Chapter 13
Surface Areas and Volumes

Exercise No. 13.1

Multiple Choice Questions:

Write the correct answer in each of the following:

1. The radius of a sphere is 2*r*, then its volume will be

(A) $\frac{4}{3}\pi r^3$

(B) $4\pi r^3$

(C) $\frac{8}{3}\pi r^3$

(D) $\frac{32}{3}\pi r^3$

Solution:

Given:

Radius(R) = 2r

Now, the volume of sphere is:

$$\begin{aligned}\frac{4}{3}\pi R^3 &= \frac{4}{3}\pi \times (2r)^3 \\ &= \frac{4}{3}\pi \times 8r^3 \\ &= \frac{24}{3}\pi r^3\end{aligned}$$

Hence, the correct option is (D).

2. The total surface area of a cube is 96 cm^2. The volume of the cube is:

(A) 8 cm^3

(B) 512 cm^3

(C) 64 cm^3

(D) 27 cm^3

Solution:

The formula of total surface area of cube is $6(\text{edge})^2$.

So, $6(\text{edge})^2 = 96$

$(\text{edge})^2 = \frac{96}{6}$

$(\text{edge})^2 = 16$

$\text{edge} = \sqrt{16}$

$\text{edge} = 4\text{cm}$

Therefore, the volume of cube is = $(\text{edge})^3 = (4\text{cm})^3 = 64\text{cm}^3$.

Hence, the correct option is (C).

3. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is:
(A) 4.2 cm
(B) 2.1 cm
(C) 2.4 cm
(D) 1.6 cm

Solution:

The formula of volume of the cone is $\frac{1}{3}\pi r^2 h$.

So, $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (2.1)^2 \times 8.4$

Now, volume of sphere=$\frac{4}{3}\pi r_1^3$

According to the question,

$\frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi (2.1)^2 \times 8.4$

$4r_1^3 = (2.1)^2 \times 8.4$

$r_1^3 = \frac{(2.1)^2 \times 8.4}{4}$

$r_1^3 = (2.1)^2 \times 2.1$

$r_1^3 = (2.1)^3$

$r_1 = 2.1$

Hence, the correct option is (B).

4. In a cylinder, radius is doubled and height is halved, curved surface area will be
(A) halved
(B) doubled
(C) same

(D) four times

Solution:

The formula of curved surface area of cylinder is $2\pi rh$.

According to the question, when radius is double and height is halved, then the curve surface are will be:

$= 2\pi \times (2r) \times \frac{h}{2}$

$= 2\pi rh$

Since, the curved surface area will be same.

Hence, the correct option is (C).

5. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is

(A) $2\pi r(l+r)$

(B) $\pi r\left(l+\frac{r}{4}\right)$

(C) $\pi r(l+r)$

(D) $2\pi rl$

Solution:

The formula of Total surface area of cone = Area of the base + Curved surface area of cone

$$= \pi\left(\frac{r}{2}\right)^2 + \pi\left(\frac{r}{2}\right) \times 2l$$

$$= \frac{\pi r}{2}\left(\frac{r}{2} + 2l\right)$$

$$= \frac{\pi r}{2}(r + 4l)$$

$$= \pi r\left(l + \frac{r}{4}\right)$$

Hence, the correct option is (B).

6. The radii of two cylinders are in the ratio of 2:3 and their heights are in the ratio of 5:3. The ratio of their volumes is:

(A) 10 : 17

(B) 20 : 27

(C) 17 : 27

(D) 20 : 37

Solution:

Let the radii of two cylinders be 2r and 3r respectively and their heights are in the ratio 5h and 3h. The volume of cylinders be V_1 and V_2. So,

$$\frac{V_1}{V_2}=\frac{\pi(2r)^2(5h)}{\pi(3r)^2(3h)}$$

$$=\frac{4r^2\times 5h}{3r^2\times 3h}$$

$$=\frac{20}{27}$$

Hence, the correct option is (B).

7. The lateral surface area of a cube is 256 m². The volume of the cube is
(A) 512 m³
(B) 64 m³
(C) 216 m³
(D) 256 m³

Solution:

The formula of the lateral surface area of a cube is $4(\text{edge})^2$.

So,

$$4(\text{edge})^2=256$$

$$(\text{edge})^2=\frac{256}{4}$$

$$(\text{edge})^2=64$$

$$\text{edge}=\sqrt{64}=8\text{m}$$

Therefore, volume of cube $=(\text{edge})^3=8^3=512\text{m}^3$

Hence, the correct option is (A).

8. The number of planks of dimensions (4 m × 50 m × 20 m) that can be stored in a pit which is 40 m long, 12m wide and 160 m deep is
(A) 1900
(B) 1920
(C) 1800
(D) 1840

Solution:

Volume of pit $=(16\times 12\times 4)\text{m}^3$

Volume of a plank $=(4\times 0.5\times 0.2)\text{m}^3$

Now, the required number of planks is calculated as follows:

$$\text{Required number of planks} = \frac{\text{Volume of pit}}{\text{Volume of plank}}$$
$$= \frac{16 \times 12 \times 4}{4 \times 0.5 \times 0.2}$$
$$= 1920$$

Therefore, the required number of planks is 1920.
Hence, the correct option is (B).

9. The length of the longest pole that can be put in a room of dimensions (10 m × 10 m × 5m) is
(A) 15 m
(B) 16 m
(C) 10 m
(D) 12 m

Solution:

$$\text{The formula of the longest pole } = \sqrt{l^2 + b^2 + h^2}$$
$$= \sqrt{10^2 + 10^2 + 5^2}$$
$$= \sqrt{100 + 100 + 25}$$
$$= \sqrt{225}$$
$$= 15\text{cm}$$

Hence, the correct option is (A).

10. The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratios of the surface areas of the balloon in the two cases is
A) 1 : 4
(B) 1 : 3
(C) 2 : 3
(D) 2 : 1

Solution:

As we know that balloon is hemispherical in shape.
The formula of surface area of hemispherical balloon of radius is $2\pi r^2$.
So, the ratio of the surface areas of two balloons = 1:4
Hence, the correct option is (A).

Exercise No. 13.2

Short Answer Questions with Reasoning:

Write True or False and justify your answer in each of the following:

1. The volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere.

Solution:
We consider the radius of the sphere be r.
Given in the question, that height and diameter of cylinder are equal to the diameter of sphere.
Therefore, the radius of cylinder is r and its height be 2r.

As, volume of sphere = $\frac{2}{3}$ Volume of cylinder

$$\frac{4}{3}\pi r^3 = \frac{2}{3}\left(\pi r^2 \times 2r\right)$$

$$= \frac{4}{3}\pi r^3$$

Hence, the given statement is true.

2. If the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged.

Solution:
We consider the original radius of the cone be r and height ne h.

The volume of cone = $\frac{1}{3}\pi r^2 h$

As we know that the radius of a height circular cone is halved and height is doubled. So,

$$V = \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times 2h$$

$$= \frac{1}{3}\pi \times \frac{r^2}{4} \times 2h$$

$$= \frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right)$$

Since, the volume become half of the original volume.
Hence, the given statement is false.

3. In a right circular cone, height, radius and slant height do not always be sides of a right triangle.

Solution:
We consider that in a right circle cone , height (h), radius (r), and slant height(l) are always the sides of a right triangle that is $l^2 = r^2 + h^2$.
Hence, the given statement is false.

4. If the radius of a cylinder is doubled and its curved surface area is not changed, the height must be halved.

Solution:
We consider that radius and height of the cylinder be r and h respectively.
So, the curved surface area of cylinder = $2\pi rh$
Now, according to the question, when radius is doubled and the curved surface area is not changed that means the height must be halved. So,

The formula of curved surface area = $2\pi \times (2r) \times \frac{h}{2} = 2\pi rh$

Hence, the given statement is true.

5. The volume of the largest right circular cone that can be fitted in a cube whose edge is 2*r* equals to the volume of a hemisphere of radius *r*.

Solution:
Given in the question, edge of cube = 2r, then height of cube becomes h = 2r.

The formula of volume of a cone = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\pi r^2 (2r)$

= $\frac{2}{3}\pi r^3$

The formula of volume of a hemisphere = $\frac{2}{3}\pi r^3$

Therefore, the volume of a cone is equal to the volume of a hemisphere.
Hence, the given statement is true.

6. A cylinder and a right circular cone are having the same base and same height. The volume of the cylinder is three times the volume of the cone.

Solution:
We consider that the radius of the base of a cylinder and a right circular cone be r and height be h. So,
The formula of volume of a cylinder = $\pi r^2 h$

The formula of volume of a cone = $\frac{1}{3}\pi r^2 h$

Since, Volume of a cylinder = 3 $\times$ Volume of a cone
Therefore, the volume of a cylinder is three times the volume of the right circular cone.
Hence, the given statement is true.

7. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is 1 : 2 : 3.

Solution:
Let radius of hemisphere is r.

The formula of volume of a cone, = $V_1 = \frac{1}{3}\pi r^2 h$

$V_1 = \frac{1}{3}\pi r^2 (r)$ [As h = r]

$= \frac{1}{3}\pi r^3$

Volume of a hemisphere, $V_2 = \frac{2}{3}\pi r^3$

volume of cylinder, $V_3 = \pi r^2 h = \pi r^2 \times r = \pi r^3$ [As h = r]

$V_1 : V_2 : V_3 = \frac{1}{2}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 = 1 : 2 : 3$

Therefore, the ratio of their volumes is 1 : 2 : 3.
Hence, the given statement is true.

8. If the length of the diagonal of a cube is $6\sqrt{3}$ cm, then the length of the edge of the cube is 3 cm.

Solution:
Given, the length of the diagonal of a cube = 6√3 cm
we consider the edge (side) of a cube be a cm.
So, diagonal of a cube = a√3
6√3 = a√3
a = 6 cm
Therefore, the edge of a cube is 6 cm.
Hence, the given statement is false.

9. If a sphere is inscribed in a cube, then the ratio of the volume of the cube to the volume of the sphere will be 6 : π.

Solution:
We consider a be the edge of the cube.

As the sphere is inscribed in a cube, the radius of the sphere is $\frac{a}{2}$.

$V_1 = \text{Volume of cube} = (\text{edge})^3 = a^3$

$V_2 =$ Volume of sphere $= \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{1}{3}\pi a^3$

$V_1 : V_2 = a^3 : \frac{1}{3}\pi a^3 = 1 : \frac{\pi}{6} = 6 : \pi$

Hence, the given statement is true.

10. If the radius of a cylinder is doubled and height is halved, the volume will be doubled.

Solution:

Let the cylinder have radius r and height h.

So, the volume of cylinder (V_1) = $\pi r^2 h$

According to the question, when radius of cylinder is doubled and height is halved$\left(V_2\right)$. So,

$$V_2 = \pi\left(2r\right)^2 \times \frac{h}{2}$$
$$= \pi \times 4r^2 \times \frac{h}{2}$$
$$= \pi \times 2r^2 \times h$$
$$= 2\pi r^2 h$$
$$= 2V_1$$

Hence, the given statement is true.

Exercise No. 13.3

Short Answer Questions:

1. Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. Give your answer to the nearest integer. [Use π = 3.14]

Solution:

Given, radius of each metal sphere = 2cm

So, volume of a metallic sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times 3.14\times(2\text{cm})^3$$

$$= \frac{100.48}{3}\text{cm}^3$$

$$= 33.49\text{cm}^3$$

Now, the volume of 16 such a sphere $= 16\times 33.49\text{cm}^3 = 535.84\text{cm}^3$

Dimensions of internal box is 16 cm × 8 cm × 8 cm.

Now, internal volume of a rectangular box =16 cm × 8 cm × 8 cm=1024cm^3

Volume of the preservative liquid = $1024\text{cm}^3 - 535.84\text{cm}^3 = 488.16\,\text{cm}^3$

2. A storage tank is in the form of a cube. When it is full of water, the volume of water is 15.625 m^3.If the present depth of water is 1.3 m, find the volume of water already used from the tank.

Solution:

Suppose the edge of the cube is a.

So, the volume of cube= a^3

Now, the volume of water when the cube is full of water is 15.625 m^3.

According to the question,

a^3 =15.625 m^3

$$a = \sqrt[3]{15.625m^3}$$

$$= \sqrt[3]{(2.5)^3\,m^3}$$

$$= 2.5m$$

So, edge of cube = 2.5cm

Now, present depth of water in the tank = 1.3m [Given]

So, remaining depth=2.5m – 1.3m=1.2m

Therefore, Volume of water already used in the tank = 2.5m$\times$2.5 m$\times$1.2m=7.5 m^3

3. Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm, when it is completely immersed in water.

Solution:

Given: Diameter of spherical ball=4.2cm

Now, radius of spherical ball (r) = $\frac{4.2}{2}$ cm=2.1cm

Amount of water displaced by solid spherical ball = Volume of solid spherical ball

So, volume of spherical ball = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times(2.1)^3$

$$= \frac{88}{21}\times\frac{21}{10}\times\frac{21}{10}\times\frac{21}{10} = 38.808cm^3$$

Therefore, the amount of water displaced by solid spherical ball when it completely immersed in water is $38.808cm^3$.

4. How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of the base is 12 m?

Solution:

Given: Height is h=3.5 m and the radius of the base is r=12 m.

Now, Slant height (l) will be:

$$\sqrt{h^2+r^2} = \sqrt{(3.5m)^2+(12m)^2}$$
$$= \sqrt{12.25m^2+144m^2}$$
$$= \sqrt{156.25m^2}$$
$$= 12.5m$$

So, area of canvas required:

$= \pi rl$

$= \frac{22}{7}\times 12m\times 12.5m$

$= 471.42m^2$

5. Two solid spheres made of the same metal have weights 5920 g and 740 g, respectively. Determine the radius of the larger sphere, if the diameter of the smaller one is 5 cm.

Solution:

Let weight of one solid sphere is $m_1 = 5920g$ and its radius is r_1. Similarly, weight of another solid sphere is $m_2 = 740g$ and its radius is r_2 .

Now, diameter of the smaller sphere: $r_1 = 5\text{m}$

So, it's radius $= r_2 = \frac{5}{2}$

As we know that:

$$\text{Density}(\text{D}) = \frac{\text{Mass}}{\text{Volume}} \text{ or } \text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

Then,

$$V_1 = \frac{5920}{D}\text{cm}^3 \qquad \ldots \text{(I)}$$

And: $V_2 = \frac{740}{D}\text{cm}^3 \qquad \ldots$ (II)

Now, dividing equation, (I) and (II), get:

$$\frac{V_1}{V_2} = \frac{\frac{5920}{D}\text{cm}^3}{\frac{740}{D}\text{cm}^3}$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{5920}{740} \text{ [Volume of a sphere is } \frac{4}{3}\pi r^3 \text{]}$$

$$\frac{r_1^3}{r_2^3} = \frac{592}{74}$$

$$\left(\frac{r_1}{\frac{5}{2}}\right)^3 = \frac{592}{74}$$

$$\begin{aligned} r_1^3 &= \frac{592}{74} \times \frac{125}{8} \\ &= \frac{74000}{592} \\ &= 125 \\ r_1^3 &= 125 \\ r_1 &= 5cm \end{aligned}$$

Hence, the radius of larger sphere is 5 cm.

6. A school provides milk to the students daily in a cylindrical glasses of diameter 7 cm. If the glass is filled with milk upto an height of 12 cm, find how many litres of milk is needed to serve 1600 students.

Solution:

Given: diameter of cylinder glass = 7cm

Glass is filled with milk upto an height of 12cm.

Radius of cylinder glass = $\frac{\text{Diameter}}{2} = \frac{7}{2} = 3.5\text{cm}$

Volume of cylinder glass (V) $= \pi r^2 h$

$$= \frac{22}{7} \times (3.5)^2 \times 12$$

$$= \frac{22}{7} \times 12.25 \times 12$$

$$= 22 \times 1.75 \times 12$$

$$= 462\text{cm}^2$$

Quantity of milk needed for 1600 students = $(462 \times 1600)\, cm^3$

$$= 739200 cm^3$$

$$= 739.2\text{litres}$$

7. A cylindrical roller 2.5 m in length, 1.75 m in radius when rolled on a road was found to cover the area of 5500 m2. How many revolutions did it make?

Solution:

Given: Length of cylinder roller(h) = 2.5 m

Radius of cylinder roller(r) = 1.75 m

Total area on road covered by cylinder roller=5500 m^2

Now, area covered in one revolution = lateral surface area of the cylinder

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 1.75m \times 2.5m$$

$$= 27.5m^2$$

Since, the number of revolution (n) made by the roller is:

$$= \frac{\text{Total area covered}}{\text{Area covered in one revolution}}$$

$$= \frac{5500}{27.5}$$

$= 200$ revolutions.

8. A small village, having a population of 5000, requires 75 litres of water per head per day. The village has got an overhead tank of measurement 40 m × 25 m × 15 m. For how many days will the water of this tank last?

Solution:

Water contained in overhead tank = 40 m × 25 m × 15 m.

$= (40\times25\times15)\,m^3$

$= (40\times25\times15\times1000)$ litres

Now, water needed for 5000 villages for one day$=(5000\times75)$ litres $= 375000$litres

So, total number of days the water of the tank last:

$= \dfrac{40\times25\times15\times1000}{375000}$

$= \dfrac{15000}{375}$

$= 40$days

9. A shopkeeper has one spherical laddoo of radius 5cm. With the same amount of material, how many laddoos of radius 2.5 cm can be made?

Solution:

Number of ladoos $= \dfrac{\text{Volum of spherical ladoo of radius 5cm}}{\text{Volume of one spherical ladoo of radius 2.5cm}}$

$= \dfrac{\frac{4}{3}\times\frac{22}{7}\times5^3}{\frac{4}{3}\times\frac{22}{7}\times\left(\frac{5}{3}\right)^3}$

$= 8$

10. A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm. Find the volume and the curved surface of the solid so formed.

Solution:

According to the question, the solid formed is a cone whose height of a cone, h = 8 cm and radius of a cone, r = 6 cm. Slant height of a cone, l = 10 cm. So,

Volume of a cone = $\frac{1}{3}\pi r^2h = \frac{1}{3}\times\frac{22}{7}\times6\times6\times8$

$\dfrac{6336}{21} = 301.7\,cm^3$

Now, curved surface of the area of cone = πrl

$$= \frac{22}{7} \times 6 \times 10$$

$$= \frac{1320}{7}$$

$$= 188.5\ cm^2$$

Therefore, the volume and surface area of a cone are 301.7 cm^3 and 188.5 cm^2, respectively.

Exercise No. 13.4

Long Answer Questions:

1. A cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick. If the outer diameter is 16 cm and its length is 100 cm, find how many cubic centimeters of iron has been used in making the tube ?

Solution:

Given:

Outer diameter of cylinder tube(d) = 16 cm

Thickness of the iron sheet = 2cm

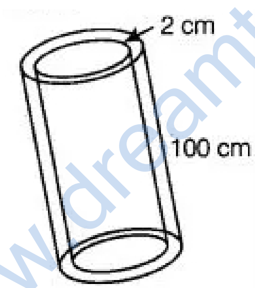
Height of the cylindrical tube (h) = 100cm

Outer radius of a cylindrical tube $\left(r_1\right)=\frac{d}{2}=\frac{16}{2}cm=8cm$

Inner radius of a cylindrical tube=$\left(r_1-\text{thickness of the iron sheet}\right)$

= 8-2

= 6cm



Now, volume of metal used in making cylindrical tube = Outer volume of a cylindrical tube – Inner volume of cylindrical tube

$$=\pi r_1^2h-\pi r_2^2h$$
$$=\pi h\left(r_1^2-r_2^2\right)$$
$$=\frac{22}{7}\times100\left(8^2-6^2\right)$$
$$=\frac{22}{7}\times100\times(8-6)\times(8+6)$$
$$=2200\times4$$
$$=8800\text{cm}^2$$

Hence, 8800cm^2 of iron has been used in making the tube.

2. A semi-circular sheet of metal of diameter 28cm is bent to form an open conical cup. Find the capacity of the cup.

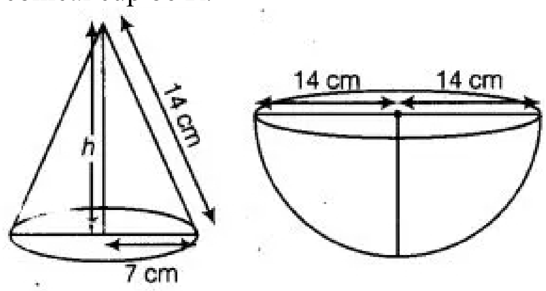
Solution:

Given: Diameter of a semi-circular sheet is 28 cm. So,

Radius(r) = $\frac{28}{2}$=14cm

$=14\text{cm}$

Suppose the radius of a conical cup be R.



So, Circumference of base of cone = Circumference of semi-circle

$2\pi R=\pi r$

$2\pi R=\pi\times 14$

$R=7\text{cm}$

Now,

$h=\sqrt{l^2-R^2}$

$=\sqrt{14^2-7^2}$

$=\sqrt{196-49}$

$=\sqrt{147}$

$=12.1243\text{cm}$

Volume of conical cup = $\frac{1}{3}\pi R^2 h$

$=\frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 12.1243$

$=622.38cm^3$

Hence, the capacity of an open conical cup is $622.38cm^3$.

3. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 m

(i) How many students can sit in the tent if a student, on an average, occupies $\frac{5}{7}$ m^2 on the ground?

(ii) Find the volume of the cone.

Solution:

(i) Given:

Radius of the base of a conical tent = 5cm

Area needs to sit a student on the ground $=\frac{5}{7}\text{m}^2$

So, area of the base of a conical tent $=\pi r^2$

$$=\frac{22}{7}\times 5\times 5\text{m}^2$$

Now, number of students $=\dfrac{\text{Area of the base of a conical tent}}{\text{Area needs to sit a student on the ground}}$

$$=\frac{\frac{22\times 5\times 5}{7}}{\frac{5}{7}}$$

$$=\frac{22}{7}\times 5\times 5\times\frac{7}{5}$$

$$=110$$

Hence, 110 students can sit in the conical tent.

(ii) Given: area of the cloth to form a conical tent = $165m^2$

Radius of the base of a conical tent(r) = 5m

Now, Curved surface area of a conical tent = Area of cloth to form a conical tent

$$\pi rl=165$$

$$\frac{22}{7}\times 5\times l=165$$

$$l=\frac{165\times 7}{22\times 5}$$

$$=\frac{33\times 7}{22}$$

$$=10.5m$$

Now, height of the conical tent is calculated as:

$$h=\sqrt{l^2-r^2}$$

$$=\sqrt{(10.5)^2-(5)^2}$$

$$=\sqrt{110.25-25}$$

$$=\sqrt{85.25}$$

$$=9.23$$

Volume of a cone $=\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 923$

$= \frac{1}{3} \times \frac{1550 \times 923}{7}$

$= \frac{50765}{7 \times 3}$

$= 241.7 m^3$

$\approx 242 m^3$

Hence, the volume of the cone is $242 m^3$.

4. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill to its capacity. Calculate the volume of water pumped into the tank.

Solution:

Given: The tank contains 50 kilolitres of water.

Internal diameter of a hemispherical tank= 14 cm

So, internal radius of hemispherical tank = $\frac{\text{Diameter}}{2} = \frac{14}{2} m = 7m$

Now, Volume of hemisphere tank = $\frac{2}{3} \pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 7^3$

$= \frac{44 \times 49}{3}$

$= 718.66 m^3$

The tank contains 50 kilolitres of water = 50,000litres = $\frac{50,000}{1,000} m^3 = 50 m^3$

Volume of water pumped into the tank = $718.66 m^3 - 50 m^3 = 668.66 m^3$

5. The volumes of the two spheres are in the ratio 64 : 27. Find the ratio of their surface areas.

Solution:

Suppose V_1 and V_2 be the volume of two sphere.

So, according to the question:

$\frac{V_1}{V_2} = \frac{64}{27}$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27} \quad \text{[As volume of sphere is } \frac{4}{3}\pi r^3\text{]}$$

$$\frac{r_1^3}{r_2^3} = \frac{64}{27}$$

$$\frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$

$$\frac{r_1}{r_2} = \frac{4}{3} \qquad \ldots \text{(I)}$$

Let surface area of both spheres are SA_1 and SA_2 respectively. So,

$$\frac{SA_1}{SA_2} = \frac{4\pi r_1^2}{4\pi r_2^2} \quad \text{[As surface area of sphere is } 4\pi r^2\text{]}$$

$$\frac{SA_1}{SA_2} = \frac{r_1^2}{r_2^2}$$

$$\frac{SA_1}{SA_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$\frac{SA_1}{SA_2} = \left(\frac{4}{3}\right)^2 \quad \text{[Using equation (I)]}$$

$$\frac{SA_1}{SA_2} = \frac{16}{9}$$

Hence, the ratio of the surface area of the two sphere is 16 : 9.

6. A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between.

Solution:

Side of a cube = 4cm

As cube contains a sphere touching its sides. So, the diameter of the sphere= 4cm

Side of cube = Diameter of sphere

4 = **Radius of sphere**

Radius of sphere $= \frac{4}{2} = 2$

Volume of the gap = Volume of cube – Volume of sphere

$$= (\text{Side})^3 - \frac{4}{3}\pi r^3$$

$$= (4)^3 - \frac{4}{3}\pi \times 2^3 \quad \text{[Since, side of cube=diameter of sphere]}$$

$$= \left(64 - \frac{4}{3} \times \frac{22}{7} \times 8\right)$$
$$= 64 - 33.52$$
$$= 30.48 cm^3$$
Hence, the volume of the gap between a cube and a sphere is $30.48 cm^3$.

7. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceed its height?

Solution:

Suppose the radius of sphere = r = Radius of a right circular cylinder
According to the question,
Volume of right circular cylinder = Volume of a sphere [Given]

$$\pi r^2 h = \frac{4}{3} \pi r^3$$
$$h = \frac{4}{3} r$$

As diameter of the cylinder = 2r

Increased diameter from height of the cylinder = $2r - \frac{4r}{3} = \frac{2r}{3}$

Now, percentage increase in diameter of the cylinder $= \dfrac{\frac{2r}{3} \times 100}{\frac{4}{3} r}$

$= 50\%$
Hence, the diameter of the cylinder exceeds its height by 50%.

8. 30 circular plates, each of radius 14 cm and thickness 3 cm are placed one above the another to form a cylindrical solid. Find:
(i) the total surface area
(ii) volume of the cylinder so formed.

Solution:

Given: radius of a circular plate(r) = 14cm
Thickness of the circular plates = 3cm
The height of the cylinder solid (h) = Thickness of 30 circular plates = 30×3 = 90 cm
(i) Total surface area of the cylinder solid so formed

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 14 \times (90 + 14)$

$= 44 \times 2 \times 104$

$= 9152 cm^2$

Hence, the total surface area of the cylinder solid is $9152 cm^2$.

(ii) Volume of the cylinder so formed = $\pi r^2 h$

$= \frac{22}{7} \times 14^2 \times 90$

$= \frac{22}{7} \times 14 \times 14 \times 90$

$= 22 \times 28 \times 90$

$= 55440 cm^3$

Hence, the volume of the cylinder so formed is $55440 cm^3$.