Theme 1: Real Numbers

Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Euclid's division algorithm
- The Fundamental Theorem of Arithmetic
- Finding HCF and LCM by prime factorisation method
- Rational and Irrational numbers

Water Tankers

In many cities across the country, we often get to see water tankers transporting water from one part of the city to another. Tractors are often used for doing this job. Tractors are heavy-duty vehicles custom designed to move without getting stuck in soft or damp soil. This makes them suitable for agricultural use. These tractors have now been adapted for alternate uses, like transporting water.

Case Study A - Tractor Tyres

Tractors have rear wheels, which are bigger in diameter than the front wheels. The rear wheels have diameters in the range of 45 to 55 inches (around 115 cm to 140 cm). Note that this is the outer diameter, i.e. it includes the height of the tyre.

Fig. 1.1, Tractor (Water tanker); Image by Rakesh.5suthar via Wikimedia Commons

The front wheels have outer diameters in the range of 25 to 30 inches (around 65 cm to 75 cm). This difference in the size of wheels is not commonly seen in other vehicles. In tractors, the larger rear wheel shifts the centre of gravity closer to the rear wheel, where the driver's seat is located. This gives it the stability it needs while driving on uneven surfaces, like fields or uneven roads. The short front wheel gives good visibility to the driver from the high positioned seat.

When any object comes in physical contact with another and a movement of either of the two objects occurs, the two objects undergo some form of wear and tear due to the rubbing at the surface level. More frequently a surface comes in contact with another, more is the wear and tear. The tractor tyres come in frequent contact with land, and with hard and uneven surfaces, they are more likely to experience wear and tear. The uneven surfaces exert high pressure on specific points of the tyres. The tractor tyres are extremely strong by design to minimise the rate of wear and tear.

Will the wear and tear experienced by the front tyres be the same as the rear tyres?

Question 1

i. Consider a tractor with the rear wheel having an outer diameter of 49 inches and front wheel 28 inches. When the tractor is stationary, each of its wheels is touching the road at one point. Let this point on one out of the two front wheels be point A. Let us name the point on one of the two rear wheels as point B. As the tractor starts moving, so do the points A and B.

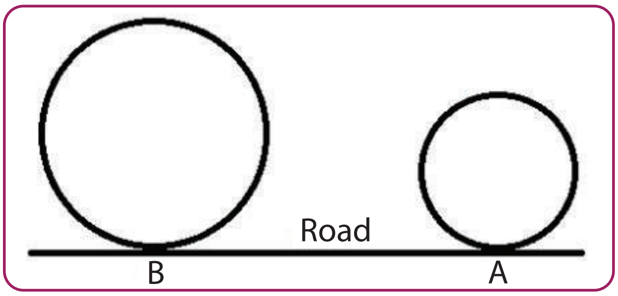


Fig. 1.2, Tractor wheels and point of contact with road

After how many rotations of the rear wheel and front wheel will both these two points touch the road again at the same time? Write your answer in the space provided below.

a. Number of rotations of rear-wheel: ______________________

b. Number of rotations of front-wheel: ______________________

ii. While solving a mathematical problem, using a method that involves the least number of steps may save time & effort. Which of the following steps can be skipped for answering the previous question?

a. Calculate the radius of the front wheel and rear wheel

b. Calculate the circumference of the front wheel and rear wheel

c. Finding the LCM of two numbers

d. Dividing one number by another number

Answer

Question 2

The number of times point A touches the road in one rotation is the same as the number of times other points on the front wheel's tyre touch the road (in one rotation). Same for point B on the rear wheel's tyre. (Use $\pi = \frac{22}{7}$).

i. After the tractor has moved a distance of 1 km, how many times would point A have touched the road? What about point B? Write your answer in the space provided below. Note that 1 km = 39,370 inches.

Answer

ii. Which tyres are likely to undergo more wear and tear - rear tyres or front tyres? Assume that the thickness of the front and rear wheel tyres are the same, and so is the quality of rubber used to manufacture them. Write your answer in the space provided below.

Answer

Case Study B - School Auditorium

A school auditorium has a rectangular stage, which is 9.6 metres long, 15.2 metres wide and is to be paved. The tiles available are all square-shaped. A masonry team is asked to select the most suitable tiles. The team reasoned that the effort required to lay bigger tiles is less than that for laying smaller tiles. The team was informed that the material used to make these tiles has the property of splitting unevenly if one tries to break the tile into smaller pieces. So, in order to minimise wastage and also to maintain a smooth transition from the front stage to the backstage, the masonry team has to work with the constraint of selecting a size, which covers the entire stage exactly, without the need to break any tile.

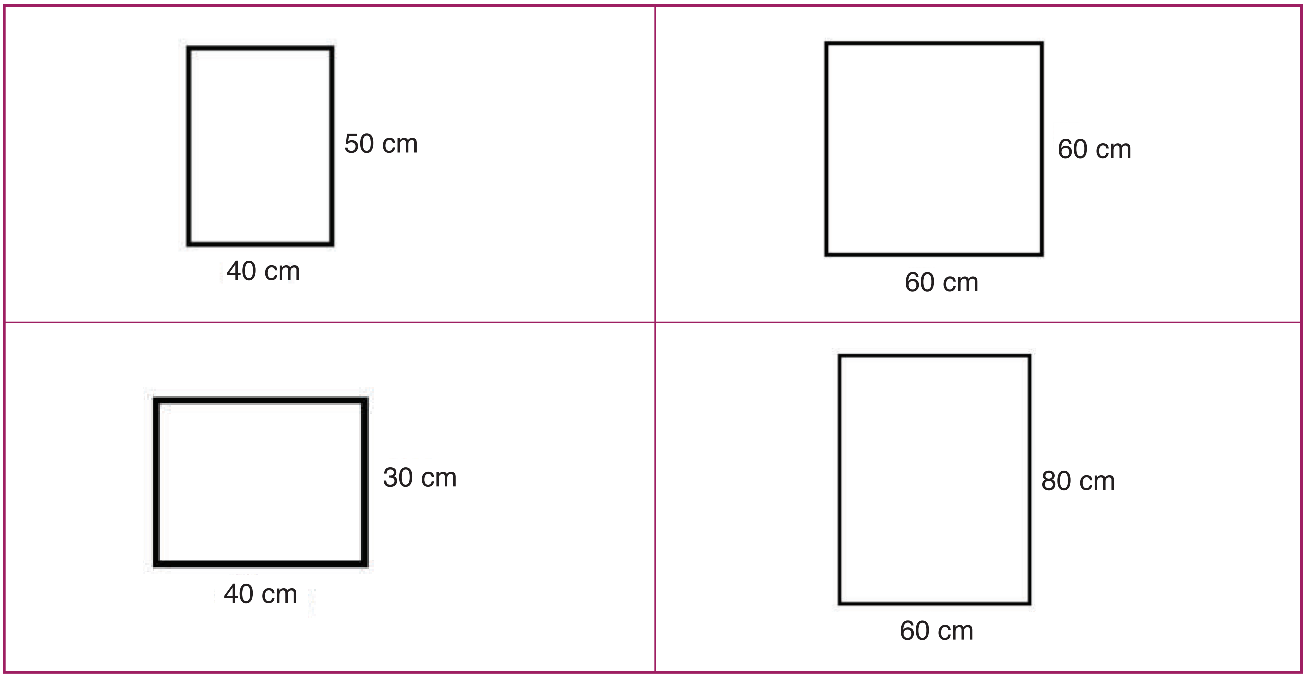
Question 3

i. The team has to select one size from the following. Which one should they select? Only one side of each size is specified (in cm), since these are square-shaped tiles.

a. 20	b. 10
c. 40	d. 50

Answer

ii. On researching further, the team found another vendor, who offered a few more options, which included rectangle-shaped tiles.



Do you think they should continue with the decision to use the previously chosen square tile or change it? Explain your answer in the space given below and include the new size in your explanation, if you think they should change their decision. Assume that the total cost does not change significantly if they change the type of the tile. Also, assume that the constraint of not breaking the tiles into smaller pieces continues to hold.

Answer

Question 4

The school invites parents as well as students from a neighbouring school, during the annual function. The seating of parents and students was arranged in separate rows. The teachers tried the following combinations:

i. 16 students per row, they were left with 11 students
ii. 20 per row, left with 15
iii. 25 per row, left with 20
iv. 32 per row, left with 27

The idea was to have an equal number of students from their own school in each row so that the rest of the seats from each row can be allocated to the students from the neighbouring school.

If you include 5 students from the neighbouring school while looking for the seating arrangement, how many students will be left for each combination? Write your answer in the blank spaces provided in Table 1.1.

Number of students per row	Number of remaining students
16	______________
20	______________
25	______________
32	______________

Table 1.1, Number of students per row and number of students remaining

Question 5

i. Based on the information available, if you want to guess the total number of students in this school, what process would you follow?

a. Find the highest factor of 32, which is also a factor of 16, 20 and 25

b. Find the first common multiple of 16, 20, 25 and 32 to arrive at the answer

c. Find the first multiple of 16, which is divisible by 20, 25 and 32; then subtract 5 from this value to arrive at the final answer

d. Find the first multiple of 16, which is divisible by 20, 25 and 32; then add 5 to this value to arrive at the final answer

Answer

ii. If the team who is looking after seating arrangement wants to continue to work with the rule of the same number of students from their school seated in each row, what is the largest number which will satisfy this constraint? Each row in the auditorium has a maximum of 60 seats. Write your answer in the space provided below.

Answer

Case Study C - Practice Schedule

Many countries have practice grounds where football and cricket are played on the same site. Consider one such ground where cricket and football are practised, but the cricket team has more practices scheduled, due to an upcoming match. These two teams start their practices on March 9th. The cricket team meets for practice every 2 days, and the football team meets every 4 days.

Question 6

For how many days will the teams not meet each other before they share the field again, and how many times will they have to share the field till the end of March, including Sundays and excluding March 9th?

a. 3, 6	b. 2, 6
c. 3, 5	d. 2, 7

Answer

Question 7

The organising committee decided to distribute free goodies on the occasion of 100th match being played on the ground for online tickets only. Everyone who purchases the ticket will receive a bag with a small gift. Online booking programs are written in such a way that everyone does not receive a referral code to claim the gift. Table 1.2 shows which ticket numbers will receive a gift.

Items	Ticket number
Pen	Every 2nd ticket
Cap	Every 7th ticket
Key chain	Every 10th ticket

Table 1.2, Gift items as per the ticket number

A family book their tickets online and they are allotted seat numbers - 74, 75, 76, 77. Would anyone from this family get all the three gift items? If yes, which seat number is the lucky one? If not, how many types of items will they get in total as a family?

a. No, they get only two types of items in total

b. No, they get only one type of item in total

c. Yes, seat number 77

d. Overall they will have all 3 types of items but not through a single ticket

Answer

Case Study D - Patterns

Here is an interesting activity, which can lead to beautiful pattern(s).

1. Take a sheet and draw the x and y axis such that the origin is at the centre of the page (approximately).
2. Draw an arc intersecting the x-axis with the origin as the centre, in the positive direction. The radius of the arc should be 2 cm.
3. Now draw a line perpendicular to the x - axis at the point where the arc meets the x-axis. On this perpendicular line draw another arc which has a radius 2 cm.
4. Join the point where the arc intersects the perpendicular line to the origin. This line ($\overline{OC}$), the perpendicular line ($\overline{BC}$) and the line on the x-axis ($\overline{OB}$) form a right angled triangle $\triangle OBC$, as shown in Fig. 1.3.

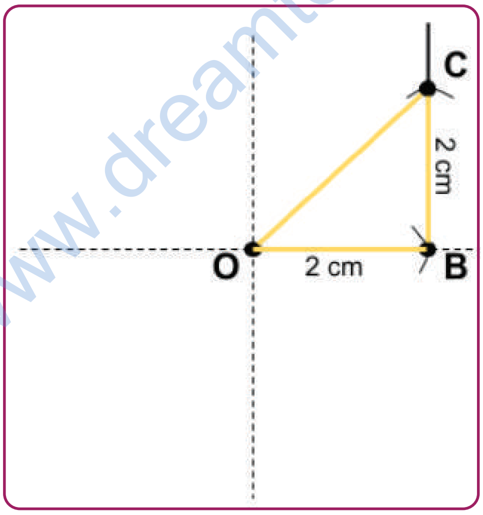


Fig. 1.3, Triangle constructed by arcs of equal length

5. Now draw a line perpendicular to $\overline{OC}$ at C and draw an arc 2 cm long on this line with C as the centre, as shown in Fig. 1.4. This will form another right angled triangle $\triangle$ OCD.

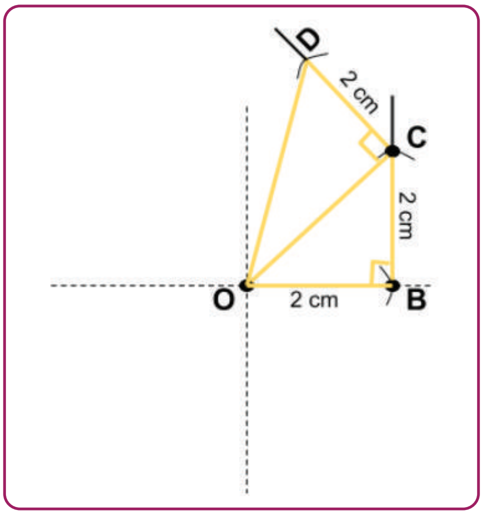


Fig. 1.4, Triangles constructed by arcs of equal length

6. Follow this rule and draw another 23 triangles to form a pattern with 25 triangles. The next triangle in this series will have the line $\overline{OD}$ as its base and a 2 cm long line perpendicular to $\overline{OD}$.
7. We will refer to the $\triangle OBC$ as Triangle 1, $\triangle OCD$ as Triangle 2 and so on.

A larger pattern with several triangles is given in Fig. 1.5

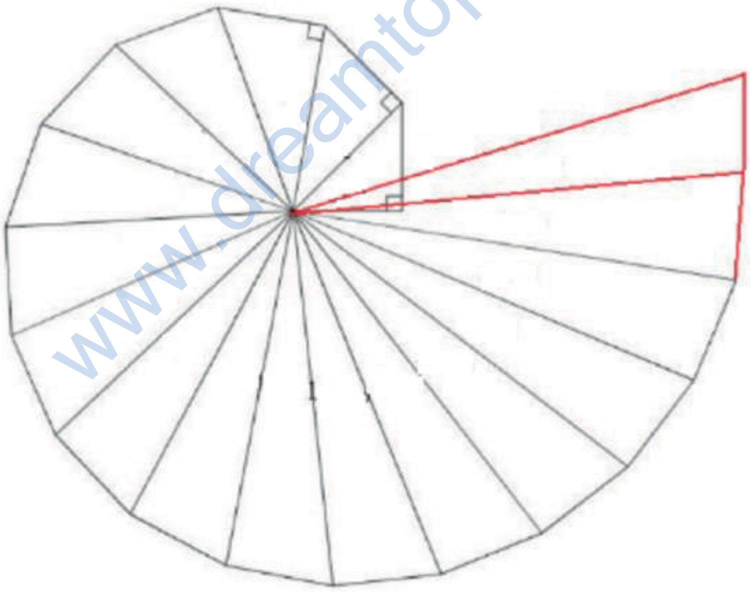


Fig. 1.5, A larger pattern with many triangles

Question 8

In $\triangle OBC$ we shall call the line $\overline{OB}$ as the base and line $\overline{BC}$ as the perpendicular. Let us follow the same naming convention for all the 25 triangles. The length of the base and perpendicular of Triangle 1 ($\triangle OBC$), is a rational number. What about the base and perpendicular of Triangle 2 and all the remaining triangles?

Fill Table 1.3 with your prediction about the hypotenuse, base and perpendicular of all the 25 triangles. Eg: If you think that the hypotenuse of the triangles will be a rational number for all the 25 triangles, then write "Y" under "Always Rational". If you think it will be a rational number for some triangles but irrational for others, write "Y" under "Sometimes Rational, Sometimes Irrational".

	Always Rational	Always Irrational	Sometimes Rational, Sometimes Irrational
Hypotenuse			
Perpendicular			
Base			

Table 1.3, Prediction about the type of sides of the triangles

Question 9

As we add more triangles to this pattern, will the pattern grow or shrink? To understand this, let us check if the sides and angles of the triangles increase as we add more triangles or decrease.

For each triangle in the pattern discussed above, let us call the angle formed by the base and hypotenuse as θ. In $\triangle OBC$ the θ will be $\angle COB$. As you go from one triangle to the next one in the series of triangles, what happens to the length of the sides of the triangle - increases/ decreases/ remains constant? What happens to θ? Write the answers to these questions in the blank space next to each side and angle.

i. Base ______________________________ (increases/decreases/remains constant)

ii. Hypotenuse ______________________________ (increases/ decreases/remains constant)

iii. Perpendicular ______________________________ (increases/decreases/remains constant)

iv. θ ______________________________ (increases/decreases/remains constant)

Question 10

Analyse the following statements and mark them as always true, never true and some times true.

Answer

i. If the radius of a circle is rational, then the area must be irrational. ____________

ii. If the area of a circle is rational, then the circumference is rational. ____________

iii. If the circumference of a circle is rational, then the area is rational. ____________

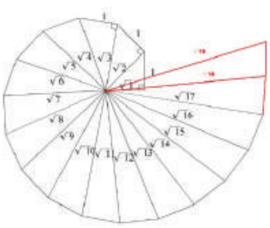
iv. The diagonal of a square is irrational. ____________

v. The hypotenuse of an isosceles right triangle with a whole number as its base, is always irrational ____________

Exploration Pathway

Area, Perimeter & Factorisation

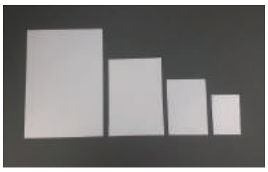
Pick any number smaller than 50 and make rectangles using matchsticks such that the area of the rectangle is equal to the number you have selected. Repeat this exercise for 5 numbers and record the length of sides of the rectangles you made. Identify the numbers for which you can't make more than one size of rectangles.



Square Root Spiral

On a graph paper, draw the x and y axis. From the origin, draw on arc of length 2 cm on the x axis. Draw a line from the point where the arc intersects the x axis. The line should be perpendicular to the x axis. Now draw another arc on this perpendicular line, of length 2 cm. Join the point of intersection of this arc to the origin. This would form a right angled triangle with two sides 2 cm long and the hypotenuse $2\sqrt{2}$ cm long.

Now draw the next right angled triangle by following the same steps and using this hypotenuse as the base of the next triangle. Continue these steps and draw 20 triangles. Observe and record the length of the base, perpendicular and hypotenuse of any 5 triangles. Do these three lengths follow a pattern? Are they always rational/irrational or only sometimes rational/irrational? Which of these lengths are growing/shrinking as you go to the next triangle?



Irrational Numbers - Paper Sizes

When we fold an A4 standard sized paper into half along its longer length, we get the A5 standard size. This pattern continues as you go from A5 to A6 or from A0 to A1. This pattern in the standards emerges from a requirement to enlarge/reduce the size of an image at the printer.

In this TACtivity we use a bunch of A4 papers to visualise all the standards in the A series of papers starting from A0 to A8. We also observe the ratio of the sides of each standard and identify the pattern.