## Theme 4: Quadrilaterals

## Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Properties of different types of quadrilaterals
- Properties of triangles
- Pythagoras' theorem or the right angle triangle properties
- Area of a trapezium



## Quadrilateral

Think of the different types of 4-sided shapes we see in our daily lives, namely a chess board, kite, Indian flag, tennis or basketball court, cricket pitch and all the other images that come to your mind. Do they all have the exact same shape? What differences do you notice in their shapes? While in some shapes, all sides are equal in length, some have only the opposite sides equal, some have opposite sides parallel and equal to each other while others have only two sides parallel to each other and so on. Such figures are known as quadrilaterals. What is a quadrilateral?


Fig. 4.1, A family tree of quadrilaterals
It is a four sided polygon. It has four sides, four angles and four vertices.

| SI. No. | Type of the quadrilateral | Properties |
| :---: | :---: | :---: |
| 1 | Parallelogram: <br> A parallelogram is a quadrilateral whose pair of opposite sides are each parallel | 1. Opposite sides are parallel and equal <br> 2. Opposite angles are equal <br> 3. Adjacent angles are supplementary <br> 4. Diagonals bisect each other <br> 5. Each diagonal bisects the parallelogram into two congruent triangles <br> 6. Sum of all the interior angles is $360^{\circ}$ |
| 2 | Rhombus: <br> A rhombus is a parallelogram whose sides are all of equal length | 1. All the properties of a parallelogram <br> 2. Diagonals are perpendicular to each other |
| 3 | Rectangle: <br> A rectangle is a parallelogram where one angle is a right angle | 1. All the properties of a parallelogram <br> 2. All angles are $90^{\circ}$ <br> 3. Diagonals are equal |
| 4 | Square: <br> A square is a rectangle whose sides are all of equal length | 1. All the properties of a parallelogram, rhombus and a rectangle. |
| 5 | Kite: <br> A kite is a quadrilateral which has exactly two pairs of equal and consecutive sides | 1. The diagonals are perpendicular to each other <br> 2. One of the diagonals bisect the other <br> 3. The two angles formed where the unequal sides meet are equal to each other |
| 6 | Trapezium: <br> A trapezium is a quadrilateral which has a pair of parallel opposite sides. | 1. The two parallel sides are called bases and the other two non-parallel sides are called legs. <br> 2. The diagonals intersect each other <br> 3. Sum of all the interior angles is $360^{\circ}$ |

Table 4.1, Properties of different types of quadrilaterals.
Think of all the 4-sided images again and try to identify the shape they represent, based on the descriptions above!

## Case Study A - Farmland

Faisal Ali is a perfume manufacturer. He farms various kinds of flowers to make perfumes. He has a piece of land on which he wants to farm jasmine and rose shrubs, each occupying equal area. As the farmland lies between two parallel roads, its shape could be approximated to a parallelogram (ABCD), as shown in Fig. 4.2. Mr. Ali thought of two methods for dividing the farmland equally, i.e. to draw a boundary line either along BD or AC.


Fig. 4.2, Dividing the land $A B C D$ in two different ways

## Question 1

If Mr. Ali chooses the blue shaded area for planting jasmine shrubs, how much area is left for planting the rose shrubs?
a. Less than the blue shaded area
b. More than the blue shaded area
c. Both are equal in area

## Question 2

Mr. Ali wants to fence the two parts separately. Which method of partitioning will result in more expense for him?
a. Method 1 by having the boundary line along BD
b. Method 2 by having the boundary line along AC
c. Both are the same

## Case Study B - Growing a Lawn

Mrs. Radha had a rectangular patch of land in her farmhouse to be used as a lawn. She also wants a part of the land to be used for planting trees and have sitting areas. She tells her workers to mark the midpoints of each side of the rectangular patch and use the area obtained by joining the midpoints for creating the lawn. At two opposite corners of the rectangular patch, she decides to have sitting areas with big shady trees. And at the other two opposite corners, she decided to have ornamental plants. The workers fixed small poles to mark out the midpoints of the rectangular patch. They joined the consecutive midpoints with multicoloured pebbles to fix the boundary of the lawn and sow grass seeds in that area.

## Question 3

Assuming the grass growth is uniform throughout, after a few months, what shape does the grass lawn look like?

| answer |
| :---: | :---: | :---: | :---: |

## Question 4

Also, there is a rectangular patch of land unutilised in the backyard. Mrs. Radha decides to use the patch as a vegetable garden, but her daughter insists on growing some herbs too. Which partitioning described below does not satisfy the criterion of equal area allocation for both the purposes?
a. Partition the land by the imaginary line connecting the midpoint of the longer sides.
b. Partition the land by the imaginary line connecting the midpoint of the shorter sides.
c. Partition the land by the imaginary line connecting one pair of opposite vertices.
d. Partition the land by the imaginary line connecting the midpoint of adjacent sides.

## Case Study C - Exploring Parallelograms

Shubhajit is exploring parallelograms and their area. In order to get a better feel for the area occupied by any polygon, Shubhajit usually draws them on a sheet of paper, cuts them out, wraps/covers them over different objects etc. He does this instead of just memorising the formula.
As a further step of his exploration of parallelograms, he decided to fold the parallelograms along a diagonal and also cut it along that. He drew parallelogram ABCD on a sheet of paper for this purpose (Fig. 4.3).


Fig. 4.3, Parallelogram drawn on a sheet of paper

## Question 5

Which of the following is true about $\triangle A B C$ and $\triangle A D C$ ? Choose all that apply.
a. They are congruent to each other
b. Both of them have a common hypotenuse, which is AC
c. They lie between the same parallels and have equal base lengths
d. The area of each of them is half of the area of the parallelogram $A B C D$

## Question 6

Shubhajit folds the parallelogram ABCD along AC.
i. Will the the triangles ABC and ADC overlap exactly on each other?

## Answer

ii. Which of the following quadrilaterals will result in complete overlap of the triangles when folded along the diagonals? Choose all that apply.

| a. Square | b. Rectangle | Answer |
| :---: | :---: | :---: |
| c. Rhombus | d. Trapezium |  |

## Question 7

Shubhajit cuts the parallelogram along AC and obtains two triangular pieces, T1 and T2. He wants to completely cover T2 with T1. Among the different steps given below, choose the ones which will be useful to Shubhajit and arrange them in the correct order.


Fig. 4.4, Parallelogram is cut into two triangles
Note: While answering this question, assume the distance between the longer sides of the parallelogram to be H 1 and the distance between the shorter sides to be H 2 . Also, in the answer options, the words 'left', 'right', 'top', 'bottom' etc. stand for the respective sides of the paper.
I. Move T 1 rightwards by a distance of H 1
II. Rotate T 1 by $90^{\circ}$, clockwise, about the top-right corner of the parallelogram
III. Move T1 downwards by a distance of H1
IV. Rotate T1 by $180^{\circ}$, clockwise/anti-clockwise, about the top-left corner of the parallelogram
V. Move T1 downward by a distance of H 2
VI. Rotate T1 by $270^{\circ}$ clockwise about the bottom-left corner of the parallelogram
VII. Move T1 upwards by a distance of H2
VIII. Move T1 leftwards by a distance of H1

## Answer

## Case Study D - Project Wheelbarrow

The gardening club students of a school wanted to repair their rusted wheelbarrow. They came up with the idea to fix the wheelbarrow, instead of buying a new one, and also to minimise the wastage to make the project as economical as possible. The students first designed a wheelbarrow, using cardboard pieces, and later drew the same pattern on a sheet of metal and cut it accordingly.


Fig. 4.5, Wheelbarrow
They observed that each side of the wheelbarrow was in the shape of an isosceles trapezium. The shorter parallel side was in the ratio of 1:2 with respect to the longer side. The length of the longer side was 110 cm and the height was 45 cm , as shown in Fig. 4.6. The students calculated the area to buy the sheet of metal required.


Fig. 4.6, Measurement of the wheelbarrow drawn on the sheet metal

## Question 8

i. How much metal sheet is required to make this wheelbarrow, excluding the wastage?
$\square$
A. Capacity will be higher.
B. Weight of the wheelbarrow will be less and hence easier to carry.
C. Emptying would be easier since you will get a downward slope on tilting it.
D. Looks better.

| a. $A$ and $B$ | b. B and C | Answer |
| :---: | :---: | :---: |
| c. A and D | d. Only C |  |

## Question 9

The metal sheet is available in a roll with a width of 145 cm . The shopkeeper will give the required length cut from this sheet, charging ₹315 per square metre.
i. Calculate the total area of the metal sheet they need to purchase.

## Answer

ii. What is the cost of the sheet of metal required for one such wheelbarrow?

## Answer

iii. In order to minimise the wastage, the students created a design for their school gate by utilising the leftover pieces, as shown in Fig. 4.7. How much area of the sheet is required to make the design shown in Fig. 4.7.?


Fig. 4.7, Design created for the school gate

## Case Study E - Harvesting Solar Energy

Solar energy is an eco-friendly and cost-efficient renewable power source. Solar panels convert the energy in solar radiation (sunlight) to electricity. Mrs. Iyengar is planning to install solar panels on the roof of her house. Fig. 4.8 shows the side view of the roof-top.


Fig. 4.8, Side view of the roof-top of Ramesh's house; Image via chiefarchitect.com
Mrs. lyengar decides to mount the solar panels on the roof covering the front portion of her house because it is exposed to the Sun for the longest duration in a day. Fig. 4.9 shows the front view of the roof. In order to generate sufficient electricity she wants to install as many solar panels as possible on this roof.


Fig. 4.9, Solar panel on a roof; Image by Gray Watson via Wikimedia Commons

## Question 10

Fig. 4.10 shows the schematic diagram of the top-view of the area on the roof where solar panels need to be installed:


Fig. 4.10, Top-view of the entire roof (not to scale)
Solar panels with dimensions of $1.65 \mathrm{~m} \times 1 \mathrm{~m}$ need to be placed on the rectangular area EFKI.
i. While mounting the solar panels on the roof, a gap of 20 cm from the edges of the roof is left blank on all sides, in order to ease the maintenance of the panels in the future. If the solar panels are arranged in such a way that the longer edge of the roof is aligned with the shorter edge of the panel, how many panels can be installed?

## Answer

ii. How many panels will fit if the panels are rotated by $90^{\circ}$ and placed, i.e. if the longer edge of the roof is aligned with the longer edge of the panels?

Answer

## Question 11

Mrs. lyengar wants the garage roof to be tiled. Fig. 4.11 shows the schematic diagram of the garage roof.


Fig. 4.11, Top-view of the roof of the garage
Tiles have the dimension of $30 \mathrm{~cm} \times 15 \mathrm{~cm}$ and each box has 100 tiles. How many boxes of tiles should be purchased to cover the entire roof?
While calculating the required number of tiles, add 10\% extra tiles to account for the wastage around the corners and edges.


## Exploration Pathway



Wheelbarrow

A wheelbarrow is an age-old cart, usually propelled manually, that has a small wheel in the front, a load-bearing receptacle in the middle and two handles at the back to hold and push.

In this TACtivity, we make a simple wheelbarrow, using cardboard and bottle caps that can be steered as you explore the geometry of the shapes involved.


Tangram
The Tangram is a puzzle consisting of seven flat polygons, which you can play around with to make a large variety of shapes. The idea of the puzzle is to use all the 7 pieces at once to make specific shapes, like other geometric shapes or even representations of plants and animals!

