# Theme 4: Coordinate Geometry

## Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Interpreting a point on a given axis, distance of a point from the given axis, and collinear points
- Distance Formula
- Section Formula
- Area of triangle using co-ordinates of its vertices

## Sky Ways

Consider two planes A and B travelling on different flight paths, which possibly intersect. Are they in danger of colliding midair? How would one know? The important parameters to be considered with air traffic are location, distance, speed, and separation distances with other planes or common points in the path. The location in space is described using 3 co-ordinates (x,y,z), and co-ordinate geometry helps us to understand these parameters.

In the vicinity of airports — especially large airports, where at peak traffic as many as three aeroplane landing or takeoff operations may occur every minute — the control of aircraft in the air is a challenging but extremely important operation. The graph in Fig. 4.1 shows the leading airports worldwide in 2019, based on aircraft movements.

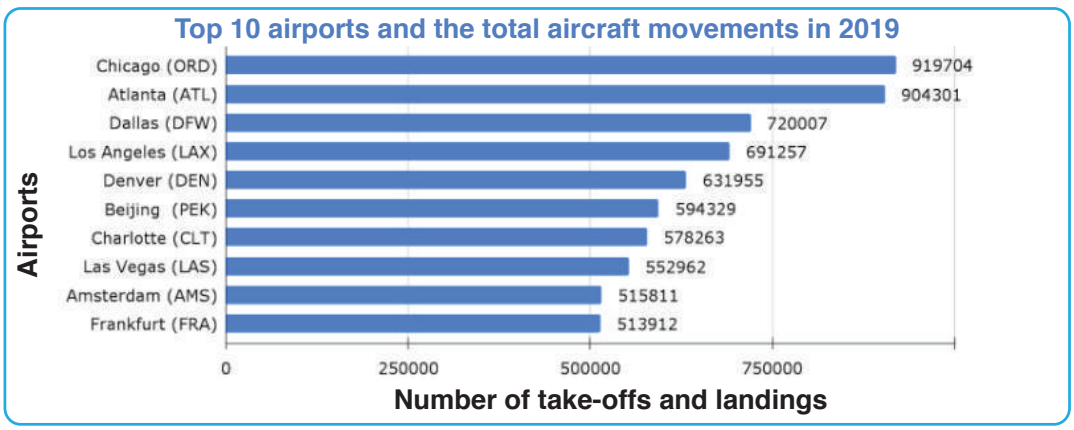


**Fig. 4.1,** Leading airports worldwide in 2019 based on aircraft movements data via https://www.internationalairportreview.com/

The altitude at which planes fly is used to make sure that planes are separated vertically too. Planes are expected to fly within a certain altitude range due to several possible factors, e.g. aircraft type, distance to the destination, weight of the airplane, type of engine, wind speeds, etc. Co-ordinating several arrivals, departures in an airport and so many flights in the sky without collisions requires masterful planning; sometimes the x and y co-ordinate seem to be close by, but the third co-ordinate, z, which is the height, keeps the flight at safe separation.

### Case Study A - Air Traffic Controller

All the air traffic is controlled by the air traffic controller (ATC). ATC co-ordinates the movement of air traffic to make certain that planes stay a safe distance apart. Co-ordinate systems allow them to keep track of an aircraft's position and orientation in space. A controller system must know the location of every aircraft at any instant of time in the sky. Even if an aircraft moves a small distance (up, down, forward), its co-ordinates are updated in the system. The ATC monitors the location of aircraft by radar and communicates with pilots using radio. It's the ATC's job to ensure that planes have their own minimum airspace and that separation rules are followed between aeroplanes to prevent collisions. Consider one such air traffic controller analysing the positions of flights in an airport, using the co-ordinate system, and answer the questions below.



## Question 1

Assume that each unit is 1 km to answer the following questions. The co-ordinates given below are given from a base station of XYZ airport. Find the distance between two aeroplanes, in kilometres, which are at location A and B, if A lies on the y-axis and has an ordinate 1.5, while B lies on the x-axis and has an abscissa 0.5

| | | |
|---|---|---|
| a. $\sqrt{2.5}$ km | b. 2 km | Answer |
| c. $\sqrt{2}$ km | d. 1 km | |

## Question 2

Three parking points are marked with co-ordinates (2a, 4a), (2a+$\sqrt{3}$a, 5a) and (2a, 6a), such that these three points form the vertices of an equilateral triangle. When three planes are at these locations, what will be the distance between any two planes?

| | | |
|---|---|---|
| a. $4a^2$ | b. $\sqrt{(2a)}$ | Answer |
| c. 2a | d. 2a+$\sqrt{3}$ | |

## Question 3

As you have understood, location of a plane, direction in which it is moving and speed are all important to know the possible meeting at a common point on their route. Look at the table given below (table 4.1) in which 5 scenarios are sketched out for 2 planes that are flying at the same altitude (see column IV). In each of the case the speeds of these two planes remains the same or varies (see II); in each of the case, the distance of the planes from the intersection of the routes also remains the same or varies.

The table is followed by four statements a to d. Analyse the cases and select the incorrect statement.

| Possibility # | Plane Speeds | Distance from Intersection of Routes | Pictorial Representation |
|---|---|---|---|
| A | Different | Different |  |
| B | Different | Same |  |
| C | Same | Different |  |
| D | Same | Same |  |
| E | Same route and planes at different distance | |  |

**Table 4.1**

a. Planes may meet at the intersection in Case A and E depending on the speed

b. Planes will meet for sure in Case D

c. Planes will never meet in Case B and C

d. Planes will meet for sure in Case B but not in Case D

Answer

### Case Study B - Treasure Map

Disha loves reading story books. She has started reading a new series of books, which is about a pirate searching for a hidden treasure chest. She has finished reading part 2 of the book in which the location of the key of the treasure chest has been hidden in the map. Part 3 of the book will be delivered in a day or two, which will have the location given. But Disha is too impatient to wait and decides to find out the location of the key by herself. Can you help Disha find the location of the key by answering the questions below?

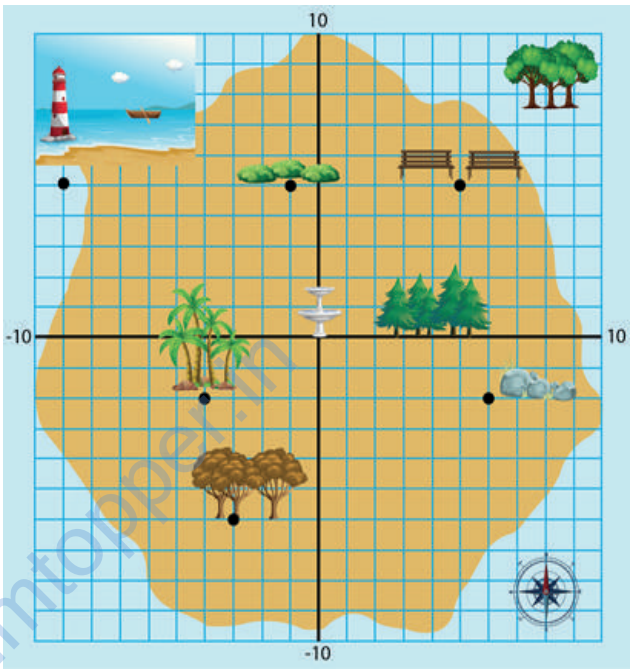


**Fig. 4.2,** Treasure map

**Scale**: 1 unit = 100 m on both the axes, with the fountain at the centre being taken as the origin



## Question 4

What is the location of the coconut trees?

| | |
|---|---|
| a. (-4, -2) | b. (-4, 2) |
| c. (4, -2) | d. (4, 2) |

Answer

## Question 5

Which of the following is the wrong information, as per the map?

a. Stones are at a distance of 600 m from the vertical axis.

b. Stones and coconut trees are at the same distance from the x-axis.

c. We have to walk an equal distance along both the axes to reach the benches.

d. Pine trees have their x co-ordinate as zero.

Answer

## Question 6

The key is buried at a point two-fifths of the distance from the point marked near the bench to the coconut trees. Then the co-ordinates of the key are_____.

| | |
|---|---|
| a. (-1.4, 2.2) | b. (1.4, 2.2) |
| c. (1/5, 1) | d. (7/5, 19/5) |

Answer

## Question 7

The pirate is sitting in his boat, which is 6 units to the west and 7 units to the north from the fountain and is also chasing the key. How far is he from the key?

Answer



### Case Study C - Designing a Board Game

Arjun's sister, Ritika, is learning about collinear points. Arjun designs a game, using a handmade chess board. In this game, Player 1 puts two pins at any two points on the chess board, where the lines intersect. Player 2 has to put the third pin, so that all the pins make a line.

To demonstrate this, Arjun placed two pins on the board and asked Ritika to place the third pin by bringing her eye to the level of the pins. A little while later, Arjun's friend, Ali, shows up. So Arjun challenged Ali to place the third pin without bringing his eye to the level of the pins.

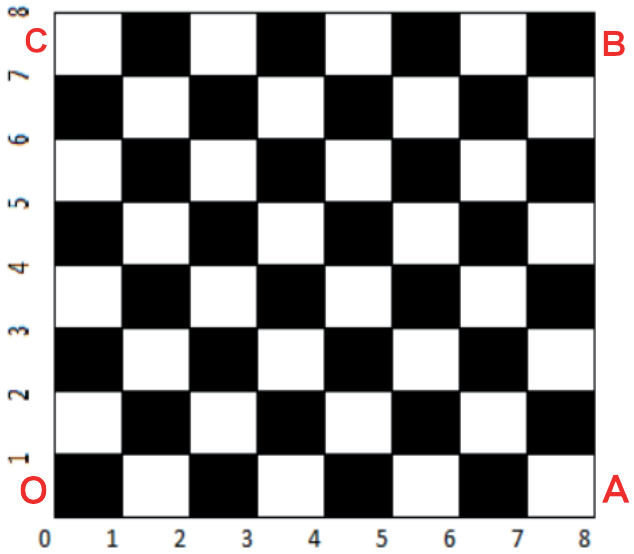


**Fig. 4.3,** Hand made chess board

## Question 8

Arjun puts his pins (P1 and P2) at the following locations: 1 space right and 3 spaces forward from the bottom/near-left corner (the Origin), 3 spaces right and 4 spaces forward from the same corner. Ritika puts her pin (P3) 1 space left and 1 space backward from the top/far-right corner. Are the 3 pins collinear? Justify your answer with mathematical reasoning.

a. Yes, they are collinear

b. No, they are not collinear

Answer

## Question 9

Ritika in turn challenges Arjun to place his pin (P4) on the bottom reference line (OA), such that P2 is at the same distance from P3 and P4. Where should Arjun place his pin, P4, to satisfy this condition? [Remember that the previously placed pins are not disturbed]

| | |
|---|---|
| a. at the origin | b. (6, 0) |
| c. (0, 6) | d. either a or b |

Answer

## Question 10

Ritika challenges Ali to place a pin (P5) so that the top right corner is the midpoint of the distance between pin P3 and P5. What will be the location of pin P5? Fig.4.4 shows the same chess board with extended axes on all four sides by 2 units. The position of P5 may also be one of the newly formed points.

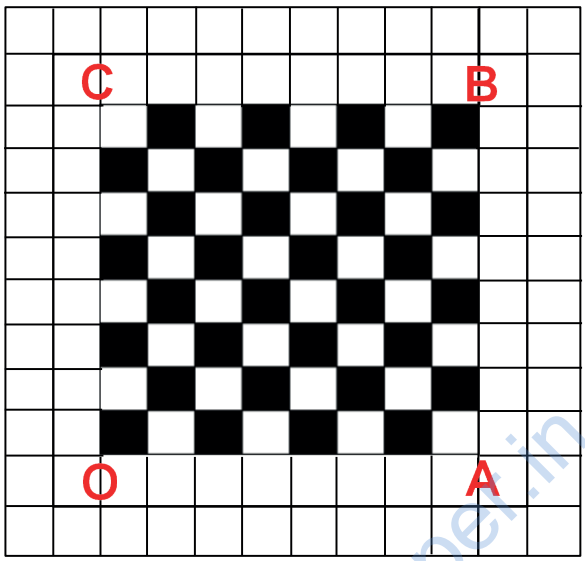


**Fig. 4.4,** Chessboard with extended axes

| | |
|---|---|
| a. (9, 0) | b. (9, 9) |
| c. (1, 1) | d. (8, 8) |

Answer

## Question 11

Ritika noted the location of pins P1, P2, P3, P5 and calculated the distance between these pins. Which of the statements below is correct?

a. P1 to P3 distance = P2 to P5 distance

b. P1 to P3 distance > P2 to P5 distance

c. P1 to P3 distance < P2 to P5 distance

d. P1 to P3 distance = P2 to P5 distance = 6 units

Answer

## Exploration Pathway

Pantograph

Using a few cardboard pieces, screws and ruler templates, make your own pantograph, which can enlarge/shrink a shape you draw. If you treat the pivot of the pantograph as the origin, predict, observe and validate the position of the points on the bigger and the smaller shapes.

Co-ordinates - Chess Board

Co-ordinate geometry, also known as analytic geometry or Cartesian geometry, is a field in mathematics where geometry is studied using a co-ordinate system. It has wide applications in the real world: in physics, engineering, aviation, spaceflight etc. It is a simple system, whereby you have co-ordinate axes, and are able to define any single point in space in terms of these co-ordinates. Here, we use a graph sheet to replicate a chess board to validate the distance and section formulas