Chapter 3

Understanding Quadrilaterals

Introduction to Understanding Quadrilaterals

What is Plane Surface?

A plane surface is a flat surface having two dimensions (length and breadth) and no thickness.

What is Plane Figure?

A plane figure obtained by joining a number of points without lifting pencil from the paper is called a curve.

What is Open Curve?

An open curve is a curve which does not cut itself.

What is Closed Curve?

A closed curve is a curve which cuts itself.

What is Simple Closed Curve?

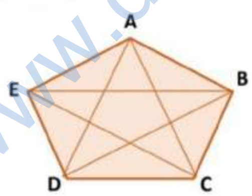
A closed curve is a simple closed curve, if it does not pass through a point more than once.

Polygons

What is a Polygon?
A polygon is a simple closed curve made up of line segments only.

Polygons are classified according to the number of sides they have.

Figure				
No. of sides	3	4	5	6
Name	Triangle	Quadrilateral	Pentagon	Hexagon



What are Adjacent Sides?

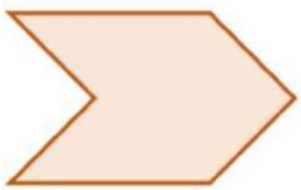
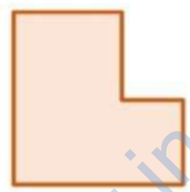
If the two sides of a polygon have a common vertex then they are called the adjacent sides of the polygon. In the given figure, AE and AB are adjacent sides of the Polygon ABCDE as they have the common vertex A.

What are Adjacent Vertices?
The two endpoints of the same side of the polygon are called adjacent vertices. In the given figure, A and E are adjacent vertices as they are the two endpoints of the side AE.

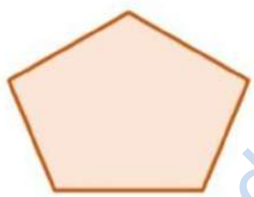
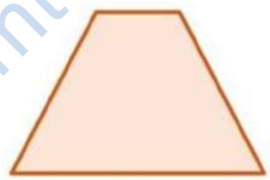
What are Diagonals?
The line segment joining two non-adjacent vertices are called diagonals of the polygon. In the figure, AD, AC, EB, EC, and BD are the diagonals of the polygon ABCDE.

Concave and Convex Polygon

• Concave Polygon – A polygon is a concave polygon if a line segment joining two different points does not lie completely inside the polygon.

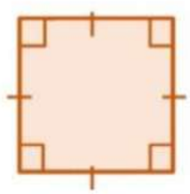
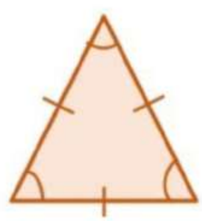



• Convex Polygon – A polygon is a convex polygon if the line segment joining any two points lies completely inside the polygon.

Regular and Irregular Polygon

• Regular Polygon – A polygon whose all sides and all angles are equal is called a Regular Polygon. A square has sides of equal length and angles of equal measure, i.e., 90°

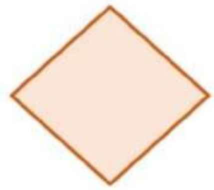



Similarly, an Equilateral Triangle has sides of equal length and angle of equal measure i.e., 60°.

Therefore, a square and an equilateral triangle are Regular Polygons.

• **Irregular Polygons –** Polygons which do not have all the sides and angles of equal measure is called an Irregular Polygon. Examples, Rhombus and Rectangle.

Rectangle



Rhombus

Angle Sum Property

Angle Sum Property of a quadrilateral

The sum of all the interior angles of a quadrilateral is 360°.

Let us prove the same.

Let us consider a quadrilateral ABCD.

In Δ ABC,

By angle sum property of triangle, we have

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$....... (1)

By angle sum property of triangle, we have

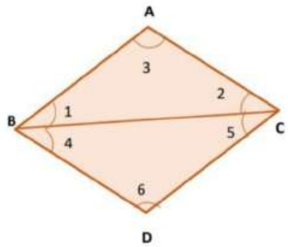
In Δ DBC,

$m\angle 4 + m\angle 5 + m\angle 6 = 180°$....... (2)

On adding equation (1) and (2), we get

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6$

$= 180° + 180°$
$= 360°$
$\angle A + \angle B + \angle C + \angle D = 360°$
$\therefore$ The sum of the measure of the four angles of a quadrilateral is 360°



Sum of the angles of a quadrilateral is 360^0

In the table given below, each figure is divided into triangles and the sum of the angles is deduced from that.

Figure			
Side	3	4	5
Angle Sum	180°	2×180^0 $= (4 - 2) \times 180^\circ$	3×180 $= (5 - 2) \times 180^\circ$

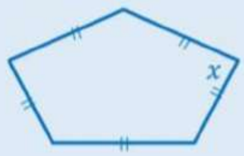
What will be the angle sum of a convex polygon with number of sides as:

i) n
ii) 8

i) Angle Sum = $(n - 2) \times 180^\circ$
(where n is the no. of sides)

ii) Angle Sum = $(n - 2) \times 180^\circ$
(where n is the no. of sides)
$n = 8$
Angle Sum = $(8 - 2) \times 180^\circ = 6 \times 180^\circ = 1080^\circ$

Find the angle measure x in the following figure:



As the given pentagon is a regular pentagon

The measure of all the angles will be x.

Angle Sum = $(n-2) \times 180^{\circ}$ (where n is the no. of sides)

Here, n = 5

So,

Angle Sum = $(5-2) \times 180^{\circ} = 3 \times 180^{\circ} = 540^{\circ}$

$x + x + x + x + x = 540^{\circ}$

$5x = 540^{\circ}$

$x = \frac{540}{5}$

$x = 108^{\circ}$

Sum of Exterior Angles of a Polygon

Sum of the Measures of the Exterior Angles of a Polygon

The sum of the measures of the external angles of any polygon is 360°.

Let, the exterior angles of the quadrilateral ABCD be ∠1, ∠2, ∠3 and ∠4.

∠1 and ∠DAB forms a linear pair and the sum of the angles of a linear pair is 180°

$\angle 1 + \angle DAB = 180°$ **(1)**

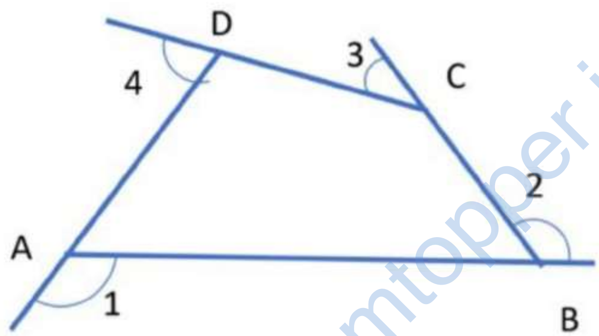
Similarly,

$\angle 2 + \angle CBA = 180°$ **(2)**

$\angle 3 + \angle DCB = 180°$ **(3)**

$\angle 4 + \angle ADC = 180°$ **(4)**

Adding (1), (2), (3) and (4)



$\angle 1 + \angle DAB + \angle 2 + \angle CBA + \angle 3 + \angle DCB + \angle 4 + \angle ADC =$

$180° + 180° + 180° + 180°$

$(\angle 1 + \angle 2 + \angle 3 + \angle 4) + (\angle DAB + \angle CBA + \angle DCB + \angle ADC) = 72$

$(\angle 1 + \angle 2 + \angle 3 + \angle 4) + 360° = 720°$

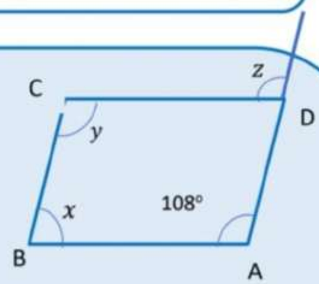
(Sum of the interior angles of the quadrilateral is equal to 360°)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 720° - 360°$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$

Each exterior angle of the of a regular polygon of n sides is equal to $(\frac{360}{n})$.

Find the value of x, y, z:



$y = 108°$

(Opposite angles of a parallelogram are equal)

$y + x = 180°$

(Adjacent angles of a parallelogram form linear pair)

$108° + x = 180°$

$x = 180° - 108° = 72°$

$\angle ADC = x$ (Opposite angles of a parallelogram are equal)

$\angle ADC = 72°$

$z + \angle BCD = 180°$

$z = 180° - 72° = 108°$

Kinds of Quadrilaterals

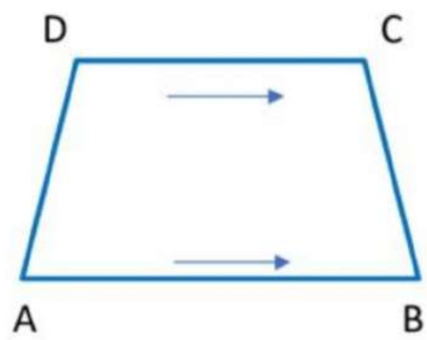
Kinds of Quadrilaterals

1) Trapezium

- Trapezium is a quadrilateral with a pair of opposite parallel sides and the other pair of opposite sides is non – parallel.

ABCD is a trapezium, where AB ||CD

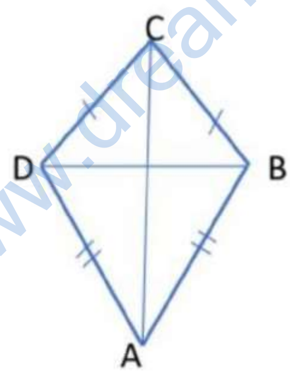
AD and BC are its non-parallel sides.



2) Kite

- Kite is a special type of quadrilateral.
- The two pairs of adjacent sides are equal.
- The opposite sides are unequal.
- Diagonals of a kite intersect each other at right angles.

ABCD is a kite, where AB = AD and BC = CD.



3) Parallelogram

- A quadrilateral with each pair of opposite sides parallel.
- Opposite sides are equal.
- Diagonals bisect each other.
- Adjacent angles are supplementary.

Each diagonal divides parallelogram into a set of congruent triangles.

Let us prove the above property

In the parallelogram ABCD,

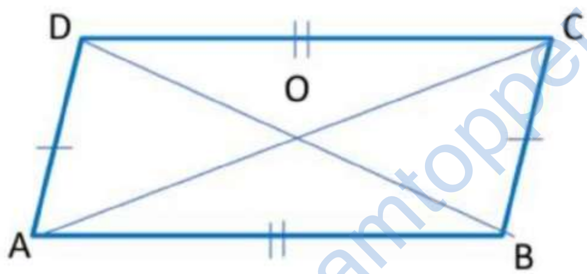
AB = CD and BC = AD

(Opposite sides are equal)

∠A =∠C and ∠B =∠D (Opposite angles are equal)

AO = OC and BO = OD (Diagonals bisect each other)

ΔABC ≅ ΔCDA and ΔBCD ≅ ΔDAB (Each diagonal divides a parallelogram into two congruent triangles)



Some Special Parallelograms

Rhombus

- It is a parallelogram with sides of equal length.
- Opposite angles are equal.
- Diagonals bisect each other at right angles.

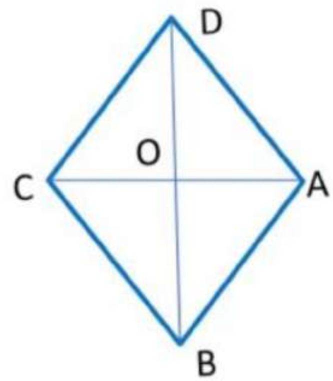
In Rhombus ABCD,

AB = BC = CD = DA (All the sides of a rhombus are equal)

∠A =∠C and ∠B =∠D (Opposite angles are equal)

AO = OC and BO = OD (Diagonals bisect each other)

∠AOB = ∠BOC = 90° (Diagonals bisect each other at right angles)



A Rectangle

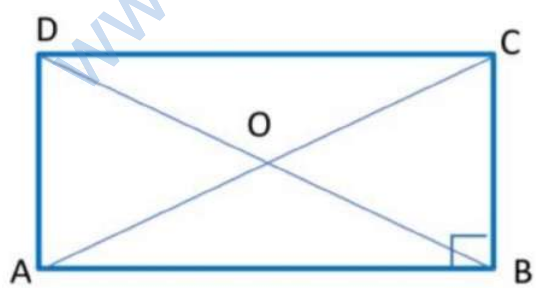
- A rectangle is a parallelogram whose each angle is a right angle.
- The diagonals of a rectangle are equal and bisect each other perpendicularly.

In rectangle ABCD,

$\angle A = \angle B = \angle C = \angle D = 90°$

(Each angle of a rectangle is a right angle)

AC = BD (Diagonals of a rectangle are equal)



A Square

- A rectangle with equal sides.
- The diagonals are of equal length.
- The diagonals are equal.
- The diagonals bisect each other perpendicularly.

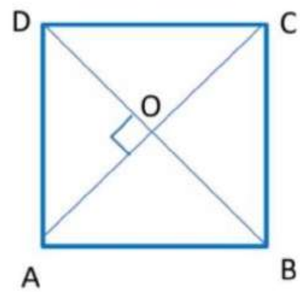
In square ABCD,

$\angle A = \angle B = \angle C = \angle D = 90°$

(Each angle of a square is a right angle)

AC = BD (Diagonals of a rectangle are equal)

$\angle AOB = \angle BOC = 90°$ (Diagonals bisect each other at right angles)



Find the value of x:

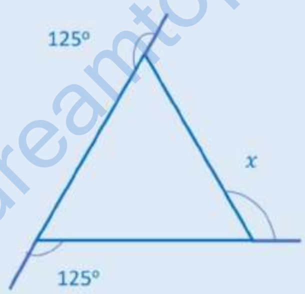
The sum of exterior angles is equal to 360°.

$125° + 125° + x = 360°$

$250° + x = 360°$

$x = 360° - 250°$

$x = 110°$



Find the value of x, y, z:

$y = 108°$

(Opposite angles of a parallelogram are equal)

$y + x = 180°$

$108° + x = 180°$

$x = 180° - 108° = 72°$

$\angle BCD = x$ (Opposite angles of a parallelogram are equal)

$\angle BCD = 72°$

$z + \angle BCD = 180°$

$z = 180° - 72° = 108°$

$x = 72°, y = 108°, z = 108°$