Chapter 1

Integers

Introduction to Integers

Ethan is going for a picnic with his friends. He wants to carry cupcakes with him, but he has got only 3 cupcakes and there are 4 friends. What is he going to do now?

So, Ethan decided to borrow one cupcake from his sister, which he would return later.

How many cupcakes does he have now?

After borrowing one cupcake from his sister, he has got 4cupcakes, which he would give to his four friends.

He goes for the picnic, where he gave away the 4 cupcakes to his friends.

Now, how many cupcakes are left with him?

Is your answer zero (0)?

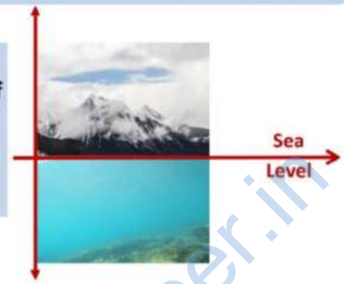
We can say that there are no or 0 cupcakes left with him, but we also have to keep in mind that he has borrowed one cupcake from his sister.

So, in actual Ethan has (-1) cupcake, which means that 1 cupcake is borrowed and did not belong to him.

If he buys 3 more cupcakes the next day, he will have to return 1 cupcake to his sister and will be left with 2 cupcakes only.

Numbers with a negative sign are less than zero and they are called negative numbers.

+	−
The height of a mountain above sea level is denoted by a positive number.	Below the surface of the sea level, depth is denoted by a negative number



+	−
Temperature above 0^oC is denoted by a '+' sign.	Temperature below 0^oC is denoted by a '-' sign.

Temperature in Iceland can drop down to -10^oC during winter

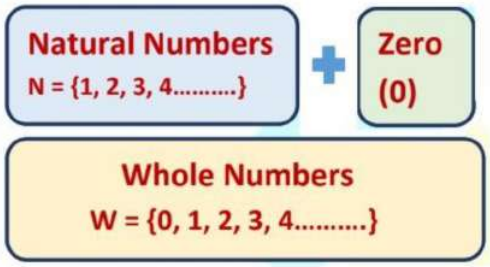
Temperature in Rajasthan can go beyond 50^oC during summer

Natural Numbers: Natural Numbers is a set of counting numbers. They are denoted by N.

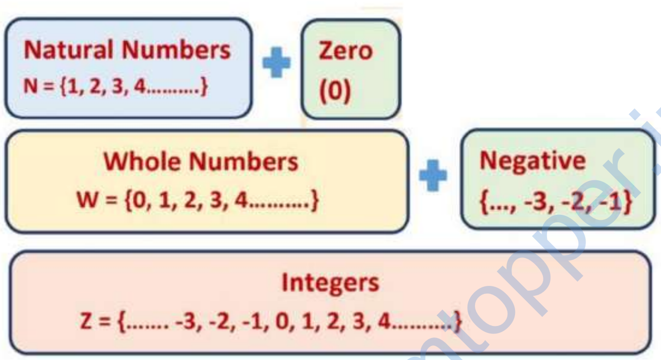
Natural Numbers

N = {1, 2, 3, 4.........}

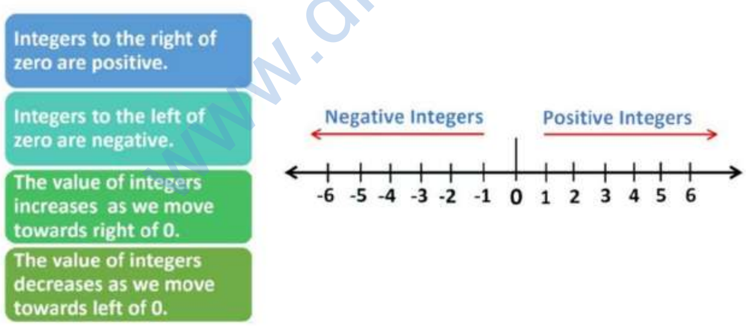
Whole Numbers: If zero is included in the collection of naturalnumbers, we get a new collection of numbers known as whole numbers.



Integers: Integers are a set of whole numbers and negative of all natural numbers.

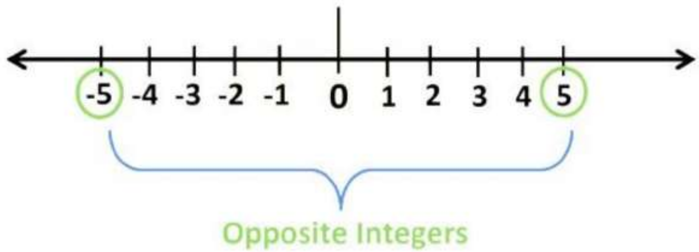


Integers on a Number Line



Opposite Integers/ Additive Inverse

The opposite of an integer is at the same distance from zero but on the opposite sides of the number line. Therefore, one integer will have a positive sign and the other will have a negative sign.



So, we can say that 5 and -5 are opposite integers.

Opposite of any integer a is $-a$ and opposite integer of $-a$ is $-(-a) = a$

Example: Write the opposite of the following integers:

-25, 16, 7, -100

a) -25

The given integer is negative.

The opposite integer of -25 = -(- 25) = 25

b) 16

The given integer is positive.

The opposite integer of 16 = -(16) = -16

c) 7

The given integer is positive.

The opposite integer of 7 = -(7) = -7

d) -100

The given integer is negative.

The opposite integer of -100 = -(-100) = 100

Absolute Value of Integers

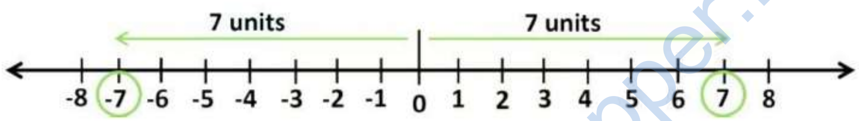
Absolute Value of Integers

The absolute value of an integer is its distance from 0 on a number line or its numerical value without taking its sign into consideration.The symbol for absolute value is ||.

1. As opposite integers are at the same distance from 0, their absolute values are same.
2. The absolute value of an integer cannot be negative.

Consider the integers 7 and -7 on the number line.

How far are 7 and -7 from zero?



Both 7 and -7 are at the same distance from 0.

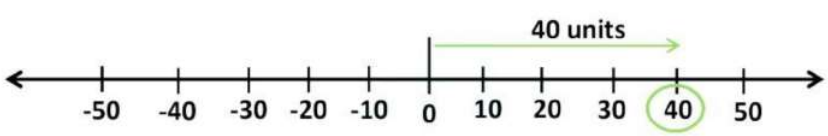
Absolute value of 7 = |7 | = 7

Absolute value of -7 = |-7|= 7

Example: Write the absolute values of the following integers.

a) 40 b) -21

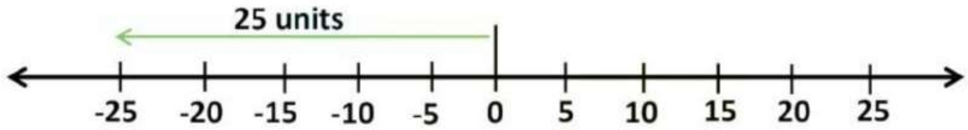
a) We know that the absolute value of an integer is its distance from 0 on a number line.



Therefore, the absolute of 40 = |40| = 40

b) -25

Now, the absolute value of an integer is its distance from 0 on a number line.



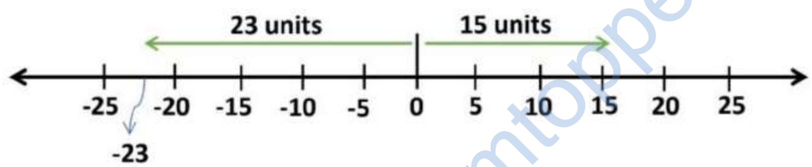
The absolute value of -25 = |-25| = 25

Example: Which is greater?

|-23| or |15|

We know that the absolute value of an integer is its distance from 0 on a number line.

First, we will locate -23 and 25 on the number line and compare their distance from 0.



Distance of 0 from -23 is more than its distance from 15.

Absolute value of -23 = |-23| = 23

Absolute value of 15 = |15| = 15

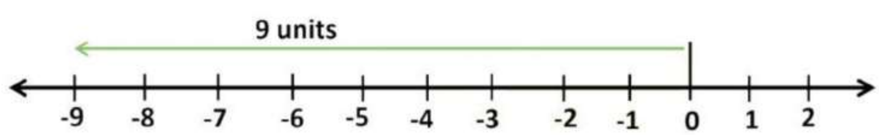
So, |-23|>|15|

Example: Find the value of,

a) |8 – 17| b) |6| + |-7|

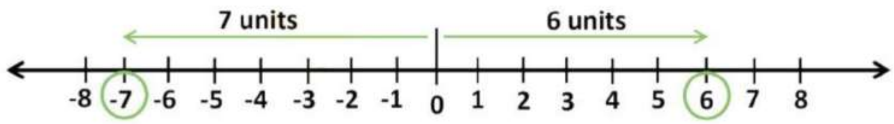
a) |8 – 17|

|8 – 17| = |-9|



So, |-9| = 9

b) |6|+ |-7|



We see, |6| = 6 and |-7| = 7

|6|+ |-7| = 6 + 7 = 13

Ordering of Absolute Values

Example: Arrange the following integers in descending order.

|3|, |-12|, |3 – 12|, |3|-|-12|, |3|+|12|
We know that the absolute value of an integer is the distance of the integer from 0.

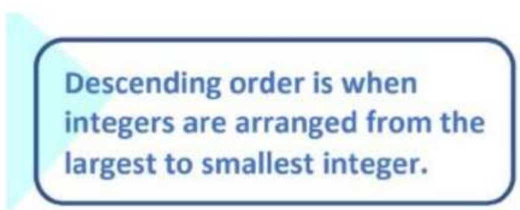
|3| = 3

|-12| = 12

|3 – 12|= |-9| = 9

|3|- |-12| = 3 – (12) = 3 – 12 = -9

|3| + |12| = 3 + 12 = 15

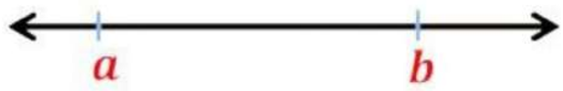


Now, arranging the integers in descending order we get,

15 > 12 > 9 > 3 > -9

Absolute values as the distance between Numbers

Let a and b be the two numbers marked on the number line as shown below.



As b lies to the right of a on the number line, so b > a.

Distance between a and b = b − a

Now, we interchange the position of a and b on the number line.

Here, a > b

Distance between a and b = a − b

If we don't know which of the two numbers is greater, we find the absolute value of the distance between the two numbers.

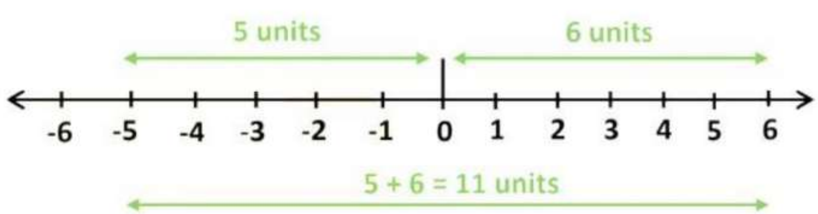
$|a - b| = |b - a|$

Example: Find the distance between -5 and 6.

a) Distance between -5 and 6

$|-5 - 6| = |-11| = 11$

$|6 - (-5)| = 6 + 5 = 11$



Distance between -5 and 6 = 11

Addition of Integers

Rule 1: When we add two positive integers, we add their values and the result will take the positive sign (common sign of both the integers)

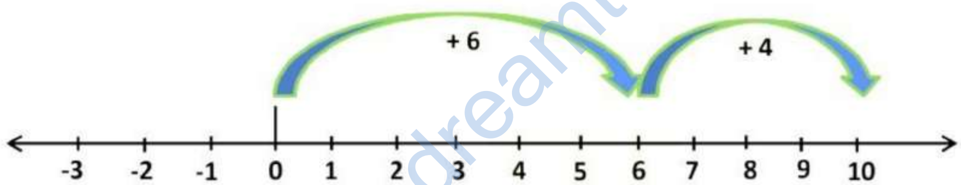
If we add 6 and 4 (both are positive integers), we add their values and the result will also be a positive integer.

So, $6 + 4 = 10$

Let's do this addition on a number line also.

When we add two or more integers on the number line then we move towards the right of any one of the given numbers.

So we start from 0 and jump 6 units towards the right and then again jump 4 units towards right from 6.



Example: Add 25 and 46

25 and 46 are positive integers. So, we add their values.

$25 + 46 = 71$

The result will be a positive integer. (a common sign of both the integers)

Rule 2: When we add a positive and a negative integer, we find the difference of their numerical values, regardless of their signs and give the sign of the integer which is greater.

Add: $6 + (-9)$

Here, one integer is positive and the other integer is negative.

So, we find the difference of the integers, (9 – 6 = 3)

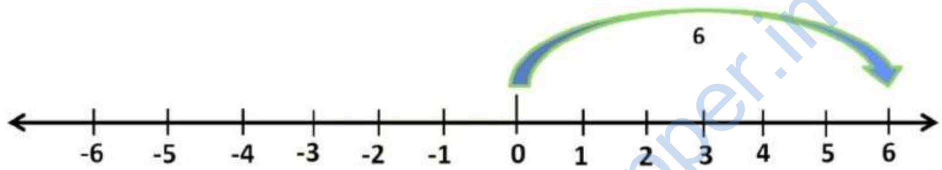
Out of the two integers, which one is greater (do not consider the sign of the integers here)?

9 is the greater integer, but it is a negative integer.

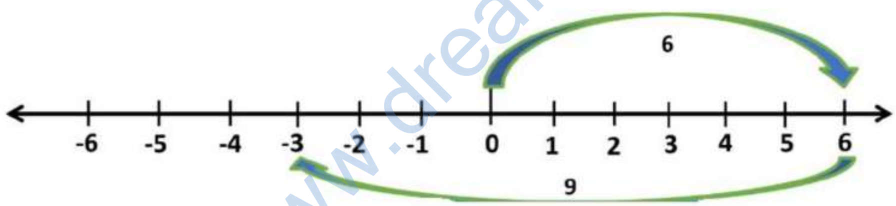
Therefore, result 3 will have a negative sign.

So, -9 + 6 = -3

When we add -9 and 6 on a number line, we first start from 0 and jump 6 places to the right of zero.



We reach 6 on the number line, and then we jump 9 places to the left of 6. (When we add a negative integer on a number line, we move towards left)



We reach -3 on the number line.

So, -9 + 6 = -3

Example: Add -67 and 32

-67 + 32

Here, one integer is positive and the other is negative so we find the difference of their numerical values.

67 – 32 = 35

Now, 67 is the greater integer (signs of the integers are not considered).

As 67 is a negative integer, the result will take the negative sign.

-67 + 32 = -35

Example: Write down a pair of integers whose:
i) sum is -7
iii) sum is 0

i) sum is -7
Consider the pair of integers (-10, 3).
As one of the integers is negative, we find the difference between their numerical values and put the sign of the greater integer.
Sum of -10 and 3 = -10 + 3 = -7

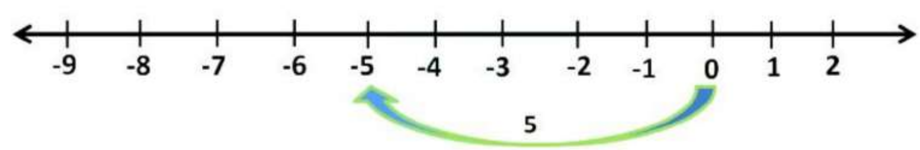
ii) sum is 0
Consider the pair of integers (-7, 7)
One of the integers is negative, we find the difference between the numerical values.
Sum of -7 and 7 = -7 + 7 = 0

Rule 3: If two negative integers are added then we add their values and the result will take the negative sign.

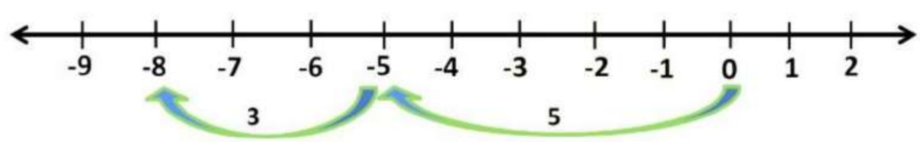
Add: -5 and -3

-5 + (-3) = -5 – 3 = -8

If we do the addition of -5 and -3 on the number line, then we start from 0 and jump 5 places to the left of 0.



We reach -5 on the number line, and then we jump 3 places to the left of -5.
(As we are adding a negative integer on a number line, we move towards left)



We reach -8 on the number line.

-5 + (-3) = -8

Example: Add -78 and -36

Both the integers are negative, so we add their values and the result will take the negative sign.

-78 + (-36) = -78 – 36 = -114

Subtraction of Integers

When we subtract one integer from the other, we convert the integer to be subtracted to its negative and then add the two integers.

> Subtracting an integer from the other is same as adding the additive inverse of the integer.
> $a - b = a +$(additive inverse of b) $= a + (-b)$

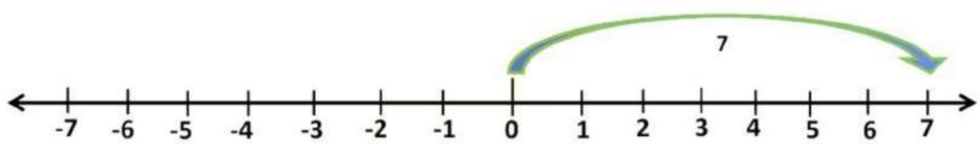
Subtract 3 from 7.

7 – 3
Additive inverse of 3 is -3

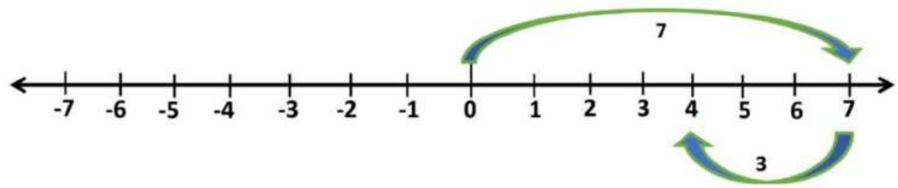
So we add 7 to the additive inverse of 3 that is -3.

7 -3 = 7 + (-3) = 4

On a number line, we jump 7 places to the right of zero.



We reach 7 on the number line, and then we jump 3 places to the leftof 7. As we are adding a negative integer we move towards left.



We reach 4 on the number line.

So, 7 + (-3) = 4

Example: Subtract 18 from 76

76 – 18

Additive inverse of 18 is -18. So, we add 76 to the additive inverse of 18, which is -18.

= 76 + (-18) = 58

Example: Subtract 45 from 34

34 – 45

Now, we add 34 to the additive inverse of 45, which is -45.

= 34 + (-45) = -11

When we subtract a negative integer from another integer, it is the same as adding the two integers.

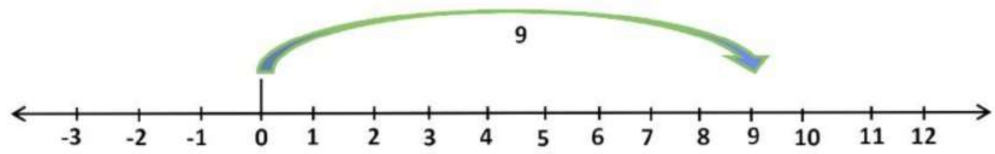
a and b are two integers, where a is positive and b is negative.

$$a - (-b) = a + b$$

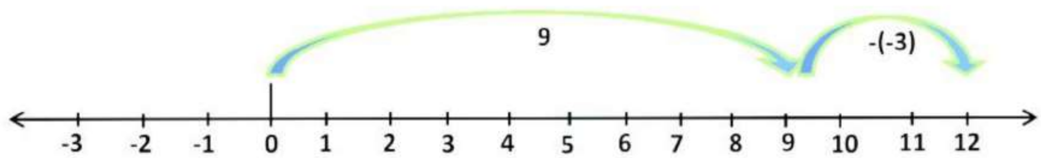
Subtract -3 from 9.

9 – (-3) = 9 + 3 = 12

On a number line we jump 9 places to the right of 0.



We reach 9 on the number line. When we subtract a negative integer from any integer we move towards the right. So we jump 3 places to the right of 9.



We reach 12 on the number line.

So, 9 – (-3) = 9 + 3 = 12

Example: Subtract -26 from 48

Here, we are subtracting the negative integer, -26 from 48. So we simply add the two integers.

48 – (-26)

48 – (-26) = 48 + 26 = 74

Example: -89 from -67

Now, we have to subtract a negative integer, -89 from another negative integer, -67.

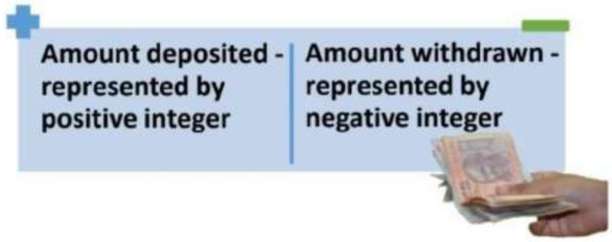
-67 – (-89) = -67 + 89

We know that when we add a positive and a negative integer, then we find their difference and put the sign of the greater integer.

-67 + 89 = 22 (sign will be positive as the greater integer, 89 has a positive sign)

Example: Shyam deposits Rs 2,000 in his bank account and withdraws Rs 1,542 from it, the next day. If the withdrawal of the amount from the account is represented by a negative integer, then how will you represent the amount deposited? Find the balance in Shyam's account after the withdrawal.

Amount deposited = Rs 2,000
Amount withdrawn = Rs 1,542



If the withdrawal of the amount from the account is represented by a negative integer then the amount deposited will be represented by a positive integer.

Balance amount = Rs 2,000 – Rs 1,542 = Rs 458

Closure Property of Integers

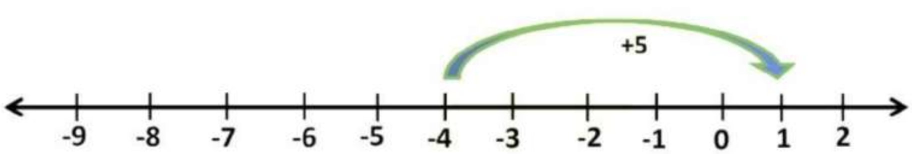
Closure property of Addition of Integers:

If a and b are two integers, then a + b is also an integer.

When we add any two integers, the result will always be an integer.
This is true for all integers.

a	b	$a + b$
-4	5	-4 + 5 = 1, an integer
-4	0	-4 + 0 = -4, an integer
-4	-5	(-4) + (-5) = -9, an integer

If we add -4 to 5 on the number line, we start from -4 and jump 5 places to the right of -4. We reach 1 on the number line.



So, -4 + 5 = 1, an integer

Under addition, integers are closed.

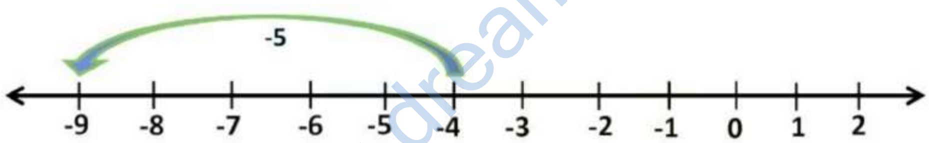
Closure Property of Subtraction of Integers

If a and b are two integers, then a − b is also an integer.

When we subtract an integer from another, the result will always be an integer. This is true for all integers.

a	b	$a-b$
-6	5	-6 - 5 = -11, an integer
-6	0	-6 - 0 = -6, an integer
-6	-5	(-6) - (-5) = -1, an integer

If we subtract4 from-5,(-5 -4) on the number line, we start from -4 and jump 5 places to the left of -4. We reach-9 on the number line.



So, -5 - 4 = -9, which is an integer

Under subtraction, integers are closed.

Commutative Property of Integers

Commutative property of addition of integers
If a and b are two integers, then a + b = b + a

a	b	$a+b$	$b+a$	Is $a+b=b+a$?
8	9	8 + 9 = 17	9 + 8 = 17	Yes
8	-9	8 + (-9) = -1	-9 + 8 = -1	Yes
-8	-9	-8 + (-9) = -17	-9 + (-8) = -17	Yes

Hence, we can add two integers in any order.

Addition is commutative for integers

Commutative property of subtraction of integers

If a and b are two integers, then $a - b \neq b - a$

a	b	$a - b$	$b - a$	Is $a - b = b - a$?
1	7	1 - 7 = -6	7 – 1 = 6	No
1	-7	1 – (-7) = 8	-7 - 1 = -8	No
-1	-7	-1 – (-7) = 6	-7 – (-1) = -6	No

Subtraction is not commutative for integers

Example: Verify the following and state the property used.

(-5) + (-8) = (-8) + (-5)

LHS

-5 + (-8) = -5 – 8 = -13

RHS

-8 + (-5) = -8 – 5 = -13

LHS = RHS

Here, we have used the commutative property of addition of integers which states that, if a and b are two integers, then $a + b = b + a$

Associative Property of Integers

Associative property of Addition of Integers

If a, b & c are any three integers, then
(a + b) + c = a + (b + c)

a	b	c	$(a+b)+c$	$a+(b+c)$	Is $(a+b)+c = a+(b+c)$?
3	6	8	(3 + 6) + 8 = 17	3 + (6 + 8) = 17	Yes
-3	6	8	(-3 + 6) + 8 = 11	-3 + (6 + 8) = 11	Yes
-3	-6	8	[-3 + (-6)] + 5 = -4	-3 + (-6 + 5) = -4	Yes

When we are adding integers, they can be grouped in any order and the result remains the same.

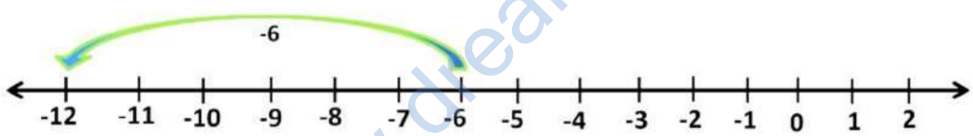
Consider the three integers, -2, -4 and -6

Case 1: [-2 + (-4)] + (-6)

In the first case, we group -2 and -4.

[-2 + (-4)] + (-6) = -6 + (-6)

On a number line, we start from -6 and jump 6 places to the left of -6.



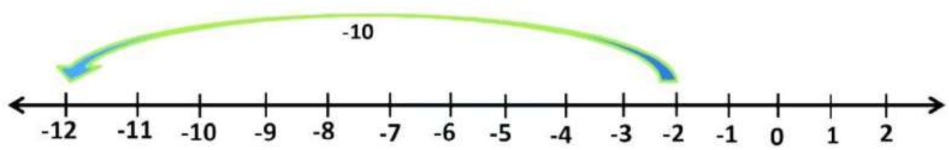
We reach -12 on the number line.

Case 2: (-2) + [-4 + (-6)]

In the second case we group together -4 and -6.

(-2) + [-4 + (-6)] = -2 + (-10)

On a number line, we start from -2 and jump 10 places to the left of -2.



We reach -12 on the number line.

We see that the result is the same in both cases.

Addition of Integers is Associative

Associative property of Subtraction of Integers

For any three integers a, b and c,

$(a - b) - c \neq a - (b - c)$

a	b	c	$(a-b)-c$	$a-(b-c)$	Is $(a-b)-c$ $= a-(b-c)$?
2	3	5	(2 - 3) – 5 = -6	2 - (3 - 5) = 4	No
2	-3	5	[2 – (-3)] – 5 = 0	2 - (-3 - 5) = 10	No
-2	3	-5	(-2 – 3) - (-5) = 0	-2 – [3 –(-5)] = -10	No

Consider the integers, -3, -5 and -6

Case 1: [-3 - (-5)] - (-6)

In the first case, we group together -3 and -5.

[-3 - (-5)] - (-6) = 2 + 6 = 8

Case 2: (-3) – [-5 – (-6)]

In the second case, we group together -5 and -6.

(-3) – [-5 – (-6)] = -3 – 1 = -4

[-3 - (-5)] - (-6) ≠ (-3) – [-5 – (-6)]

Subtraction of Integers is not Associative

Example: Fill in the blanks to make the following statements true.

i) [13 + (-12)] + (__) = 13 + [(-12) + (-7)]

ii) (-4) + [15 + (-3)] = [-4 + 15] + (__)

i) [13 + (-12)] + (___) = 13 + [(-12) + (-7)]

We have used the associative property of addition of integers which states that, if a, b & c are any three integers, then

(a + b) + c = a + (b + c)

If a = 13, b = -12 and c = -7 then,

[13 + (-12)] + (-7) = 13 + [(-12) + (-7)]

ii) (-4) + [15 + (-3)] = [-4 + 15] + (__)

We use the associative property of addition of integers which states that, if a, b & c are any three integers, then

(a + b) + c = a + (b + c)

If a = -4, b = 15 and c = -3 then,

(-4) + [15 + (-3)] = [-4 + 15] + (-3)

Additive Identity

Additive Identity Property:

If a is any integer, then a + 0 = a = 0 + a

a	0	$a + 0$	Is $a + 0 = a$?
1	0	1 + 0 = 1	Yes
-15	0	-15 + 0 = -15	Yes
196	0	196 + 0 = 196	Yes

The number 'zero' has a special role in addition. When we add zero to any integer the result is the same integer again. Zero is the additive identity for integers.

Example: Fill in the blanks

i) (-23) + 0 = ____
ii) 0 + ___ = -43
iii) 8 + ___ = 8
i) (-23) + 0 = ____

If we add zero to any integer the result is the same integer again. This property is known as additive identity property.

So, (-23) + 0 = -23

ii) 0 + ___ = -43

We again use the additive identity property.

So, 0 + (-43) = -43

iii) 8 + ___ = 8

Using the additive identity property, we get, 8 + 0 = 8

Multiplication of Integers

Rule 1: To find the product of two integers with the same sign, we find the product of their values and put the positive sign before the product.

(-ve) × (-ve) = (+ve)

(+ve) × (+ve) = (+ve)

For any two positive integers a and b,

$a \times b = ab$

For any two negative integers $(-a)$ and $(-b)$

$(-a) \times (-b) = ab$

Consider two positive integers 6 and 8

$6 \times 8 = +48$

Now, consider the two negative integers -6 and -8

$(-6) \times (-8) = +48$

We see that the product is positive in both cases.

Example: Find
i) (-11) × (-100)
ii) 25 × 250
iii) (-60) × (-21)

i) (-11) × (-100)
The two integers have the same sign (negative), so we find the product of their values and put the positive sign before the product.
(-11) × (-100) = +1100
ii) 25 × 250
As the two integers are positive (same sign) we find the product of their values and give a positive sign to the product.
25 × 250 = 6250

iii) (-60) × (-21)
Now, both the integers are negative (same sign) so we find the product of their values and put the positive sign before the product.
(-60) × (-21) = -1260

Rule 2: To find the product of two integers with unlike signs, we find the product of their values and put the negative sign before the product.

For any two integers a and b

$$a \times (-b) = (-a) \times b = -(a \times b)$$

Example: Find
i) (-31) × 30
ii) 26 × (-13)
iii) (-60) × 14

i) (-31) × 30
The two integers have different signs, one is positive and the other is negative.

So we find the product of their values and give the product a negative sign.
(-31) × 30 = -930

ii) 26 × (-13)
Here, one integer is positive and the other integer is negative. So, we find the product of their values and put a negative sign before the product.
26 × (-13) = -338

iii) (-60) × 14
As the two integers have unlike signs, we find the product of their values and put a negative sign before the product.
(-60) × 14 = -840

Product of three or more Negative Integers

We know that the product of two negative integers is a positive integer. What happens if we have to find the product of more than two negative integers?

Negative integers	Product	Result
-3, -5	(-3) × (-5) = +15	Positive
-3, -5, -7	(-3) × (-5) × (-7) = -105	Negative
-3, -5, 7, -9	(-3) × (-5) × (-7) × (-9) = 945	Positive
-3, -5, -7, -9, 10	(-3) × (-5) × (-7) × (-9) × (-10) = 9450	Positive

We see that when the number of negative integers in a product is even, then the product is an even integer and if the number of negative integers in the product is odd, then the product is a negative integer.

Number of Negative Integers being multiplied	Result
Even	Positive
Odd	Negative

Example: The product of (-9) × (-5) × (-6) × (-3) is positive whereas the product of (-9) × (-5) × 6 × (-3) is negative. Why?

The product of (-9) × (-5) × (-6) × (-3) is positive because the number of negative integers in the product is 4, which is an even number.
(-9) × (-5) × (-6) × (-3) = [(-9) × (-5)]×[(-6) × (-3)]
= 45×18 = 810
The product of (-9) × (-5) × 6 × (-3) is negative because the number

of negative integers in the product is 3, which is an odd number.
(-9) × (-5) × 6 × (-3) = [(-9) × (-5)]×[6 × (-3)]
= 45 × (-18) = -810

Example: What will be the sign of the product if we multiply together,
i) 8 negative integers and 3 positive integers?
ii) 5 negative integers and 4 positive integers?

i) 8 negative integers and 3 positive integers
When we consider the sign of the product, we count the number of negative integers. Here, the number of negative integers in the product is 8, which is an even number.
So the product of 8 negative integers and 3 positive integers is an even integer.

ii) 5 negative integers and 4 positive integers
We know that if the number of negative integers in the product is odd, then the product is a negative integer. Here, the number of negative integers in the product is 5, an odd number. So, the product of 5 negative integers and 4 positive integers is odd.

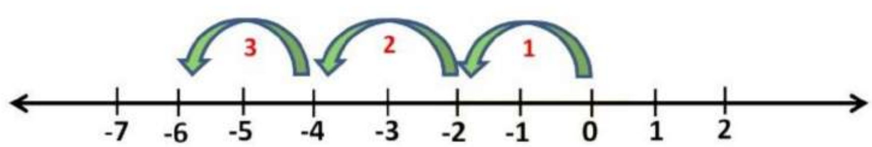
Properties of Multiplication of Integers

Closure Property:
If a and b are two integers, then a × b is an integer.

a	*b*	*a* × *b*	Integer
2	3	2× 3 = 6	Yes
-2	3	(-2) × 3 = - 6	Yes
-2	-3	-2×(-3) = 6	Yes

(-2)× 3 = (- 6)

We start from 0 and jump 2 places to the left of 0. We make 3 such jumps. We reach -6 on the number line.



If we multiply two integers, the product is also an integer.

Integers are closed under multiplication

Commutative Property:

If a and b are two integers, then $a \times b = b \times a$

a	b	$a \times b$	$b \times a$	Is $a \times b = b \times a$?
3	4	$3 \times 4 = 12$	$4 \times 3 = 12$	Yes
-3	4	$(-3) \times 4 = -12$	$4 \times (-3) = -12$	Yes
-3	-4	$(-3) \times (-4) = 12$	$(-4) \times (-3) = 12$	Yes

The value of the product does not change even when the order of multiplication is changed.

Multiplication is commutative for Integers

Associative Property

If a, b&c are any three integers, then

$(a \times b) \times c = a \times (b \times c)$

a	b	c	$(a \times b) \times c$	$a \times (b \times c)$	Is $(a \times b) \times c = a \times (b \times c)$?
1	-2	3	$[1 \times (-2)] \times 3 = -6$	$1 \times [(-2) \times 3)] = -6$	Yes
1	-2	-3	$[1 \times (-2)] \times (-3) = +6$	$1 \times [(-2) \times (-3)] = +6$	Yes
-1	-2	-3	$[(-1) \times (-2)] \times (-3) = -6$	$-1 \times [(-2) \times (-3)] = -6$	Yes

When we multiply three or more integers, the value of the product remains the same even if they are grouped in any manner and this is called the associative property for multiplication of integers.

Multiplication of Integers by Zero (0)

If a is any integer, then $a \times 0 = 0 \times a = 0$

$15 \times 0 = 0$

(-100)× 0 = 0

0×(-25) = -25

The product of a negative integer and zero is always zero.

Multiplicative Identity

If a is any integer, then a × 1 = a = 1 × a

a	1	$a \times 1$	Is $a \times 1 = a$?
1	1	1 × 1 = 1	Yes
(-15)	1	(-15)× 1 = -15	Yes
205	1	205× 1 = 205	Yes

If we multiply any integer by 1, the product is the integer itself.

So, 1 is the multiplicative identity of integers.

Distributive Property of Multiplication over Addition:

If a, b & c are any three integers, then

$$a \times (b + c) = a \times b + a \times c$$

a	b	c	$a \times (b + c)$	$a \times b + a \times c$	Is $a \times (b + c)$ $= a \times b + a \times c$?
2	3	5	2 × (3 + 5) = 16	2 × 3 + 2 × 5 = 16	Yes
2	-3	-5	2×[(-3) + (-5)]= -16	2×(-3) +2×(-5) = -16	Yes
-2	-3	-5	-2× [(-3) +(-5)] = 16	(-2)×(-3)+(-2)×(-5)= 16	Yes

Example: Find the product using suitable property.

i) 26 × (-48) + (-48)× (-36)

ii) 8× 53× (-125)

iii) (-41)× 101

i) 26× (-48) + (-48)× (-36)

= (-48)×26 + (-48)×(-36)

(by commutative property,$a \times b = b \times a$)

$= (-48)\times[26 + (-36)]$

$= (-48)\times[26 - 36]$

$= (-48)\times(-10) = 480$

ii) $8\times53\times(-125)$

$= 53\times[8\times(-125)]$

(by associative property of multiplication $(a \times b) \times c = a \times (b \times c)$)

$= 53\times(-1000) = -53000$

iii)(-41)×101

$= (-41)\times(100 + 1)$

$= (-41)\times100 + (-41)\times 1$

(By the Distributive Property of Multiplication over Addition,$a \times (b + c) = a \times b + a \times c$)

$= -4100 + (-41)$

$= -4100 - 41 = -4141$

Example: A certain freezing process requires that room temperature be lowered from 50°C at the rate of 6°C every hour. What will be the room temperature 12 hours after the process begins?

Initial room temperature = 50°C

Decrease in temperature in 1 hour = -6°C

Decrease in temperature in 12 hours = 12 × (-6) = - 72°C

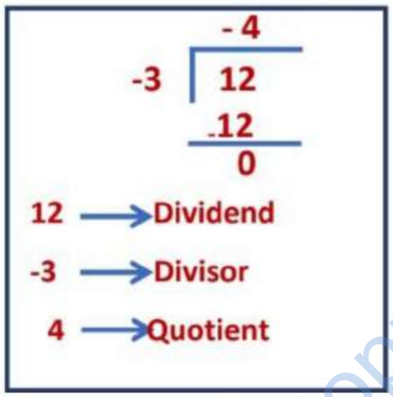
Final temperature = 50°C + (- 72°C) = -22°C

Division of Integers

Rule 1: If two integers of different signs are divided, then we divide them as whole numbers and give a negative sign to the quotient.

$$a \div (-b) = -\frac{a}{b}$$

$$(-a) \div b = -\frac{a}{b}$$



Consider the two integers, 12 and -3.

If we divide 12 by -3, we get,

$$12 \div (-3) = \frac{12}{-3} = -4$$

(+ve)÷(-ve) = (-ve)

(-ve)÷(+ve) = (-ve)

Example: Evaluate each of the following:

i) $(-30) \div 10$
ii) $49 \div (-49)$
iii) $13 \div [(-2) + 1]$

i)$(-30) \div 10$

$(-a) \div b = -\frac{a}{b}$

$(-30) \div (10) = -\frac{30}{10} = -3$

ii) $49 \div (-49)$

$a \div (-b) = -\frac{a}{b}$

$49 \div (-49) = -\frac{49}{49} = -1$

iii) $13 \div [(-2) + 1]$
$= 13 \div (-1)$

$a \div (-b) = -\frac{a}{b}$

$= 13 \div (-1) = -\frac{13}{1} = -13$

Rule 2: If two integers of the same signs are divided, then we divide them as whole numbers and give a positive sign to the quotient.

$(-a) \div (-b) = \frac{a}{b}$

$a \div b = \frac{a}{b}$

On dividing 25 by 5 we get,

$25 \div 5 = \frac{25}{5} = 5$

If we divide (-25) by (-5) we get,

$(-25) \div (-5) = \frac{(-25)}{(-5)} = 5$

We see that the result is the same in both cases.

Example: Evaluate each of the following:

i) $(-36) \div (-4)$

ii) $(-31) \div [(-30) + (-1)]$

iii) $[(-6) + 5] \div [(-3) + 2]$

i) $(-36) \div (-4)$

$(-a) \div (-b) = \frac{a}{b}$

$= (-36) \div (-4) = \frac{36}{4} = 9$

ii) $(-31) \div [(-30) + (-1)]$

$= (-31) \div [(-30) + (-1)]$

$= (-31) \div (-31)$

$(-a) \div (-b) = \frac{a}{b}$

$(-31) \div (-31) = \frac{31}{31} = 1$

iii) $[(-6) + 5] \div [(-3) + 2]$

$= (-1) \div (-1)$

$(-a) \div (-b) = \frac{a}{b}$

$(-1) \div (-1) = \frac{1}{1} = 1$

Example: Write five pairs of integers (a, b) such that $a \div b = -4$.

Five pairs of integers are,

i) $(8, -2)$

$a \div (-b) = -\frac{a}{b}$

$8 \div (-2) = -\frac{8}{2} = -4$

ii) $(-4, 1)$

$(-a) \div b = -\frac{a}{b}$

$= (-4) \div 1 = -\frac{4}{1} = -4$

iii) $(-16, 4)$

$(-a) \div b = -\frac{a}{b}$

$(-16) \div 4 = -\frac{16}{4} = -4$

iv) $(-24, 6)$

$(-a) \div b = -\frac{a}{b}$

$(-24) \div 6 = -\frac{24}{6} = -4$

v) $(36, -9)$

$a \div (-b) = -\frac{a}{b}$

$36 \div (-9) = -\frac{36}{9} = -4$

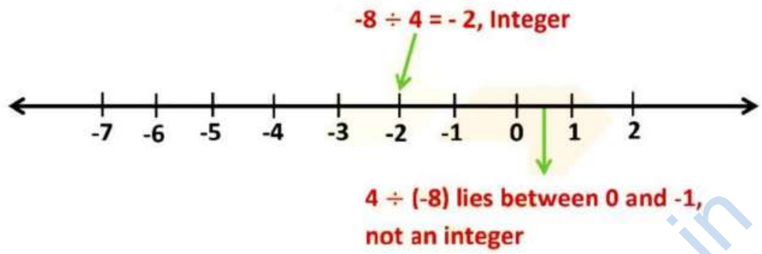
Properties of Division of Integers

i) Closure Property

If a and b are two integers, then $a \div b$ is not always an integer.

a	b	a ÷ b	Integer
-8	4	-8 ÷ 4 = -2	Yes
4	-8	4 ÷ (-8) = Not an integer	No
-10	27	-10 ÷ 27 = Not an integer	No

Integers are not closed under division



ii) Commutative Property

If a and b are two integers, a ÷ b ≠ b ÷ a

a	b	a ÷ b	b ÷ a	Is a ÷ b = b ÷ a ?
18	-6	18 ÷ (-6) = -3	-6 ÷ 18 = not an integer	No
-25	5	(-25) ÷ 5 = -5	5 ÷ (-25) = not an integer	No
36	9	36 ÷ 9 = 4	9 ÷ 36 = not an integer	No

d5

iii) Associative Property

For any 3 integers a, b and c, (a ÷ b) ÷ c ≠ a ÷ (b ÷ c)

a	b	c	(a ÷ b) ÷ c	a ÷ (b ÷ c)	Is (a ÷ b) ÷ c = a ÷ (b ÷ c)?
24	-4	2	[24 ÷ (-4)] ÷ 2 = -3	24 ÷ [(-4) ÷ 2] = -12	No
40	10	-2	(40÷ 10)÷ (-2) = -2	40 ÷ [10 ÷ (-2)] = -8	No
-48	12	4	[(-48)÷ 12]÷ 4 = -1	-48÷ (12÷ 4) = -16	No

Division is not Associative for integers

iv) Division of 0 by any integer

If a is any integer other than zero, then a ÷ 0 is not defined but 0 ÷ a = 0,

a	0	$0 \div a$
-5	0	0 ÷ (-5) = 0
7	0	0 ÷ 7 = 0
-12	0	0 ÷(-12) = 0

iv) Division by 1

If a is an integer, then a ÷ 1 = a

a	1	$a \div 1$	Is $a \div 1 = a$?
-5	1	(-5) ÷ 1 = -5	Yes
15	1	15 ÷ 1 = 15	Yes
-150	1	-150 ÷ 1 = -150	Yes

Example: Fill in the blanks

i) ___ ÷ 25 = 0

ii) (-206) ÷ _____ = 1

iii) _____ ÷ 1 = -87

i) ____ ÷ 25 = 0

If we divide 0 by any integer, the result is always zero.

So, 0 ÷25 = 0

ii) (-206) ÷ _____ = 1

If we divide any -206 by -206, the result is one.

(-206) ÷ (-206) = 1

iii) _____ ÷ 1 = -87

If any integer is divided by 1 the result is the same integer.

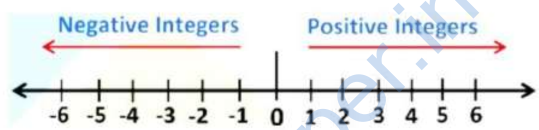
(-87) ÷ 1 = -87

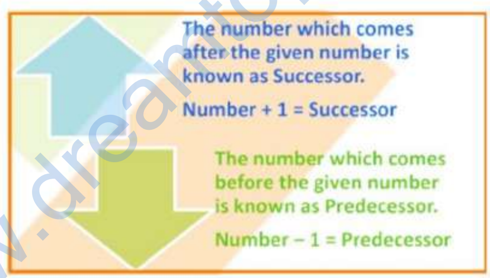
Summary Notes - Integers

The collection of numbers .... -3, -2, -1, 0, 1, 2, 3 .... is called integers.

So, ... -3, -2, -1 these negative numbers are called negative integers and 1,2,3, ... are called positive integers.

- Integers to the right of zero are positive.
- Integers to the left of zero are negative.
- The value of integers increases as we move towards right of 0.
- The value of integers decreases as we move towards left of 0.





The absolute value of an integer is its distance from 0 on a number line or its numerical value without taking its sign into consideration. The symbol for absolute value is | |.
E.g. |-8|=8, |8|=8, |0|=0 etc.

Addition of Integers

Rule 1: When we add two positive integers, we add their values and the result will take the positive sign (common sign of both the integers).

Rule 2: When we add a positive and a negative integer, we find the difference of their numerical values, regardless of their signs and give the sign of the integer which is greater.

Rule 3: If two negative integers are added then we add their values and the result will take the negative sign.

Subtraction of Integers

When we subtract one integer from the other, we convert the integer to be subtracted to its negative and then add the two integers.

Subtracting an integer from the other is same as adding the additive inverse of the integer.
$a - b = a +$(additive inverse of b) $= a + (-b)$

Following are the properties satisfied by Integers:-
Let a, b and c be any three integers
(a) **Closure Property**: $a + b$, $a - b$ and $a\times b$ are again integers.
(b) **Commutative property**: $a + b = b + a$
$a\times b = b\times a$
(Subtraction of integers is not commutative)
(c) **Associative Property**: $(a + b) + c = a + (b + c)$
$(a \times b) \times c = a \times (b \times c)$
(Subtraction of integers is not commutative)
(d) **Additive identity**: $a + 0 = 0 + a = a$
(e) **Multiplicative identity**: $1\times a = a = a\times 1$

Distributivity of Multiplication over Addition:
If a, b & c are any three integers, then
$$a \times (b + c) = a \times b + a \times c$$

Distributivity of Multiplication over Subtraction:
If a, b & c are any three integers, then
$$a \times (b - c) = a \times b - a \times c$$