:

I. $x - 7 = 12$
II. $5w = 25$
III. $2n + 7 = 8$
IV. $\frac{y}{4} - 5 = 7$

Solution

I. A number x decreased by 7 is 12.
II. Five times a number w is 25.
III. 2 times a number n added to 7 is 8.
IV. One-fourth of a number y minus 5 is 7.

It is important to note that for a given equation, not just one, but many statements forms can be given. For example: for equation (I) above, we can say:

Subtracting 7 from x we get 12.

Or, the number x is 7 more than 12.

Or, the number x is greater by 7 is 12.

Or, the difference between x and 7 is 12.

Example 8: Ram's father is 40 years old. He is 4 years older than three times Ram's age. Form an equation for this case.

Solution: Suppose, Ram's age is x years.

Now, Ram's father's age = 4 years older than three times Ram's age is $3x + 4$.

According to question, $3x + 4 = 40$.

Solving an equation by Trial-Error-Method

In this method, we substitute different values of the variables and check whether LHS = RHS for that specific value and the value for which it satisfies the given equation will be our answer.

The trial-and-error method for finding the solution of an equation may be time-consuming and is not a direct method to find the solution.

The following example will illustrate the applications of the above method.

Example 1: Solve the simple equation $x + 5 = 8$ by the trial and error method.

Solution: We have $x + 5 = 8$.

To solve this equation,

We will take various values of x.

- If $x = 1$

Then, RHS = 8

LHS $= x + 5 = 1 + 5 = 6 \neq$ RHS

- If $x = 2$

Then,

RHS = 8

LHS $= x + 5 = 2 + 5 = 7 \neq$ RHS

- If $x = 3$

Then, RHS = 8

LHS $= x + 5 = 3 + 5 = 8 =$ RHS.

So, $x = 3$ is the solution for the given equation.

Solving an equation by Systematic Method

The trial-and-error method for finding the solution of an equation may be time-consuming and is not a direct method to find the solution. So, we shall study the Systematic method or Balancing method of solving simple equations.

A simple equation can be compared with a weighing balance. The two sides of an equation are two pans and the equality symbol '=' tells us that the two sides are in balance as shown in the figure below.

When we remove weights from both the pans, we find that the pans still remain in balance.

Multiplying a number by 2 means adding it two times and dividing a number by 4 means subtracting the same number 4 times from it. Thus, the pans will still remain undisturbed when we increase or decrease the weights in two pans by the same quantity.

Similarly, in the case of an equation, we have the following rules:

Rule 1: We can add the same number to both sides of the equation.
For example: If $x + 5 = 8$, then $x + 5 + 2 = 8 + 2$.

Rule 2: We can subtract the same number from both sides of the equation. For example: If $x + 5 = 8$, then $x + 5 - 2 = 8 - 2$.

Rule 3: We can multiply both sides of the equation by the same non-zero number. For example: If $\frac{x}{5} = 8$, then $\frac{x}{5} \times 4 = 8 \times 4$.

Rule 4: We can divide both sides of the equation by the same non-zero number. For example: $2x = 8$, then $\frac{2x}{2} = \frac{8}{2}$

The following examples will illustrate the applications of the above rules.

Example 1: Solve the simple equation $x + 6 = 8$ by the systematic method.

Solution: We have x + 6 = 8.

To solve this equation, we have to rearrange the equation to get only x on L.H.S. To do this, we need to subtract 5 from both sides of the equation.

∴ x + 6 – 6 = 8 – 6. (Subtracting 5 to both sides)

⇒ x+ 0 = 2.

⇒ x= 2.

Hence, x = 2 is the solution of the equation x + 6 = 8.

Example 2: Solve the simple equation $\frac{3x}{4} - 1= 5$ by the systematic method.

Solution: We have $\frac{3x}{4} - 1 = 5$.

To solve this equation, we have to rearrange the equation to get only x on L.H.S. To do this, we need to add 1 from both sides of the equation.

∴ $\frac{3x}{4} - 1 + 1 = 5 + 1$. (add 1 to both sides)

⇒ $\frac{3x}{4} + 0 = 6$.

⇒ $\frac{3x}{4} \times \frac{4}{3} = 6 \times \frac{4}{3}$ (Multiplying both sides by $\frac{4}{3}$)

⇒ x= 8.

Hence, x = 8 is the solution of the equation $\frac{3x}{4} - 1 = 5$.

Solving an equation by Transposition Method

In this method, we keep the variable on one side of the equation and transpose all other terms to the other side of the equation.

When a term of an equation is transposed to the other side, its sign gets reversed i.e. positive term becomes negative and vice-versa. If a term is in multiplication and it is transposed to the other side, then it becomes the divisor on the other side and vice-versa.

The transposition method involves the following steps:

Step 1: Identify the unknown quantity (variable).

Step 2: Simplify the L.H.S. and R.H.S. (If it is required)

Step 3: Transfer all terms containing the variable on the L.H.S. and constant terms on R.H.S. of the equation.

Step 4: Simplify L.H.S. and R.H.S. in the simplest form so that each side contains just one term.

The following example will illustrate the above procedure.

Example 1: Solve the simple equation x + 2 = 8 by the transposition method.

Solution: We have,

$x + 2 = 8$

$\Rightarrow x = 8 - 2$. (Transposing 2 to R.H.S.)

$\Rightarrow x = 6$.

Hence, x = 6 is the solution of the equation x + 2 = 8

Example 2: Solve the simple equation $\frac{x}{2} - 4 = \frac{x}{3} + 5$ by the transposition method.

Solution: We have,

$\frac{x}{2} - 4 = \frac{x}{3} + 5$

$= \frac{x}{2} - 4 = \frac{x}{3} + 5$ (Transposing $\frac{x}{3}$ on L.H.S and -4 on R.H.S.)

$\Rightarrow \frac{x}{2} - \frac{x}{3} = 5 + 4$

$\Rightarrow (\frac{1}{2} - \frac{1}{3})x = 9$

$\Rightarrow (\frac{3-2}{6})x = 9$

$\Rightarrow (\frac{1}{6})x = 9$

$\Rightarrow (\frac{1}{6})x \times 6 = 9 \times 6$ (Multiplying both sides by 6)

$\Rightarrow x = 54$

Hence, x = 54 is the solution of the equation $\frac{x}{2} - 4 = \frac{x}{3} + 5$

From Solution to Equation

Generally, we derive the solution from the equation to the solution.
Here, we derive the equation from the solution of the equation.

Equation → Solution. (Normal path)

Solution → Equation. (Reverse path)

Example: If x =7, then find the equation.

Solution: We have,

x = 7

$\Rightarrow x + 5 = 7 + 5$. (Adding 5 to both sides)

$\Rightarrow x + 5 = 12$.

Hence, x + 5 = 12 is an equation for x = 7.

Another way,

x = 7

$\Rightarrow 3x = 3 \times 7$. (Multiply 3 to both sides)

$\Rightarrow 3x = 21$.

Hence, $3x = 21$ is an equation for $x = 7$.

Application of Simple Equation

Simple equations are used to solve daily life problems. We convert the word problems into a mathematical equation and then solve it by using any one of the methods already discussed.

The procedure to translate a word problem in the form of an equation is known as the formulation of the problem. Thus, the process of solving a word problem consists of two parts, namely, the formulation of the equation and its solution.

Following steps should be followed to solve a word problem:

Step 1: Read the problem carefully and note what is given and what is required.

Step 2: Denote the unknown quantity by some letters say x, y, z, etc.

Step 3: Translate the statements of the problem into mathematical statements.

Step 4: Using the conditions given in the problem and form the equation.

Step 5: Solve the equation for the unknown.

The following examples will illustrate these steps.

Example 1: 5 added to a number gives 10. Find the number.

Formulation: Suppose, the required number be x.

It is given that when 5 added to a number, we get 10. Thus, we obtain the equation $x + 5 = 10$.

Solution: We have, $x + 5 = 10$

$\Rightarrow x = 10 - 5$. (Transposing 5 on R.H.S.)

$\Rightarrow$ x= 5.

Hence, the required number is 5.

Example 2: Ram's father is three times as old as Ram. If the sum of their ages is 40 years. Find their ages.

Formulation: Let Ram's age be x years. Then,

Ram's father's age = 3x years.

$\therefore$ Sum of their ages = x + 3x.

But the sum of their ages is given as 40 years. Therefore, we have the equation x + 3x = 40.

Solution: We have,

x + 3x = 40.

$\Rightarrow$ 4x = 40.

$\Rightarrow \frac{4x}{4} = \frac{40}{4}$. (Dividing both sides by 4)

$\Rightarrow$ x= 10.

Hence, Ram's age = 10 years

Ram's father's age = 3x years = 3 × 10 years = 30 years.