Chapter 7

Concurrence of triangles

Introduction to Congruence of Triangles

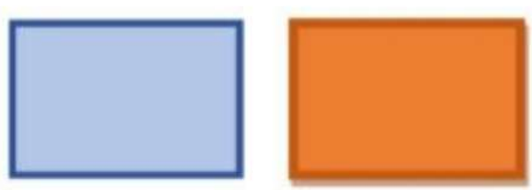
Introduction

Look at the following figures. What do you observe in them?

In the figures shown above, we see each pair is looking exactly the same with each other or if we put one object from each pair on the other it completely superimposed the other object.

This type of figures or objects which have the same shape and size such objects are to be congruent. The relation between two congruent objects is called congruence and the congruency is denoted by the symbol ≅(congruent to)

Example: Are the following two boxes congruent to each other?



The shape and size of the given boxes are the same. Hence, these given two boxes are congruent with each other

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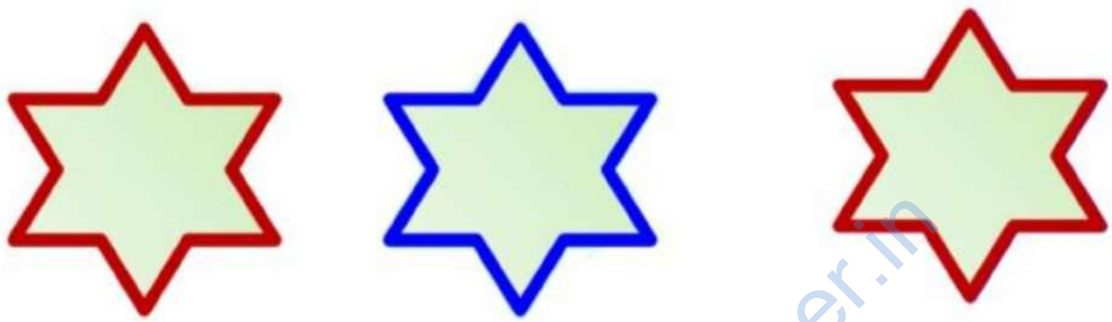
Congruence of plane figures

If one plane figure is superimposed on the other and it covers the other completely and exactly then they are said to be congruent.

Example: Consider the following star shape figures,



F_1 F_2 F_2

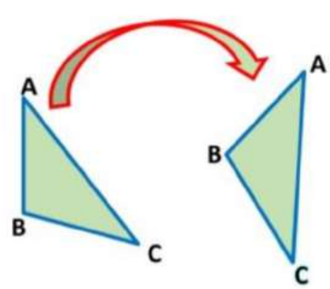
In the above figures when we superimpose figure F1 on figure F2, we see figure F_1 covers figure F_2 completely and exactly. Hence, we can say that $F_1 \cong F_2$.

Note 1:

In order to determine the given two geometric shapes are congruent we need to learn about geometric transformations.

1. Rotation or Turn

In rotation, shapes are rotated or turn. The given two triangles are similar but the second triangle is rotated. To see these two triangles are similar we get them in the same direction. Such a transformation is called rotation.



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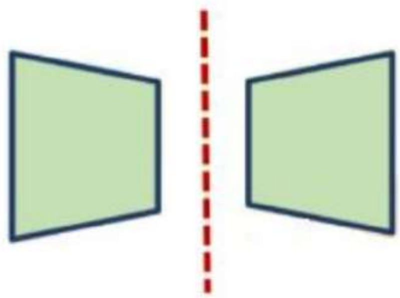
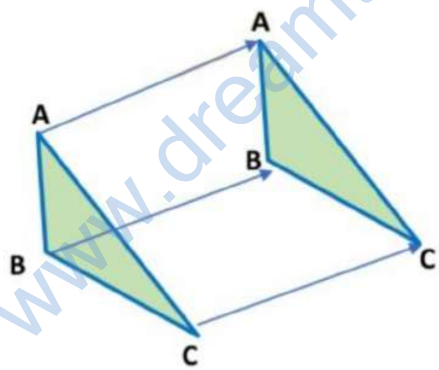
2. Reflection or Flip

In reflection, shapes are flipped across the imaginary line. Now, consider the following trapezoid in which we can easily see how the the image can reflect across the imaginary line.



3. Shifting or Translating

In shifting we slightly shift the shape without changing its direction. In translation, we can easily see two shapes are similar or not.



After any of these geometric transformations (turn, flip, or shift), the shape still has the same shape and size. Hence, we can say that these two shapes are congruent.

Note 2:

To prove the congruency of two figures you are not allowed to bend, twist, or stretch the figure. Let us see this by using examples.

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1. Stretching

The following plane figures are congruent with each other?

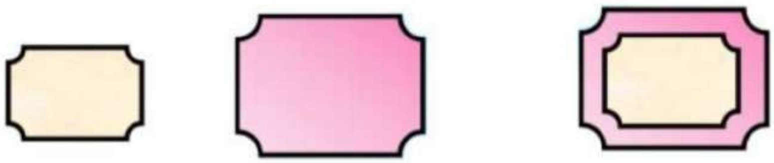

Fig. A　　Fig. B　　Fig. A B

Let us see if we superimpose figure A on figure B we see figure A is not covered figure B completely and exactly. Also, to prove the congruency in two figures, stretching is not allowed because it changes the shape and dimensions of the figures. So, we can say that the two figures have different shapes and sizes and so, they are not congruent.

2.Bending

Fig. A　　Fig. B　　Fig. A

Let us see, if we superimpose figure A on figure B we see figure A is not covered by figure B completely and exactly. If we bend figure A or stretch figure B then it could be possible to prove the congruency in two figures, but bending or stretching is not allowed because it changes the shapes and dimensions of the figures. So, we can say that the two figures have different shapes and sizes cannot be made congruent by bending or twisting.

3.Twisting

Fig. A　　Fig. B

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What do you observe in the above figures?

We observe some wires are twisted in figure A and in figure B some wires are there which are untwisted.

In both the figures same wires are present but from that can you say that wires in figure A are congruent to wires in figure B?

No, because twisting is not allowed to prove the congruency because it changes the shape and dimensions of the figures. So, we can say that the two figures have different shapes and sizes cannot be made congruent by twisting.

Congruence among line segments and angles

Any two line segments are congruent if and only if their lengths are equal.

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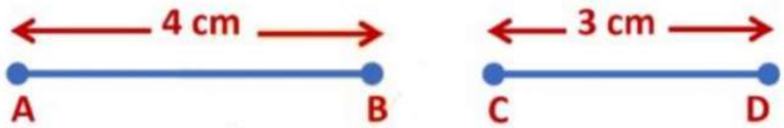
In the above figures, we see the length of the line segment AB is equal to the length of the line segment CD. Hence, two line segments AB and CD are congruent.

Also, if two line segments are congruent, we sometimes just say that the line segments are equal.

Here, $AB \cong CD$ is same as $AB = CD$

$\because AB = CD = 5$ cm

Example: Find the given line segments are congruent or not?



We know any two line segments are congruent if and only if their lengths are equal.

Here, we see the lengths of the two line segments are not equal. Hence, they are not congruent with each other.

$\because AB \neq CD$

Example: Two line segments AB and PQ are congruent. If AB = 5 cm, then what is the length of PQ?

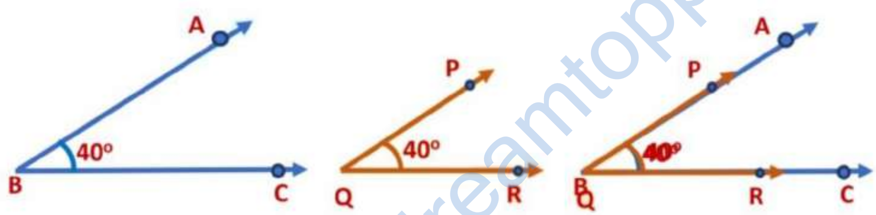
The line segment AB and PQ are congruent.

$\therefore AB = PQ$

Given that AB = 5 cm ⇒ PQ = 5 cm

Congruence of Angles

Two angles are congruent if and only if their measures are equal.



In the above figure, when we superimpose ∠PQR on ∠ABC, we see two arms of ∠PQR are completely superimpose on ∠ABC and their measures are also the same i.e.,40°

So, we can say that

$\angle ABC \cong \angle PQR$

or

$\angle ABC = \angle PQR$

Also, if the angles are congruent, their measures are the same.

Here, we know $\angle ABC \cong \angle PQR$

$\therefore \angle ABC = \angle PQR$

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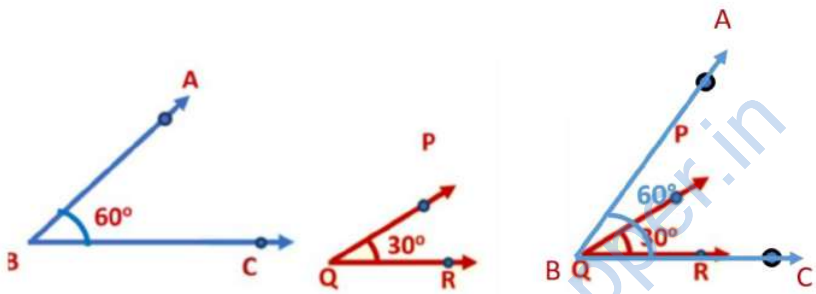
∵∠ABC = ∠PQR = 40°

In this case we see the length of arm AB and arm BC are longer than the ray arm PQ and arm QR. But, remember the arms only indicate the direction of angle not the length. So, we can say that arms of the angles do not play any part in congruency.

Example: Find the given angles are congruent or not?



In the above figures when we superimpose ∠PQR on ∠ABC, we see two arms of ∠PQR are not completely superimposed on ∠ABC and their measures are also different.

Hence, ∠ABC is not congruent with ∠PQR

Or ∠ABC ≠∠PQR

Example: Two angles ∠ABC and ∠XYZ are congruent. If ∠ABC = 75°, then what is the measure of ∠XYZ?

We have,

∠ABC ≅∠XYZ

We know if the angles are congruent, their measures are the same.

∴∠ABC = ∠XYZ

Given that ∠ABC = 65°

⇒∠XYZ = 65°

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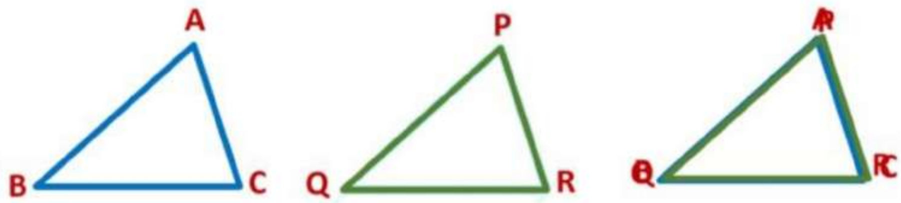
Congruence of Triangle

Two triangles are congruent if they are exact copies of each other and cover each other exactly.



In the above figure when we place ΔPQR on ΔABC we see both triangles are exactly the same shape and size also they cover each other exactly. So, they are congruent to each other. We express this as ΔABC ≅ ΔPQR.

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The alphabetical order of naming triangles is very important as they represent the corresponding parts of the triangle which means
when we place ΔPQR on ΔABC we see,

The vertex A coincides with vertex P, vertex B coincides with vertex Q and vertex C coincides with vertex R. Here correspondence is represented using ⟷.

The side AB coincides with side PQ, side AC coincides with side PR and side BC coincides with side PQ.

Here, vertex and sides are coinciding with each other so, we can say that their angles are also matching with each other.

m∠BAC = m∠QPR, m∠ACB = m∠PRQ and m∠CBA = m∠RQP

Thus, in these two congruent triangles, we have:

Corresponding vertices: A ⟷P, B ⟷Q, C ⟷R.

Corresponding sides: AB⟷PQ, BC ⟷QR, AC ⟷PR.

Corresponding angles: ∠A ⟷∠P, ∠B ⟷∠Q, ∠C ⟷∠R.

Example: If ΔPQR ≅ ΔXYZ write the part(s) of ΔPQR that corresponds to (i) PQ (ii) ∠A (iii) YZ

Here, ΔPQR ≅ ΔXYZ

ΔPQR ⟷ ΔXYZ

(i)PQ ⟷ XY (ii) ∠A ⟷∠P (iii) YZ ⟷QR

Criteria for Congruence of Triangles

In the previous section, we learnt that two triangles are congruent if all six pairs of corresponding parts of the two triangles are congruent, or in other words, we can say that all the three pairs of corresponding sides must be congruent and all the three pairs of corresponding angles must be congruent.

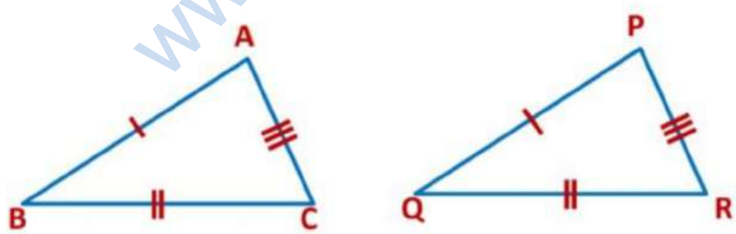
In this section, we learn how to prove two triangles are congruent using only three pairs of corresponding parts. So, there are four conditions for the congruency of two triangles. Let us see them one by on

(i) SSS (Side – Side – Side) Congruence criterion

If under a given condition, the three sides of one triangle are congruent to three sides of another triangle, then two triangles are congruent.

[Note: Here we don't need to know about angles.]

Example:



In ΔABC and ΔPQR, we have

AB = PQ, BC = QR and AC = PR
Hence, ΔABC and ΔPQR are congruent by side–side–side congruence criterion.

∴ΔPQR ≅ ΔXYZ

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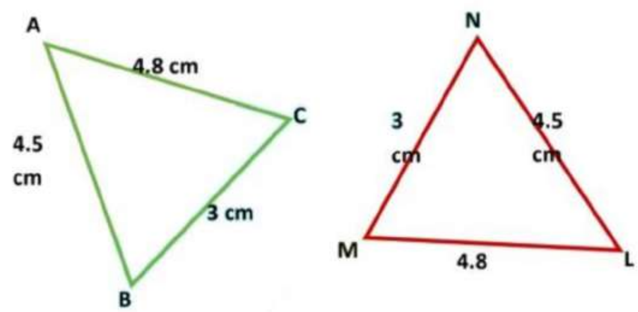
Example: Check whether two triangles ABC and triangles LMN are congruent.



Solution: In Δ ABC and ΔLMN, we have

BC = MN = 3 cm (Given)

⇒AB = LN = 4.5 cm (Given)

⇒AC = LM = 4.8 cm (Given)

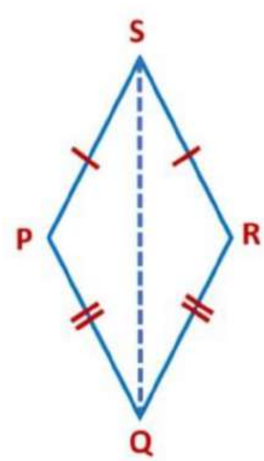
Therefore, Δ ABC ≅ ΔLMN (By SSS-criterion of congruence)

Example: In Fig, PS = RS and PQ = RQ.

(i) State the three pairs of equal parts in ΔPQS and ΔRQS.

(ii) Is ΔPQS≅ΔRQS? Why?

(i)In ΔPQS and ΔRQS, the three pairs of equal parts are as given below:



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PQ = RQ... (Given)

PS = RS ... (Given)

and

QS = QS ... (Common in both)

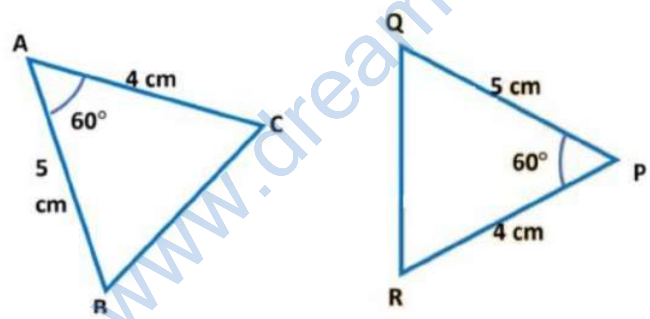
Hence, ΔPQS≅ΔRQS

(By SSS congruence rule)

(ii) SAS (Side –Angle– Side) Congruence criterion

If under a correspondence, two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, then two triangles are congruent.

Example:



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In ΔABC and ΔPQR, we have

AB = PQ = 5 cm (Given)

∠BAC = ∠QPR = 60° (Given)

AC = PR = 4 cm (Given)

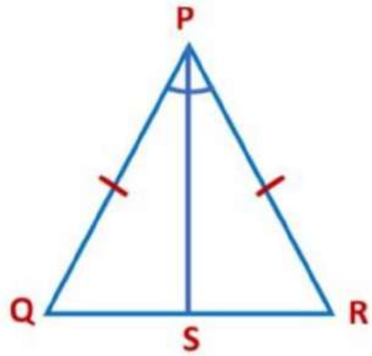
Therefore, ΔABC ≅ ΔPQR (By SAS-criterion of congruence)

Example: In Fig, PQ = PR and PS is the bisector of ∠QPR.

(i) State three pairs of equal parts in triangles PSQ and PSR.

(ii) Is ΔPSQ ≅ΔPSR? Give reasons.

(iii) Is ∠Q = ∠R?



(i) The three pairs of equal parts are as follows:

PQ = PR (Given)

∠QPS = ∠RPS (PS bisects ∠QPR) and PS = PS (common)

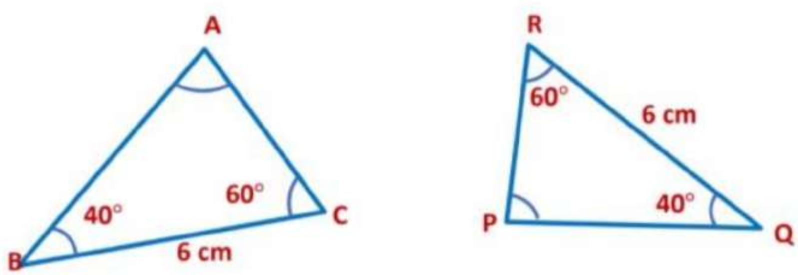
(ii) Yes, ΔPSQ ≅ΔPSR (By SAS congruence rule)

(iii) ∠Q = ∠R (Corresponding parts of congruent triangles)

(iii) ASA (Angle – Side –Angle) Congruence criterion

If under a correspondence, two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent.

Example:



In ΔABC and ΔPQR, we have

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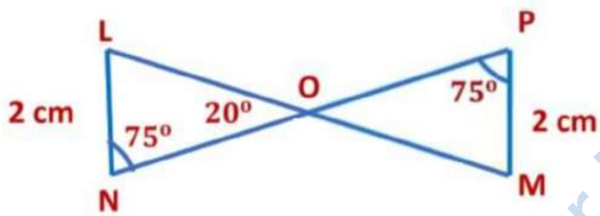
$\angle ABC = \angle PQR = 40°$... (Given)

$BC = QR = 6$ cm ... (Given)

$\angle BCA = \angle QRP = 60°$... (Given)

Therefore, Δ ABC $\cong \Delta$ PQR (By ASA-criterion of congruence)

Example: In the following figure can you say that $\Delta LON \cong \Delta MOP$



In the two triangles ΔLON and ΔMOP

$\angle LNO = \angle MPO = 75°$ ... (Given)

$\Delta LON = \Delta MOP = 20°$ ... (Vertically opposite angles)

In ΔLON,

$\angle OLN + \angle LNO + \angle NOL = 180°$... (Vertically opposite angles)

$\angle OLN = 180° - (\angle LNO + \angle NOL)$

$\angle OLN = 180° - (75° + 20°)$

$\angle OLN = 180° - (95°)$

$\angle OLN = 85°$

In ΔMOP,
$\angle OMP + \angle MPO + \angle POM = 180°$... (Vertically opposite angles)

$\angle OMP = 180° - (\angle MPO + \angle POM)$

$\angle OMP = 180° - (75° + 20°)$
$\angle OMP = 180° - (95°)$

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$\angle OMP = 85°$

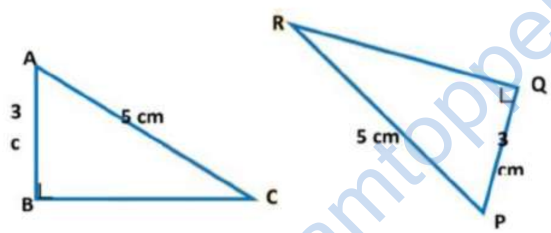
Thus, we have $\angle OLN = \angle OMP$, $LN = PM$ and $\angle LNO = \angle MPO$

Now, side LN is between $\angle OLN$ and $\angle LNO$ and side PM is between $\angle MPO$ and $\angle OMP$.

So, by ASA rule $\Delta LON \cong \Delta MOP$

(iv) RHS (Right angle – Hypotenuse – Side) Congruence criterion

If under a correspondence, hypotenuse and one side of the right angle triangle are respectively equal to the hypotenuse and one side of the other right angle triangle, then the triangles are congruent.
Example:



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In ΔABC and ΔPQR, we see

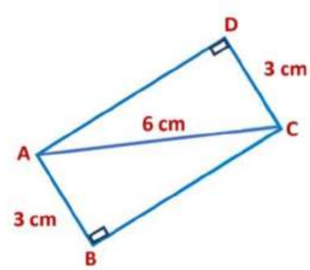
$\angle ABC = \angle PQR = 90°$(Given)

$AC = PR = 5$ cm (Given)

$AB = PQ = 3$ cm (Given)

Therefore, $\Delta ABC \cong \Delta PQR$ (By RHS-criterion of congruence)

Example: In the given diagram check whether ΔADC and ΔCBA hold the property of RHS congruence.



In ΔADC, $\angle ADC = 90°$

In ΔCBA, $\angle CBA = 90°$

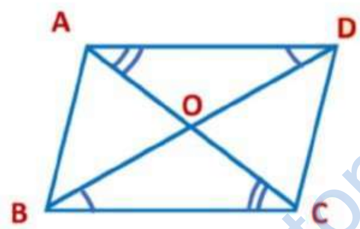
$AC = 6$ cm (hypotenuse is same for both ΔADC and ΔCBA)

$AB = BC = 3$ cm

So, ΔADC and ΔCBA hold the property of the RHS congruence rule,

$\Delta ADC \cong \Delta CBA$

Prove that the diagonals of the parallelogram bisect each other.



In the parallelogram ABCD, diagonals AC and BD intersect at O.

It is required to prove that $AO = OC$ and $BO = OD$

In ΔAOD and ΔCOB, we have

$\angle OAD = \angle OCD$ (Alternate angles as AD || BC and AC is the transversal)

Similarly $\angle ODA = \angle OBC$

$AD = BC$ (Opposite sides of a parallelogram)

$\therefore \Delta AOD \cong \Delta COB$ (ASA congruency)

$\therefore AO = OC$ and $BO = OD$ (Corresponding parts of congruent triangles)

Hence, the diagonals of the parallelogram bisect each other is proved.

Prove that the bisector of the vertical angle of an isosceles triangle bisects the base at a right angle.

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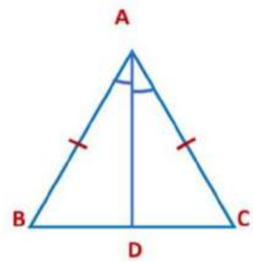


Given: AB = AC and AD is the bisector of ∠A

To prove: ∠ADB = ∠ADC = 90oand BD = DC

Proof: In ΔADB and ΔADC, we have

AB = AC ... (given)

AD = AD ... (Common)

∴ ΔADB≅ΔADC (SAS congruency)

So, BD = DC and ∠ADB = ∠ADC ....(i)

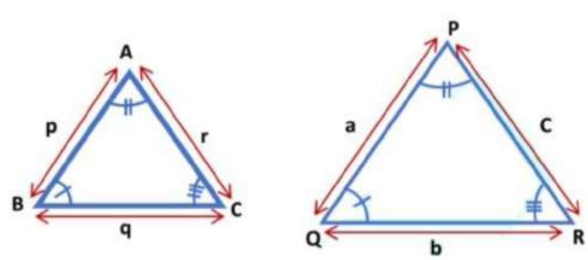
But, ∠ADB + ∠ADC = 180°

2∠ADB = 180°

$\angle ADB = \frac{180}{2}° = 90°$

∴ ∠ADB = ∠ADC = 90°

(v) AAA (Angle-Angle-Angle) Rule



In ΔABC and ΔPQR, we have

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∠BAC = ∠QPR ..... (Given)

∠ACB= ∠PRQ ..... (Given)

∠CBA= ∠RQP ..... (Given)

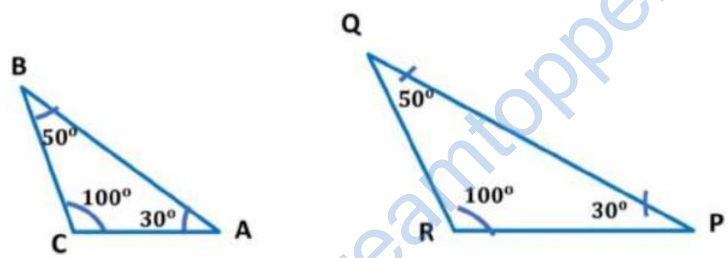
Hence, ΔABC and ΔPQR are similar but not congruent because their sizes are different.

Hence, we do not use AAA congruence criterion.

Example: In ΔABC, ∠A= 30°, ∠B = 50°∠C = 100°

In ΔPQR, ∠P = 30°, ∠Q = 50°∠R = 100°

Here ΔABC ≅ΔPQR by AAA congruence criterion. Is it true?



Here, we have

∠A = ∠P = 30°

∠B = ∠Q = 50°

∠C = ∠R = 100°

Here, all the three angles of both triangles are equal but sides may not be equal. So, triangles may not be congruent. Hence, the given statement is false.

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