Theme 2: Polynomials

Prior Knowledge

It is recommended that you revise the following topics before youstart working on these questions.

- Degree and terms of a polynomial.
- Determining the value of a polynomial for the given value of variables.
- Zero of a polynomial.
- Algebraic identities.

Applications of Polynomials

In our world where we are frequently required to measure things, such as time, length, temperature etc., there are often occasions where we cannot perform our measurements directly. For example, if one wishes to measure the height of a mountain or the depth of an ocean, one requires the help of mathematical modelling. If one wishes to measure the volume of water in a lake, then depending on its shape we need the depth, length and breadth or radius of the lake.

The expressions - ($l \times b \times h$), (πr^2) and ($l \times b$) are examples of polynomials that are used to model mathematical objects. However, polynomials are not restricted to describing area and volumes only. On the contrary, they are used to describe a wide variety of phenomena - the path of a projectile, the dynamics of a spring, the relationship between profit and price, the list goes on…

Case Study A - Terracotta Pots

Riddhi got an order to paint 100 terracotta pots. She went to get a rough estimate of the area of each pot so that she could buy the necessary quantity of paint. She found that each pot was of similar shape and size and all of them were open at the top. The height of the pots was equal to her height, which is 170 cm, and the base was square in shape. But Riddhi was unable to measure the length of the side of the square shaped base. You may ignore the thickness of the pot in your calculations.

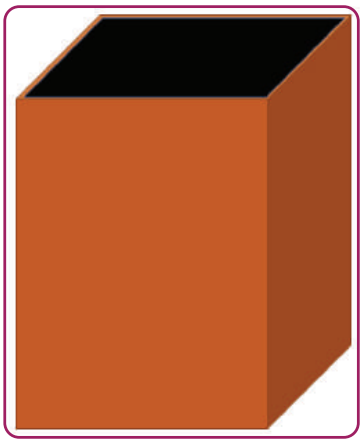


Fig. 2.1, A metal container with its base in the shape of a square

Question 1

Riddhi was supposed to paint both sides of the pot, i.e. inside and outside. If we denote the unknown side of the square shaped base of the pot by x and the total area of a single pot to be painted (in m^2) by A. Which of the following equations best represents the relation between A and expression for total area in terms of x?

a. $x^2 + 6.8x = A$	b. $x^2 + 13.6x = A$
c. $2x^2 + 13.6x = A$	d. $5x^2 = A$

Answer

Question 2

Assume that the volume of each pot that Riddhi was supposed to paint, is denoted by V. If we write down an expression for the volume in terms of the unknown side x as f(x), then what is the degree of f(x)?

Answer

Question 3

Riddhi gets another order of painting 100 more pots which are cubical in shape. But the length of each side of these pots is also equal to x. Consider the following expression,

$$g(x) = ax^4 + bx^3 + cx^2 + dx$$

If the above polynomial expression g(x) should represent the total volume of all the 200 pots, then which of the following must be true?

a. a = 100 and b = 0	b. b = 100 and c = 100	Answer
c. a = 0 and d = 0	d. c = 0 and d = 100	

Question 4

If you rewrite the above polynomial g(x) after eliminating the terms with zero coefficients, what will be the name of the new polynomial on the basis of its degree?

Answer

Case Study B - The Tiled Floors

A huge hall in Tsering's house recently got renovated. Tsering noticed that each tile was square shaped and there were exactly 60 tiles along the length and 30 tiles along the breadth by which he could calculate the total number of tiles on the floor. Additionally, a strip of tiles was fixed to the wall on all the four sides. The tiles fixed to the wall were of the same width but their height was 20 cm (Fig. 2.2).

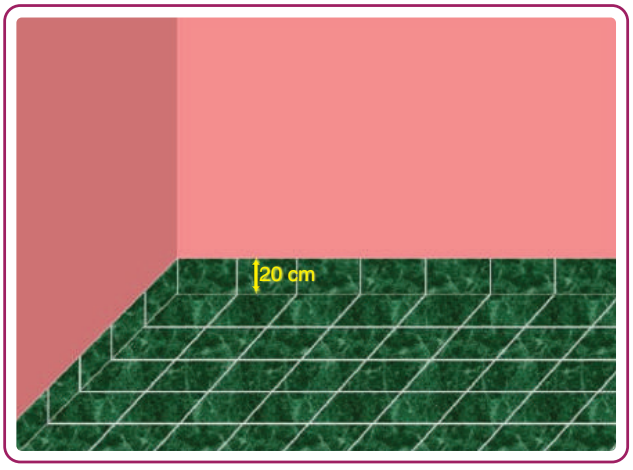


Fig. 2.2, A portion of the floor with tiles in Tsering's home, we can see the wall tiles of 20 cm attached to the wall

Question 5

Tsering got the information that the total tiled area including the strips of tiles on the walls is 468 square metres. If he assumes the side of a tile to be x metres, which of the following is the correct option of total area of the tiles installed in terms of x ?

a. $1800x^2 + 360x = 468$	b. $180x^2 + 360x = 468$	Answer
c. $1800x^2 + 36x = 468$	d. $180x^2 + 360x = 46$	

Question 6

Later on, Tsering wants to calculate the dimensions of the tiles, being laid in another room. The area of the room is given by the polynomial

$$f(x) = 50x^2 + x - 13 = 0$$

Find the correct value of the side of a tile.

a. 0.3 m	b. 0.25 m	Answer
c. 0.4 m	d. 0.5 m	

Question 7

If the area of tiled floor (not including the ones fixed on the walls) is 450 square metres, and if you rewrite the polynomial expression representing the total area of installed tiles as $f(x)$ (Question 6), then what would be the number of terms in the polynomial, so formed?

Answer

Case Study C - Algebraic Identities using 3-D Shapes

Chaitra has a few 3D wooden geometrical shapes. She writes the volume of each block in algebraic terms as shown in Fig. 2.3.

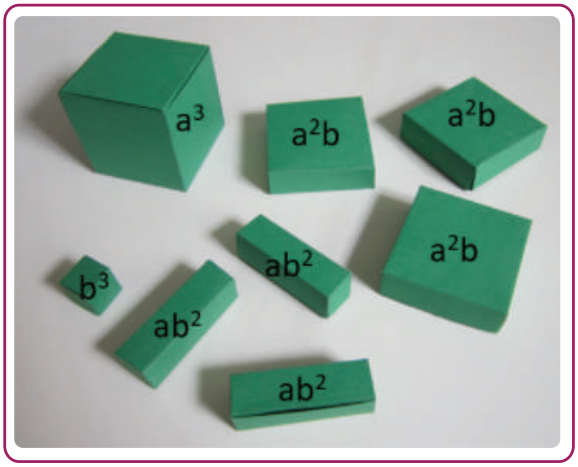


Fig. 2.3, Some 3-D models with algebraic expression

She arranges these eight wooden blocks to form a cube (Fig. 2.4).

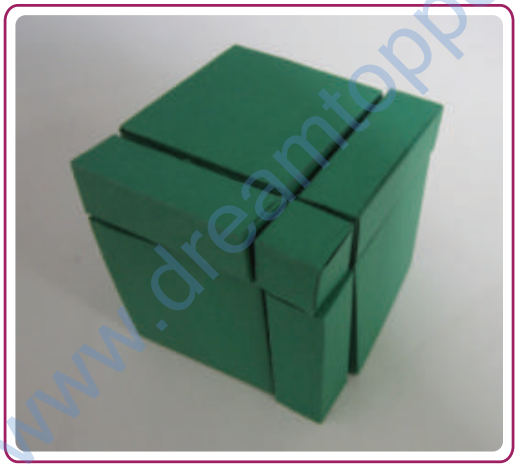


Fig. 2.4, All the models in Fig. 2.3 arranged together to make a cube

Fig. 2.5 shows some of the intermediate steps in arriving at the previous image.

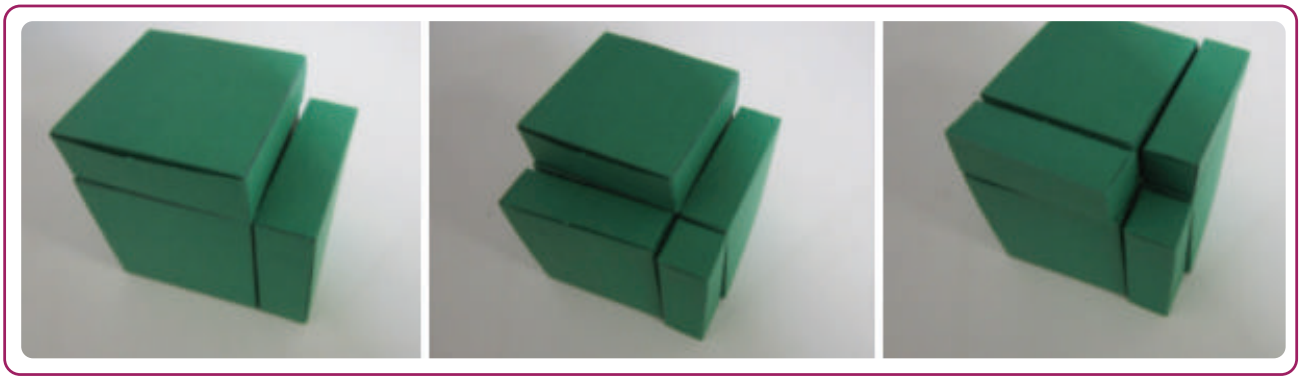


Fig. 2.5, Some intermediate steps

Question 8

Which is the correct expression that represents the volume of the cube that Chaitra made?

a. $2a^3 + b^3 + 3a^2b + 2ab^2$	b. $2a^3 + 2b^3 + 3a^2b + 3ab^2$	Answer
c. $a^3 + b^3 + 3a^2b + 3ab^2$	d. $a^3 + 2b^3 + a^2b + 3ab^2$	

Question 9

Which of the following options represents the sides of the cube so formed?

a. a, b, (a+b)	b. (a+b), (a+b), (a+b)	Answer
c. a, a, (a+b)	d. b, b, (a+b)	

Question 10

If the length b is 2 cm, and volume of the cube is 640 cm3, choose the correct polynomial to represent the situation.

a. $a^3 + 6a^2 + 12a = 640$	b. $a^3 + b^3 + 3a^2b + 3ab^2 = 640$	Answer
c. $a^3 + 12a^2 + 24a = 640$	d. $a^3 + 6a^2 + 12a = 632$	

Case Study D - Factorisation

Alex and Prajwal are exploring polynomials using algebra tiles. They have plenty of the following tiles in their collection along with a frame.

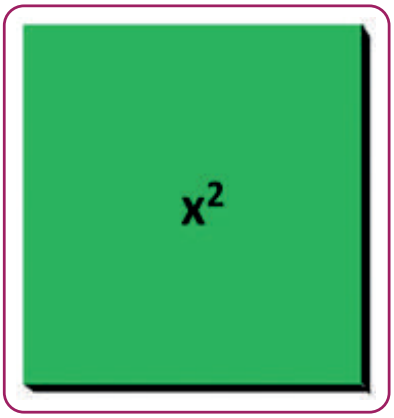


Fig. 2.6, Green coloured big square tile which represents x^2

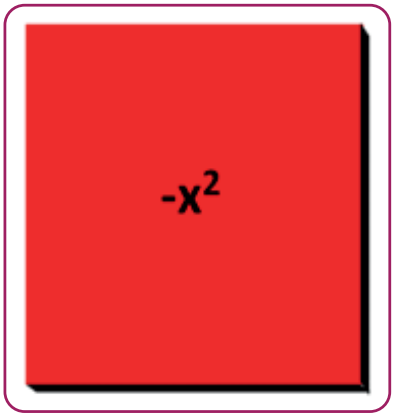


Fig. 2.7, Red coloured big square tile which represents $-x^2$

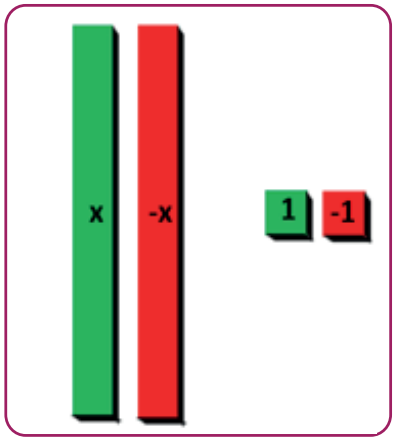


Fig. 2.8, Rectangular green tiles which represent x and red tiles which represent -x. The picture also shows green coloured small square tiles which represent +1 and red tiles which represent -1.

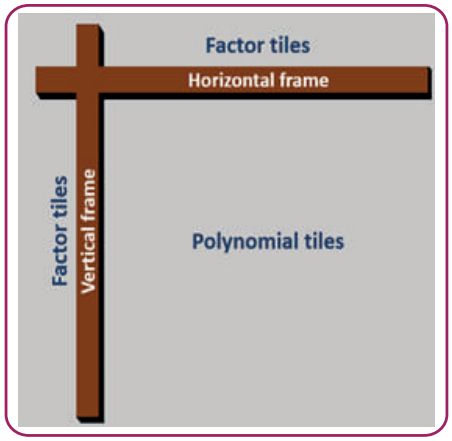


Fig. 2.9, A frame inside which the tiles corresponding to a polynomial need to be arranged so that the tiles representing the factors will be placed outside the edges of two frames.

As an initial attempt, they have factorised $x^2 - x - 6$. The arrangement of the tiles are shown below.

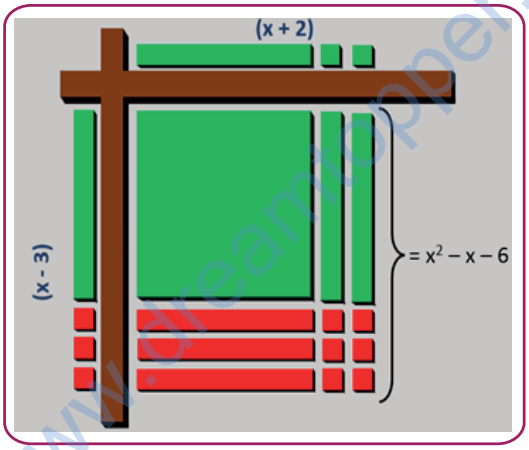


Fig. 2.10, factorising $x^2 - x - 6$ using algebra tiles.

Question 11

Prajwal asked Alex if he can arrange the tiles to factorise $x^2 + 9x + 14$ without changing the factor tiles arranged in the horizontal frame in Fig. 2.10. Alex wants to answer Prajwal's question by making use of the factor theorem instead of arranging the polynomial tiles inside the frame.

i. Can you describe the method by which Alex can answer Prajwal's question using the factor theorem?

Answer

ii. What should Alex's answer to Prajwal's question be? If the answer is 'yes', then write down the factor that will appear on the vertical frame.

Answer

Question 12

Alex challenged Prajwal to find out the factors of $-2x^2 + 9x - 7$ using the algebra tiles. Prajwal solved the problem and arranged the factor tiles corresponding to two factors along the vertical and horizontal frame. Which of the following is a possible combination of tiles along the two frames in Prajwal's solution?

a.

Horizontal	
Vertical	

b.

Horizontal	
Vertical	

c.

Horizontal	
Vertical	

d.

Horizontal	
Vertical	

Answer

Question 13

Prajwal asked Alex to find out if $(x + 3)$ is a factor of $-2x^2 + 9x - 7$; and if the answer is no, then find the remainder. Can you help Alex answer this question using the factor theorem and remainder theorem?

Answer

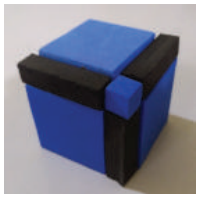
Question 14

Which of the following polynomials will have the same factor tiles along both the horizontal and vertical frames? (Show the solution to this question using one of the algebraic identities).

a. $9x^2 + 42x + 48$	b. $4x^2 + 12x + 8$
c. $16x^2 + 56x + 49$	d. $25x^2 + 50x + 24$

Answer

Exploration Pathway



Visualising Algebraic Identities

Make different 3D shapes with the sides equal to either *a* or *b* or a combination of the two and arrange them to visualise how $(a+b)^3$ is equal to $a^3 + b^3 + 3a^2b + 3ab^2$