

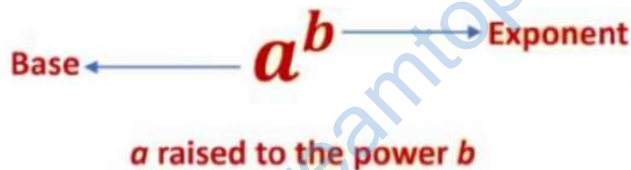
Chapter 13

Exponents and Powers

Introduction to Exponents and Powers

Exponents and powers help in writing the large numbers in a shorter form which we can read, understand and compare easily. A number placed in a superscript position to the right of another number or variable indicate repeated multiplication.

The short notation 10^4 stands for the product $10 \times 10 \times 10 \times 10$. Here '10' is called the base and '4' the exponent. The number 10^4 is read as 10 raised to the power of 4 or simply as the fourth power of 10. 10^4 is called the exponential form of 10,000.



For example:

a^2 indicates $a \times a$

a^3 indicates $a \times a \times a$

In expression 2^5 , 2 is called the base and 5 is called the exponent or power.

$$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

i) Express 256 as a power 2.

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8 = 256$$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

ii) Which one is greater 6^2 or 2^6 ?

$$6^2 = 6 \times 6 = 36$$

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

So, 2^6 is greater than 6^2 .

Laws of exponents

Laws of Exponents

(a) First Law

If a is any non-zero integer (base) and m, n are integers (powers),

then $a^m a^n = a^m \times a^n = a^{m+n}$

for example,

(i) $(2)^5 \times (2)^{10} = (2)^{5+10} = (2)^{15}$

(ii) Multiply 3^2 and 3^4 .

$$\Rightarrow 3^2 \times 3^4$$

$$\Rightarrow 3^{2+4}$$

$$\Rightarrow 3^6$$

$$\Rightarrow 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$\Rightarrow 729$$

(iii) Multiply -3^2 and -3^3 .

$$\Rightarrow (-3)^2 \times (-3)^3$$

$$\Rightarrow (-3)^{2+3}$$

$$\Rightarrow (-3)^5$$

$$\Rightarrow (-3) \times (-3) \times (-3) \times (-3) \times (-3)$$

$$\Rightarrow -243$$

(b) Second Law

If a is any non-zero integer (base) and m, n are whole numbers (powers).

If $m > n$, then $a^m \div a^n = a^{m-n}$.

If $m < n$, then $a^m \div a^n = \frac{1}{(a)^{n-m}}$.

For example,

(i) $3^6 \div 3^5 = 3^{6-5} = 3^1$

(ii) Divide 3^6 by 3^2 .

$$\Rightarrow 3^6 \div 3^2$$

$$\Rightarrow 3^{6-2}$$

$$\Rightarrow 3^4$$

$$\Rightarrow 3 \times 3 \times 3 \times 3$$

$$\Rightarrow 81$$

(c) Third Law

If a and b are two non-zero integers (base) and m is a whole number (power), then $(a^m)^n = (a^{m \times n})$

For example,

(i) $(5^2)^3 = 5^{2 \times 3} = 5^6$

(ii) Calculate $(3^3)^2$.

$$\Rightarrow 3^3 \times 3^3$$

$$\Rightarrow 3^3 + 3$$

$$\Rightarrow 3^6$$

$$\Rightarrow 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$\Rightarrow 729$$

iii) Which one is greater $(5^2) \times 3$ or $(5^2)^3$?

$(5^2) \times 3$ means 52 is multiplied by 3

i.e., $5 \times 5 \times 3 = 75$

but $(5^2)^3$ means 5^2 is multiplied by itself 3 times

$$5^2 \times 5^2 \times 5^2$$

$$5^{2+2+2}$$

$$5^6 = 15625$$

So, $(5^2)^3$ is greater than $(5^2) \times 3$.

(d) Fourth Law

If a and b are two non-zero integers (base) and m is a whole number (power), then $a^m \times b^m = (ab)^m$.

For example,

(i) $(3 \times 5)^4 = 3^4 \times 5^4$

$$\begin{aligned} &= (3 \times 3 \times 3 \times 3) \times (5 \times 5 \times 5 \times 5) \\ &= 81 \times 625 \\ &= 50625 \end{aligned}$$

(ii) Multiply 3^3 and 4^3 .

$$\begin{aligned} &\Rightarrow 3^3 \times 4^3 \\ &\Rightarrow (3 \times 4)^3 \\ &\Rightarrow 12^3 \\ &\Rightarrow 12 \times 12 \times 12 \\ &\Rightarrow 1728 \end{aligned}$$

(e) Fifth Law

If a and b are two non-zero integer (base) and m is a whole number (power),

then $a^m \div b^m = (ab)^m = \frac{a^m}{b^m} = (a \div b)^m$

For example,

(i) $\frac{2}{3^4}$

$$= 2^4 \div 3^4$$

$$= \frac{16}{81}$$

(ii) Divide 3^3 by 4^3 .

$$\Rightarrow 3^3 \div 4^3$$

$$\Rightarrow (3)^3 \div (4)^3$$

$$\Rightarrow (34)^3$$

(f) Sixth Law

If a is an integer (base) then $a^0 = 1$

For example,

(i) Divide 3^4 by 3^4 .

$$\Rightarrow 3^4 \div 3^4$$

$$\Rightarrow \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} = 1$$

$$\Rightarrow \frac{3^4}{3^4} = 1$$

$$\Rightarrow 3^{4-4} = 1$$

$$\Rightarrow 3^0 = 1$$

So, we can say that any number (except 0) raised to the power (or exponent) 0 is 1.

Simplify and write the answer in the exponential form

i) $(3^8 \div 3^5)^5 \div 3^5$

$$(3^{8-5})^5 \div 3^5 \quad (\text{Using the law } a^m \div a^n = a^{m-n})$$

$$= (3^3)^5 \div 3^5 \quad (\text{Using the law } (a^m)^n = a^{mn})$$

$$= (3)^{15} \div 3^5$$

$$= 3^{(15-5)} \quad (\text{Using the law } a^m \div a^n = a^{m-n})$$

$$= 3^{10}$$

Expressing Numbers in Standard Form

Decimal Number system

Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10.

For example:

1) 5678925 can be written as,

$$56789 = 50000 + 6000 + 700 + 80 + 9$$

$$= 5 \times 10^4 + 6 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 9 \times 10^0$$

2) 14678925 can be written as,

$$146789 = 100000 + 40000 + 6000 + 700 + 80 + 9$$

$$= 1 \times 10^5 + 4 \times 10^4 + 6 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 9 \times 10^0$$

Here, we can see the exponents in first example starts from 4 and go on decreasing till 0.

Similarly, we can see the exponents in first example starts from 5 and go on decreasing till 0.

Expressing numbers in Standard Form

Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its standard form.

A number is said to be in the Standard Form, if it is expressed as the product of a number between 1 and 10 and the integral power of 10.

For example,

$$153,600,000,000 = 1.536 \times 10^{11}$$

Decimal is moved 11 places towards left, therefore the power is positive.

Large numbers in Standard Form

Express the following numbers in standard form.

i) 14320000000

ii) 205000000

i) $1432000000 = 1432 \times 10000000$

$$= 1.432 \times 1000 \times 10000000$$

$$= 1.432 \times 10^3 \times 10^7$$

$$= 1.432 \times 10^{(3+7)}$$

$$= 1.432 \times 10^{10}$$



ii) $205000000 = 205 \times 1000000$

$$= 2.05 \times 100 \times 1000000$$

$$= 2.05 \times 10^2 \times 10^6$$

$$= 2.05 \times 10^{(2+6)}$$

$$= 2.05 \times 10^8$$



Express 15360000000 in standard form.

$$15360000000 = 1536 \times 10000000$$

$$= 1.536 \times 1000 \times 10000000$$

$$= 1.536 \times 10^3 \times 10^7$$

$$= 1.536 \times 10^{(3+7)}$$

$$= 1.536 \times 10^{10}$$

