# Theme 2: Arithmetic progressions

## Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Term, common difference
- General form of an AP
- $n^{th}$ term of an AP
- Sum of first n terms of an AP

## Amphitheatre



Most of us have seen an amphitheatre in some context or another: either at school, or at a performance venue, or even temporary ones at fairs and *melas*. The name itself has Greek and Roman origins, and famous amphitheatres like the Coliseum in Rome or Epidaurus in Greece come to mind.

They tend to be of a similar shape, inspiring many modern theatres too: a curved, even semicircular, arrangement of seats, tiered at different heights, giving a clear view of a stage placed opposite the seating at a slight height/plinth; and typically set in the outdoors. This shape and arrangement gives a clear view to all spectators and also enhances the acoustics of the space. This makes an amphitheatre an ideal space for music, dance and theatre performances.

It is crucial to know the total number of seats planned in each row, to decide the entrance, exit, where to leave gaps for easy access to seats and so on, during the initial design phase of building an amphitheatre.

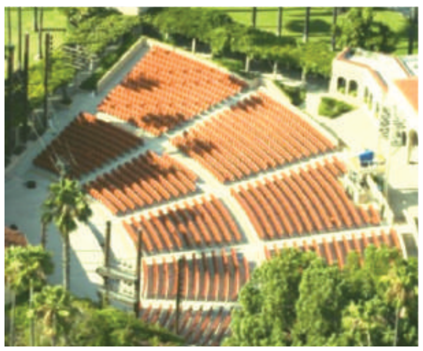


**Fig. 2.1,** Model of amphitheatre

## Case Study A - Amphitheatre

Each amphitheatre has two main sections - one where the artists perform (the stage) and one where the audience is seated. While designing an amphitheatre, the architect evaluated different possible shapes for the audience section. The goal was to design an amphitheatre with 26 seats in the first row and a total of 20 rows.

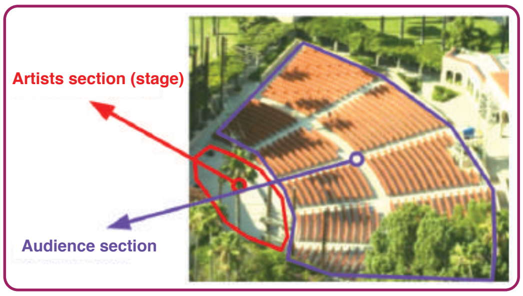
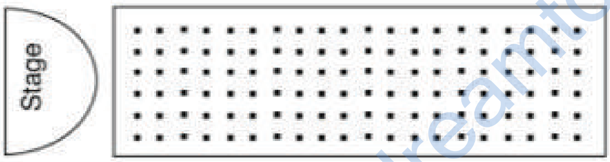
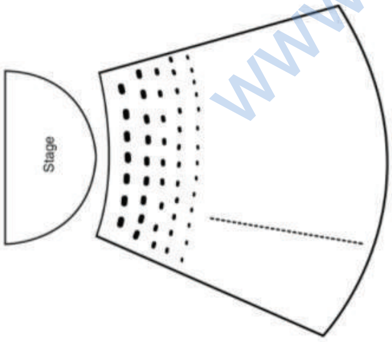
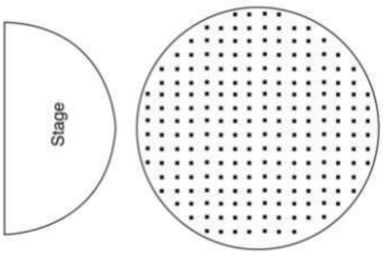


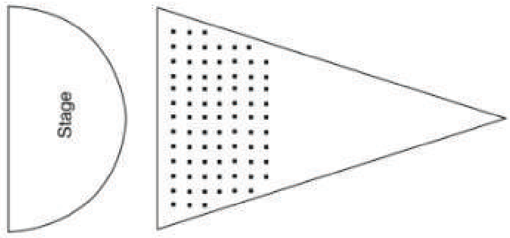

**Fig. 2.2,** Picture of amphitheatre showing audience and artist sections

## Question 1

Given below is a list of shapes (top view) for the audience section. For each, select the relevant formula representing the number of seats in the $R^{th}$ row, where row numbers start with 0 and end with 19. Assume that seats are placed from one end of the wall to another in each row.



| Shape for audience section | Formula |
|---|---|
| a.   Rectangle shaped audience section | A. {26 - 2R} |
| b.   A sector of a circle shaped audience section | B. if (R<10) then {26 + 2R} else {44 - 2(R-9)} |
| c.   Circle shaped audience section | C. {26} |

| | |
|---|---|
| d.   Triangle shaped audience section with pointed end away from the stage | D. {26 + 2R} |

Write the formula option (one out of A, B, C, D) next to each shape in the blank space given next to the shape:

i. Shape a, rectangle ⟶ Formula ____________________

ii. Shape b, sector of a circle ⟶ Formula ____________________

iii. Shape c, circle ⟶ Formula ____________________

vi. Shape d, triangle ⟶ Formula ____________________

After looking at the various options, the architect chose a shape shown in Fig. 2.3 as the shape of the audience section with 26 seats in the first row and 2 seats added in each of the subsequent rows. Answer questions 2 to 5 based on this design of the amphitheatre.

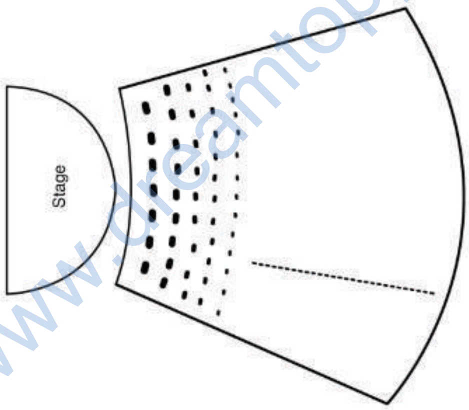



**Fig. 2.3,** Shape chosen by the architect



## Question 2

What is the total seating capacity of the theatre?

**Answer**

## Question 3

If one plotted a graph with the total seating capacity (till a given row number) on the y-axis and row number on the x-axis, what shape would the graph have?

a. straight line sloping upwards

b. flat line with zero slope

c. parabola

d. straight line sloping downwards

Answer

## Question 4

The owners wanted to double the capacity of the amphitheatre. What is the minimum number of additional rows they need to add?

a. 20

b. more than 20

c. less than 20 but more than 5

d. less than or equal to 5

Answer

## Question 5

Being open to the sky, the access to the theatre was free for birds. Bird droppings became a common phenomenon and with that came a few seeds. After a few years, 4 trees grew in the middle of the amphitheatre. The owners wanted to maintain the natural look and chose to remove the seats instead of trees. Each tree occupied the space for 2 seats per row and the trunk spread across 2 rows. (Refer Fig. 2.4)

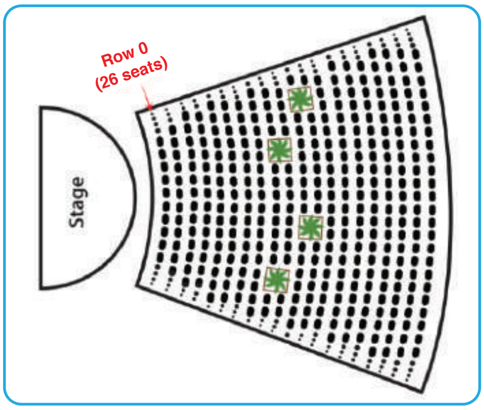
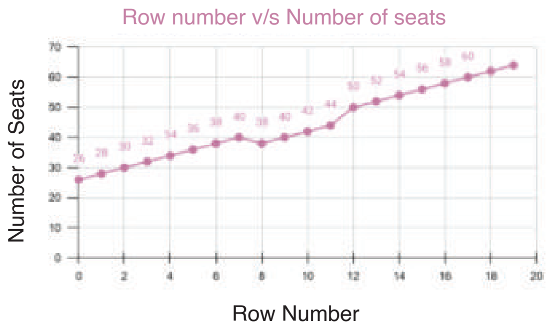


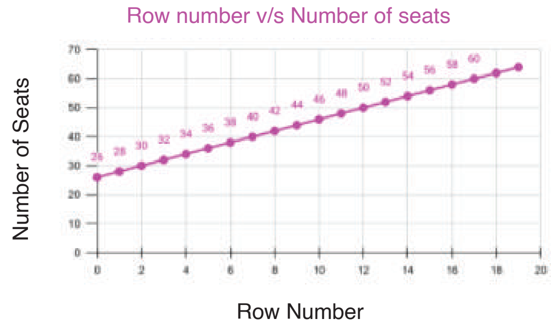

**Fig. 2.4,** Trees in the middle of the amphitheatre

Given below are graphs with the number of seats in each row on the y-axis and row number on the x-axis. Which of the 4 graphs (A to D) correctly represents the amphitheatre being discussed in Fig. 2.4?

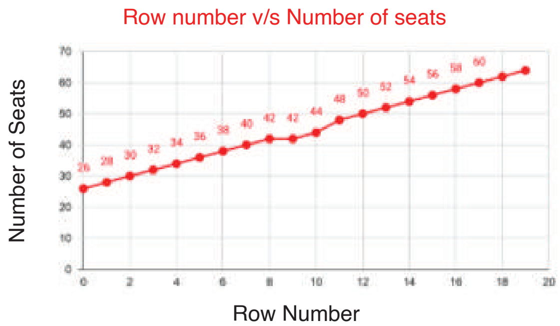
A



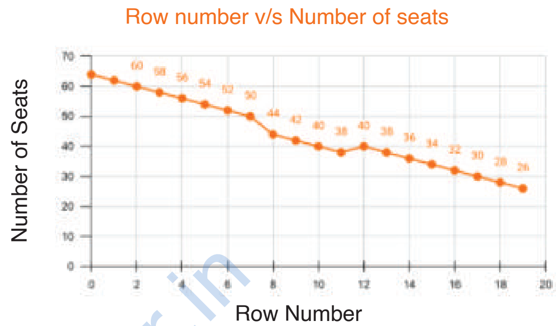

B




C




D




| Answer |
|---|
| |



### Case Study B - The Pattern Game

In a funfair, there is a counter with a 1 minute game that requires participants to create patterns with sticks. Two players participate in each batch of the game. Both the players will start at same time to make the patterns similar to the one shown in Fig. 2.12.

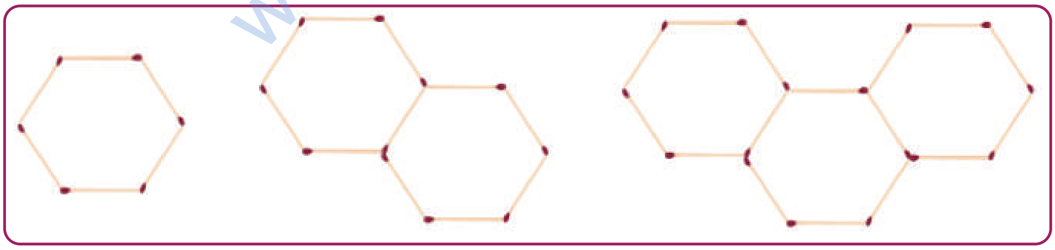


**Fig. 2.5,** Patterns from stick

Each participant can use any number of sticks from the tray. There should be only one common stick between the two connected patterns and each pattern should connect to the adjacent one only, making it a chain. The one who makes the longest complete pattern chain in 1 minute wins the game, where the length of the pattern chain is defined as below:

Chain length = Number of complete hexagons in the chain

## Question 6

Aditi and Renu participate in this game. Aditi uses 61 sticks and Renu makes a chain with 12 hexagons in one minute. Who gets the prize?

a. Aditi, because she uses more sticks

b. Renu, because she makes more patterns

c. Aditi, because she gets chain length as 13 by using 61 sticks

d. Both share the prize equally, because the chain length is the same

Answer

## Question 7

i. If adding each hexagonal pattern, including the first one, takes a minimum of 4 seconds, what is the maximum number of sticks that can be used by a player in this one minute game? Write your answer in the space provided.

Answer

ii. How will the number calculated above help the organisers of the funfair counter?

a. Estimate the maximum number of batches they can run in the 4 hours available

b. Estimate number of prizes they need to arrange

c. It is not helpful, since different players will take different time to make the patterns

d. Decide how many sticks to place in the tray

Answer

## Question 8

As more players play the game, the word starts spreading and people start queuing at the counter. The organisers have 10 people in their team, who can manage the game, and hence they decide to run 5 games in parallel batches. But they have a limited stock of sticks. What is the minimum number of sticks they need to run 5 batches in parallel?

a. Upto 500

b. More than 500 but less than 750

c. 750 to 900

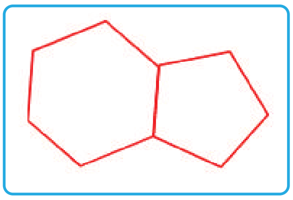
d. More than 900 but less than 1100

Answer

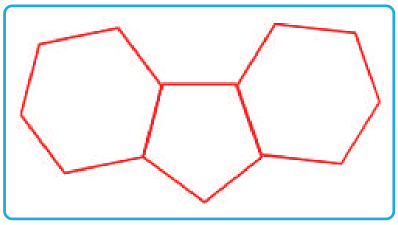
## Question 9

To give a twist, the fun fair management team made a small change after a few rounds of the competition. Every alternate shape added through the sticks will be a pentagon. This means

- a chain of 2 will have 1 hexagon + 1 pentagon

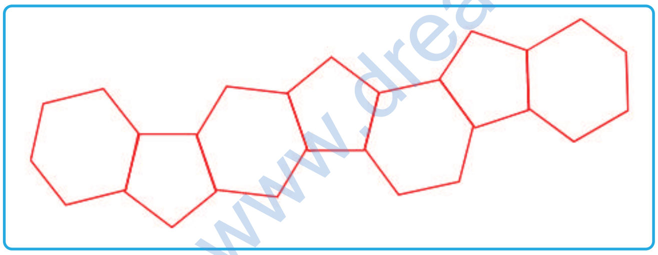


- chain of 3 will have 1 hexagon + 1 pentagon + 1 hexagon



Continuing in the same manner,

- chain of 7 will have 1 hexagon + 1 pentagon + 1 hexagon + 1 pentagon + 1 hexagon + 1 pentagon + 1 hexagon





Write a formula in the space provided below, representing the total number of sticks required for a chain of length P, where P is an:

i. odd number

Answer

ii. even number

**Answer**

**Case Study C - Regular Polygons**

Our life is surrounded by a variety of shapes. The 2D shapes, which are made of three or more straight lines joined together from end to end to enclose a space, are called polygons. The polygons, which have all their sides of equal length and all angles of equal measure, are called regular polygons. Regular polygons often look pleasing to the eyes due to their symmetry. As we go from an equilateral triangle to a square to a regular pentagon (and so on), we observe patterns in the size of angles, sum of angles, etc. Let us explore which of these patterns have an Arithmetic Progression (AP).

An interior angle of a polygon is the angle formed inside the polygon by any two adjacent sides of the polygon. An exterior angle is the angle formed outside the polygon by one side and extending one of its adjacent sides. The sum of the interior angle and the exterior angle on the same vertex is always 180°.



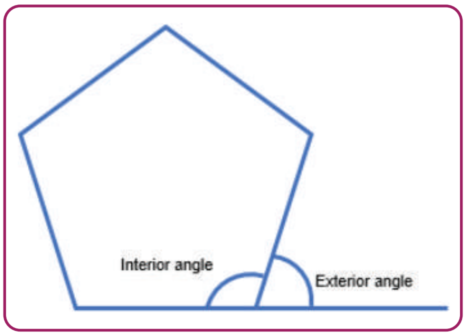



**Fig. 2.6,** Interior and exterior angle example in a pentagon

**How to calculate the interior angle of a regular polygon?**

1. If you don't know the value of an interior angle of a regular polygon, you can calculate it by considering the fact that the interior angle is the supplement of the exterior angle.
2. Recall that the sum of exterior angles of a regular polygon is 360°.
3. Each interior angle = $180° - (\frac{360°}{n})$, where n is the number of sides of the polygon, $n \geq 3$.

## Question 10

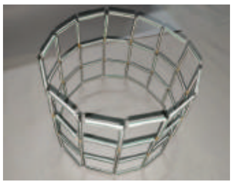
Complete table 2.1. Fill column 7, 8 and 9 only if the answer to the 6th column is "Yes" for a particular row else fill NA (not applicable).

| Column #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 |
|---|---|---|---|---|---|---|---|---|
| Regular polygon | Equilateral triangle | Square | Regular pentagon | Regular hexagon | Is this row in AP? (Yes/No) | First term (a) | Common difference (d) | $n^{th}$ term |
| | Term 1 ($a_1$) | Term 2 ($a_2$) | Term 3 ($a_3$) | Term 4 ($a_4$) | | | | |
| No. of sides | 3 | 4 | 5 | 6 | Yes | 3 | 1 | n+2 |
| Interior angle | 60 | | | | | | | |
| Exterior angle | 120 | | | | | | | |
| ∑Interior angles | 180 | | | | | | | |
| ∑Exterior angles | 360 | | | | | | | |

**Table 2.1**, Table of angles and sides of regular polygons



## Exploration Pathway



Growing Patterns

Make a hexagon, using 6 matchsticks. Add another 5 sticks to attach another hexagon, which shares one side with the first hexagon. Repeat this process and make different patterns. Can you make a pattern with more than 2 hexagons, where the last hexagon shares a side with the first hexagon? Predict and validate the number of sticks required to make 15 hexagons.

DIY Buckminsterfullerene Model

Take a few stiff straws and join them together, using loops of binding wire. Prepare hexagons and pentagons and join them together to make your own Buckminsterfullerene model. Observe the pattern in the design and predict the number of wire loops you will need to join the straws. Also predict the number of straws required to make the full model.