Theme 5: Area of Parallelograms & Triangles

Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Parallelograms on the same base and between the same parallels
- Triangles on the same base and between the same parallels
- Theorems related to the area of parallelograms and triangles

Measuring distance using triangles

In surveying, measuring the distance to a point (which is very far) by direct methods (using tape or chain) sometimes becomes difficult. In such cases, the distance is measured using a method called *Triangulation*. In this method, the distance is measured by measuring the angles from a couple of known locations to the point (whose distance needs to be measured). The length of the baseline between the known points and the angles measured allow you to then calculate the distance to the point. This method has been used for aeons, especially in fields such as surveying (like calculating the height of a tree, building or mountain!), astronomy (measuring the distance to a star), navigation, etc. It is a powerful tool to measure large distances that would be hard to measure directly.

Case Study A - Retgadh and Sonpur

Rethgad and Sonpur were two friendly countries separated by an ocean. Once the head of Sonpur decided to visit Rethgad. He had informed the head of Rethgad about the date of his visit. The head of Rethgad was excited to welcome his friend. He had given the instructions to his guards to keep watch on the ocean and asked them to send a quick information as soon as they spotted the ship with Sonpur's flag coming towards their shoreline, so that he could be prepared to welcome the guest.

On the day of visit, the guards of Retgadh spotted a ship with Sonpur's flag. They observed the ship from two end points of a straight line on their shoreline. Using a theodolite (an instrument used to measure angles which has a telescope) they measured the angle the ship was making with the straight line at those two points. They repeated this exercise after 5 minutes and captured the results in a drawing on a paper, as shown in Fig. 5.1, in order to estimate the lengths and areas. In Fig. 5.1, A and B are the positions of the guard posts at the shoreline. X and Y are the positions of the ship when it was first spotted and after 5 minutes respectively.

Though calculating the area of ΔABY was straightforward as it is a right angled triangle, ΔABX needed some extra effort. In order to calculate the area, they drew the triangles on a graph paper. Consider the side of each square on the graph paper as 1 unit long.

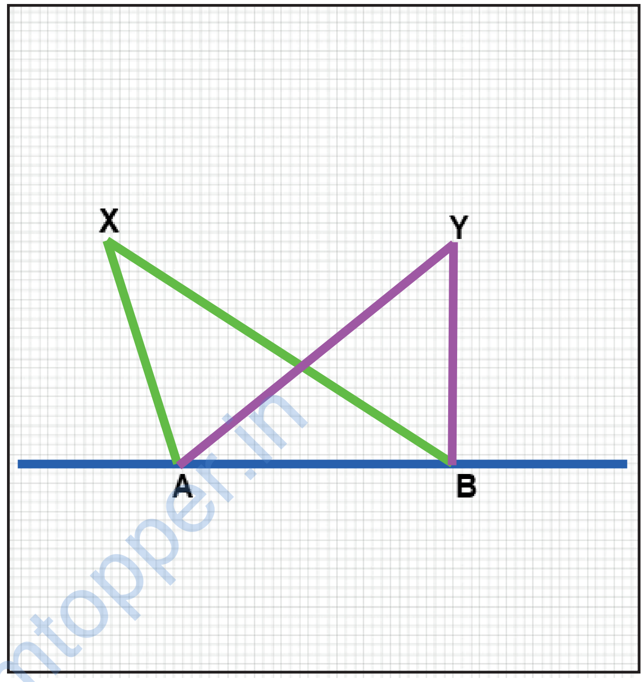


Fig. 5.1, Position of the ship before and after 5 minutes as seen from the shoreline. Grid image by Karen Arnold via publicdomainpictures.net.

Question 1

ΔABX occupied 276 completely filled unit squares and 24 half-filled unit squares on the graph paper. The length of AB is 24 units and that of BY is also 24 units. Which of the following is true about the areas of ΔABX and ΔABY ?

a. Area of ΔABX is larger
b. Area of ΔABY is larger
c. The area of both the triangles is equal
d. Data is insufficient to arrive at the conclusion

Answer

Question 2

i. In Fig. 5.1, if you join the points X and Y, will the line XY be parallel to AB? Justify your answer.

Answer

ii. Is the ship coming towards Retgadh, going away or moving parallel to Retgadh's shore-line?

Answer

Case Study B - Peripheral vision

While looking at an object even though you fix your eye at a particular point on the object, you will still be able see the area around your point of fixation. This ability of our eye is called peripheral vision. Write a letter (say V) in bold on a small placard of size 12 cm x 12 cm. Now stand at a distance of 30 cm from a wall. Hold the placard pressed against the wall at a height of your eye level. Make sure that the placard is exactly opposite to your face and you can read the letter V clearly. Now slowly slide the placard along the wall, at the same height, towards your left such that you can still read the letter V. The moment you feel that you can't read the placard, pause sliding it and mark its position. Similarly, move the placard to your right and mark the position.

Fig. 5.2, Mark the position of the placard at which you are unable to read it

If you imagine two lines joining the midpoint of your eyes with the two positions that you marked on the wall, ideally you should get an isosceles triangle. Let's call it the 'Field of Vision Triangle'. Measure the distance between the two markings on the wall and this should give you the base length of the triangle.

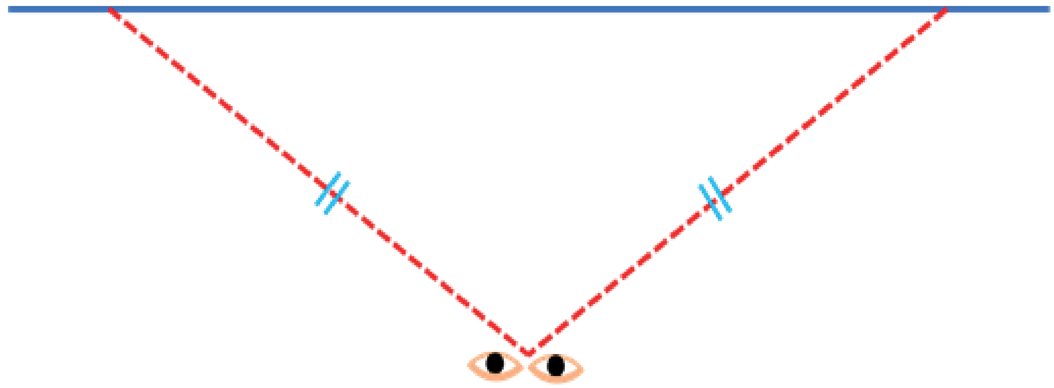


Fig. 5.3, Field of vision triangle

After the above exercise move a horizontal distance of 2 m (or more depending on the width of the wall you have chosen), either towards your left or right. Repeat the same exercise as described above. You will be surprised to notice that the base length of both the triangles will be exactly equal!

Question 3

Shyna made an attempt to record the field of vision triangle following the procedure described above. Fig. 5.4 represents two fields of vision triangles recorded by Shyna.

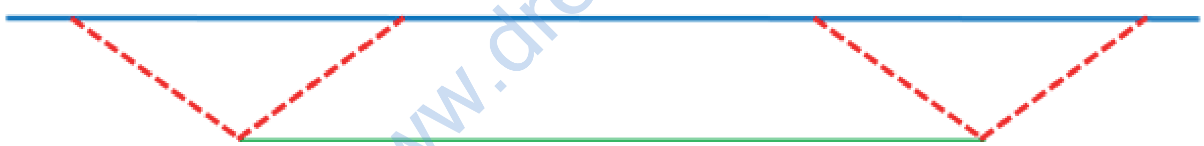


Fig. 5.4,Two fields of vision triangles recorded by Shyna

i. Apart from two triangles, which of the following shape(s) can you identify in Fig. 5.4?

a. Trapezium (s)

b. Parallelogram (s)

c. Two trapeziums and a parallelogram

d. Two trapeziums and two parallelograms

Answer

ii. Which of the following is true about different polygons that you have identified in fig. 5.4? Choose all that apply.

a. All of them lie between the same parallels

b. All of them have equal height

c. All of them have equal base

d. The triangles have equal bases

Answer

Question 4

Which of the following is not true regarding the areas of the triangles shown in Fig. 5.4?

a. Both the triangles have equal areas

b. The area of both triangles depends on the distance between the two positions where Shyna did the experiment (length of the green line in Fig. 5.4)

c. The area of each triangle depends on the distance between the wall and Shyna's eyes

d. The area of each triangle depends on the leftmost and rightmost position of the placard corresponding to that triangle

Answer

Case Study C - Equidissection

Dividing a polygon into equal area triangles is called *Equidissection*. Artists and designers make use of this method by applying it to tiling and patchwork. How can one tile an area using equal area tiles, for example?

Look at some of the equidissections of a square below:

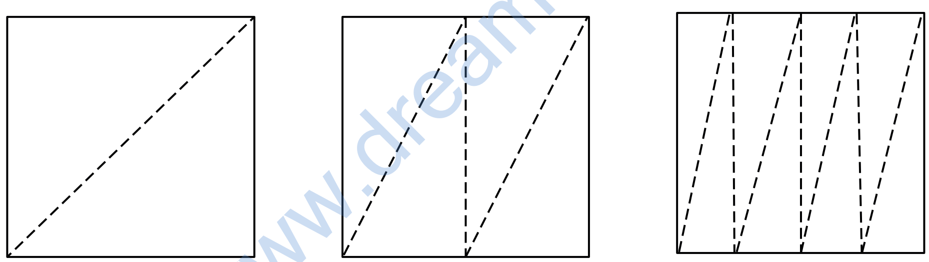


Fig. 5.5, Sample equidissection of a square

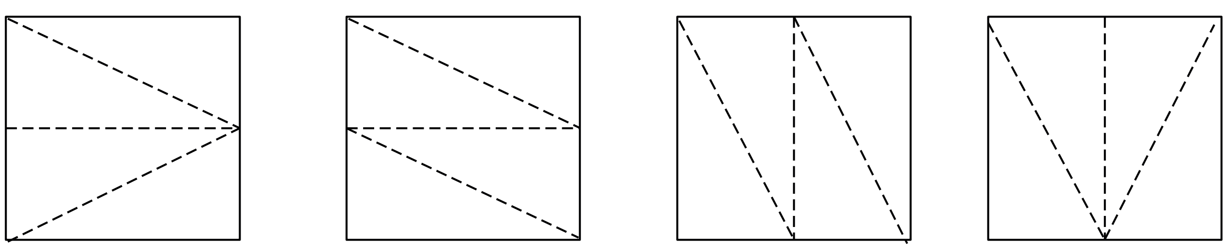


Fig. 5.6, Some more equidissections of the square into 4 triangles

There may be other possible ways, but it is possible only with an even number of triangles. Consider one such partitioning as shown in Fig. 5.7 and answer the questions below.

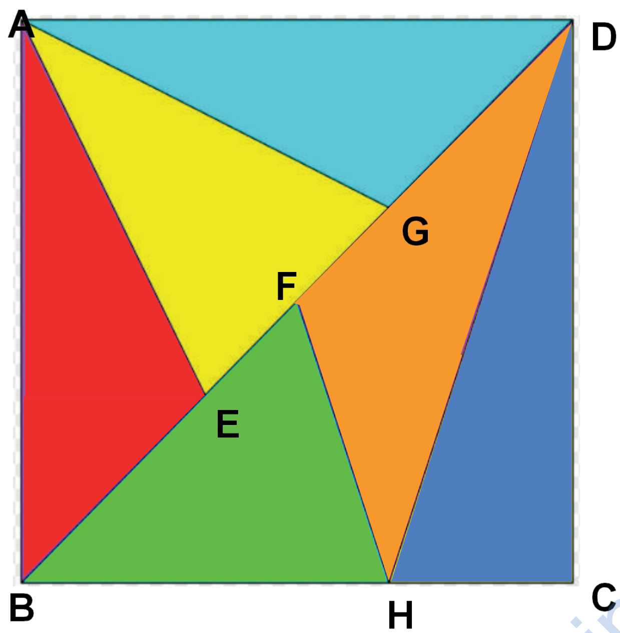


Fig. 5.7, Square dissected into 6 equal-area triangles; Image by Melchoir via Wikimedia Commons

Question 5

Consider the equidissection of a square as shown in Fig. 5.7. Is there any relation between the lengths BE, EG and GD? Justify your answer along with stating the definition of the relevant theorem statement.

Answer

Question 6

Based on the definition of equidissection, all the six triangles (partitions) have an equal area. Which of the below statements is valid with respect to these triangles.

A. All these triangles will have equal height.

B. All these triangles will have an equal base length.

C. All these triangles are congruent to each other.

Choose the correct option from the following:

a. Only C	b. Both A and C
c. All three statements	d. None of these statements

Answer

Question 7

i. As the areas of green triangle (ΔFHB) and orange triangle (ΔFHD) are equal by the definition of equidissecton, write down the appropriate relation between the length BF and FD.

Answer

ii. In ΔBHD, the line HF is the___________ of the triangle and it divides the triangle into ____________.

a. Median, two congruent triangles

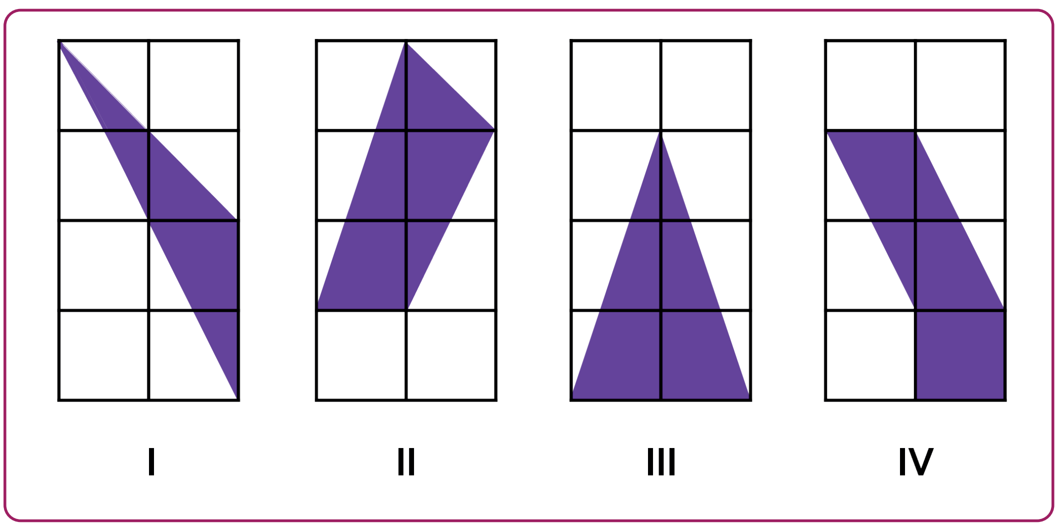
b. Median, two triangles with equal area

c. Altitude, two triangles with equal area

d. Height, two congruent triangles

Question 8

In Fig. 5.9, which purple polygon doesn't have the same area as the others? Assume that the sides of all the small squares in the grid are equal.



Answer

Fig. 5.9, Different polygons inside square grids

Question 9

Observe the different parallelograms in Fig. 5.10 carefully. Which parallelograms are equal in area?

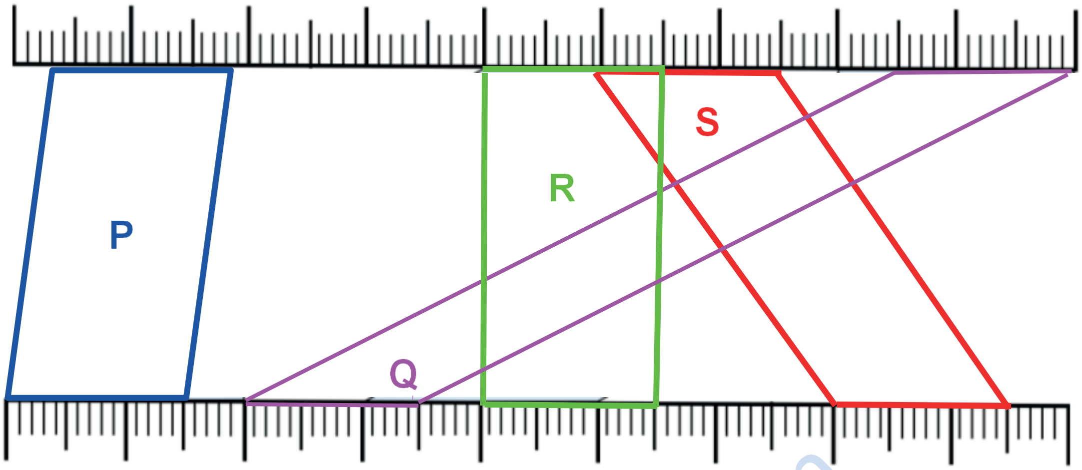


Fig. 5.10, Parallelograms P, Q, R and S

a. P & R	b. Q & S	Answer
c. P, Q & R	d. P, Q, R & S	

Case Study D - Patchwork

Patchwork is the technique of stitching together brightly coloured fabric pieces of geometrical shapes, like squares and triangles, to form a pattern that is pleasing to the eye and looks good on textiles.

One such design has to be stitched for a made-to-order double bedsheet of size 256 cm x 240 cm along with 2 pillow covers of size 48 cm x 96 cm. The design of the rectangular patch is shown in Fig 5.12. These rectangular patches get stitched on one side of the pillow covers and the bedsheet. Fig. 5.14 shows a portion of the fabric (bedsheet/pillow cover) after the patches are stitched together.

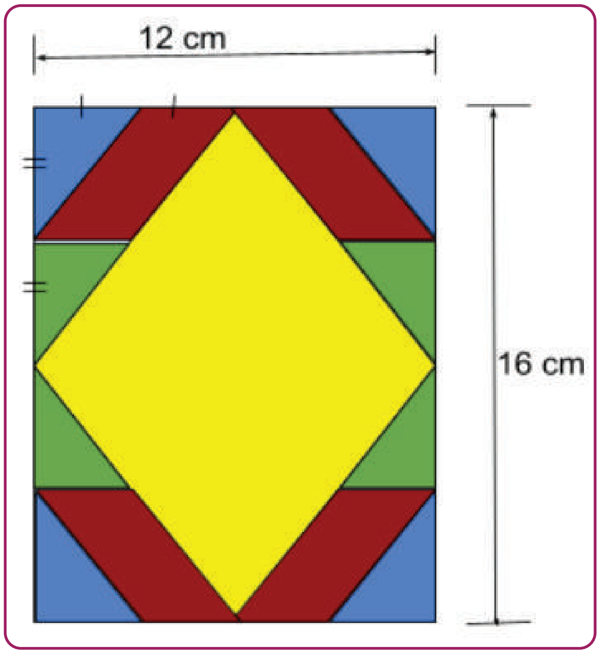


Fig. 5.12, One patchwork

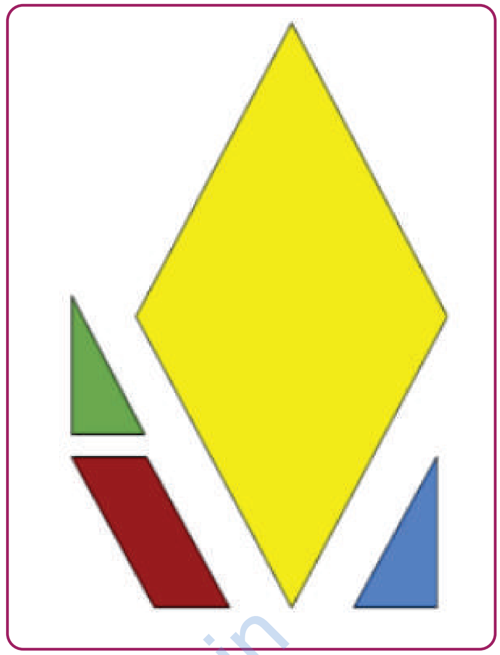


Fig. 5.13, Patchwork pieces

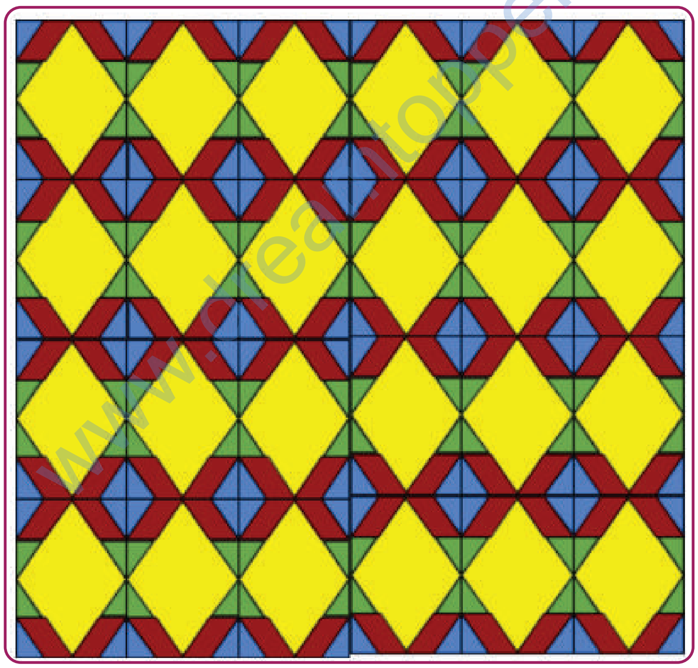


Fig. 5.14, Portion of the bedsheet with patchwork

Question 10

How many such rectangular patches are required to make one bedsheet and 2 pillow covers? Write your answer in the box below.

Answer

Question 11

Calculate the total area of fabric of each colour used to make one bedsheet and two pillow covers. Refer to Fig. 5.13 for the 4 shapes and their colours. Note that the patchwork has 4 pieces of all the shapes except the bigger rhombus at the centre, which is only one. All the pieces of one colour used in a patch are identical to each other. Fill the values of the total area of different fabrics in the blank spaces provided next to each fabric colour below.

i. Total area of maroon parallelogram patches is _______ square metres

ii. Total area of blue triangular patches is _______ square metres

iii. Total area of green triangular patches is _______ square metres

vi. Total area of the yellow rhombus patch is _______ square metres

Question 12

A shopkeeper had to deliver a large number of bedsheets stitched with colourful patches to fulfil an order but there was a sudden shortage of green and blue coloured fabric in the market. He decided to make a small change in the design of a patch as shown in Fig. 5.15.

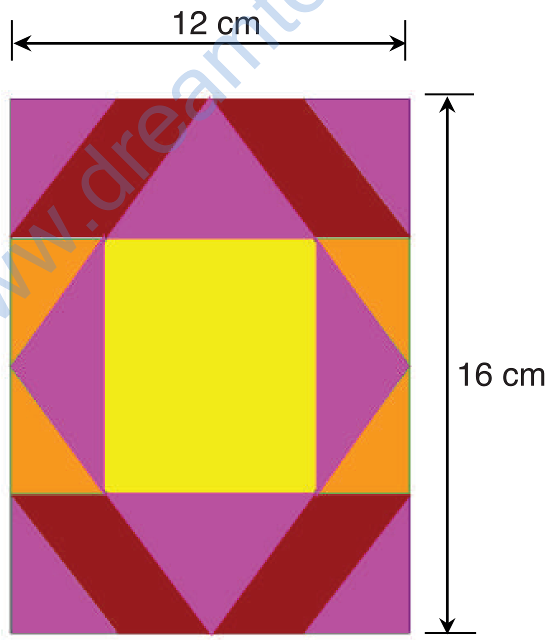
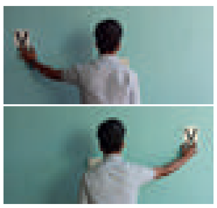


Fig. 5.15, New design of the patchwork

He could get fabrics of all the colours present in the new design. In order to prepare a bedsheet of the same size as described above, will he require the same number of patches as with the earlier design or will he require a different number of patches? Write your answer in the box below.

Answer

Exploration Pathway



Field of vision triangle

Draw the field of vision triangle corresponding to your eyes. Explore the two triangles you get by applying different formulas and theorems related to the triangles.