Theme 3: Linear Equations in Two Variables

Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Linear equation in two variables
- Graph of linear equation in two variables
- Graphs of $x = a$ and $y = b$
- Equation of a straight line on the graph

Mathematics Facilitates Decision Making

What would you like to eat today for breakfast: parathas, idlis, upma or dosa? Which subject should you study first - science or mathematics? Should I play football or cricket today? If we pay attention, there are hundreds of decisions which we take on a daily basis. Ever wondered what process our mind goes through while making these decisions?

Decision making is one of the highest order thinking abilities humans are gifted with. While we make many decisions without putting a numerical value for each option, it is very relieving for our mind when we manage to put a numerical value. Imagine the effort required to explain and convince people if we couldn't use numbers while selecting/rejecting someone in a competition or choosing the right material in a design based on its cost and properties.

The fun starts when we are able to replace the numbers by names. This approach gives us a way to capture patterns and hence generalise behaviours of a system without knowing the actual numbers. Consider the previous example of judging a competition. It is common to have more than one parameter as the basis for selection, but all need not be equally important. A singing competition may opt to give 30% weightage to synchronising with the instruments and only 10% weightage to voice quality while giving 20% weightage to interaction with the audience while singing. A smart way to communicate the level of importance to the judges would be to replace each parameter by a name. Something like Final score = $0.3x + 0.1y + 0.2z$....It would be a fascinating exercise to replace numbers by names in your daily life, especially when it reveals patterns and interconnections. You may start with simple quantities, like the number of hours you sleep, the amount of water you drink per day and correlating them to the relevant quantities, like body weight (monthly average) and energy level. In this chapter, we will attempt using this approach in a couple of situations from daily life.

Case Study A - Sweet Tooth

Japneet went to the market to buy his favourite sweets. He had ₹ 380/- in his pocket and he was allowed to spend all the money if he wished. He decided to buy Burfis whose price was ₹ 400/- per kg and Besan Ladoos whose price was ₹ 300/- per kg. Let x represent the quantity of Burfis that Japneet could buy, in kgs, and y represent the quantity of Besan Ladoos that Japneet could buy, in kgs.

Fig. 3.1, Different sweets arranged in the sweet shop;
Image by Mark Kobayashi - Hillary via Wikimedia.org

Question 1

If Japneet spent all the money he had, then which of the following is the correct representation of the amount of money spent by Japneet?

		Answer
a. $400x + 300y + 380 = 0$	b. $400x + 300y - 380 = 0$	
c. $400x - 300y + 380 = 0$	d. $400x - 300y - 380 = 0$	

Question 2

Assuming that you have chosen the correct answer for the above question, if you plot that in the graph, then what does the x-intercept and the y-intercept represent?
(In the last column of the table 3.1 below, enter 'Yes' if the mentioned statement is correct, 'No' otherwise)

The x-intercept means the distance between the origin and the point where a line in the graph intersects the x-axis, or abscissa.

The y-intercept means the distance between the origin and the point where the line intersects the y-axis, or ordinate.

Statement	Answer
x-intercept represents the quantity of 'Burfi' Japneet will get for ₹ 400	
y-intercept represents the quantity of 'Burfi' Japneet will get for ₹ 300	
x-intercept represents the quantity of 'Burfi' Japneet will get for ₹ 380	
y-intercept represents the quantity of 'Besan Ladoo' Japneet will get for ₹ 380	

Table 3.1

Question 3

Japneet visited the same shop with the same amount of money after 2 weeks. But this time the price of Burfis had increased by 50 rupees per kg. If you rewrite the equation for the quantity of sweets that Japneet could buy for the money he had, then which of the following is correct?

a. Coefficients of both x and y will be different

b. Coefficient of x will be different but coefficient of y will be same

c. Coefficient of y will be different but coefficient of x will be same

d. There will be no change in the coefficients of x and y

Answer

Case Study B - The Tortoise and the Rabbit

Once a tortoise and a rabbit ran a race which was won by the rabbit because the rabbit was much faster than the tortoise. They decided to race again. The destination was 500 m away from the starting point. As the rabbit had won in the previous race by a large margin this time the tortoise was given a concession of 200 m. This means though both of them start at the same time, the rabbit will have to cover 500 m and the tortoise has to cover only 300 m. When the race began, the tortoise had moved with a constant speed of 15 metres per minute, till she reached the destination. The rabbit had covered the initial 300 metres in 0.5 minutes and then fell asleep for 30 minutes. After waking up, the rabbit again continued with the same speed that he had before falling asleep.

Question 4

Consider a graph with distance covered from the common starting point in metres along the y-axis and time taken in minutes along the x-axis. Then which of the following equations will accurately represent the distance of the tortoise from the common starting point on the graph?

a. $x = \dfrac{y + 200}{15}$

b. $x = 15y + 200$

c. $x = \dfrac{y - 200}{15}$

d. $y = 15x - 200$

Answer

Question 5

Which of the following graphs (I to IV) accurately represents the distance travelled by the tortoise with respect to time?

I

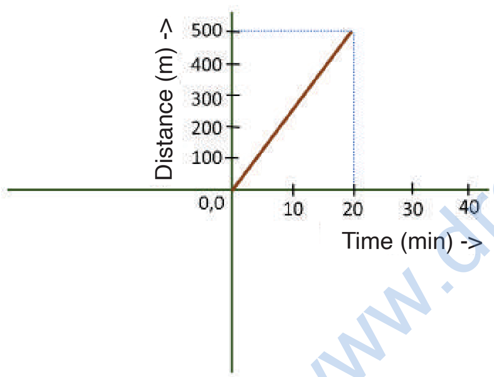


Fig. 3.2

II

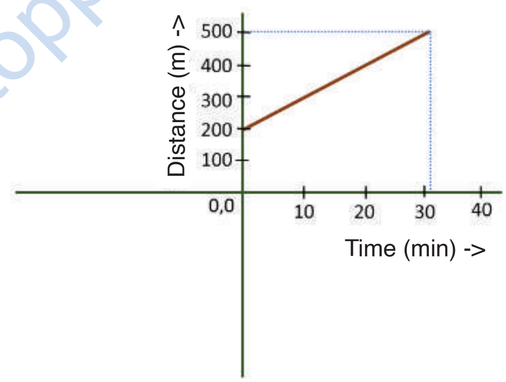


Fig. 3.3

III

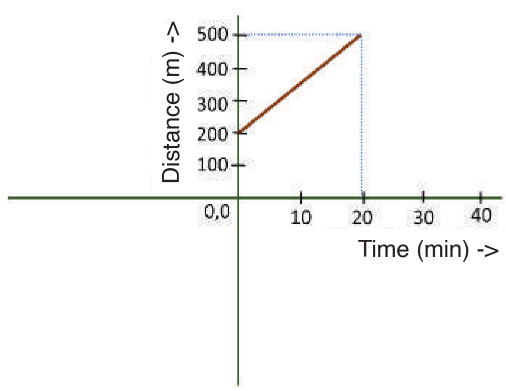


Fig. 3.4

IV

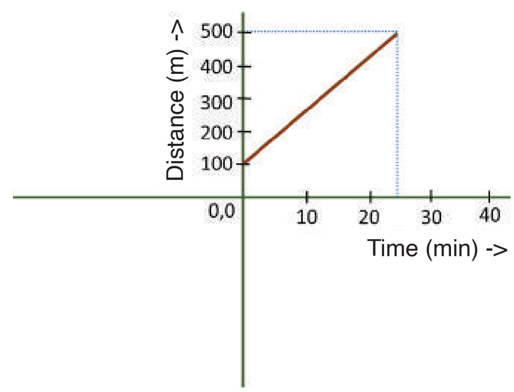


Fig. 3.5

Answer

Question 6

The graph in Fig. 3.6 represents the distance covered by the rabbit with respect to time. As the rabbit had slept for some time during the race, that portion of the graph is denoted by “sleeping”.

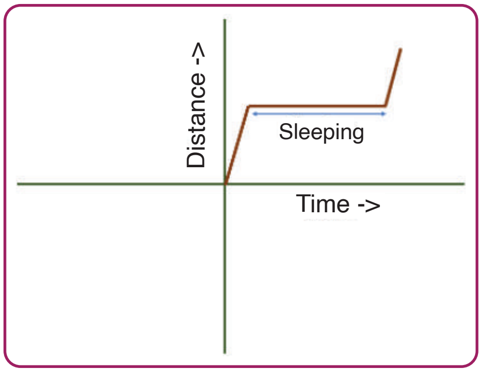


Fig. 3.6, Distance covered by the rabbit with respect to time

Which of the following equations represent the “sleeping” part of the graph?

a. x = 0.5	b. x = 20
c. y = 200	d. y = 300

Answer

Question 7

i. Who is the winner of the race?

a. Rabbit
b. Tortoise
c. Both reach at the same time
d. Can’t conclude with the given information

Answer

Case Study C - Electricity Bill

The following table represents the electricity tariff of Bengaluru.

#	Number of units	Price
01	0 - 30	₹ 130/- (minimum)
02	31 - 50	₹ 4.5 per unit
03	51 - 100	₹ 7 per unit
04	101 and above	₹ 10 per unit

Table 3.2, Electricity bill tariff of a city for a month

Question 8

If we plot the price along the y-axis and number of units along the x-axis, which of the following graphs (I - IV) represents the variation of price with number of units?

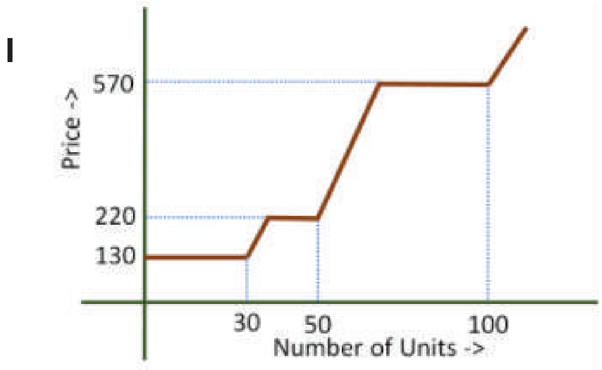


Fig. 3.7

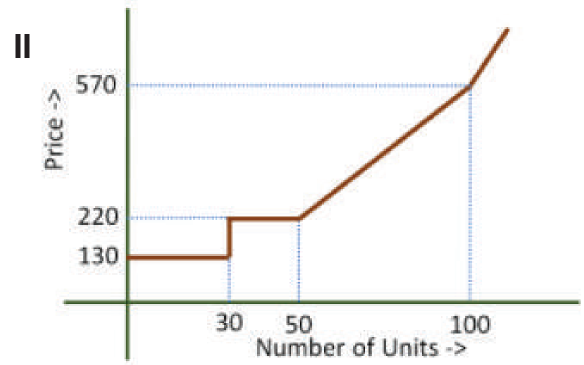


Fig. 3.8

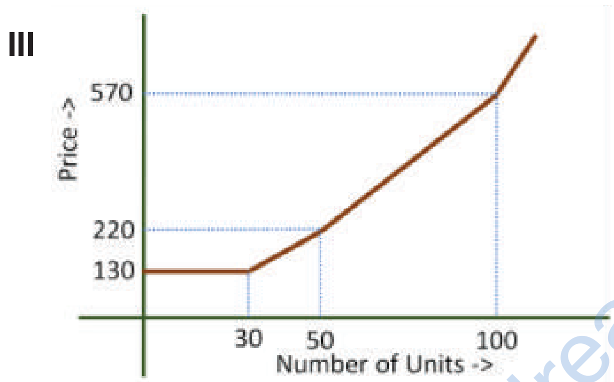


Fig. 3.9

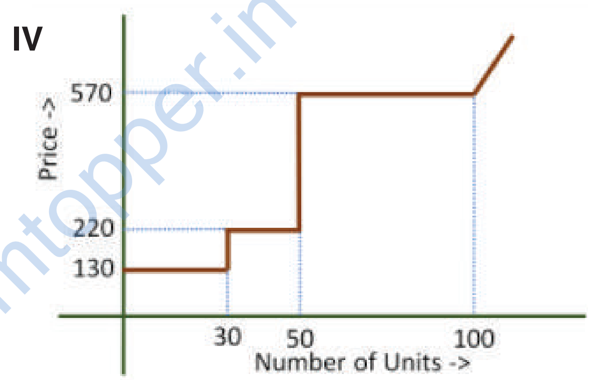


Fig. 3.10

Answer

Question 9

In the Price vs Number of Units graph (right answer for Question 8), which of the following equations represents the part of the graph corresponding to the initial 30 units?

a. $x = 30$	b. $y = 130$
c. $x = 130$	d. $y = 210$

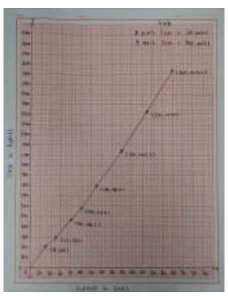
Answer

Question 10

Write down an expression, which represents the relationship between the y co-ordinate and the x co-ordinate of the graph, beyond 100 units.

Answer

Exploration Pathway


Standard Curve for Utility Bill

Plot a standard curve on a graph sheet corresponding to the given electricity bill tariff so that by noting the corresponding y co-ordinate for the number of units consumed in a month one can figure out the electricity bill for that month.