# Theme 3: Triangles and Similarity

## Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Basics of Geometry
- Types of Triangles
- Lines and Angles
- Triangle Theorems

## Scientific Modelling



Other than microscopes, telescopes and projectors, how often and where do we see magnification in real life? Imagine an Engineer is constructing a house for one of her clients. She wants to take her client's inputs and decide on the size of the rooms, the shape of the building, etc. Now imagine she constructs some sample rooms and a sample building so that her client can make a selection. Does that sound absurd? Be it constructing a new building or designing a new product or the replacement of a body part or analysing the route from one point to another, modelling the actual product helps in planning, analysing and evaluating before making the product or instead of physically visiting the route. One type of model that we work with are called maps. You may draw the map of your hand by placing your palm on a piece of paper. This map can be carried around by someone to say buy hand gloves, which fit your hand, without taking you or your hand around. This would be a map, which is as big as your hand but you may as well draw maps which are bigger/smaller than the actual object/area they are mapping, like the example of a building or a route map. In such cases, we use a scale to magnify or minify and still represent the physical object/area and communicate the information effectively.

### Case Study A - Scale Factor

A company acquires 6 hectares of agricultural land for cultivating 5 types of cash crops. The shape of the land is a perfect rectangle with one side being 300 m long. To plan the distribution of the 5 types of crops on this land, a map has to be prepared. Imagine that the plan is prepared on an A4 paper.

**Note:**

1. Consider the size of an A4 paper as 30 cm x 21 cm.
2. The goal is to utilise as much part of the A4 paper as possible so as to get the maximum clarity in the map.
3. 1 Hectare is an area occupied by a square with the side equal to 100 m.
4. A scale for a map is specified as a ratio, something like 1:10. The first number here (1 in this case) indicates the length in the map and the second (10 in this case) indicates the length in the physical world. For this example, a line/curve which is 1 cm long on the map will be 10 cm long in the physical world.

## Question 1

i. Which of the following scales should be used? Select the best option out of those given below:

| | | |
|---|---|---|
| a. 1:120 | b. 1:10 | Answer |
| c. 1:1200 | d. 10:1 | |

ii. What is the ratio of the area of the actual land bought by the company to the area of its map depicted on an A4 sheet? Assume that the map of the land is drawn in such a way that the maximum possible area of the A4 sheet is utilized to draw the map.

| | | |
|---|---|---|
| a. 1:100 | b. 1:1000 | Answer |
| c. 1:10000 | d. 1:1000000 | |



## Question 2

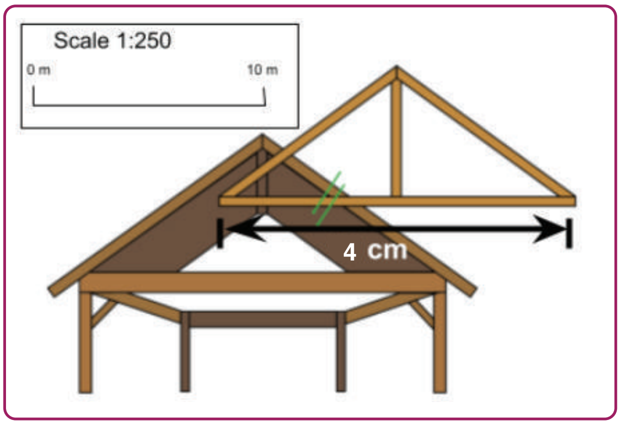



**Fig. 3.1,** House with sloping roof

Fig. 3.1 shows a model of a house, which has a sloping roof. It has a triangular shaped frame below the slope. Find the actual width of the roof, in metres. Write your answer in the space provided.

Answer

## Question 3

The triangular section of the wall right under a roof is called a 'gable' as shown in Fig.3.2.


**Fig. 3.2,** A building with a gable; Image by P Flannagan via Wikimedia Commons



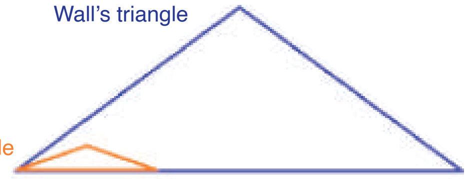
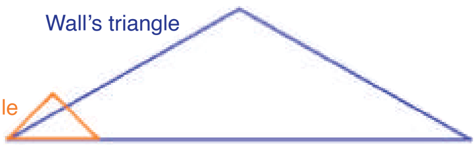
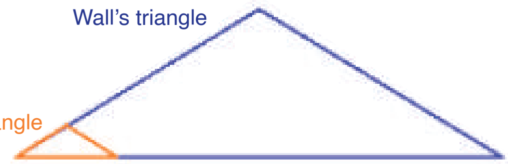
i. If you compare the gable of the actual house mentioned in Question 2, with its model in the map, how do you think the two triangles are related?

a. These two triangles are congruent
b. They both are similar
c. Neither similar nor congruent

Answer

ii. Predict what you will see if you cut out the triangle from the map and stick it on any one corner of the actual triangular wall. Refer to the picture next to each option (Fig. 3.3-3.5) to visualise each option.

Note that we do not rotate the map's triangle before sticking, which means the apex of the triangle would always be on the top. In our case, the apex is the vertex of the triangle representing the corner where the two roofs connect to each other.

| | |
|---|---|
| a. The map's triangle will not fully cover the corner of the wall where it is being pasted. |   **Fig. 3.3,** Answer option a |
| b. Some part of the map's triangle will go outside the wall. |   **Fig. 3.4,** Answer option b |
| c. Map's triangle will exactly overlap with the wall's corner. |   **Fig. 3.5,** Answer option c |

| Answer |
|---|
| |

## Pinhole Camera

A pinhole camera is a simple device where light enters a box through a small aperture and an image is projected on a screen without the need for a lens. You can easily make a pinhole camera using a chart paper, butter paper and tape to see lovely inverted images. This camera doesn't have a lens but just a tiny aperture. When light passes through this tiny aperture, it projects an image of the object being observed through the aperture. You can change the size of the image by varying the relative distance between the aperture (pinhole) and the screen.

**Fig.3.6,** Using the pinhole camera and sample images as seen through the camera

Implement a pinhole camera yourself and experience the change in the size of the image (if any) as you move the outer box of the camera. To access a quick guide for this activity please visit ThinkTac's website or YouTube channel.

### Case Study B - Light Through Small Apertures

One of the standard ways of drawing diagrams to understand the behaviour of light is to draw rays coming from the topmost point of an object and those coming from the bottom.

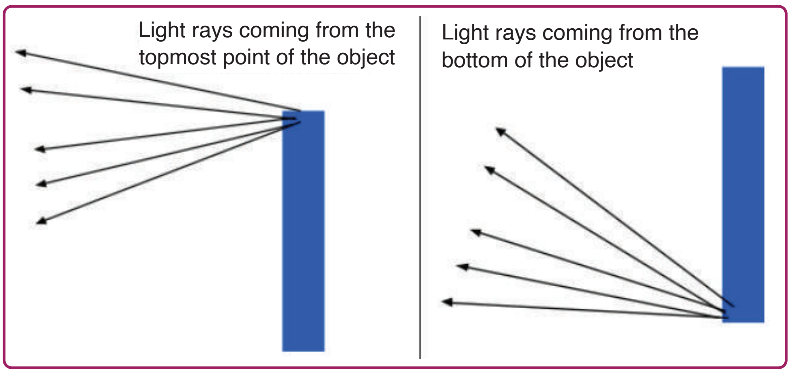



**Fig.3.7,** Rays of light coming from an object represented as straight lines

Now, imagine placing an opaque sheet in front of the object, which will stop light from passing through and then make a hole in the sheet which is so small that only one ray of light can pass through.

Here, if we include the object and image, then we see two triangles on the two sides of the opaque sheet. Let us look at this type of camera through the lens of the triangles formed with the light rays.

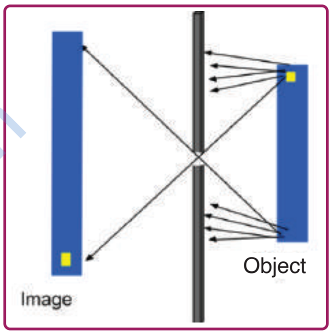



**Fig.3.8,** Rays of light passing through a hole in an opaque sheet forming two triangles



## Question 4

In the pinhole camera diagram (Fig. 3.9), H is the actual size of the object, D is the distance from the pinhole camera aperture to the object, h is the size of the object's image in the pinhole camera, and d is the distance from the pinhole camera aperture to the image projection surface.

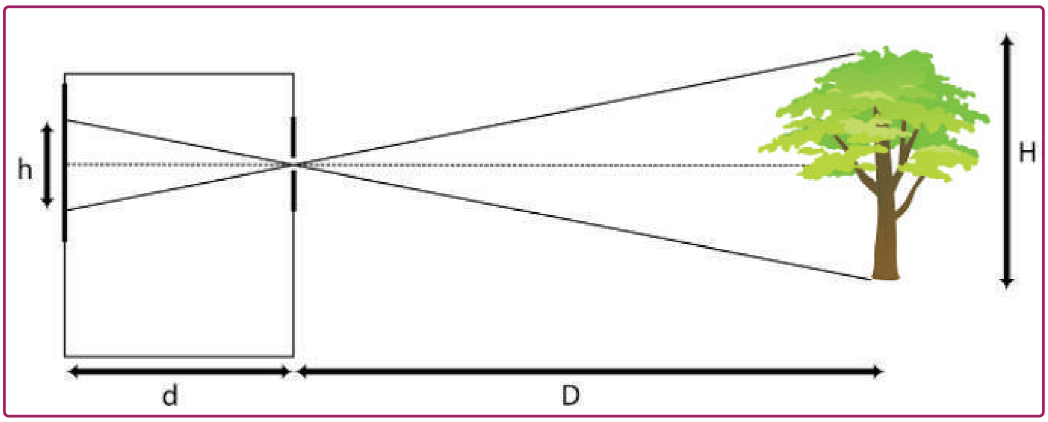



**Fig.3.9,** Pinhole camera schematic

Which of the following equations correctly describes the relationship between image height h, projection distance d, object distance D, and object height H?

| | |
|---|---|
| a. H x h = d x D | b. H x d = h x D |
| c. H x D = h x d | d. $\frac{h}{D} = \frac{H}{d}$ |

| Answer |
|---|
| |

## Question 5

Suppose the distance, d, between the screen and the pinhole is doubled from 15 cm to 30 cm. How will it affect the image of an object kept at a distance of 20 cm in front of the pinhole?

a. Image size doubles

b. Image size reduces by a factor of 2

c. No change of the image size as long as the object is the same

d. Image size will be two-third of the original size

| Answer |
|---|
| |

## Question 6

Magnification tells us if the image is of the same size, smaller or larger than the object. It is represented by the unitless quantity, m, and it is defined as the ratio of the image height to the object height. If the image-to-object distance is double of the pinhole-to-object distance, then m will be

| | |
|---|---|
| a. Less than 1 | b. More than 1 |
| c. Equal to 1 | d. Depends on the hole size |

| Answer |
|---|
| |



### Case Study C - Robotics Prelims

Four grade 10 students from Pragati school participated in the preliminary round of a robotics competition. The model diagram (Fig. 3.10) was shared with all participants a week before the event, which included the details of the paths and angles in the path of the robot's movement.

The robot placed at point A is tasked to pick the can from anywhere in its path from A to C and put it in the bin located at B. Each of the four students came up with different models, using different design components. In the registration form, each one of them updated the battery life and speed at which their robot can move. Table 3.1 shows these details updated by the students.

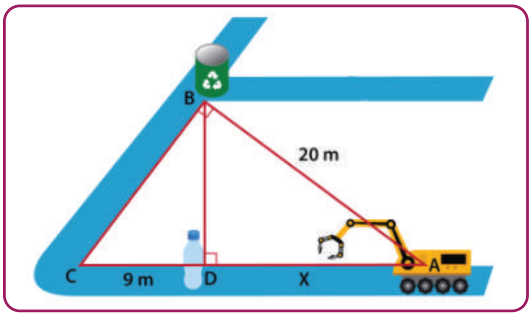



**Fig. 3.10,** Route map for the robot

| Student name | Battery life (hours) | Robot's speed (m/s) |
|---|---|---|
| Sanjay | 5 | 0.5 |
| Renu | 3 | 1.0 |
| Joseph | 4 | 0.5 |
| Syed | 6 | 0.2 |

**Table 3.1,** Battery life and speed of robot designed by the four students

## Question 7

What is the maximum and the minimum distance the robot has to cover from its starting position A to pick the bottle anywhere in the path A to C and drop it at the bin positioned at B? Write your answer in the space provided below. Note: The bottle cannot be placed at A and the robot cannot travel backward.

Answer



## Question 8

Robots are kept switched ON all the time from the start of the competition at 10 a.m. For each participant, the competition starts with a quick interview with the judges. It is followed by a demo, where the robot is expected to start from point A and come back after finishing the task in less than 3 minutes. After this, to test the sensor design, the robots are made to go around the path multiple times by slightly altering the position of the can, without changing the position of the bin. With multiple schools participating in the competition, Pragati School students get a chance to demonstrate only at 12:30 p.m. Who all (if any) will get disqualified because of their robot not coming back within the 3 minute time limit? Write your answer in the space provided. If nobody misses the chance, write "None". Justify your answer.

Answer

## Case Study D - Three Triangle Logo

The logo of a company comprised of a series of 3 similar triangles. Fig. 3.11 shows the 3 triangles. Note that this figure has not been drawn to scale.

All the angles at A are equal. Also, the ratio of some sides of the triangles are equal:

$$\frac{AB}{AG} = \frac{AG}{AF} = \frac{AF}{AH} = \frac{3}{5}$$

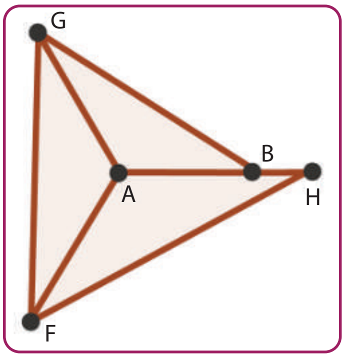



**Fig. 3.11,** Spiral logo of the company with 3 similar triangles

## Question 9

Unlike what is shown in Fig. 3.11, the company wanted to redesign their logo to showcase their 70% growth. The company's graphic designer provided a constraint that the logo must fit within a circle of radius 7 cm. Do you think this constraint can be satisfied if the design team chooses to have the length of the side $\overline{AB}$ of the triangles as 2.7 cm?

a. Yes, the constraint of fitting the logo within a circle of radius 7 cm can be satisfied

b. Yes, actually the logo can be fit inside the circle of radius 6 cm

c. No, it is not possible to fit the logo inside the circle of radius 7 cm

d. No, it can't be fit inside of the circle of radius 7 cm, but it can be fit inside of the circle of radius 8 cm

| Answer |
|---|
| |



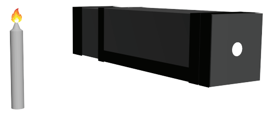
## Question 10

Given that the logo has three similar triangles, each will have an angle equal to another angle in the other triangle. Table 3.2 has a list of the 3 angles in $\triangle ABG$ (in Fig. 3.11). Identify two more triangles, which are similar to $\triangle ABG$, and write the name of the corresponding angles, which are equal to the respective angle in $\triangle ABG$.

| Triangle 1 | Triangle 2 | Triangle 3 |
|---|---|---|
| $\triangle ABG$ | | |
| $\angle ABG$ | | |
| $\angle AGB$ | | |
| $\angle BAG$ | | |

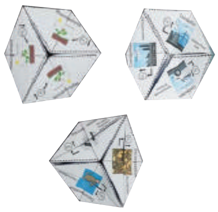
**Table 3.2,** Table placing the equal angles of the 3 similar triangles in a single row

## Exploration Pathway



Pinhole camera

A pinhole camera is a simple camera without a lens but with a tiny aperture. When light passes through this tiny aperture, it projects an upside down image of the object being observed through the aperture. In this TACtivity, we will make a pinhole camera using a chart paper, butter paper, and tape to see lovely inverted images.



Flexagon

1) Make a 3D flexagon from a rectangle with sides in the ratio of 1:2. Observe the sizes of the 6 triangles in the final flexagon. Are they equilateral/isosceles/scalene?

2) Change the ratio of the rectangle to 1:4 and predict the type & angles of the 6 triangles. Make and validate your prediction.