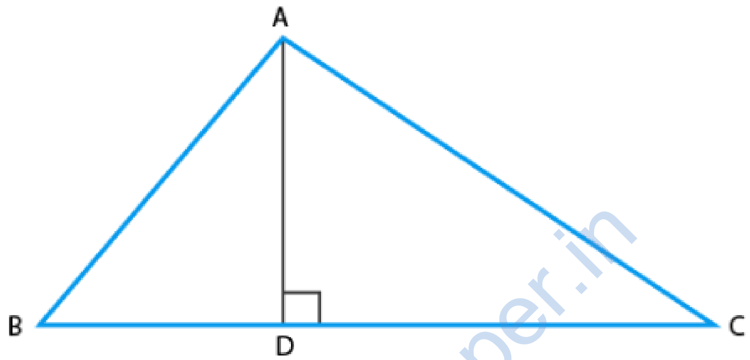
Chapter 6
Triangles

Exercise No. 6.1

Multiple Choice Questions:

Choose the correct answer from the given four options:

1. In figure, if ∠BAC =90°and $AD \perp BC$. Then,



If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

(A) $BD \cdot CD = BC^2$

(B) $AB \cdot AC = BC^2$

(C) $BD \cdot CD = AD^2$

(D) $AB \cdot AC = AD^2$

Solution:
(C) BD.CD=AD^2

In ΔADB and ΔADC,
We have,
∠D = ∠D = 90° (∵ AD ⊥ BC)
∠DBA = ∠DAC [each angle = 90°- ∠C]

From AAA similarity rule,
ΔADB ~ ΔADC
Therefore,

$$\frac{BD}{AD} = \frac{AD}{CD}$$

BD.CD = AD^2

2. If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

(A) 9 cm

(B) 10 cm

(C) 8 cm

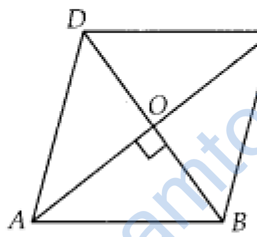
(D) 20 cm

Solution:

(B) 10 cm

We have,

A rhombus is a simple quadrilateral whose four sides are of same length and diagonals are perpendicular bisector of each other.



Now,

AC = 16 cm and

BD = 12 cm

$\angle AOB = 90°$

AC and BD bisects each other

$AO = \frac{1}{2} AC$

$BO = \frac{1}{2} BD$

So,

AO = 8 cm

BO = 6 cm

In right angled ΔAOB,

By Pythagoras theorem,

We have,

$AB^2 = AO^2 + OB^2$

$AB^2 = 8^2 + 6^2$

$= 64 + 36$

$= 100$

$AB = \sqrt{100}$

$= 10$ cm

As the four sides of a rhombus are equal.

So, one side of rhombus = 10 cm.

3. If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true?

(A) BC · EF = AC · FD

(B) AB · EF = AC · DE

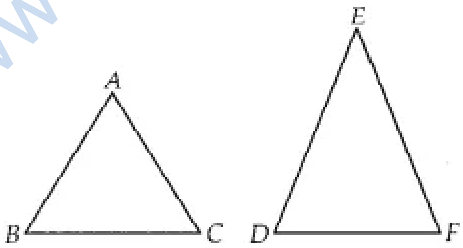
(C) BC · DE = AB · EF

(D) BC · DE = AB · FD

Solution:

(C) BC · DE = AB · EF

If sides of one triangle are proportional to the side of the other triangle, and the corresponding angles are also equal, then the triangles are similar by SSS similarity.



So, ΔABC ~ ΔEDF

By similarity rule,

$$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$$

At first we take,

$$\frac{AB}{ED} = \frac{BC}{DF}$$

$$\frac{AB}{ED} = \frac{BC}{DF}$$

AB.DF = ED.BC

Hence, option (D) BC · DE = AB · FD is true

Now taking,

$\frac{BC}{DF} = \frac{AC}{EF}$, we get

BC.EF = AC.DF

Hence, option (A) BC · EF = AC · FD is true

Now if,

$\frac{AB}{ED} = \frac{AC}{EF}$, we get,

AB.EF = ED.AC

Hence, option (B) AB · EF = AC · DE is true.

4. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(A) $\Delta PQR \sim \Delta CAB$

(B) $\Delta PQR \sim \Delta ABC$

(C) $\Delta CBA \sim \Delta PQR$

(D) $\Delta BCA \sim \Delta PQR$

Solution:

(A) Δ PQR ~ Δ CAB

We have, from ΔABC and ΔPQR,

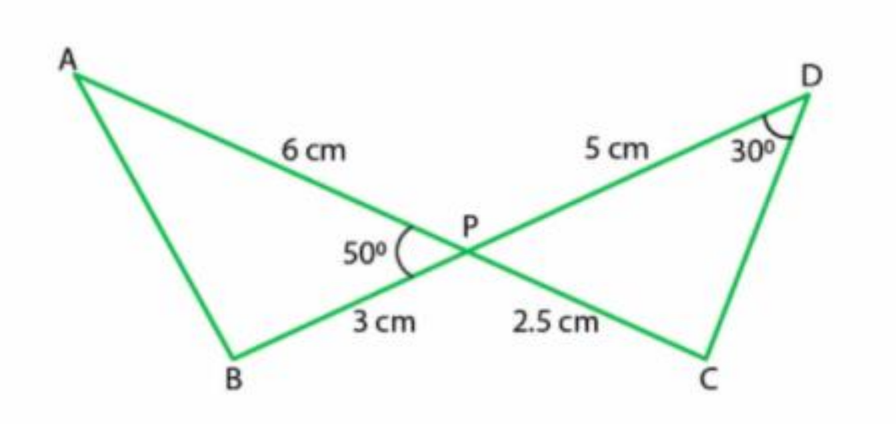
$$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$$

If sides of one triangle are proportional to the side of the other triangle, and their corresponding angles are also equal, then both the triangles are similar by SSS similarity.

So, we can say that,

Δ PQR ~ Δ CAB

5. **In fig. 6.3, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm,** $\angle APB = 50°$ **and** $\angle CDP = 30°$**. Then,** $\angle PBA$ **is equal to**



(A) 50°

(B) 30°

(C) 60°

(D) 100°

Solution:

(D) 100°

In ΔAPB and ΔCPD,

$\angle APB = \angle CPD = 50°$ (vertically opposite angles)

$$\frac{AP}{PD} = \frac{6}{5} \quad \dots (i)$$

And,

$$\frac{BP}{CP} = \frac{3}{2.5}$$

$$\frac{BP}{CP} = \frac{6}{5} \quad \dots (ii)$$

From equations (i) and (ii),

$$\frac{AP}{PD} = \frac{BP}{CP}$$

Therefore,

$\Delta APB \sim \Delta DPC$ [using SAS similarity rule]

$\angle A = \angle D = 30°$ [Corresponding angles of similar triangles]

As,
Sum of angles of a triangle = 180°

From ΔAPB,
$\angle A + \angle B + \angle APB = 180°$
$30° + \angle B + 50° = 180°$
$\angle B = 180° - (50° + 30°)$
$\angle B = 180 - 80°$
$= 100°$

So,
$\angle PBA = 100°$

6. If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?

(A) $\frac{EF}{PR} = \frac{DF}{PQ}$

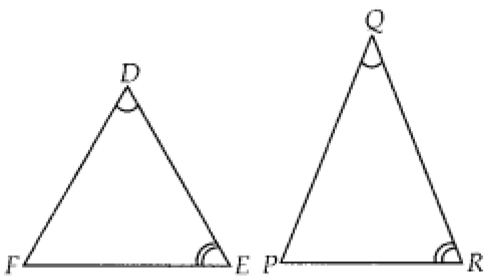
(B) $\frac{DE}{PQ} = \frac{EF}{RP}$

(C) $\frac{DE}{QR} = \frac{DF}{PQ}$

(D) $\frac{EF}{RP} = \frac{DE}{QR}$

Solution:
(B)

We have,



In ΔDEF and ΔPQR,

$\angle D = \angle Q$,

$\angle R = \angle E$

$\Delta DEF \sim \Delta QRP$ [using AAA similarity criterion]

$\angle F = \angle P$ [corresponding angles]

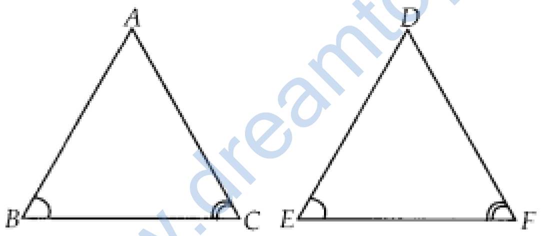
$$\frac{DF}{QP} = \frac{ED}{RQ} = \frac{FE}{PR}$$

7. In triangles ABC and DEF, $\angle B = \angle E, \angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are

(A) Congruent but not similar
(B) Similar but not congruent
(C) Neither congruent nor similar
(D) Congruent as well as similar

Solution:

(B)
In ΔABC and ΔDEF,
$\angle B = \angle E$,
$\angle F = \angle C$ and
$AB = 3DE$



We know that, if in two triangles corresponding two angles are same, then they are similar by AA similarity criterion.
But,
$AB \neq DE$
Therefore ΔABC and ΔDEF are not congruent.

8. It is given that $\Delta ABC \sim \Delta PQR$, with $\frac{BC}{QR} = \frac{1}{3}$. Then, $\frac{ar(PRQ)}{ar(BCA)}$ is equal to

(A) 9
(B) 3
(C) $\frac{1}{3}$
(D) $\frac{1}{9}$

Solution:

(A)

We have,
$\Delta ABC \sim \Delta PQR$
$$\frac{BC}{QR}=\frac{1}{3}$$

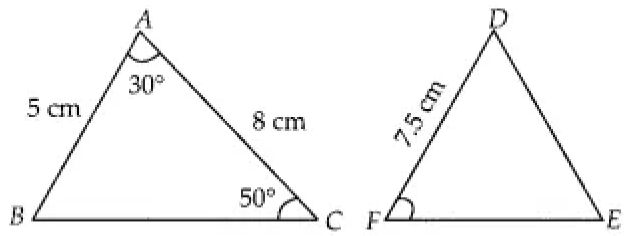
We know that, the ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

Therefore,
$$\frac{ar(PRQ)}{ar(BCA)}=\frac{QR^2}{BC^2}$$
$$\frac{QR^2}{BC^2}=\frac{3^2}{1^2}$$
$$=9$$

9. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 30°, \angle C = 50°, AB = 5\text{ cm}, AC = 8\text{ cm}$ and $DF = 7.5\text{ cm}$. Then, the following is true:

(A) $DE = 12\text{ cm}, \angle F = 50°$
(B) $DE = 12\text{ cm}, \angle F = 100°$
(C) $EF = 12\text{ cm}, \angle D = 100°$
(D) $EF = 12\text{ cm}, \angle D = 30°$

Solution:
We have,
$\Delta ABC \sim \Delta DFE$,
$\angle A = \angle D = 30°$,
$\angle C = \angle E = 50°$



$\angle B = \angle F$
$= 180^\circ-(50^\circ+30^\circ)$
$= 100^\circ$

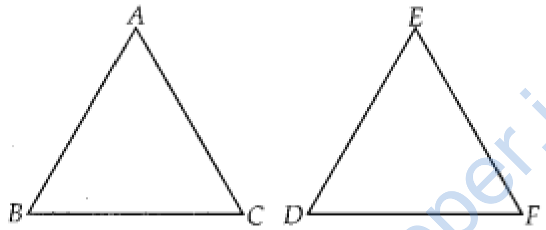
Now,
$$\frac{AB}{DF}=\frac{AC}{DE}$$

$$\frac{5}{7.5}=\frac{8}{DE}$$
DE = 12cm

10. If in triangles ABC and DEF, $\frac{AB}{DE}=\frac{BC}{FD}$, then they will be similar when

(A) $\angle B=\angle E$
(B) $\angle A=\angle D$
(C) $\angle B=\angle D$
(D) $\angle A=\angle F$

Solution:



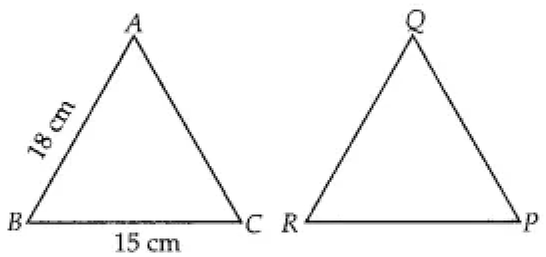
Given, in ΔABC and ΔEDF,
$$\frac{AB}{DE}=\frac{BC}{FD}$$

Therefore,
$\Delta ABC \sim \Delta EDF$ if , $\angle B = \angle D$ [By SAS similarity criterion]

11. If $\Delta ABC \sim \Delta QRP$, and $BC = 15$ cm, then PR is equal to

(A) 10 cm
(B) 12 cm
(C) $\frac{20}{3}$ cm
(D) 8 cm

Solution:
In given question,



We know that the ratio of area of two similar triangles is equal to the ratio of square of their corresponding sides.

$$\frac{ar(ABC)}{ar(QRP)}=\frac{(BC)^2}{(RP)^2}\ Also, \frac{ar(ABC)}{ar(QRP)}=\frac{9}{4}$$

Therefore,

$$\frac{(BC)^2}{(RP)^2}=\frac{9}{4}$$

$$\frac{(15)^2}{(RP)^2}=\frac{9}{4}$$

$$RP \quad =10cm$$

12. If S is a point on side PQ of a ΔPQR such that PS = QS = RS, then

(A) $PR \cdot QR = RS^2$

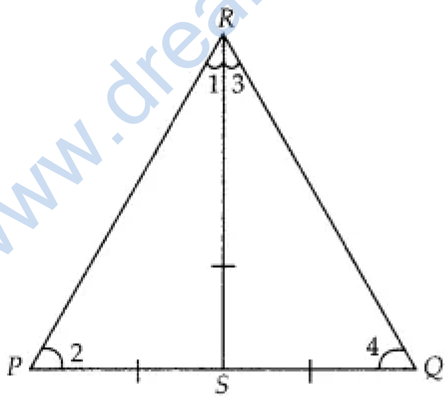
(B) $QS^2 + RS^2 = QR^2$

(C) $PR^2 + QR^2 = PQ^2$

(D) $PS^2 + RS^2 = PR^2$

Solution:

In given question,



In ΔPQR,
PS = QS = RS .... (i)

Now,
In ΔPSR,
PS = RS (By eqn (i))
$\angle 1 = \angle 2$ .... (ii)
[Angles opposite to equal sides are equal]

Also, in ΔRSQ,
RS = SQ
$\angle 3 = \angle 4$ .... (iii)
[angles opposite to equal sides are equal]

We know that, in ΔPQR, sum of angles = 180°

$$\angle P + \angle Q + \angle P = 180°$$
$$\angle 2 + \angle 4 + \angle 1 + \angle 3 = 180°$$
$$\angle 1 + \angle 3 + \angle 1 + \angle 3 = 180°$$
$$2(\angle 1 + \angle 3) = 180°$$
$$= 90°$$

So, $\angle R = 90°$

In ΔPQR, by Pythagoras theorem,

$$PR^2 + QR^2 = PQ^2$$

Exercise No. 6.2

Short Answer Questions with Reasoning:

Question:

1. Is the triangle with sides 25 cm, 5 cm and 24 cm a right triangle? Give reason for your answer.

Solution:

It is not true.
Taking,
a = 25 cm,
b = 5 cm and
c = 24 cm

Now,
$b^2 + c^2 = (5)^2 + (24)^2$
$= 25 + 576$
$= 601$
$\neq (25)^2$

Therefore, given sides do not make a right triangle because it does not satisfy the property of Pythagoras theorem.

2. It is given that ΔDEF ~ ΔRPQ. Is it true to say that ∠D = ∠R and ∠F = ∠P? Why?

Solution:

It is not true
We know that, if two triangles are similar, then their corresponding angles are equal.
∠D = ∠R,
∠E = ∠P and
∠F = Q

3. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB || QR? Give reason for your answer.

Solution:

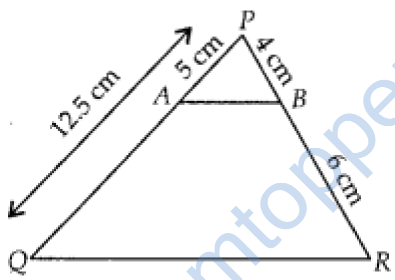
It is correct.
Given,
PQ = 12.5 cm,
PA = 5 cm,
BR = 6 cm and

PB = 4 cm
Also,

$$\frac{PB}{BR} = \frac{4}{6}$$
$$= \frac{2}{3}$$

So, QA=QP-PA
=12.5-5
=7.5 cm

$$\frac{PA}{AQ} = \frac{5}{7.5}$$
$$= \frac{2}{3}$$

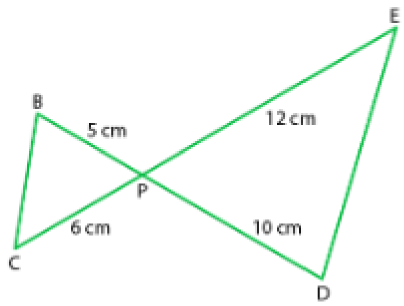


Therefore,

$$\frac{PA}{AQ} = \frac{PB}{BR}$$

So by converse of basic proportionality theorem, AB || QR.

4. In figure, BD and CE intersect each other at the point P. Is ΔPBC ~ ΔPDE? Why?



Solution:
It is correct.

In ΔPBC and ΔPDE,
∠BPC = ∠EPD

[vertically opposite angles]

$\frac{PB}{PD} = \frac{5}{10}$

$= \frac{1}{2}$

$\frac{PC}{PE} = \frac{6}{12}$

$= \frac{1}{2}$

So,

$\frac{PB}{PD} = \frac{PC}{PE}$

As, one angle of ΔPBC is equal to one angle of ΔPDE and the sides including these angles are proportional, so both triangles are similar.

So, ΔPBC ~ ΔPDE, by SAS similarity criterion.

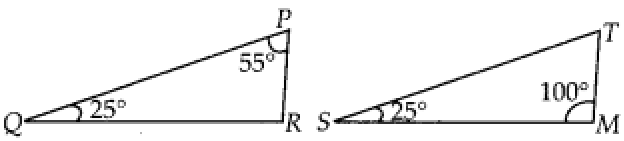
5. In ΔPQR and ΔMST, ∠P = 55°, ∠Q =25°, ∠M = 100° and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?

Solution:
It is not true.

As, the sum of three angles of a triangle is 180°.

In ΔPQR,
∠P +∠Q +∠R = 180°
55° + 25 ° + ∠R = 180°
∠R = 180° – (55° + 25 °)
= 180° – 80° = 100°

In ΔTSM,
∠T + ∠S + ∠M = 180°
∠T + ∠25° + 100° = 180°
∠T = 180° – (25° + 100°)
= 180° – 125°
= 55°



So,
In ΔPQR and ΔTSM,

∠P = ∠T,
∠Q = ∠S and
∠R = ∠M
∠PQR = ∠TSM [As, all corresponding angles are equal]

Therefore,
ΔQPR is not similar to ΔTSM, because correct correspondence is P ↔ T, Q ↔ S and R ↔ M.

6. Is the following statement true? Why? "Two quadrilaterals are similar, if their corresponding angles are equal".

Solution:

It is not true.
Two quadrilaterals are similar if their corresponding angles are equal and corresponding sides must also be proportional.

7. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

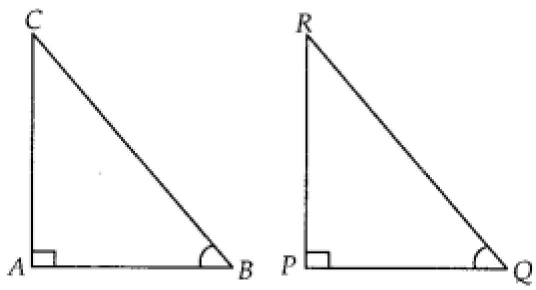
Solution:

Yes, It is true.
The corresponding two sides and the perimeters of two triangles are proportional, then the third side of both triangles will also in proportion.

8. If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles will be similar? Why?

Solution:

It is false.
Let two right angled triangles be ΔABC and ΔPQR



Where,
$\angle A = \angle P = 90°$ and
$\angle B = \angle Q$ = acute angle (Given)

So, by AA similarity criterion, $\Delta ABC \sim \Delta PQR$

9. The ratio of the corresponding altitudes of two similar triangles is $\frac{3}{5}$. Is it correct to say that ratio of their areas is $\frac{6}{5}$? Why?

Solution:
It is false.
Ratio of corresponding altitudes of two triangles having areas A1 and A2 respectively is $\frac{3}{5}$.
Using the property of area of two similar triangles,

$$\frac{A_1}{A_2} = \left(\frac{3}{5}\right)^2$$

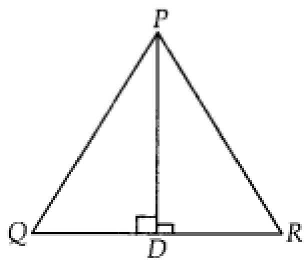
$$\frac{6}{5} \neq \frac{9}{25}$$

So, the given statement is not correct.

10. D is a point on side QR of ΔPQR such that $PD \perp QR$. Will it be correct to say that $\Delta PQD \sim \Delta RPD$? Why?

Solution:
No, it is false statement.
In given ΔPQD and ΔRPD,

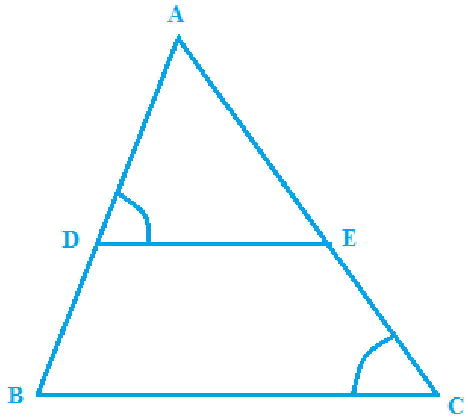
$PD = PD$ [common side]
$\angle PDQ = \angle PDR$ [each 90°]



Also, no other sides or angles are equal, so we can say that ΔPQD is not similar to ΔRPD.
But if $\angle P = 90°$, then

$\angle DPQ = \angle PRD$
[each equal to $90° - \angle Q$ and by ASA similarity criterion, $\Delta PQD \sim \Delta RPD$]

11. In Fig. 6.5, if $\angle D = \angle C$, then is it true that $\Delta ADE \sim \Delta ACB$? Why?



Solution:

True
In ΔADE and ΔACB,
$\angle A = \angle A$ [common angle]
$\angle D = \angle C$ [given]
$\Delta ADE \sim \Delta ACB$ [using AA similarity criterion]

12. Is it true to say that if in two triangles, an angle of one triangle is equal to an angle of another triangle and two sides of one triangle are proportional to the two sides of the other triangle, then the triangles are similar? Give reasons for your answer.

Solution:

False
As, according to SAS similarity criterion, if one angle of a triangle is equal to an angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
In the above question, one angle and two sides of two triangles are equal but these sides does not includes equal angle, so given statement is not true.

Exercise No. 6.3

Short Answer Questions:

Question:

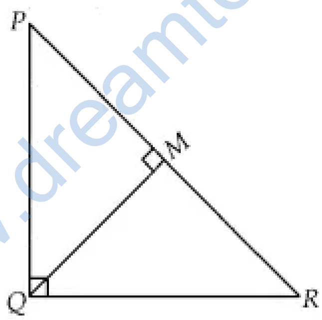
1. In a ΔPQR, $PR^2 - PQ^2 = QR^2$ and M is a point on side PR such that QM ⊥ PR.
Prove that:
$QM^2 = PM \times MR$.

Solution:

In ΔPQR,

$PR^2 = QR^2$ and

QM⊥PR



Using Pythagoras theorem, we have,

$PR^2 = PQ^2 + QR^2$

ΔPQR is right angled triangle at Q.

From ΔQMR and ΔPMQ, we get,

∠M = ∠M

∠MQR = ∠QPM [each 90°-∠R]

So, using the AAA similarity criteria,

We have,

$\Delta QMR \sim \Delta PMQ$

Also,

Area of triangles $= \frac{1}{2} \times$ base $\times$ height

So, by property of area of similar triangles,

$$\frac{ar(QMR)}{ar(PMQ)} = \frac{QM^2}{PM^2}$$

$$\frac{ar(QMR)}{ar(PMQ)} = \frac{\frac{1}{2} RM \times QM}{\frac{1}{2} PM \times QM}$$

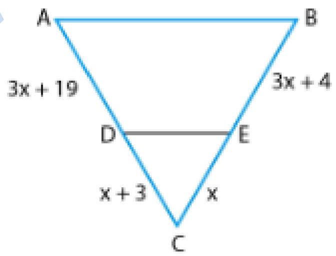
So,

$$\frac{QM^2}{PM^2} = \frac{\frac{1}{2} RM \times QM}{\frac{1}{2} PM \times QM}$$

$QM^2 = PM \times RM$

Hence proved.

2. Find the value of x for which DE||AB in given figure.



Solution:

As given in the question,

DE || AB

Using basic proportionality theorem,

$$\frac{CD}{AD} = \frac{CE}{BE}$$

If a line is drawn parallel to one side of a triangle such that it intersects the other sides at distinct points, then, the other two sides are divided in the same ratio.

Therefore, we can conclude that, the line drawn is equal to the third side of the triangle.

$$\frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$(x + 3)(3x + 4) = x(3x + 19)$

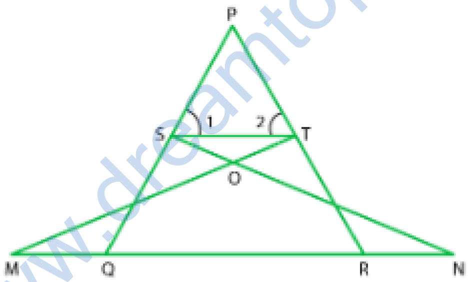
$3x^2 + 4x + 9x + 12 = 3x^2 + 19x$

$19x - 13x = 12$

$6x = 12$

$x = 2$

3. In figure, if ∠1 =∠2 and Δ NSQ ≅ ΔMTR, then prove that ΔPTS ~ ΔPRQ.



Solution:
As given in the question,

Δ NSQ ≅ ΔMTR
∠1 = ∠2
As,
ΔNSQ = ΔMTR

So,
SQ = TR ….(i)

Also,
∠1 = ∠2 so,
PT = PS ….(ii)

[As, sides opposite to equal angles are also equal]
Using Equation (i) and (ii).

$$\frac{PS}{SQ} = \frac{PT}{TR}$$

So, ST || QR

By converse of basic proportionality theorem, If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.

∠1 = PQR and∠2 = ∠PRQ

Now, In ΔPTS and ΔPRQ.

∠P = ∠P [Common angles]
∠1 = ∠PQR (proved)
∠2 = ∠PRQ (proved)
ΔPTS – ΔPRQ

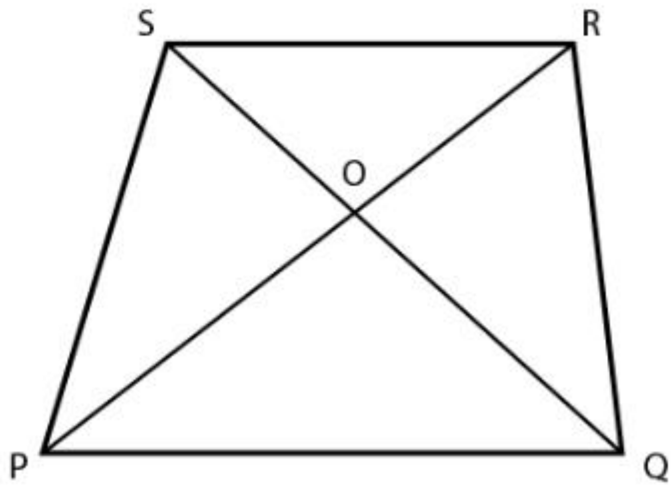
[By AAA similarity criteria]
Hence proved.

4. Diagonals of a trapezium PQRS intersect each other at the point 0, PQ || RS and PQ = 3 RS. Find the ratio of the areas of Δ POQ and Δ ROS.

Solution:
As given in the question,

PQRS is a trapezium in which PQ∥RS and PQ = 3RS

$$\frac{PQ}{RS} = \frac{3}{1} \quad \ldots(i)$$



In ΔPOQ and ΔROS,
∠SOR = ∠QOP [vertically opposite angles]
∠SRP = ∠RPQ [alternate angles]
ΔPOQ ~ ΔROS [by AAA similarity criterion]

Using property of area of similar triangle,

$$\frac{ar(POQ)}{ar(SOR)} = \frac{PQ^2}{RS^2}$$

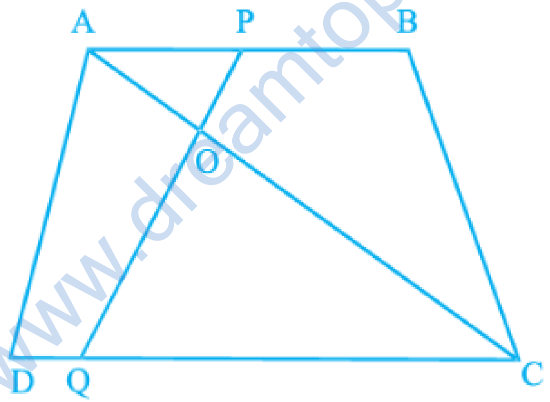
$$\frac{PQ^2}{RS^2} = \left(\frac{PQ}{RS}\right)^2$$

$$= \left(\frac{3}{1}\right)^2$$

$$= 9$$

So, the required ratio = 9:1.

5. In figure, if AB || DC and AC, PQ intersect each other at the point O. Prove that OA.CQ = OC.AP.



Solution:

As given in the question,
AC and PQ intersect each other at the point O and AB||DC.

Using ΔAOP and ΔCOQ,
∠AOP = ∠COQ [as they are vertically opposite angles]
∠APO = ∠CQO [since, AB||DC and PQ is transversal, Angles are alternate angles]

So,
ΔAOP ~ ΔCOQ [using AAA similarity criterion]
As, corresponding sides are proportional

We have,

$$\frac{OA}{OC} = \frac{AP}{CQ}$$

OA × CQ = OC × AP

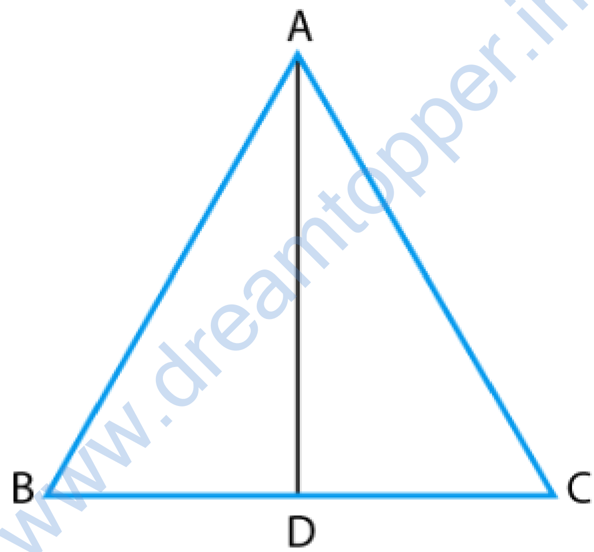
Hence Proved!!!

6. Find the altitude of an equilateral triangle of side 8 cm.

Solution:

Taking ABC be an equilateral triangle of side 8 cm.

AB = BC = CA = 8 cm (sides of an equilateral triangle is equal)



Draw altitude AD which is perpendicular to BC.

Then, D is the mid-point of BC.

$BD = CD = \frac{1}{2}$

$BC = \frac{8}{2}$

$= 4$ cm

Now,
Using Pythagoras theorem

$AB^2 = AD^2 + BD^2$
$(8)2 = AD^2 + (4)^2$
$64 = AD^2 + 16$

AD = √48
= 4√3 cm.

Therefore, altitude of an equilateral triangle is 4√3 cm.

7. If ΔABC ~ ΔDEF, AB = 4 cm, DE = 6, EF = 9 cm and FD = 12 cm, then find the perimeter of ΔABC.

Solution:
As given in the question,
AB = 4 cm,
DE = 6 cm
EF = 9 cm
FD = 12 cm

Also,
ΔABC ~ ΔDEF

We have,

$$\frac{AB}{ED} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{4}{6} = \frac{BC}{9} = \frac{AC}{12}$$

Now,

$$\frac{4}{6} = \frac{BC}{9}$$

$$BC = 6cm$$

Similarily,

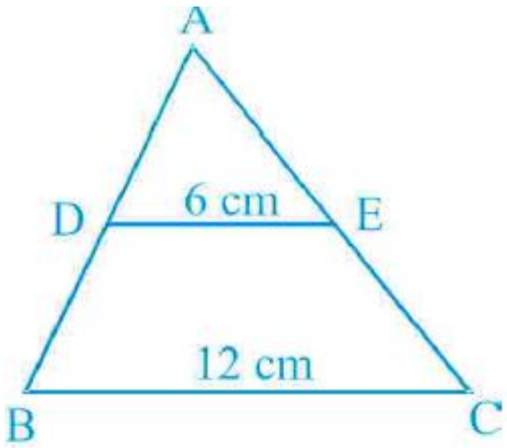
$$\frac{AC}{12} = \frac{4}{6}$$

$$AC = 8cm$$

Perimeter of ΔABC = AB + BC + AC
= 4 + 6 + 8 = 18 cm

So, the perimeter of the triangle is 18 cm.

8. In Fig. 6.11, if DE || BC, find the ratio of $\text{ar}(\text{ADE})$ **and** $\text{ar}(\text{DECB})$**.**



Solution:
We have,
DE || BC,
DE = 6 cm and
BC = 12 cm

In ΔABC and ΔADE,
∠ABC = ∠ADE [corresponding angle]
and
∠A = ∠A [common side]

ΔABC ~ ΔADE [using AA similarity criterion]

$$\frac{ar(ADE)}{ar(ABC)} = \frac{DE^2}{BC^2}$$

$$= \frac{6^2}{12^2}$$

$$= \frac{1}{4}$$

Taking,
ar(ΔADE) = k, then
ar(ΔABC) = 4k

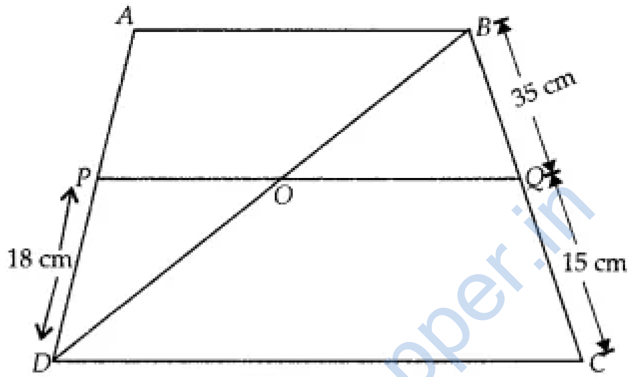
Now,
ar(ΔECB) = ar(ABC) – ar(ADE)
= 4k – k = 3k
So,
Required ratio = ar(ADE): ar(DECB)
= k : 3k
= 1 : 3

9. ABCD is a trapezium in which AB || DC and P and Q are points on ADand BC, respectively such that PQ || DC. If PD = 18 cm, BQ = 35 cm andQC = 15 cm, find AD.

Solution:

We have, a trapezium ABCD in which AB || DC. P and Q are points on AD and BC, respectively such that PQ || DC.

So, AB || PQ || DC.



$In\ \Delta ABD, \text{PO} \parallel \text{AB}$

$\frac{DP}{AP} = \frac{DO}{OB}$ .....(i)

$In\ \Delta BDC, OQ \parallel DC$

$\frac{BQ}{QC} = \frac{OB}{OD}$

$or,$

$\frac{QC}{BQ} = \frac{DO}{OB}$ ......(ii)

So, from (i) and (ii),

$\frac{DP}{AP} = \frac{QC}{BQ}$

$\frac{18}{AP} = \frac{15}{35}$

AP = 42 cm.

Also;

AD = AP + PD

= 42 + 18 = 60

So,
AD = 60 cm

10. Corresponding sides of two similar triangles are in the ratio of 2:3. If the area of the smaller triangle is 48 cm^2, find the area of the larger triangle.

Solution:
According to the question,

Ratio of corresponding sides of two similar triangles is 2 : 3 or $\frac{2}{3}$

Area of smaller triangle = 48 cm^2
Using the property of area of two similar triangles,
Ratio of area of both triangles = (Ratio of their corresponding sides)^2

$$\frac{\text{ar(smaller triangle)}}{\text{ar(larger triangle)}} = \left(\frac{2}{3}\right)^2$$

$$\frac{48}{\text{ar(larger triangle)}} = \left(\frac{2}{3}\right)^2$$

$$\text{ar(larger triangle)} = 108\,\text{cm}^2$$

11. In a triangle PQR, N is a point on PR such that $QN \perp PR$. If $PN \cdot NR = QN^2$, prove that $\angle PQR = 90°$.

Solution:
We have,

In ΔPQR, N is a point on PR, such that QN ⊥ PR and PN .
NR = QN^2

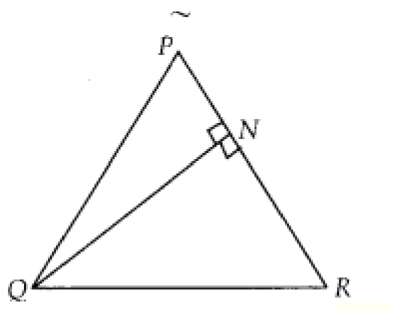
To prove: ∠PQR = 90°

Proof:
We have, PN . NR = QN^2

PN . NR = QN . QN

So,

$$\frac{PN}{QN} = \frac{QN}{NR}$$



Also,
∠PNQ = ∠RNQ [each equal to 90°]

ΔQNP ~ ΔRNQ [by SAS similarity criterion]
So we can say, ΔQNP and ΔRNQ are equiangular.
∠PQN = ∠QRN
∠RQN = ∠QPN

On adding both sides,
∠PQN + ∠RQN = ∠QRN + ∠QPN
∠PQR = ∠QRN + ∠QPN …………… (ii)

We have, sum of angles of a triangle is 180°

In ΔPQR,
∠PQR + ∠QPR + ∠QRP = 180°
∠PQR + ∠QPN + ∠QRN = 180°
[∵∠QPR = ∠QPN and ∠QRP = ∠QRN]
∠PQR + ∠PQR = 180° [using Eq. (ii)]
2∠PQR = 180°
∠PQR = 90°
Hence proved.

12. Areas of two similar triangles are 36 cm^2 and 100 cm^2. If the length of a side of the larger triangle is 20 cm, find the length of the corresponding side of the smaller triangle.

Solution:
We have,
Area of smaller triangle = 36 cm^2
Area of larger triangle = 100 cm^2
And, length of a side of the larger triangle = 20 cm

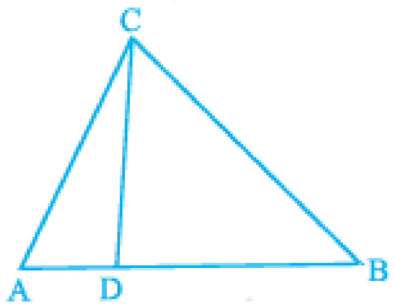
Let length of the corresponding side of the smaller triangle = x cm
By property of area of similar triangles,

$$\frac{\text{ar(larger triangle)}}{\text{ar(smaller triangle)}} = \frac{(\text{side of larger triangle})^2}{(\text{side of smaller triangle})^2}$$

$$\frac{100}{36} = \frac{20^2}{x^2}$$

$$x = 12\,\text{cm}$$

13. In the given fig., if $\angle ACB = \angle CDA$, AC = 8 cm and AD = 3 cm, find BD.



Solution:

We have,

AC = 8 cm,

AD = 3 cm

$\angle ACB = \angle CDA$

In ΔACD and ΔABC,

$\angle A = \angle A$ [Common angle]

$\angle ADC = \angle ACB$ [Given]

So,

$\Delta ADC \sim \Delta ACB$ [By AA similarity criterion]

$$\frac{AC}{AD} = \frac{AB}{AC}$$

$$\frac{8}{3} = \frac{AB}{8}$$

$$AB = \frac{64}{3}cm$$

Also,

$$AB = BD + AD$$

$$\frac{64}{3} = BD + 3$$

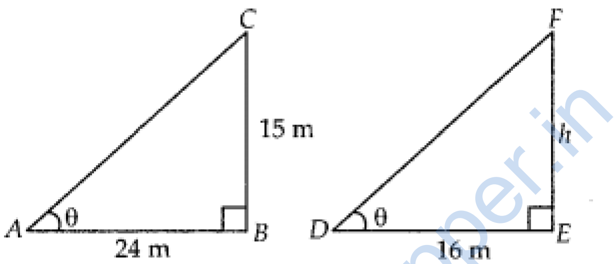
$$BD = \frac{55}{3}cm$$

14. A 15 meters high tower casts a shadow 24 meters long at a certain time and at the same time, a telephone pole casts a shadow 16 meters long. Find the height of the telephone pole.

Solution:

Taking BC = 15 m be the tower and its shadow AB is 24 m.
Let $\angle CAB = \theta$.
Again, let EF = h be a telephone pole and its shadow DE = 16 m.

At the same time $\angle EDF = \theta$.
ΔABC and ΔDEF both are right angled triangles.



$$In\ \Delta ABC\ and\ \Delta DEF,$$
$$\angle CAB = \angle EDF$$
$$\angle B = \angle E$$

So, by AA rule,

$$\Delta ABC \sim \Delta DEF$$
$$\frac{AB}{DE} = \frac{BC}{EF}$$
$$\frac{24}{16} = \frac{15}{h}$$
$$h = 10$$

Hence, the height of the point on the wall where the top of the ladder reaches is 8 m.

15. Foot of a 10 m long ladder leaning against a vertical wall is 6 m away from the base of the wall. Find the height of the point on the wall where the top of the ladder reaches.

Solution:

Let AB be a vertical wall and AC = 10 m is a ladder.
The top of the ladder reached to A and distance of ladder from the base of the wall BC is 6 m.

In right angled ΔABC

$AC^2 = AB^2 + BC^2$ [by Pythagoras theorem]

$(10)^2 = AB^2 + (6)^2$

$100 = AB^2 + 36$

$AB^2 = 100 - 36 = 64$

$AB = 8$ m

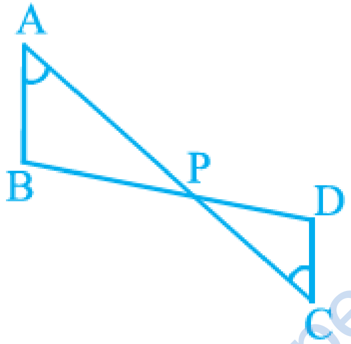
Therefore, the height of the point on th wall where the top of the ladder reaches is 8 m.

Exercise No. 6.4

Long Answer Questions:

Question:

1. In Fig., if ∠A = ∠C, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.



Solution:
We have,
∠A = ∠C,
AB = 6 cm,
BP = 15 cm,
AP = 12 cm and
CP = 4 cm

In ΔAPB and ΔCPD,
∠A = ∠C [given]
∠APB = ∠CPD [vertically opposite angles]
ΔAPB ~ ΔCPD [by AA similarity criterion]

$$\frac{AP}{CP} = \frac{PB}{PD} = \frac{AB}{CD}$$
$$\frac{12}{4} = \frac{15}{PD} = \frac{6}{CD}$$

So,
$$\frac{12}{4} = \frac{15}{PD}$$
$$PD = 5\,cm$$
Also,
$$\frac{12}{4} = \frac{6}{CD}$$
$$CD = 2\,cm$$

Therefore, length of PD is 5 cm and length of CD is 2 cm.

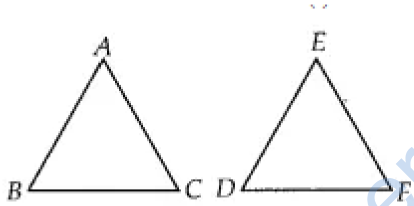
2. It is given that Δ ABC ~ Δ EDF such that AB = 5 cm, AC = 7 cm, DF= 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles.

Solution:

We have,

ΔABC ~ ΔEDF, so the corresponding sides of ΔABC and ΔEDF are in the same ratio

$\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$ ………………. (i)



Also, we have,

AB = 5 cm,
AC = 7 cm,
DF= 15 cm and
DE = 12 cm

Putting value in $\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$,

$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$

$So,$

$\frac{5}{12} = \frac{7}{EF}$

$EF = 16.8 cm$

$Also,$

$\frac{5}{12} = \frac{BC}{15}$

$BC = 6.25 cm$

So, lengths of the remaining sides of the triangles are EF = 16.8 cm and
BC = 6.25 cm.

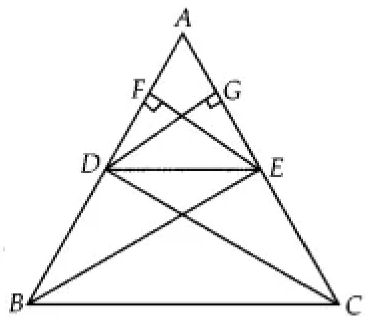
3. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.

Solution:

Let us take ΔABC in which a line DE parallel to BC intersects AB at D and AC at E.

To prove: DE divides the two sides in the same ratio.

$$\frac{AD}{DB}=\frac{AE}{EC}$$



Construction:

Join BE and CD

$EF \perp AB$

$DG \perp AC$

Proof:

$$\frac{ar(ADE)}{ar(BDE)}=\frac{\frac{1}{2}\times AD\times EF}{\frac{1}{2}\times DB\times EF}$$

$$=\frac{AD}{DB} \qquad ....(i)$$

Also,

$$\frac{ar(ADE)}{ar(DEC)}=\frac{\frac{1}{2}\times AE\times GD}{\frac{1}{2}\times EC\times GD}$$

$$=\frac{AE}{EC} \qquad .....(ii)$$

As,

ΔBDE and ΔDEC lie between the same parallel lines DE and BC and on the same base DE

So,

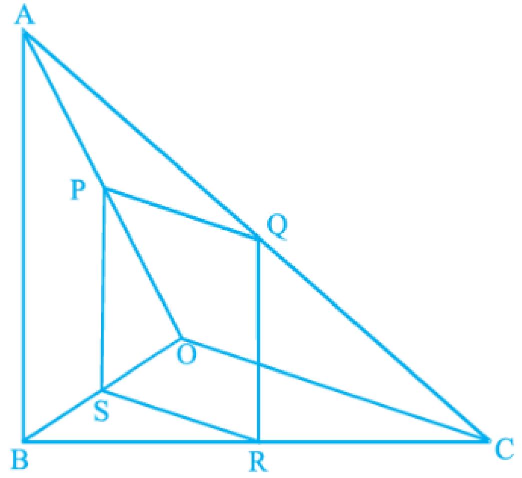
ar(ΔBDE) = ar(ΔDEC) ….. (iii)

From Eqs. (i), (ii) and (iii),

$$\frac{AD}{DB}=\frac{AE}{EC}$$

Hence proved!!!

4. In Fig., if PQRS is a parallelogram and AB||PS, then prove that OC||SR.



Solution:

We have,

PQRS is a parallelogram, so PQ || SR and PS || QR.

Also

AB || PS.

To prove: OC || SR

Proof: In ΔOPS and ΔOAB, PS || AB

∠POS = ∠AOB [common angle]

∠OSP = ∠OBA [corresponding angles]

ΔOPS ~ ΔOAB [by AA similarity criterion]

Then $\frac{PS}{AB} = \frac{OS}{OB}$ ………… (i)

In ΔCQE and ΔCAB, QR || PS || AB

∠QCR = ∠ACB [common angle]

∠CRQ = ∠CBA [corresponding angles]

So,

ΔCQR ~ ΔCAB

$$\frac{QR}{AB} = \frac{CR}{OB}$$

$$\frac{PS}{AB} = \frac{CR}{OB} \qquad \text{......}(ii)$$

$(PS = QR, \text{opposite sides of parallelogram})$

From (i) and (ii),

$$\frac{OS}{OB} = \frac{CR}{CB}$$

or,

$$\frac{OB}{OS} = \frac{CB}{CR}$$

Subtracting 1 from both sides, we get,

$$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$$

$$\frac{OB - OS}{OS} = \frac{CB - CR}{CR}$$

$$\frac{BS}{OS} = \frac{BR}{CR}$$

By using converse of basic proportionality theorem, SR || OC.

Hence proved

5. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Solution:

Taking AC be the ladder of length 5 m and BC = 4 m be the height of the wall, which ladder is placed.

If the foot of the ladder is moved 1.6 m towards the wall so, AD = 1.6 m, then the ladder is slide upward i.e., CE = x m.

In right angled ΔABC,

$AC^2 = AB^2 + BC^2$ [using Pythagoras theorem]

$(5)^2 = (AB)^2 + (4)^2$

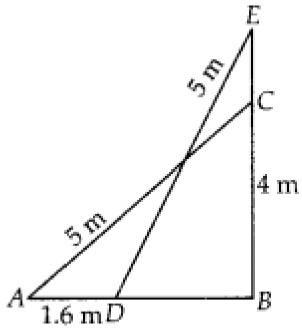
$AB^2 = 25 - 16$
$= 9$

AB = 3m

Now,

DB = AB – AD

$= 3 – 1.6$
$= 1.4$ m



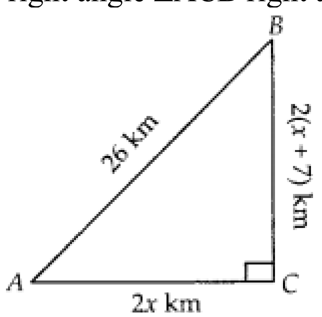
In right angled ΔEBD,

$ED^2 = EB^2 + BD^2$ [using Pythagoras theorem]
$(5)^2 = (EB)^2 + (1.4)^2$ [∵ BD = 1.4 m]
$25 = (EB)^2 + 1.96$
$(EB)^2 = 25 – 1.96$
$= 23.04$
$EB = 4.8$
Now,
$EC = EB – BC$
$= 4.8 – 4$
$= 0.8$
Therefore, the top of the ladder would slide upwards on the wall at distance is 0.8 m.

6. For going to a city B from city A, there is a route via city C such that AC⊥CB, AC = 2 x km and CB = 2 (x + 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway.

Solution:
We have,
$AC \perp CB$,
$AC = 2x$ km,
$CB = 2(x + 7)$km and $AB = 26$ km

On drawing the figure, we get the right angle ΔACB right angled at C.



Now,
In ΔACB, by Pythagoras theorem,
$AB^2 = AC^2 + BC^2$
$(26)^2 = (2x)^2 + \{2(x + 7)\}^2$
$676 = 4x^2 + 4(x^2 + 49 + 11x)$
$676 = 4x^2 + 4x^2 + 196 + 56x$
$676 = 8x^2 + 56x + 196$
$8x^2 + 56x - 480 = 0$

On dividing by 8, we get,
$x^2 + 7x - 60 = 0$
$x^2 + 12x-5x-60 = 0$
$x(x + 12) - 5(x + 12) = 0$
$(x + 12)(x - 5) = 0$
$x = -12,$
$x = 5$
As, distance cannot be negative.
$x = 5$ $[\because x \neq 12]$

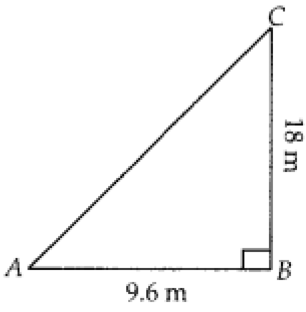
Now,
$AC = 2x$
$= 10$ km and
$BC = 2(x + 7)$
$= 2(5 + 7)$
$= 24$ km
The distance covered to reach city B from city A via city C = AC + BC
= 10 + 24
= 34 km
Distance covered to reach city B from city A after the construction of the highway is
BA = 26 km
So, the required saved distance is 34 – 26= 8 km.

7. A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.

Solution:

Let BC = 18 m be the flag pole and its shadow be AB = 9.6 m.

The distance of the top of the pole, C from the far end which is A of the shadow is AC



In right angled ΔABC

$AC^2 = AB^2 + BC^2$ [using Pythagoras theorem]

$AC^2 = (9.6)^2 + (18)^2$

$AC^2 = 92.16 + 324$

$AC^2 = 416.16$

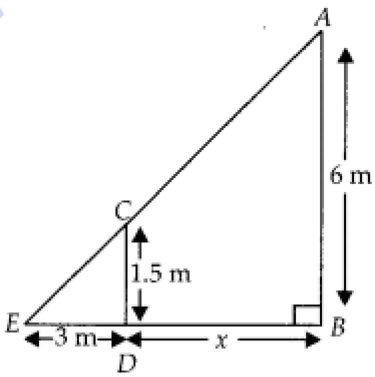
$AC = 20.4$ m

So, the required distance is 20.4 m.

8. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3m, find how far she is away from the base of the pole.

Solution:

Taking A be the position of the street bulb fixed on a pole AB = 6 m and CD = 1.5 m be the height of a woman and her shadow be ED = 3 m. And distance between pole and woman be x m.



In this question, woman and pole both are standing vertically

So,

CD ∥ AB

In ΔCDE and ΔABE,

∠E = ∠E [common angle]

∠ABE = ∠CDE [each equal to 90°]

ΔCDE ~ ΔABE [by AA similarity criterion]

Then,

$$\frac{ED}{EB} = \frac{CD}{AB}$$

$$\frac{3}{3+x} = \frac{1.5}{6}$$

$3 \times 6 = 1.5(3 + x)$

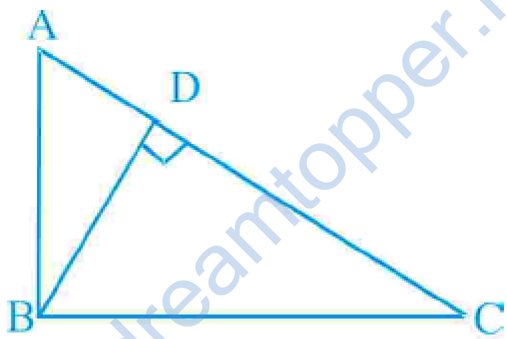
$18 = 1.5 \times 3 + 1.5x$

$1.5x = 18 - 4.5$

$x = 9$ m

So, she is at the distance of 9 m from the base of the pole.

9. In Fig., ABC is a triangle right angled at B and $BD \perp AC$. If AD = 4 cm, andCD = 5 cm, find BD and AB.



Solution:

Given,

ΔABC in which ∠B = 90° and

BD ⊥ AC

Also, AD = 4 cm and

CD = 5 cm

In ΔDBA and ΔDCB,

∠ADB = ∠CDB [each equal to 90°]

and

∠BAD = ∠DBC [each equal to 90° – ∠C] ;.

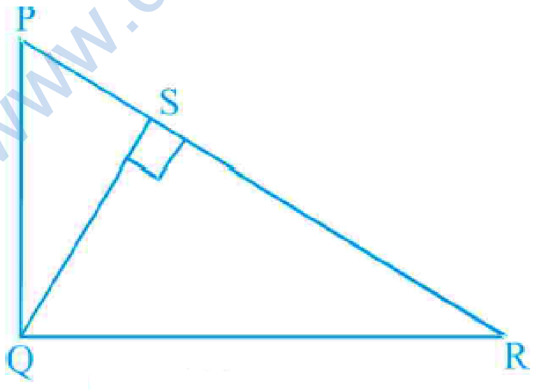
ΔDBA ~ ΔDCB [by AA similarity criterion]

So,

$$\frac{DB}{DA}=\frac{DC}{DB}$$
$$DB^2 = DA \times DC$$
$$= 4 \times 5$$
$$DB = 2\sqrt{5}\,cm$$

$In\ \Delta BDC,$
$BC^2 = BD^2 + CD^2$ (Using pythagoras theorem)
$$= \left(2\sqrt{5}\right)^2 + 5^2$$
$$= 3\sqrt{5}$$
$Also,$
$\Delta DBA \sim \Delta DBC$
$$\frac{DB}{DC}=\frac{BA}{BC}$$
$$\frac{\left(2\sqrt{5}\right)}{5}=\frac{BA}{\left(3\sqrt{5}\right)}$$
$$AB = 6\,cm$$

10. In Fig., PQR is a right triangle right angled at Q and $QS \perp PR$. If PQ = 6 cm and PS = 4 cm, find QS, RS and QR.



Solution:
We have,

In ΔPQR,
∠Q = 90°,
QS ⊥ PR and
PQ = 6 cm,
PS = 4 cm

In ΔSQP and ΔSRQ,

$\angle PSQ = \angle RSQ$ [each equal to 90°]

$\angle SPQ = \angle SQR$ [each equal to $90° - \angle R$]

$\Delta SQP \sim \Delta SRQ$ [By AA similarity criterion]

Then, $\frac{SQ}{PS} = \frac{SR}{SQ}$

$SQ^2 = PS \times SR$ ………….. (i)

In right angled ΔPSQ,

$PQ^2 = PS^2 + QS^2$ [using Pythagoras theorem]

$(6)^2 = (4)^2 + QS^2$

$36 = 16 + QS^2$

$QS^2 = 36 - 16$

$= 20$

$QS = 2\sqrt{5}$ cm

From eqn (i),

Putting value of PS and QS we get,

RS = 5cm

Now, In QSR,

$QR^2 = QS^2 + SR^2$

So, putting value of QS and SR we get,

$QR = 3\sqrt{5}$ cm

11. In $\Delta PQR, PD \perp QR$ such that D lies on QR. If PQ = *a*, PR = *b*, QD = *c* and DR = *d*, prove that $(a+b)(a-b)=(c+d)(c-d)$.

Solution:

Given:

In ΔPQR, $PD \perp QR$,

PQ = a,

PR = b,

QD = c and

DR = d

To prove: $(a + b)(a - b) = (c + d)(c - d)$

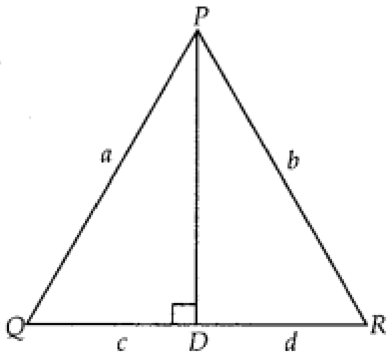
Proof:

In right angled ΔPDQ,

$PQ^2 = PD^2 + QD^2$ [using Pythagoras theorem]

$a^2 = PD^2 + c^2$

$PD^2 = a^2 - c^2$ .... (i)



In right angled ΔPDR,

$PR^2 = PD^2 + DR^2$ [using Pythagoras theorem]
$b^2 = PD^2 + d^2$
$PD^2 = b^2 - d^2$ .... (ii)

From Eqs. (i) and (ii)
$a^2 - c^2 = b^2 - d^2$
$a^2 - b^2 = c^2 - d^2$
$(a - b)(a + b) = (c - d)(c + d)$

Hence proved.

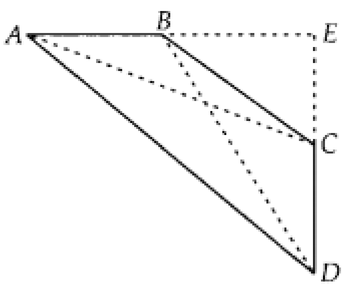
12. In a quadrilateral ABCD, $\angle A + \angle D = 90°$. Prove that $AC^2 + BD^2 = AD^2 + BC^2$ [Hint: Produce AB and DC to meet at E.]

Solution:

Given:
Quadrilateral ABCD,
$\angle A + \angle D = 90°$

To prove: $AC^2 + BD^2 = AD^2 + BC^2$

Construct: Produce AB and CD to meet at E
Also join AC and BD



Proof:
In ΔAED,
$\angle A + \angle D = 90°$ [given]

$\angle E = 180° - (\angle A + \angle D)$
$= 90°$ [sum of angles of a triangle = 180°]

So, by Pythagoras theorem,
$AD^2 = AE^2 + DE^2$

In ΔBEC, by Pythagoras theorem,
$BC^2 = BE^2 + EC^2$

Adding both equations, we get
$AD^2 + BC^2 = AE^2 + DE^2 + BE^2 + CE^2$ ………… (i)

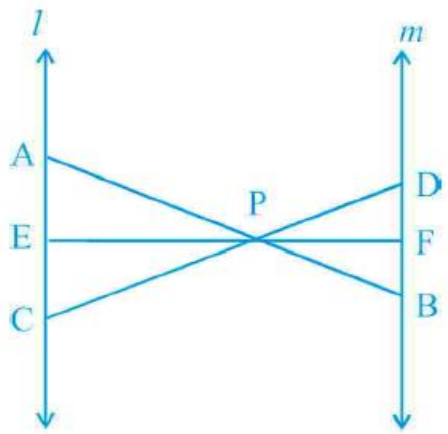
In ΔAEC, by Pythagoras theorem,
$AC^2 = AE^2 + CE^2$

In ΔBED, by Pythagoras theorem,
$BD^2 = BE^2 + DE^2$

Adding both equations, we get
$AC^2 + BD^2 = AE^2 + CE^2 + BE^2 + DE^2$ ………… (ii)

From Eqs. (i) and (ii)
$AC^2 + BD^2 = AD^2 + BC^2$
Hence proved.

13. In Fig., *l* || m and line segments AB, CD and EF are concurrent at point P.
Prove that
$$\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$$



Solution:
We have, $l \parallel m$ and line segments AB, CD and EF are concurrent at point P

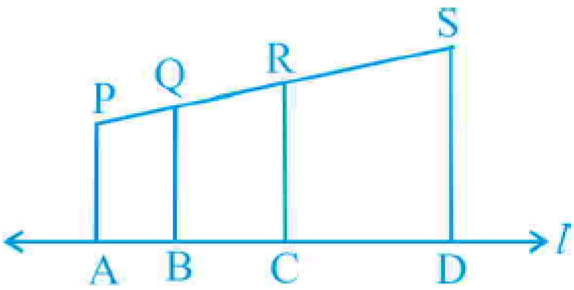
To Prove,
$$\frac{AE}{BF}=\frac{AC}{BD}=\frac{CE}{FD}$$

In ΔAPC and ΔBPD,
$APC = BPD$ (vertically opposite angles)
$PAC = PBD$ (Alternate angles)
so,
$\Delta APC : \Delta BPD$ (By AA Similarity)
$$\frac{AP}{PB}=\frac{AC}{BD}=\frac{PC}{PD}$$
Now,
In ΔAPE and ΔBPF,
$APE = BPF$ (vertically opposite angles)
$PAE = PBF$ (Alternate angles)
so,
$\Delta APE : \Delta BPF$ (By AA Similarity)
$$\frac{AP}{PB}=\frac{AE}{BF}=\frac{PE}{PF}$$

Now,
In ΔPEC and ΔPFD,
$APC = BPD$ (vertically opposite angles)
$PAC = PBD$ (Alternate angles)
so,
$\Delta PEC : \Delta PDF$ (By AA Similarity)
$$\frac{PC}{PD}=\frac{PE}{PF}=\frac{EC}{FD}$$

So, from above equations,
$$\frac{AP}{PB}=\frac{AC}{BD}=\frac{PE}{PF}=\frac{EC}{FD}=\frac{AE}{BF}$$
$$\frac{AE}{BF}=\frac{AC}{BD}=\frac{EC}{FD}$$
Hence proved!!

14. In Fig., PA, QB, RC and SD are all perpendiculars to a line *l*, AB = 6 cm,BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS.



Solution:
We have,
AB = 6 cm,
BC = 9 cm,
CD = 12 cm and
SP = 36 cm

Also, PA, QB, RC and SD are all perpendiculars to line l,

PA || QB || RC || SD

Using Basic proportionality theorem,
PQ : QR : RS = AB : BC : CD
= 6 : 9 : 12

Taking,
PQ = 6x,
QR = 9x and
RS = 12x
As,

Length of PS = 36 cm
PQ + QR + RS = 36
6x + 9x + 12x = 36
27x = 36

$$x = \frac{4}{3}$$

Now,
PQ = 6x

$$= 6 \times \frac{4}{3}$$

= 8 cm

QR = 9x

$= 9 \times \frac{4}{3}$
$= 12$ cm

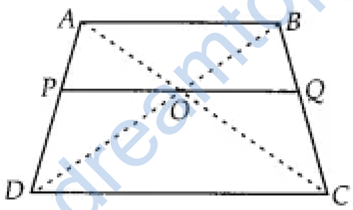
RS = 12x
$= 12 \times \frac{4}{3}$
$= 16$ cm

15. O is the point of intersection of the diagonals AC and BD of a trapezium ABCD with AB || DC. Through O, a line segment PQ is drawn parallel to AB meeting AD in P and BC in Q. Prove that PO = QO.

Solution:

Given ABCD is a trapezium. Diagonals AC and BD are intersect at O.
PQ || AB || DC

To prove: PO = QO



Proof:
In ΔABD and ΔPOD,
PO || AB [as, PQ || AB]

∠D = ∠D [common angle]
∠ABD = ∠POD [corresponding angles]
ΔABD ~ ΔPOD [by AA similarity criterion]

Then,

$$\frac{OP}{AD} = \frac{PD}{AD} \quad \text{............... (i)}$$

In ΔABC and ΔOQC, OQ || AB
∠C = ∠C [common angle]
∠B AC = ∠QOC [corresponding angles]
∴ ΔABC ~ ΔOQC [by AA similarity criterion]

$$\frac{OQ}{AB} = \frac{QC}{BC}$$

$\text{Also, In } \Delta\text{ADC}, \text{OP} \parallel \text{DC}$

$$\frac{AP}{PD} = \frac{OA}{OC}$$

$\text{In } \Delta\text{ABC}, \text{OQ} \parallel AB$

$$\frac{BQ}{QC} = \frac{OA}{OC}$$

Therefore,

$$\frac{AP}{PD} = \frac{BQ}{QC}$$

Adding 1 on both sides,

$$\frac{AP}{PD} + 1 = \frac{BQ}{QC} + 1$$

$$\frac{AP + PD}{PD} = \frac{BQ + QC}{QC}$$

$$\frac{AD}{PD} = \frac{BC}{QC}$$

or,

$$\frac{PD}{AD} = \frac{QC}{BC}$$

Also,

$$\frac{OP}{AB} = \frac{QC}{BC} \; and \; \frac{OP}{AB} = \frac{OQ}{AB}$$

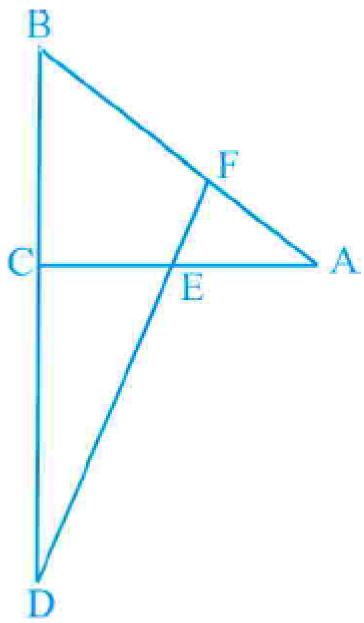
Therefore,

$$OP = OQ$$

16. In Fig., line segment DF intersect the side AC of a triangle ABC at the point E such that E is the mid-point of CA and $\angle\text{AEF} = \angle\text{AFE}$. Prove that

$$\frac{\text{BD}}{\text{CD}} = \frac{\text{BF}}{\text{CE}}$$

[Hint: Take point G on AB such that CG || DF.]



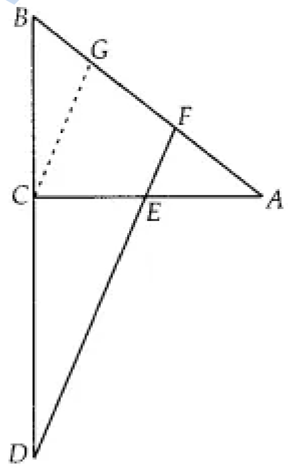
Solution:

Given ΔABC, E is the mid-point of CA and $\angle AEF = \angle AFE$

To prove: $\frac{BD}{CD} = \frac{BF}{CE}$

Construction: Take a point G on AB such that CG || DF

Proof: As, E is the mid-point of CA



CE = AE …(i)

In ΔACG, CG || EF and E is mid-point of CA

So, CE = GF … (ii) [by mid-point theorem]

Now, in ΔBCG and ΔBDF, CG || DF

$$\frac{BC}{CD} = \frac{BG}{GF}$$
$$\frac{BC}{CD} = \frac{BF-GF}{GF}$$
$$\frac{BC}{CD} = \frac{BF}{GF} - 1$$
$$\frac{BC}{CD} + 1 = \frac{BF}{CE} \qquad (from\,(ii))$$
$$\frac{BC+CD}{CD} = \frac{BF}{CE}$$
$$\frac{BD}{CD} = \frac{BF}{CE}$$

17. Prove that the area of the semicircle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semicircles drawn on the other two sides of the triangle.

Solution:

ABC is a right triangle, right angled at B in which

AB = y,

BC = x

We will draw three semi-circles are drawn on the sides AB,BC and AC, respectively with diameters AB,BC and AC, respectively.

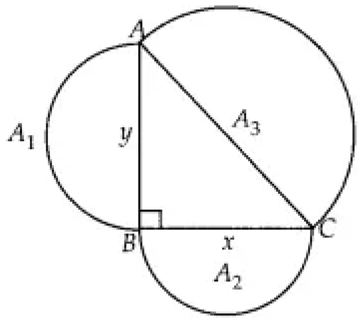
Again,

Taking area of circles with diameters AB, BC and AC are respectively A_1, A_2 and A_3

To prove : $A_3 = A_1 + A_2$

Proof :

In ΔABC, by Pythagoras theorem,



$$AC^2 = AB^2 + BC^2$$
$$AC^2 = y^2 + x^2$$
$$AC = \sqrt{y^2 + x^2}$$

$$Also, area\ of\ semicircle\ drawn\ on\ AC = \frac{\pi r^2}{2}$$
$$= \frac{\pi}{2}\left(\frac{AC}{2}\right)^2$$
$$A_3 = \frac{\pi(y^2 + x^2)}{8}$$

Now,

$$area\ of\ semicircle\ drawn\ on\ AB = \frac{\pi r^2}{2}$$
$$= \frac{\pi}{2}\left(\frac{AB}{2}\right)^2$$
$$A_1 = \frac{\pi(y^2)}{8}$$

Now,

$$area\ of\ semicircle\ drawn\ on\ BC = \frac{\pi r^2}{2}$$
$$= \frac{\pi}{2}\left(\frac{BC}{2}\right)^2$$
$$A_2 = \frac{\pi(x^2)}{8}$$

From above equations, we see that,

$$A_3 = A_1 + A_2$$

Hence Proved!!!

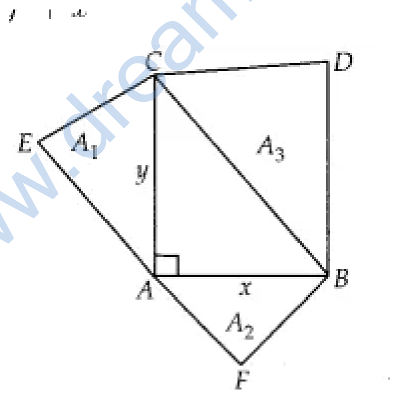
18. Prove that the area of the equilateral triangle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the equilateral triangles drawn on the other two sides of the triangle.

Solution:

BAC is a right triangle in which $\angle A$ is right angle and
AC = y,
AB = x

Now we draw three equilateral triangles on the three sides of ΔABC,
ΔAEC,
ΔAFB and
ΔCBD

Let us assume area of triangles made on AC, AB and BC are A_1, A_2 and A_3 respectively.
We need to prove that,
$A_3 = A_1 + A_2$



Proof :
In ΔCAB, using Pythagoras theorem,
$BC^2 = AC^2 + AB^2$
$BC^2 = y^2 + x^2$
$BC = \sqrt{y^2 + x^2}$

Also Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

Now we calculate the area A_1, A_2, and A_3 respectively

$$ar(\Delta AEC) = A_1$$

$$A_1 = \frac{\sqrt{3}}{4} AC^2$$

$$A_1 = \frac{\sqrt{3}}{4} y^2$$

Now,

$$ar(\Delta AFB) = A_2$$

$$A_2 = \frac{\sqrt{3}}{4} AB^2$$

$$A_2 = \frac{\sqrt{3}}{4} x^2$$

$$ar(\Delta CBD) = A_3$$

$$A_3 = \frac{\sqrt{3}}{4} CB^2$$

$$A_3 = \frac{\sqrt{3}}{4} (y^2 + x^2)$$

$$A_3 = \frac{\sqrt{3}}{4} x^2 + \frac{\sqrt{3}}{4} y^2$$

$$A_3 = A_1 + A_2$$

Hence Proved!!