Theme 1: Number System

Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Rational and irrational numbers
- Representing real numbers on the number line
- Operations on rational and irrational numbers
- Laws of exponents for real numbers

The Modern Number System

Numbers have been a core part of human history, with most of the original numerica being tally systems. Roman numerals became one of the most commonly used numerical systems across Europe during the ancient period, i.e. around the 9th century BC. One of the problems with the Roman numeral system was that large numbers were very difficult to write. By the 8th century AD, India had developed a system of numbers that compensated for the problem, though we weren't the first civilisation to come up with this kind of a solution.

India developed the use of the decimal, ten different symbols to represent numbers, and combined it with positional notation in their number system. The decimals were 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0; 0 being a unique aspect of the Indian numerical system, where there was a symbol to denote nothing.

Positional notation was a system where the value of a number is dependent on the position it takes in a sequence. For example, in the number 365, the position 3 tells the reader that its value is 3x100, the position of 6 tells the reader its value is 6x10 and the position of 5 tells the reader that its value is 5x1. The arrangement of the numbers in the order 3, 6 and then 5 tells the reader that the total value of the number is three hundred and sixty five.

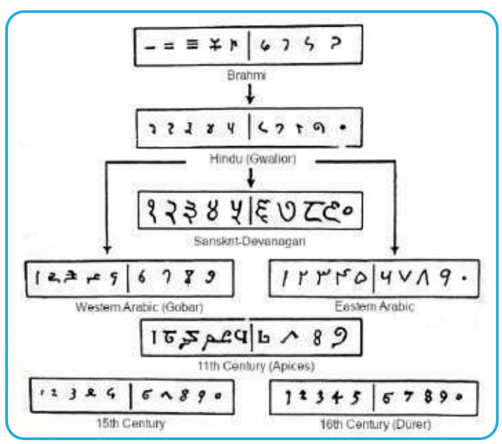

Fig. 1.1, Symbols used in different number systems
Image by Tobus via Wikimedia Commons

This number system was adopted by Arab merchants trading with India as it proved to be an extremely convenient way of keeping track of records. Hence these numerals are known as “Indo-Arabic numerals”.

Case Study A - The Real Numbers Game

Here is a two player game with rational and irrational numbers. First player will pick either a rational or irrational number along with one of the 4 basic mathematical operations (addition, subtraction, multiplication, division); the second player should pick a number, either a rational or irrational number, so that the answer obtained with the selected operation belongs to the number set other than what is selected by the first player.

E.g.: If the first player selects a rational number and addition, the second player should select a number from the real number set so that adding them results in an irrational number. In the same example, if the first one had selected an irrational number, now the answer should be rational.

Question 1

Fill in the blanks with the correct numbers to match with the operation, and by obeying the rules of the game.

Sl.No.	No. given by Player 1	Number is	Operation	No. selected by Player 2	Answer	Answer is
1	$\frac{2}{3}$	Rational	$\div$	$\sqrt{2}$	$\frac{\sqrt{2}}{3}$	Irrational
2	$\sqrt{36}$		$\times$			
3	$-\sqrt{21}$		$+$			
4	0.15		$-$			
5	$\frac{8\pi}{5}$		$\times$			
6	$\frac{\sqrt{225}}{7}$		$\div$			

Table 1.1, Rational and irrational numbers

Case Study B - Spreading News

As a result of the internet and modern technology, news spreads much faster today than it did a few decades ago. Mr. Jacob has won a car in a lucky draw at a shopping mall. He wants to keep it a secret, till he completes all the paperwork and receives the car keys. He just shares this information with 3 of his closest friends within an hour. It so happens that each of these three friends tell another set of three different people within the next hour, and so on. Therefore, the number of people who hear about Mr. Jacob's good fortune grows by the hour.

The tree diagram shown in Fig. 1.2 explains this scenario.

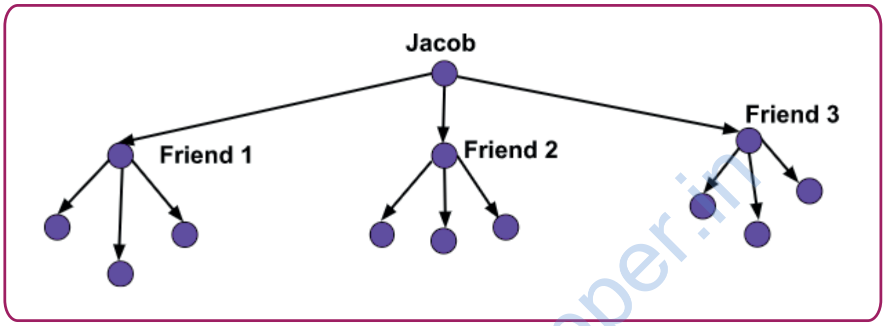

Fig. 1.2, Tree diagram showing the news getting spread to three people at a time

Question 2

i. Complete Table 1.2 with the help of the given data.

Hour (n)	No. of new people to hear the news in the n^{th} hour	Total no. of people who know the news
1	3	3
2	9	12
3	27	39
4	?	?

Table 1.2, News spreading in each hour

ii. Mr.Jacob stays in an apartment complex, which houses around 2000 people. How many new people will come to know about Mr. Jacob winning the car in the 6^{th} hour, assuming that each person forwards the news to three more people per hour?

a. Less than 100

b. More than 100 but less than 250

c. More than 250 but less than 750

d. Entire apartment!

Answer

Question 3

If the news spreads at the same rate, then how long will it take for all the occupants of the apartment block to hear about Mr. Jacob's good fortune?

		Answer
a. 6 hours	b. 7 hours	
c. 12 hours	d. 24 hours	

Question 4

Consider that Mr. Jacob shares the news of winning a car to, say, 10 people. These 10 people then go and tell 10 more of their friends, each, and so on. Let us call each time news spreads as one level. So, in level 1 only 1 person knows; in level 2, 10 new people come to know about the car; in level 3 the news spreads to 100 people and so on. Which option best expresses the rate of spreading of news in terms of the level?

		Answer
a. News spreading = 10^{level}	b. News spreading = $10^{2*level}$	
c. News spreading = $(level)^{10}$	d. News spreading = $10^{level-1}$	

Case Study C - Mathematics in Nature Studies

I. *Paramecium:*

Studying single celled organisms, like bacteria, is one of the best ways to understand the concept of a growing population. Consider the population of *Paramecium* in a small laboratory depression slide, as pictured in Fig. 1.3. It so happens that here, the individual cells divide once in one day. So if we start with one individual on Day 0, then the subsequent days will have 2, 4, 8, 16, 32 and 64 individuals, respectively.

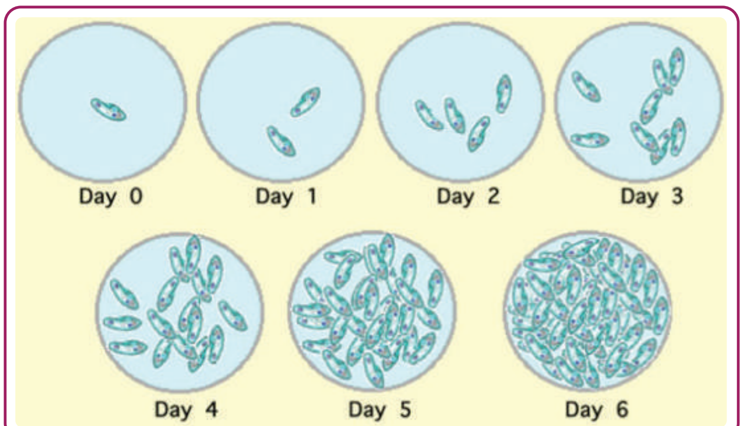

Fig. 1.3, Changes in the population of *Paramecium* over a six day period; Image via nature.com

Question 5

What can you comment about the assertion and reason by referring to the above information about the population of *Paramecium?*

Assertion (A): If a laboratory technician needs to know when a *Paramecium* culture reaches a certain population, he can use the concept of exponents and predict exactly when that population size will be reached.

Reason (R): On any particular day, the size of the Paramecium population is simply twice what the number was the day before; so on day x (for $x > 0$), the total count is given by x^2, starting with 1 *Paramecium* on day zero.

a. Both A and R are true

b. Both A and R are false

c. A is true but R is not the correct reason for A

d. A is false but R is the correct reason for the given data

Answer

II. *Wolffia:*

Also known as Asian watermeal and duckweed, *Wolffia Globosa* is a species of flowering plant that grows on calm water surfaces of freshwater ponds, lakes and marshes. It is native to Asia and is also found in some parts of the Americas, as a native or naturalised species. These floating, rootless plants are the world's smallest flowering plants. An average individual plant is 0.6 mm long and 0.3 mm wide. It weighs about 150 micrograms, the approximate weight of 2-3 grains of table salt.

Fig. 1.4, Wolffia; Image by Christian Fischer via Wikimedia Commons

Question 6

A water molecule is of the order of size 10^{-9}m; this means that a *Wolffia* plant is about 10^5 times larger than the water molecule and the Earth is about 10^{16} times larger than the water molecule. Which one of the statements below correctly relates the size of *Wolffia* and the Earth?

a. The Earth is about 10^{11} times larger than a *Wolffia* plant

b. The *Wolffia* is about $(\frac{1}{1000})^{th}$ the size of the Earth

c. The Earth is 10^{21} times the size of *Wolffia*

d. The Earth is about 10^{80} times larger than a *Wolffia* plant

Answer

Case Study D - The Golden Ratio

The real number $\frac{1+\sqrt{5}}{2}$, denoted by ϕ (Phi) and approximately equal to 1.62, is known as the golden ratio. Geometrically, it is the ratio of a line segment cut into two pieces of different lengths, such that the ratio of the whole segment to that of the longer segment is equal to the ratio of the longer segment to the shorter segment.

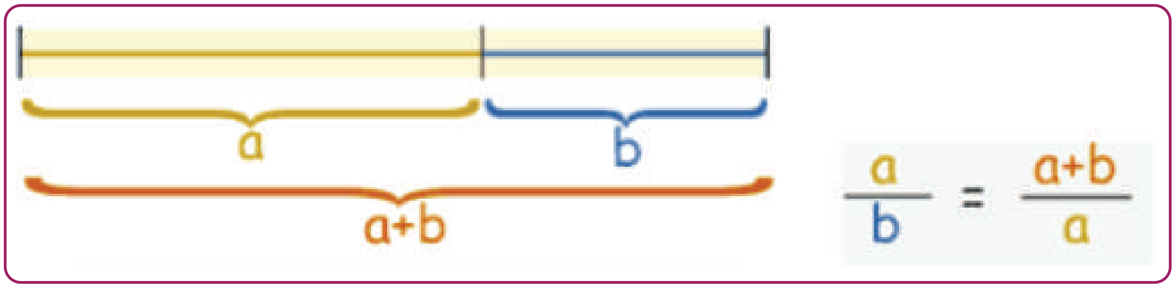

Fig. 1.5, Golden ratio

Question 7

Which option is the correct representation of ϕ (Phi), the golden ratio?

A

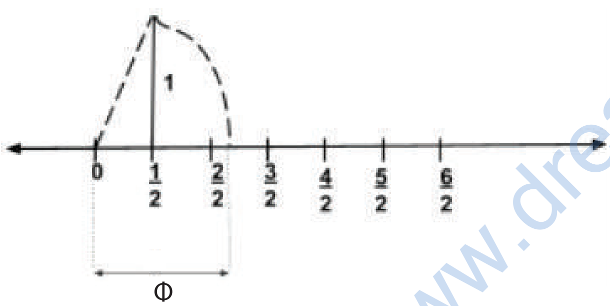

Fig. 1.6

B

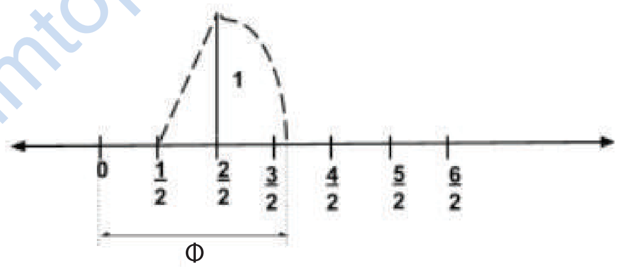

Fig. 1.7

C

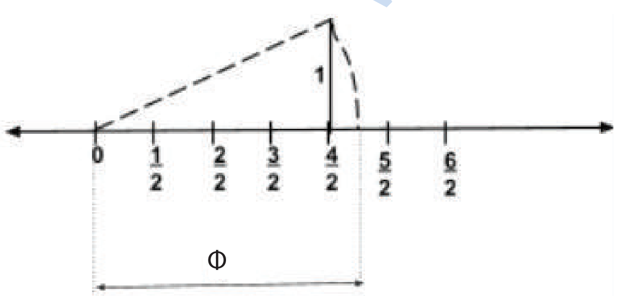

Fig. 1.8

D

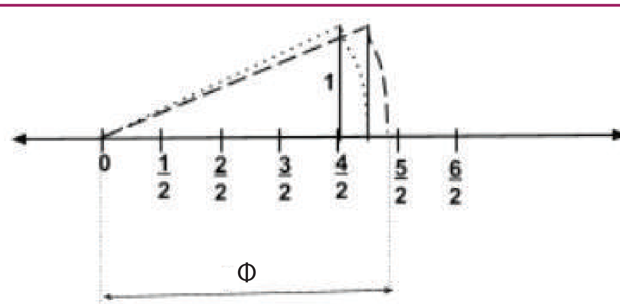

Fig. 1.9

Answer

Question 8

Which statement gives the correct nature of ϕ, according to the number system?

a. It is a rational number because we are representing it in the $\frac{p}{q}$ form with $q \neq 0$.

b. It is an irrational number because we are adding a rational and an irrational number in the numerator of ϕ.

c. It is an integer because we are able to represent it on a number line.

d. It is a whole number because the value of ϕ is positive.

Answer

Question 9

Some eminent artists and architects, such as Leonardo da Vinci, believed that the Golden Ratio makes the most pleasing and beautiful shapes in nature, especially in rectangles. Calculate the ratio of the longer length to the shorter length for each of the given images. Fill the last column by rounding off your answer to one decimal place and write "YES" if the ratio is between 1.5 and 1.7 or "NO" otherwise.

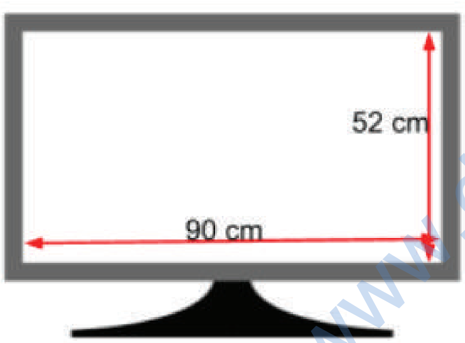

Fig. 1.10, Television screen; Image by Pedalito via Wikimedia Commons

Fig. 1.11, Laptop screen; Image by Elemenager via Wikimedia Commons

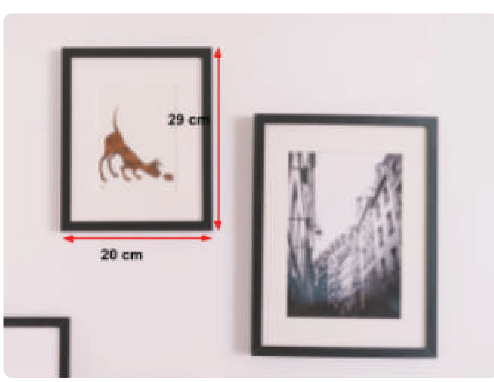

Fig. 1.12, Photo frame; Image by Crew via Wikimedia Commons

Fig. 1.13, Desktop screen; Image by EroticDesktops via Wikimedia Commons

Fig. 1.14, Microwave oven; Image by Pavel Ševela via Wikimedia Commons

Fig. 1.15, Mobile screen; Image by Ilya750 via Wikimedia Commons

Rectangular surface	Length (cm)	Width (cm)	Ratio of longer side to shorter side	Golden ratio?
Television screen				
Laptop screen				
Photo frame				
Desktop screen				
Microwave oven				
Mobile screen				

Table 1.3, Length and breadth of rectangular surfaces

The Golden Ratio in Cartoons

Design and animation industries use Golden Ratio dimensions most often. An animation firm is looking for artists to design their new cartoon series. Each artist submitted a sample design.The final selection by the panel was based on the usage of the golden ratio in the sample designs.

Given below is a sample image for reference.

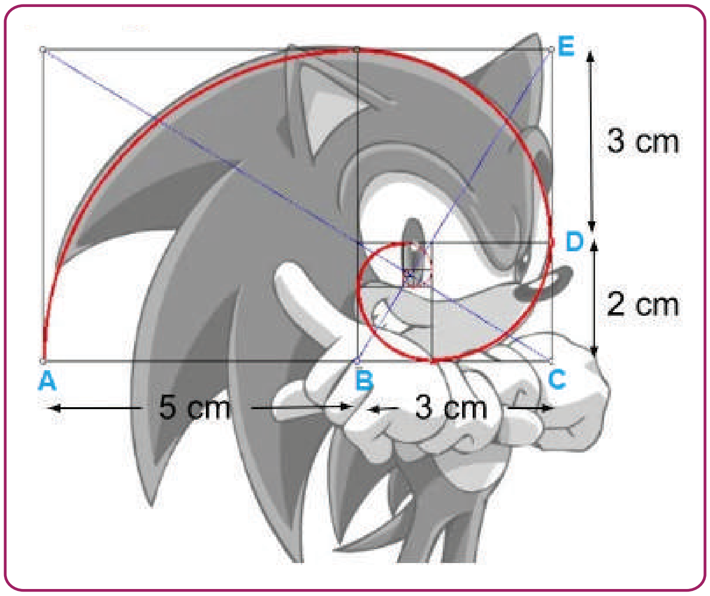

Fig. 1.16, Sonic Cartoon; Image by Doublecompile via flickr.com

Use this image to do a sample check for the golden ratio in the cartoon by taking two facial measures lengthwise (refer to the lengths AC and AB from Fig. 1.16), and width-wise (CE and DE), then finding the ratio (longer measure to shorter measure). For the Sonic cartoon image shown in Fig. 1.16,

lengthwise ratio $= \frac{AC}{AB} = \frac{8}{5} = 1.6$

widthwise ratio $= \frac{CE}{DE} = \frac{5}{3} = 1.66$

Question 10

Table 1.4 shows the two facial measures listed from each of the shortlisted candidates' cartoon drawing. Who do you think will get selected by this animation firm? Support your selection with the golden ratio calculation. (You may round off to a single decimal place to compare with 1.6)

Candidate name	Two measures taken from the cartoons drawn by the candidates			
	Lengthwise (cm)		Widthwise (cm)	
	AC	AB	CE	DE
Sahil Faiz	19	14	26	22
Samanvita.H.	15	10	17	12
Gurpreet Handa	28	17.5	40	25
Aman Gupta	13	8.5	12	8.5

Table 1.4, Finding the golden ratio in cartoons

Answer

Exploration Pathway

Golden Ratio

The golden ratio is a special number approximately equal to 1.618 that appears many times in mathematics, geometry, art, architecture and other areas.

In this TACtivity, we make a divider, using ice cream sticks, to explore the things around us with the golden ratio.