Theme 6: Surface Areas and Volumes

Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Surface Areas and Volume Formulae for Basic Solids - Cube, Cuboid, Cylinder, Cone, Frustum of Cone, Sphere, and Hemisphere
- Surface Area and Volume of an Object formed by combining two or more Basic Solids.

Case Study A - Greenhouse Design

Have you ever wondered why a car kept in the Sun is super-hot inside? Why glass buildings waste a lot of money on electricity bills due to air conditioning? Well, it's because glass traps heat and makes the interiors even hotter than outside. Farmers have used this technique - especially in cooler climes - to grow, say, tropical plants in temperate regions. These transparent structures where these plants are grown, indoors, are called greenhouses. Now you also might make the link why we call the warming of the Earth due to certain gases in the atmosphere as the 'greenhouse effect', or why the culprit gases are called 'greenhouse gases'!

Anyway, back to greenhouses themselves: since we can control the temperature within them and keep them especially warm, it allows botanists and farmers to grow crops or ornamental plants year-round, and keep them safe from a lot of the natural factors outdoors. The greenhouse structure is typically constructed using aluminium or wood, and while glass provides the best 'greenhouse effect', it also weighs and costs a lot, so plastic substitutes are also used for the transparent covering.

So the next time you go to a nursery or a botanic garden, and see a greenhouse, go in and enjoy the micro-climate experienced inside and also notice the kind of structure and materials used to make them.

The questions below are related to the design of greenhouses.

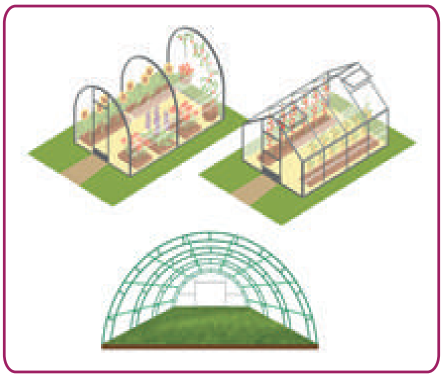


Fig. 6.1, Sample designs of greenhouse

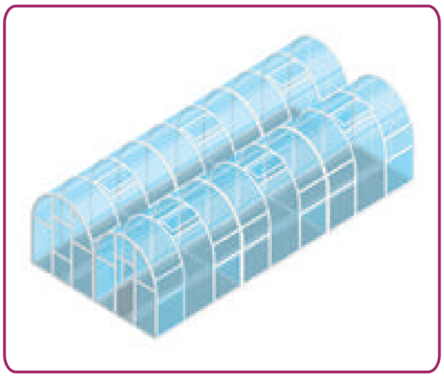


Fig. 6.2, Two or more greenhouses are sometimes joined side by side so that they have fewer external walls, and the material costs are consequently less. A greenhouse has a wide area of glazing on its sides and roof so that the plants are exposed to natural light for much of the day.

Question 1

A farmer constructed a greenhouse with the dimensions, as shown in Fig. 6.3. He needs to buy transparent tarpaulin to cover it. Transparent tatpaulin comes in 2 m x 20 m rolls and most vendors do not provide partial rolls. How many rolls does the farmer need to buy? Note that he keeps the entire front portion open to access the greenhouse. Except the front portion, he wants to cover the entire structure with tarpaulin. He assumes that there won't be any wastage when he fixes the transparent tatpaulin.

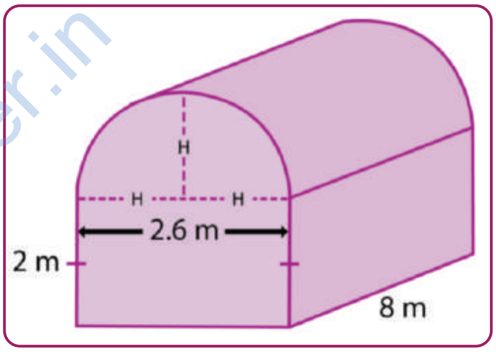


Fig. 6.3, Front view of the design of a greenhouse. Note that H = 1.3 m here.

a. 1 roll	b. 2 rolls	Answer
c. 3 rolls	d. 4 rolls	

Question 2

As the farmer started fixing the transparent tatpaulin on the greenhouse, he realised that he had to cut the rolls into small pieces to cover some parts of the greenhouse, which helped him minimise wastage. This was also necessary, because some parts required both lengths of the sheet to be more than 2 m, whereas the transparent tatpaulin rolls had a maximum length of 2 m on one side. He labelled different parts of the greenhouse as shown in Fig. 6.4 & 6.5

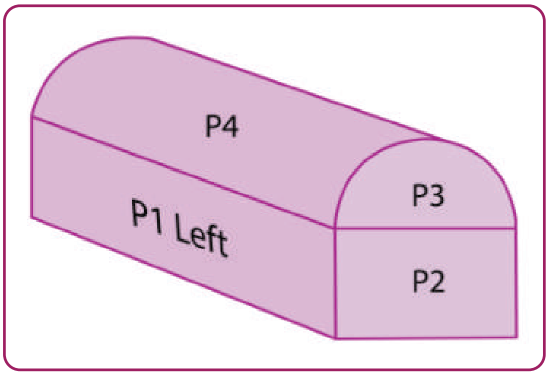


Fig. 6.4, Parts of the greenhouse labelled

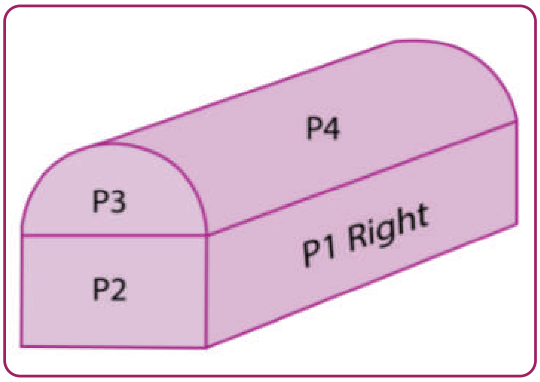


Fig. 6.5, P1 Right labelled separately

Note that he labelled the left and right side of P1 as two separate parts, so that separate sheets are cut out for both parts.

He then decided to cut rectangle-shaped pieces for each of these parts. The dimensions of the pieces P1 Left, P1 Right and P2 have been filled in boxes next to the respective sides of the rectangles. In the same manner, fill the dimensions of the remaining pieces.

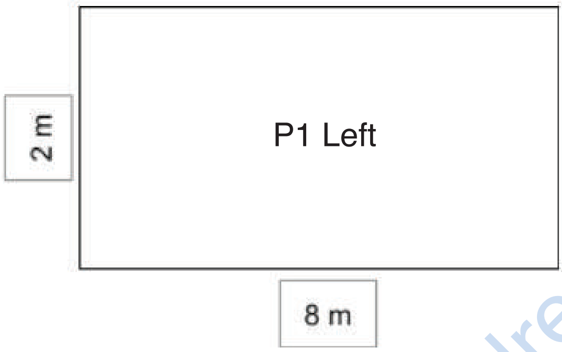


Fig. 6.6, Parts of the greenhouse - P1 Left

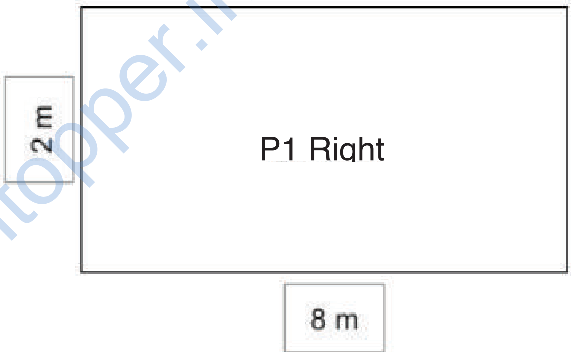


Fig. 6.7, Parts of the greenhouse - P1 Right

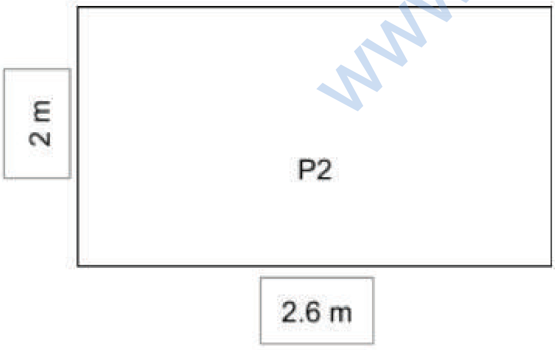


Fig. 6.8, Parts of the greenhouse - P2

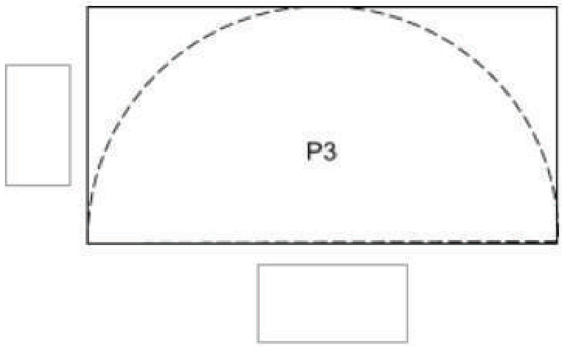


Fig. 6.9, Parts of the greenhouse - P3

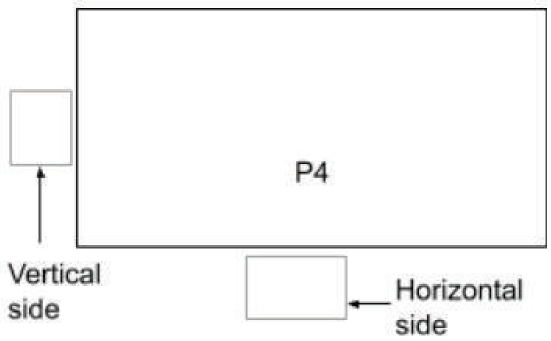


Fig. 6.10, Parts of the greenhouse - P4

i. P3 vertical side: ____________________

ii. P3 horizontal side: ____________________

iii. P4 vertical side: ____________________

iv. P4 horizontal side: ____________________

Question 3

As he details the plan further, he chose to leave a 5 cm gap on both sides, between the roof and the wall, for ventilation. He realises that this reduces the required length of the P4 vertical side by 10 cm.

He now started laying out these 5 parts on the rolls. An example layout has been shown in Fig. 6.11. Note that this is only an example.

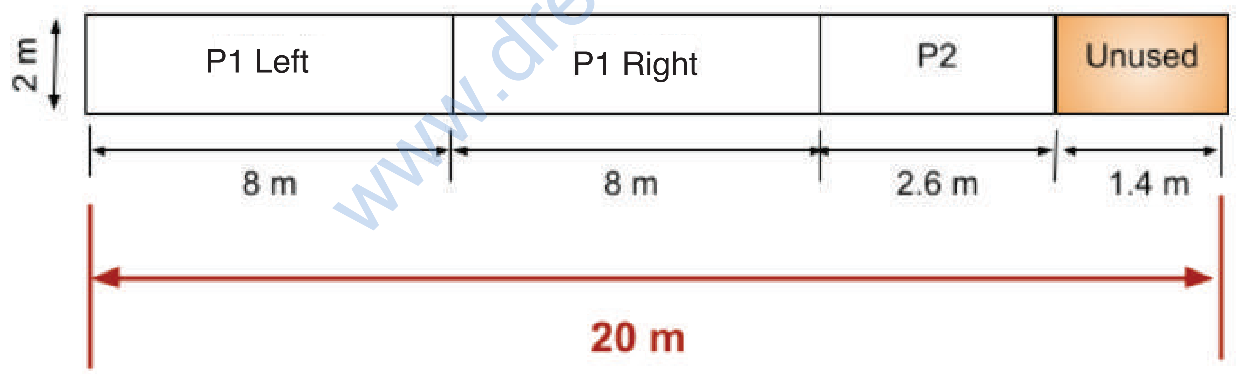


Fig. 6.11, Example layout of greenhouse parts on the transparent tatpaulin sheet

Some parts had to be cut into smaller sub-parts so as to fit into the dimensions of the roll and to minimise wastage.

i. Going by his method of cutting rectangular pieces, what would be the minimum number of rolls he would need?

Answer

ii. What is the area of the transparent tatpaulin sheet which would go to waste?

Answer

Question 4

Apart from controlling the temperature, and the watering of the plants, greenhouses also need to be well-ventilated. Plants, like us, need a constant supply of fresh air, and having a good ventilation system allows you to control temperature and humidity better and reduce the proliferation of pests. Effective ventilation can be achieved naturally by having more vents and windows, or via a powered system that circulates air. In a powered system, there has to be a combination of fans and vents so that the outside air is able to replace the inside air within minutes. The power and number of these fans is determined by the size of the greenhouse.

Calculate the maximum volume of air in this empty greenhouse structure, shown in Fig. 6.3?

Answer

Question 5

A hollow metal pipe with an external diameter of 28 cm is 1 cm thick and is placed at the centre of this greenhouse along its longest length. The farmer plans to use it as a partitioning line by shifting it in between two varieties of plants. How many people will be required to move the pipe? Assume one adult can lift a maximum of 50 kg. [Consider the density of this metal as 8.05 g/cm^3]

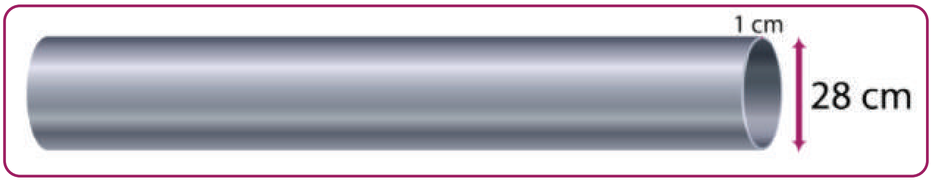


Fig. 6.12, Metal pipe used as separator in the greenhouse

a. 5 or less than 5	b. 6 to 9	Answer
c. 11 to 12	d. More than 12	

Case Study B - The Peanut Vendor

India has a large number of small entrepreneurs who choose to run their own small businesses instead of taking up a job in a larger organisation. A typical Indian market would have multiple small streets with a large number of small shops on each street. Many of these are mobile shops, with the vendor moving her/his products on small carts and reaching out to the customers. One such unique and often sought after vendor is the peanut vendor, who carries the peanuts as well as the equipment required to roast the peanuts on a cart. Some of these carts are two- or three-wheeled carts. Other than the hot and crispy roasted peanuts, these vendors are a source of attraction, especially for the younger crowd, because of the perfect cones they make from waste papers to serve the peanuts to their customers. No glue is used while making these cones. Why do they make cones and not any other shape, say a cylinder or cuboid?

Question 6

Lakshmi and Ira buy some roasted peanuts from a street vendor who makes cones from papers, which have an almost circular shape. As shown in Fig. 6.13, around 1/4th of the circle is used to overlap with another 1/4th part of the circle. So the surface area of the cone would be 3/4th of the circle.

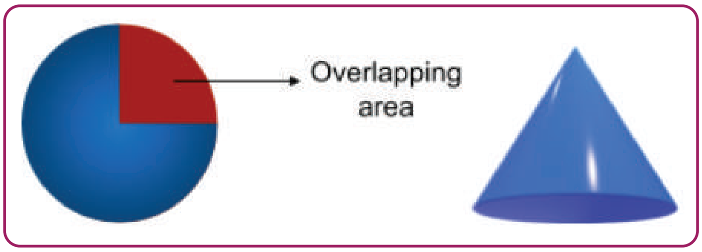


Fig. 6.13, Circular paper used to make a cone

Lakshmi and Ira insisted on being served the peanuts in two separate cones. The vendor cut the circle into two semicircles and taping the edges together he made two cones.

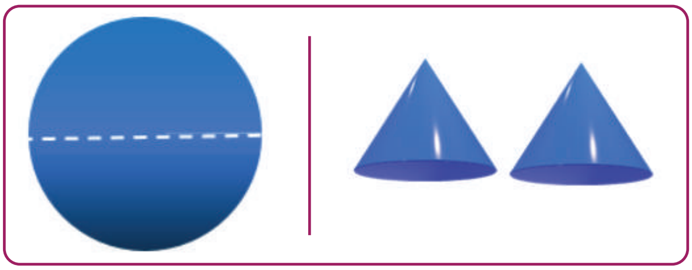


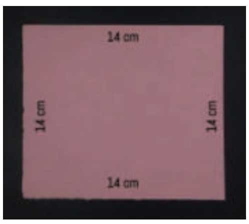
Fig. 6.14, Circle cut into half to make two smaller cones

Do you think he gave more peanuts when compared to a single cone or less or equal? Assume that the peanuts are packed as tightly as possible in both the cases.

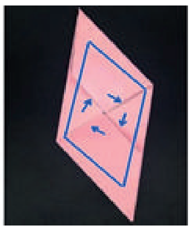
a. More	b. Less	Answer
c. Same	d. Depends on the size of the circle	

Case Study C - Activity to Visualise Volume

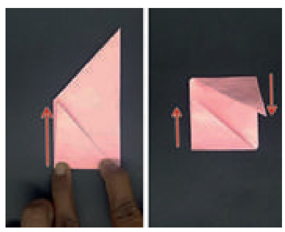
So as to visualise volume, one can make a regular cube. Given below is one of the ways of doing this, using 6 square papers.



Step 1 - Take a square sheet of paper of size 14 cm x 14 cm.

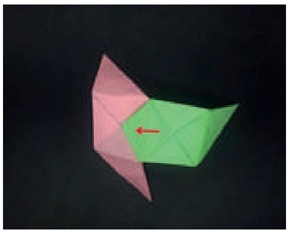


Step 2 - Fold the sheet to make one parallelogram shaped module



Step 3 - Fold triangle shaped ends to create flaps of the module.

Step 4 - Repeat the steps with 6 square sheets to create 6 modules.



Step 5 - insert one module's flap into the pocket of another, as shown.

Step 6 - Link other 4 modules in a similar manner to make a 6 sided cube.

Question 7

The volume of the cube hence made was measured using the process shown in Fig. 6.15 and Fig. 6.16.

Fig. 6.15, Cube opened and a plastic cover arranged inside

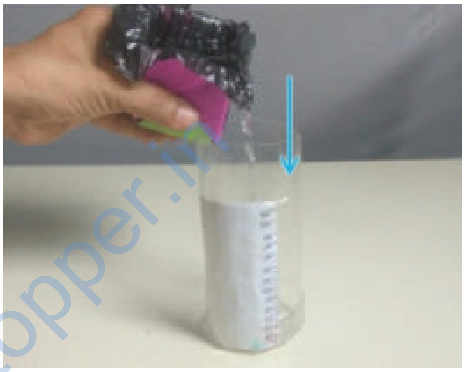


Fig. 6.16, Fill the cube upto the brim and measure the volume of collected water using a volume measuring beaker

Following observations were made and recorded, using the cube thus made.

Volume of the cube	125 ml
Length of the side of the cube	5 cm
Length of each of the square papers from which we made the faces of the cube	14 cm

i. If we want to make a cube-shaped box, which has a volume of 1 litre, using the above process, what should be the length of the side of the box when compared to the length of the box made above?

a. 8 times more	b. double	Answer
c. 4 times more	d. half	

ii. What should be the length of the 6 square papers for making the 1 litre cube? Note that the length of each square paper used to make the above cube was 14 cm. The process of making one module from a square sheet has been shown in Fig. 6.17.

a. 112 cm	b. 28 cm	Answer
c. 56 cm	d. 7 cm	

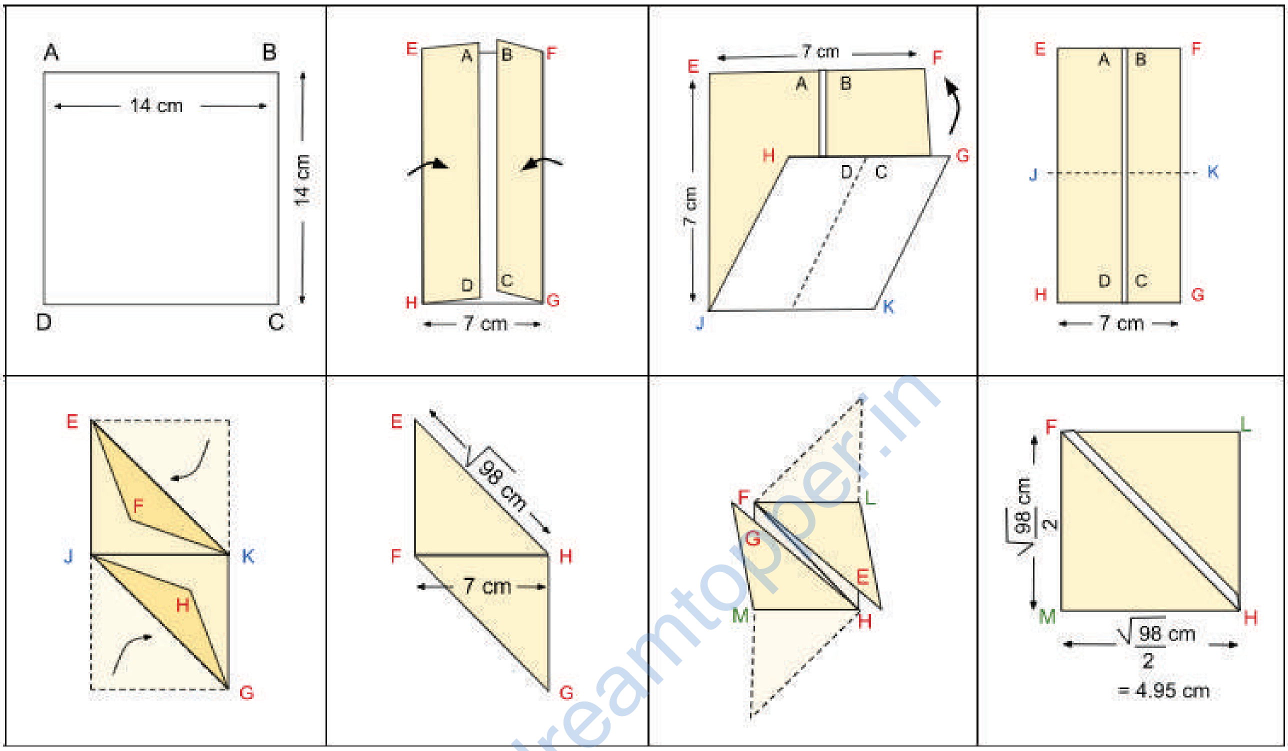


Fig. 6.17, Square sheet of 14 cm x 14 cm folded to make one module with square in the middle and triangular flaps. The side of the square in the middle would be 4.95 (can be rounded off to 5) cm long.

Case Study D - The Selling Trick

Way back in the 1950s, a toothpaste manufacturing company is said to have conducted a competition among their employees to come up with ideas to improve their sales. After looking at multiple ideas, they received a note from an outsider who offered to sell his idea for $100,000. At that time this was a huge sum of money, so the company ignored it initially. However, the stranger showed up at the office and confidently claimed that his idea was infallible. The company was convinced and decided to buy the idea. Next day the company received a brown envelope from this gentleman, which contained a small slip of paper saying, “Make the opening bigger”.

Question 8

Consider a toothpaste tube, which contains around 50 ml of toothpaste. The inner diameter of the tube's opening is 5 mm and the company decides to increase the diameter by 1 mm. By what percentage would the amount of toothpaste coming out in one usage increase? Assume that the length of toothpaste squeezed out in each usage remains the same as before. Write your answer in the box below.

Answer

Question 9

Most consumers don't pay attention to the volume of toothpaste squeezed out in each usage. They tend to just fill the entire length of the toothbrush. This could be the effect of habit or the visual cues from advertisements showing the entire length of the toothbrush being filled with toothpaste.

Fig. 6.49, Toothbrush with toothpaste

In a family of 3 members, how many days will a 50 ml toothpaste last? Consider the length of the toothbrush to be 2 cm for all the three family members. Also assume they brush twice in a day and fill the entire length of the brush during each usage. Consider the inner diameter of the toothpaste tube to be 5 mm. Write your answer in the box below. Note that 1 cm^3 = 1 ml.

Answer

Question 10

The toothpaste company tied up with a popular toothbrush company and increased the length of the toothbrush by 25%. Now, with the increase in the toothpaste tube's inner diameter to 6 mm and increase in the length of the toothbrush, how many days will a toothpaste tube of the same volume last for the 3-member family? Assume that all the family members replace their toothbrushes, thanks to the free toothbrush offer on the new toothpaste. Write your answer in the box below.

Answer

Exploration Pathway

Volume and Surface Area

Take two cardboard pieces of rectangular shape with the same dimensions and one side being longer than the other. Roll one rectangle along the shorter side and another along the longer side and make two cylinders. Predict and validate which cylinder has a larger volume and which one has more surface area. Pour soil in both cylinders to compare which one has more volume. Repeat this exercise by cutting one rectangle into two equal parts and comparing the total volume of two cylinders made from the half rectangles with that made from the full rectangle.

Visualise Volume

As our brains go from grasping one dimension to two to three, certain aspects of our understanding become less and less clear. However, volume is one of our most routine day-to-day units, but causes innumerable conceptual misunderstandings. In this incredibly visual TACtivity, you make your own cubes of fixed volume, using coloured paper to gain a better grasp of the concept of volume.