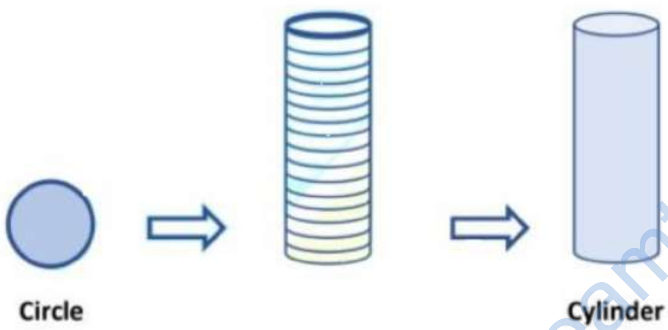
# Chapter – 13

# Surface Areas and Volumes

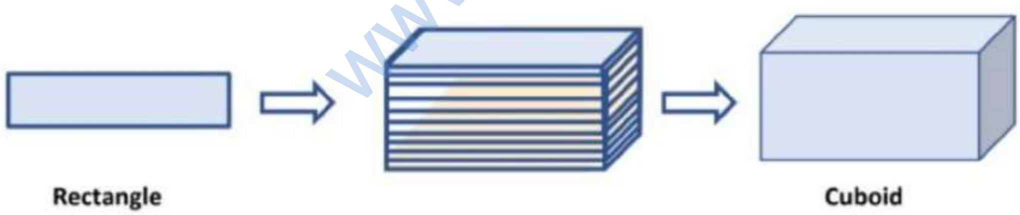
## Introduction to Area and Volume

The shapes that we have studied in previous classes like rectangles, squares, triangle, and circles are the plane figures which can be easily drawn on our notebooks. Suppose, we cut out a circle shape from a cardboard sheet. And then we cut many such circles identical to the first one and then pile them up in a single column. [Shown in the figure] Then the shape we obtained will be a 3-dimensional shape



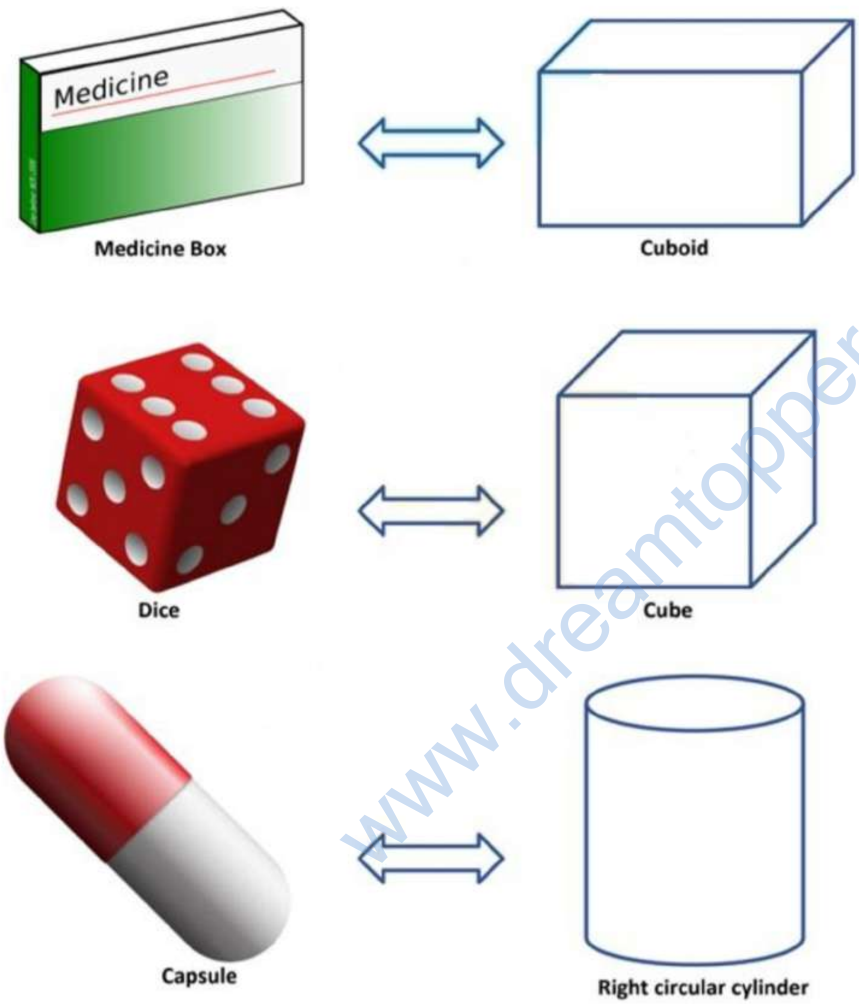



By this process, we shall obtain some solid figures such as a cuboid and cube as shown below.




All solid objects occupy some space and have three dimensions - length, breadth, and height or depth. Three dimensional (3-d)shapes have four parts that set them apart from 2-d shapes viz. faces, vertices, edges, and volume.

Some real-life examples which resemble solid shapes as shown below.



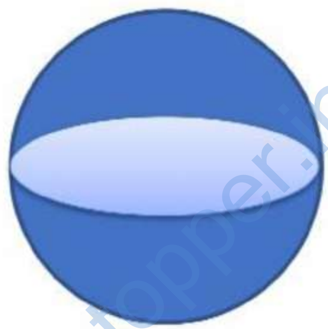
Ice-cream cone

⇔

Right circular cone

Football

⇔



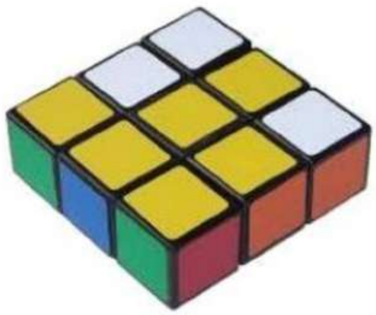
Sphere

## Surface Area of a Cuboid

Cuboid: In our day to day life, we come across various objects which are in the shape of a cuboid. Some examples are shown below.

Wooden Box

Match Box

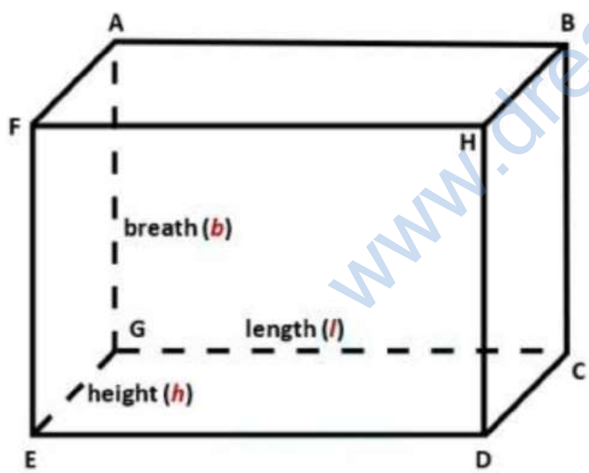


Floppy Cube

Book

If we observe closely, we find that all these objects are made up of six rectangular plane regions. A cuboid is a three-dimensional shape that has length, width, and height. The cuboid shape has six surfaces called faces. Each face of a cuboid is a rectangle, two adjacent faces of a cuboid meet in a line segment is called edge and all of a cuboid's corners (called vertices) are right angles.

Ultimately, a cuboid has the shape of a rectangular box.



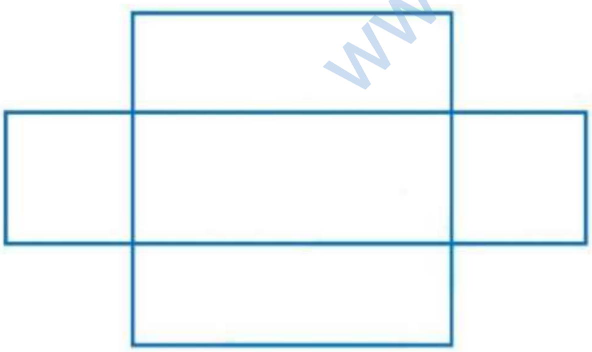

A cuboid has six faces (FHDE, HBCD, CDEG, GEFA, FABH & ABCG), eight vertices (A, B, C, D, E, F, G & H), twelve edges (AB, BH, FH, AF, BC, CD, DH, DE, FE & GC).

If we want to cover cuboid by gift paper, first we would need a rectangular piece to cover the bottom of the cuboid.

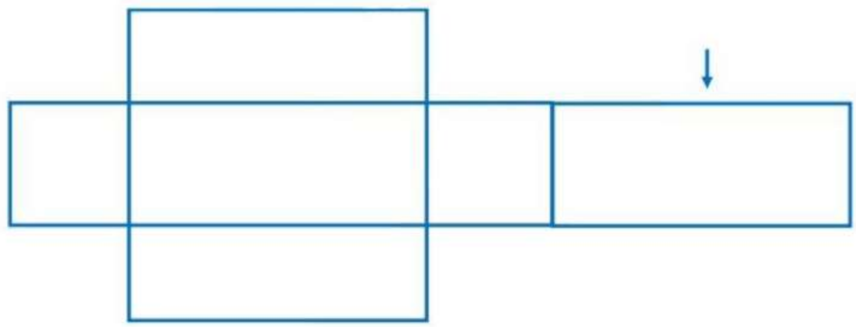
Then we would need two long rectangular pieces to cover the two side ends. Now, it would look like as shown below.

Now to cover the front and back end, we would need two more rectangle pieces of the same size as them, we get a figure as shown below.

This figure, when opened out, would look like as shown below.



Finally, to cover the top of the cuboid, we would require another rectangular piece exactly like the one at the bottom, which if we attached on the right side, it would look like as shown below.



Therefore, we have used six rectangular pieces to cover the complete outer surface of the cuboid.

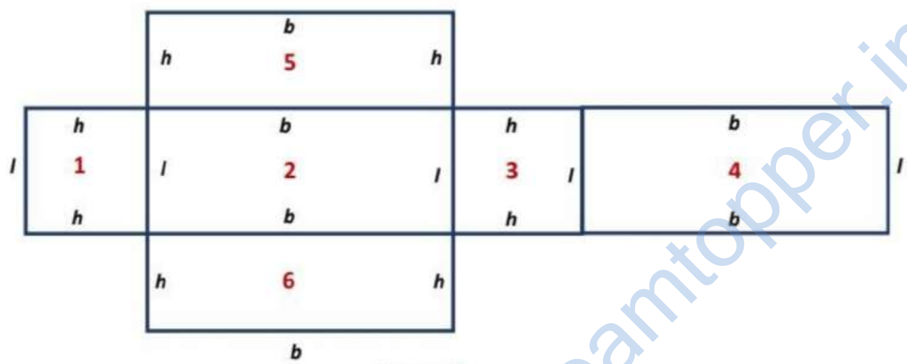



Figure: 1



This shows us that the outer surface of a cuboid is made up of six rectangular surfaces, called faces of the cuboid, whose area can be formed by multiplying the length by breath for each of them separately and then adding the six areas together.

Now, if we take the length of the cuboid as l, breadth as b and the height as h, then the figure with these dimensions would be like the shape shown in the figure: 1.

So, the sum of the areas of the six rectangles is:

Area of rectangle 1 = $(l \times h)$ + Area of rectangle 2 = $(l \times b)$ +
Area of rectangle 3 = $(l \times h)$ + Area of rectangle 4 = $(l \times b)$ +
Area of rectangle 5 = $(b \times h)$ + Area of rectangle 6 = $(b \times h)$

$\Rightarrow$Surface Area of Cuboid = $2(l \times b) + 2(b \times h) + 2(l \times h)$.

$$= 2[(l \times b) + (b \times h) + (l \times h)].$$

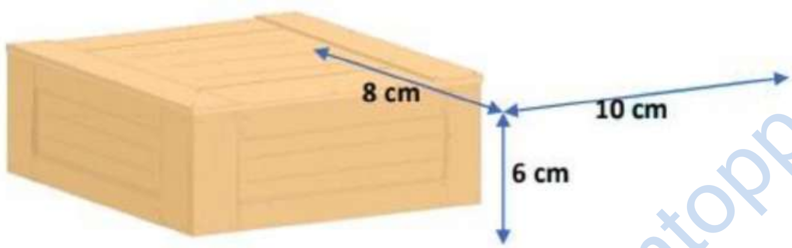
$= 2(lb + bh + lh)$.

Where l, b and h are respectively the three edges of the cuboid.

**Surface Area of Cuboid = $2(lb + bh + lh)$**

Note: Surface Area of Cuboid is also known as the Total Surface Area of Cuboid.

Example: Find the surface area of a wooden box whose length, breadth and height are 10 cm, 8 cm, and 6 cm, respectively.






Solution: Clearly, this wooden box is in the form of a cuboid.

Here, l = 10 cm, b = 8 cm and h = 6 cm.

Therefore, surface area of the wooden box $= 2(lb + bh + lh)$.

$= 2[(10 \text{ cm} \times 8 \text{ cm}) + (8 \text{ cm} \times 6 \text{ cm}) + (10 \text{ cm} \times 6 \text{ cm})]$.

$= 2(80 \text{ cm}^2 + 48 \text{ cm}^2 + 60 \text{ cm}^2)$.

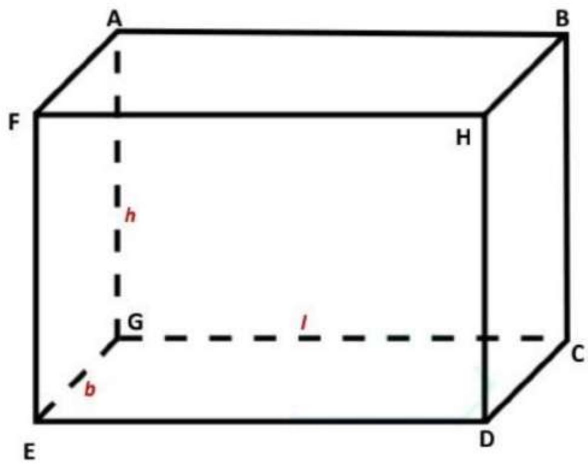
$= 2 \times 188 \text{cm}^2$.

$= 376 \text{ cm}^2$.

Hence, the surface area of the wooden box $= 376 \text{ cm}^2$.

Lateral Surface Area of a Cuboid: If out of the six faces of a cuboid, we only find the sum of the areas of four faces leaving the bottom and top faces. This sum is called the lateral surface area of a cuboid.

Consider a cuboid of length l, breadth b and height h as shown in the figure below.




The lateral surface area of a cuboid

= Area of face HBCD + Area of face ABCG + Area of face GEFA + Area of face FHDE.

$= (b \times h) + (l \times h) + (b \times h) + (l \times h)$.

$= 2(bh + lh)$.

$= 2h\ (b + l)$.

Lateral surface area of a cuboid $= 2h\ (b + l)$.

Example: Find the lateral surface area of a wooden box whose length, breadth and height are 10 cm, 8 cm, and 6 cm, respectively.

Solution: Clearly, this wooden box is in the form of a cuboid.

Here, $l = 10$ cm, $b = 8$ cm and $h = 6$ cm.

Therefore, lateral surface area of the wooden box $= 2h\ (b + l)$.

$= 2\times 6$ cm(8 cm +10 cm) $= 2\times 6$ cm(18 cm) $= 12$ cm $\times$ 18cm

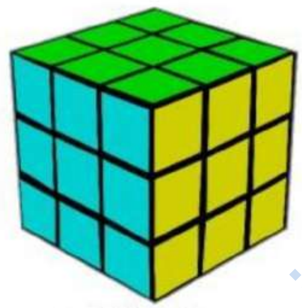
$= 216\ cm^2$

Hence, the lateral surface area of the wooden box $= 216\ cm^2$

## Surface Area of a Cube

Cube: In our everyday life, we come across objects like a dice, Rubik's cube, Sugar cube, and Ice cube, etc. These objects are in the shape of a cube. All these objects are made of six square plane regions.

Dice



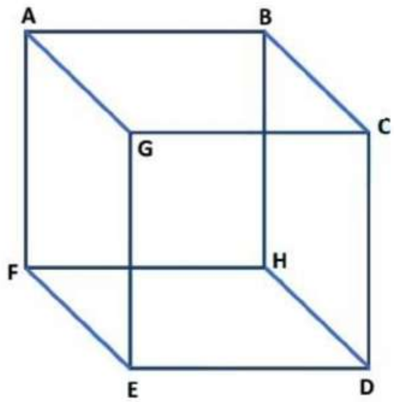
Rubik's cube

Sugar cubes

Ice cube



A cube is a three-dimensional shape whose length, breadth and height are all equal. The cube has six surfaces called faces. Each face of a cube is a square, two adjacent faces of a cube meet in a line segment called edge and all of a cube's corners (called vertices) are right angles. Ultimately, a cube has the shape of a square box.

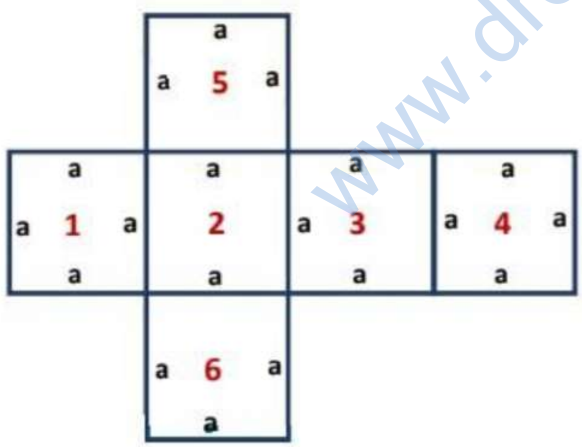



A cube has six faces (FHDE, FAGE, ABCG BCHD, ABFH & GCDE), eight vertices (A, B, C, D, E, F, G & H), twelve edges (AB, BC, CG, GA, BH, HD, DC, DE, FE, FH, AF, & GE).

In the case of a cube, its length, breadth, and height are equal.

Then, side of cube = Length = Breadth = Height

So, the figure with these dimensions would be like the shape shown below. Here, side of cube = a.




So, the sum of the areas of the six squares is:

Area of square 1 = (a × a) + Area of square 2 = (a × a) + Area of square 3 = (a × a) + Area of square 4 = (a × a) + Area of square 5 = (a × a) + Area of square 6 = (a × a).

⇒Surface Area of Cube = 2(a × a) + 2(a × a) + 2(a × a).

$$= 2[(a \times a) + (a \times a) + (a \times a)].$$

$$= 2(a^2 + a^2 + a^2).$$

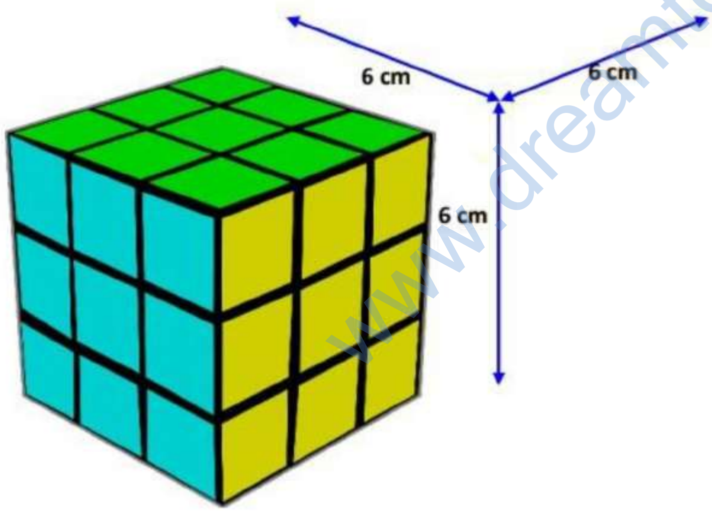
$$= 2(3a^2).$$

$$= 6\,a^2.$$

Where a is the length of edges of the cube.

$$\text{Surface Area of Cube} = 6\,a^2$$

Example: Find the surface area of a Rubik's cube whose edge is 6cm.






Therefore, the surface area of Rubik's cube = $6a^2$.

$= 6(6^2)\text{cm}^2$.

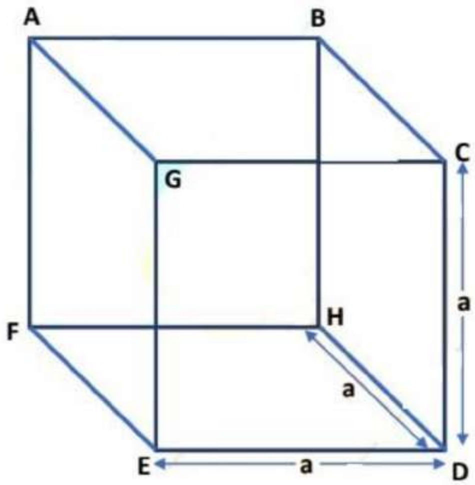
$= 6 \times 36\ \text{cm}^2$.

$= 216\ \text{cm}^2$.

Hence, the surface area of Rubik's cube = 216 $cm^2$.

Lateral Surface Area of a Cube: If out of the six faces of a cube, we only find the sum of the areas of four faces leaving the bottom and top faces. This sum is called the lateral surface area of the cube.

Consider a cube of the side as 'a' which is shown in the figure below.




Lateral surface area of cube

= Area of face HBCD + Area of face CDEG + Area of face GEFA + Area of face ABHF.

= (a × a) + (a × a) + (a × a) + (a × a).

= $a^2 + a^2 + a^2 + a^2 = 4a^2$

Lateral surface area of a cube = $4a^2$

Example 1: Find the lateral surface area of a dice whose edge is 5cm.

Solution: Clearly, dice is in the form of a cube.

Here, length of edge of dice = a = 5 cm.

Therefore, Lateral surface area of the dice = $4a^2$.

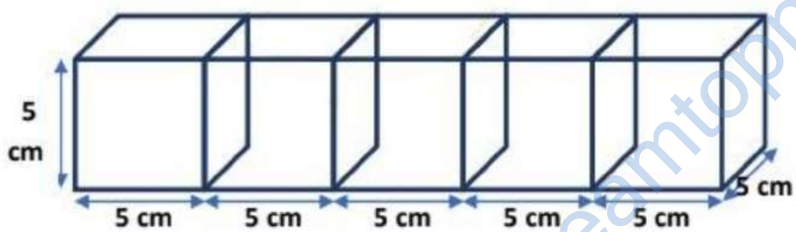
= $4(5^2)cm^2$.

= $4 \times 25\ cm^2$.

= $100\ cm^2$.

Hence, the lateral surface area of the dice = $100\ cm^2$.

Example 2:Five cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.




Solution: The dimensions of the cuboid so formed are as under :

l = length = (5+5+5+5+5) cm = 25 cm,

b = breadth = 5 cm and h = height = 5 cm.

So, surface area of the cuboid = 2(lb + bh + lh).

= $2(25\ cm \times 5\ cm + 5\ cm \times 5\ cm + 25\ cm \times 5\ cm)$.

= $2(125\ cm^2 + 25\ cm^2 + 125\ cm^2)$.

= $2(275\ cm^2)$.

= $550\ cm^2$.

Hence, Surface area of the cuboid formed = $550\ cm^2$.

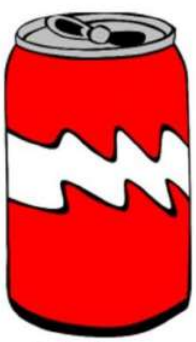
# Surface Area of a Right Circular Cylinder

Surface Area of a Right Circular Cylinder

Right Circular Cylinder: In our everyday life, we come across objects like a battery, a soda cane, a candle, chalk, a gas cylinder, a roll of paper towels, a garden roller, circular pillars, and circular pipes, etc. All these solids have a curved (lateral) surface with circular ends with an equal radius. Such objects are in the shape of right circular cylinders.


Candle


Battery

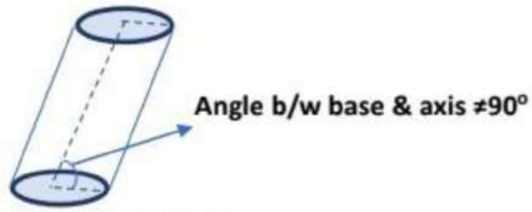

Soda Cane


Roll of paper towels



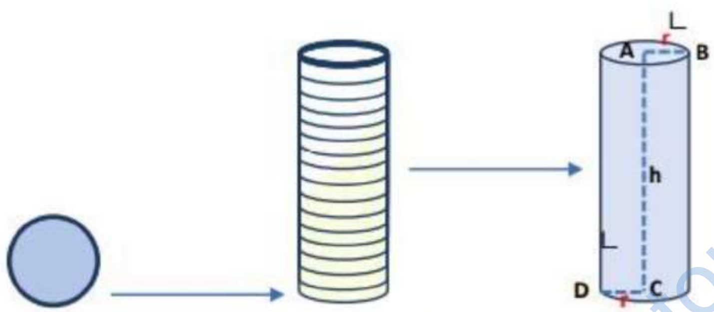
A right circular cylinder has two plane ends. Each plane end is circular, and the two plane ends are parallel. Each of the plane ends is called a base of the cylinder.

The line segment joining the centres of these two planes is called the axis of the cylinder. When this axis is not perpendicular to the circular ends, then the cylinder is not a right circular cylinder.




Not a right circular cylinder

If we take several circular sheets and stack them up vertically as shown in the below figure, then we get a right circular cylinder.




Base: Each of the circular ends of the cylinder is called its base.

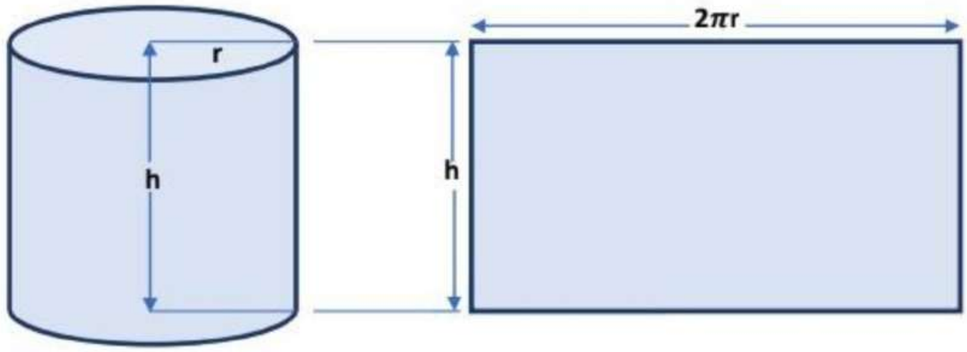
Radius: The radius of the circular bases is called the radius of the cylinder.

So, CD and AB are the radius of the cylinder. And both are equal to each other and the length of each circular edge is $2\pi r$.

Height: The shortest distance between two circular bases in a cylinder is called the height of the cylinder. So, AC is the height of the cylinder. $AC = h$.

If we take a rectangular sheet which has breadth equal to the height of the cylinder, then place the edge of the sheet of paper along with the height of the cylinder and hold it fast. Now, wrap the sheet around the cylinder then cut- off the sheet along with the height.

Remove the piece of the sheet so cut-off and spread it on a plane surface. We will find that sheet is a rectangle of length $2\pi r$ (equal to the length of each circular edge) and breadth h as shown in the below figure.




The area of the sheet gives us the curved surface area of the cylinder.

So, the curved surface area of the cylinder = area of the rectangular sheet

= length × breadth.

= perimeter of the base of the cylinder × h.

= $2\pi r \times h$.

where r is the radius of the base of the cylinder and h is the height of the cylinder.

Curved Surface Area of the Cylinder = $2\pi rh$

If the top and bottom of the cylinder are also to be covered, then we need two circles to do that, each of radius r, and thus having an area of $\pi^{2}$ each, giving us the total surface area as $2\pi rh + \pi r^2 + \pi r^2$.

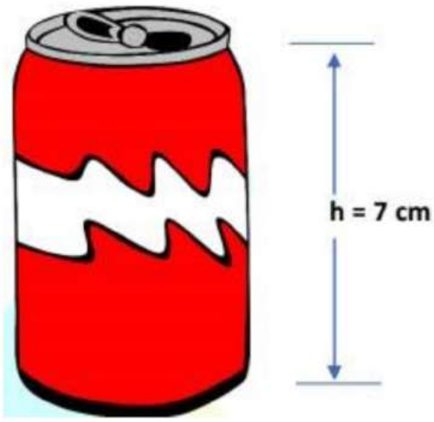
Therefore, the total surface area of the cylinder = $2\pi rh + 2\pi r^{2}$.

= $2\pi r\ (h + r)$.

Where h is the height of the cylinder and r is its radius.

Total Surface Area of the Cylinder = $2\pi r\ (h + r)$.

Example 1: The curved surface area of a soda can of height 7 cm is 44 $cm^2$. Find the radius of the base of the soda can.




Solution: Clearly, soda can is in the form of a right circular cylinder.

Here, Curved surface area of the soda can and its height is 44 $cm^2$ and 7 cm respectively.

We know that,

Curved Surface Area of Cylinder = 2πrh.

So, Curved Surface Area of the soda cane = 2πrh.

$44\ cm^2 = 2 \times \frac{22}{7} \times r \times 7$ cm.

$\Rightarrow 44\ cm^2 = 2 \times 22 \times r$ cm.

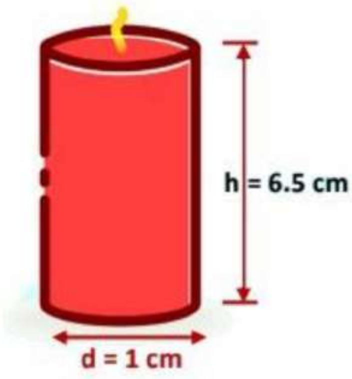
$\Rightarrow 44\ cm^2 = 44r$ cm.

$\Rightarrow r = \frac{44cm^2}{44cm}$.

$\Rightarrow r = 1$ cm.

Hence, the radius of the base of the soda cane = 1 cm.

Example 2: Find the total surface area of a candle if the height and diameter of the base of the candle are 6.5 cm and 1 cm respectively.




Solution: The candle is in the form of a right circular cylinder.

Here, the diameter of the base of the candle and its height is 1 cm and 6.5 cm respectively.

⇒ Radius of the base of the candle $= \frac{d}{2} = \frac{1cm}{2} = 0.5$ cm.

We know that,

Total Surface Area of the cylinder $= 2\pi r\,(h + r)$.

So, the total Surface Area of the candle $= 2\pi r\,(h + r)$.

$= 2 \times \frac{22}{7} \times 0.5$ cm (6.5 cm + 0.5 cm).

$= 2 \times \frac{22}{7} \times \frac{1}{2}$ cm (7 cm).

$= 22$ $cm^2$.

Here, the total Surface Area of the candle = 22 $cm^2$.

## Surface Area of a Right Circular Cone

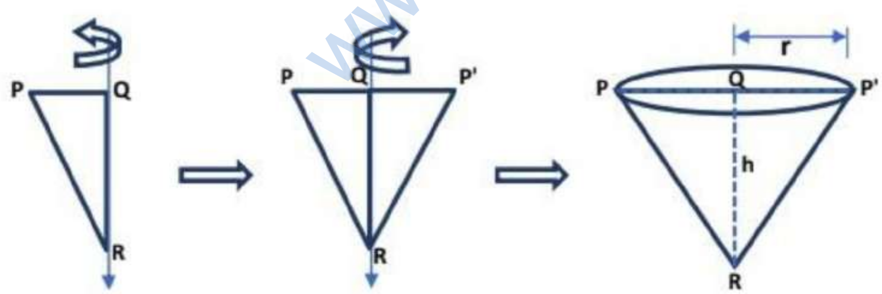
Surface Area of a Right Circular Cone

Right Circular Cone: In our everyday life, we come across conical figures almost at every step like a pastry, an ice-cream packed cone, a traffic cone, a conical vessel, a clown's cap, a tapered end of a pencil and a conical tent, etc. Such objects are in the shape of a right circular cone.


A cone has one plane end. This plane end is circular and called a base of the cone. If the height of the cone is perpendicular to the base of the cone then it is called a right circular cone.

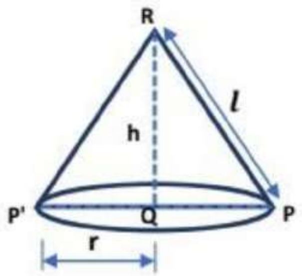
If we cut out a right-angled triangle PQR right-angled at Q. paste a long string along with one of the perpendicular sides PQ of the triangle. Hold the string with your own hands on either side of the triangle and rotate the triangle about the string several times.

We recognize the shape that the triangle is formatting as it rotates around the string, then it reminds us of a shape as shown in the below figure and we get a right circular cone.




The following are some terms related to a right circular cone.

Vertex: R is called the vertex of the cone.




Height: The shortest distance between point Q and R is called the height of the cone. It is usually denoted by h.

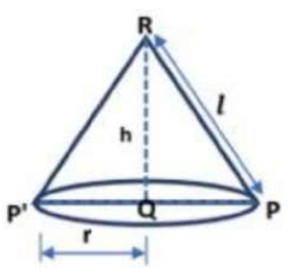
Base: A right circular cone has a circular plane end with center Q. This is called the base of the cone.

Slant Height: Line-segments PR and P'R are called slant height of the cone. It is usually denoted by l.

Radius: Radius of the base circle is also called the radius of the cone.

To obtain the surface area of a right circular cone of radius r, height h, and slant height l, let us consider the following experiment.

Consider a right circular cone of radius r, height h, and slant height l as shown in the figure below.

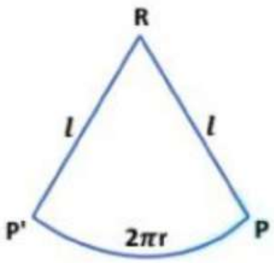





The base of the cone is a circle of radius r.

So, length of the circular edge = $2\pi r$ and Area of the plane end = $\pi r^2$.

Cut the cone along with the slant height RP and spread out it on a plane surface. We will find that the spread-out figure is a sector of a circle of radius equal to the slant height l of the cone and whose arc is equal to the circumference of the base of the cone as shown in the figure below.




The curved surface area of the cone = Area of the sector RPP'

$= \frac{1}{2} \times$ (arc length) $\times$ (radius).

$= \frac{1}{2} \times 2\pi r \times l.$

$= \pi rl.$

Hence, the area of the curved surface of a right circular cone of radius r and slant height l is given by $\pi rl$ or $\{ \frac{1}{2} \times$ (circumference of base) $\times$ (slant height)$\}$.

The curved surface area of the cone = $\pi rl$.

The total surface area of a cone = Curved surface area + Area of the base.

$= \pi rl + \pi r^2.$

$= \pi r (l + r).$

Total surface area of a cone = $\pi r (l + r)$.

Where r is the radius, l its slant height and $l^2 = r^2 + h^2$.

Example 1: The radius of an ice-cream cone is 3.5 cm and its slant height is 7 cm. Find the area of its curved surface.


Solution: Clearly, the ice cream cone is in the form of a right circular cone.

Here, the radius of the base of the ice cream cone and its slant height is 3.5 cm and 7 cm respectively.

We know that,

Curved Surface Area of the Cone =πrl.

So, Curved Surface Area of the ice cream Cone =πrl.

$= \frac{22}{7} \times 3.5$ cm$\times$ 7 cm

$= \frac{22}{7} \times \frac{35}{10}$ cm $\times$ 7 cm

=22 × 510cm × 7 cm

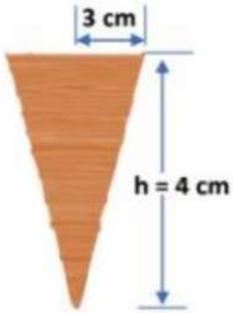
=22 × $\frac{1}{2}$ cm × 7 cm

=11 cm × 7 cm

=77 cm$^2$.

Here, Curved Surface Area of the ice cream cone = 77cm$^2$.

Example 2: Find the total surface area of the waffle cone, if its height is 4 cm and the radius of its base is 3 cm.




Solution: Clearly, the waffle cone is in the form of a right circular cone.

Here, the radius of the base of the waffle cone and its height is 3 cm and 4 cm respectively. So, l = $\sqrt{r^2 + h^2}$

= $\sqrt{(3cm)^2 + (4cm)^2}$

= $\sqrt{9cm^2 + 16cm^2}$

= $\sqrt{25cm}$ = 5 cm.

We know that,

Total Surface Area of the right circular Cone =πr (l + r).

∴Total Surface Area of the waffle cone =πr (l + r).

=3.14× 3 cm (5 cm + 3 cm).

=3.14 × 3 cm (8 cm).

= 75.36 $cm^2$.

Hence, Total Surface Area of the waffle cone = 75.36 $cm^2$.

# Surface Area of a Sphere

Surface Area of Sphere

Sphere: In our everyday life, we come across objects like the earth, a world globe, a soccer ball, a basketball, the moon, etc. These are a three-dimensional geometrical object which is in the shape of a sphere.

Earth

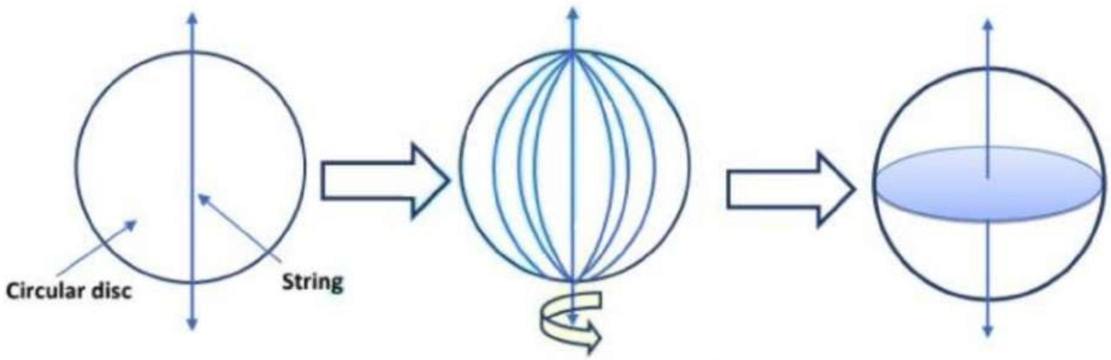
World Globe

Soccer ball

Basket ball



If we take a circle as we know that it is a plane closed figure whose every point lies at a constant distance (called radius) from a fixed point, which is called the center of the circle. Now if we paste a long string along a diameter of a circular disc. Hold the string with your own hands on either side of the disc and rotate the disc about the string a number of times. We recognize the shape that the circular disc is formatting as it rotates around the string, then it reminds us of a shape as shown in the below figure and we get a sphere.




When a circle forms a sphere on rotation, it becomes the center of the sphere. So, a sphere is a three-dimensional figure which is made up of all points in the space, which lie at a constant distance called the radius, from a fixed point called the center of the sphere. If on a sheet of paper, we draw four circles with a radius equal to the radius of the sphere as shown in the figure below.

Then start filling the circles one by one, with the string we had wound around the sphere. The string, which had completely covered the surface area of the sphere, has been used to completely fill the regions of four circles, all of the same radii as of the sphere. This suggests that the surface area of a sphere of radius

$r$ = 4 times the area of the circle of radius $r$

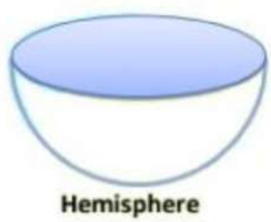
$= 4 \times \pi r^2 = 4\pi r^2$ .

Surface Area of Sphere $= 4\pi r^2$

Where $r$ is the radius of the sphere.

Let us take a sphere, and slice it exactly through the middle with a plane that passes through its center. It gets divided into two equal parts. It is called Hemisphere.

(Because 'hemi' also means 'half') Hemisphere: It has two faces one is a curved face and the other is a flat face (base).




The curved surface area of a hemisphere is half the surface area of the sphere, which is $\frac{1}{2} \times 4\pi r^2$ .

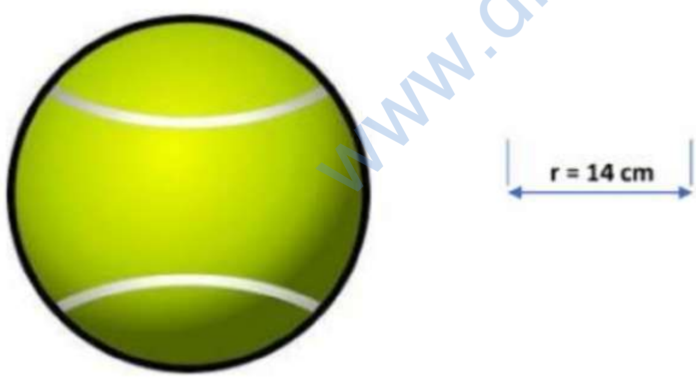
Therefore, Curved Surface Area of a Hemisphere = $2\pi r^2$ .

Where r is the radius of the sphere of which the hemisphere is a part.

Now taking the two faces of a hemisphere, its total surface area is $2\pi r^2 + \pi r^2$.

So, Total surface area of a hemisphere = $3\pi r^2$.

Example 1: Find the surface area of a tennis ball of radius 14 cm.




Solution: Clearly, the tennis ball is in the form of a sphere.

Here, the radius of the sphere is 14 cm.

We know that,

Surface Area of the sphere $=4\pi r^2$.

Therefore, Surface Area of tennis ball $=4\pi r^2$.

$$= 4 \times \frac{22}{7} \times (14\ cm)^2.$$

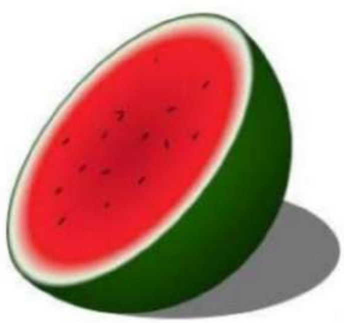
$$= 4 \times \frac{22}{7} \times 14cm \times 14\ cm.$$

$$= 4 \times 22 \times 2cm \times 14\ cm.$$

$$= 2464\ cm^2.$$

Hence, Surface area of the tennis ball is $2464\ cm^2$.

Example 2: Find the curved surface area and the total surface area of the half slice of watermelon of radius 7 cm.



Solution: Clearly, half slice of watermelon is in the form of a hemisphere.

Here, radius of hemisphere is 7 cm.

We know that,

Curved surface area of the hemisphere $=2\pi r^2$.

Therefore, Curved surface Area of half slice of watermelon $=2\pi r^2$.

$$= 2 \times \frac{22}{7} \times (7\ cm)^2.$$

$$= 2 \times \frac{22}{7} \times 7cm \times 7\ cm.$$

$$= 2 \times 22 \times 1cm \times 7\ cm.$$

$= 308\ cm^2$.

Hence, Curved Surface area of half slice of watermelon is 308 cm2.

Also, we know that

The total surface area of hemisphere $= 3\pi r^2$.

So, the total surface area of half slice of watermelon $= 3\pi r^2$.

$= 3 \times 227 \times (7\ cm)^2$.

$= 3 \times \frac{22}{7} \times 7cm \times 7\ cm$

$= 3 \times 22 \times 1cm \times 7\ cm$.

$= 462\ cm^2$.

Hence, the total surface area of half slice of watermelon $= 462\ cm^2$.

## Volume of a Cuboid and a Cube

Volume of a Cuboid and a Cube

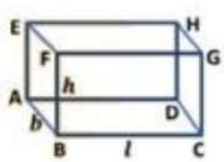
Volume of a solid object is the measure of the space occupied by it.

If the object is hollow, then its interior is empty, which can be filled with either air or some liquid that will take the shape of the hollow

object. The volume of the substance that can fill the interior of the hollow object is called it's capacity.

In other words, the volume of an object is the measure of the space it occupies, and the capacity of a hollow object is the volume of substance its interior can accommodate.

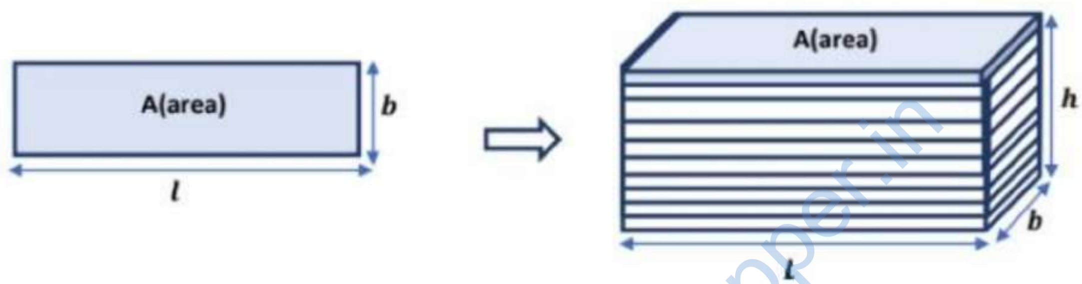
Volume of a cuboid



When we talk about the volume of a cuboid, we mean the measure of the space occupied by the cuboid.

Let there be a cuboid of length l, breadth b and height h.

The area of the rectangular base ABCD of the cuboid is $(l \times b)$. If we take rectangular sheets congruent to the base ABCD of the cuboid and the sheets are put one over the other as shown in the figure. Then, the height to which the sheets are stacked to form the cuboid is h.




The measure of the space occupied by the cuboid = the area of the rectangular sheet × height.

$= A \times h = V$. where A and V are respectively area and volume of a cuboid.

That is, the Volume of Cuboid = base area × height

= length × breadth × height.

$= l \times b \times h$.

Volume of Cuboid = base area × height = length × breadth × height.

Where l, b and h are respectively the length, breadth, and height of the cuboid.

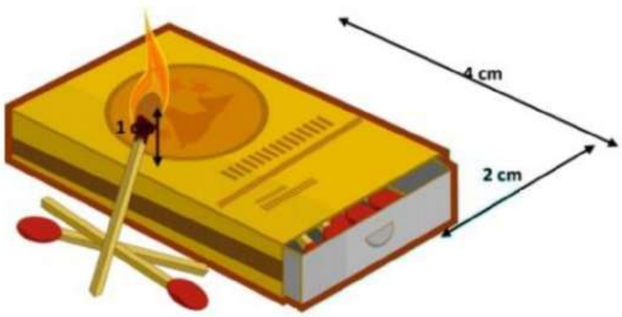
Volume of a Cube

We know that a cube is a special type of cuboid whose length, breadth and height are all equal i.e. length = breadth = height.

So, the volume of a cube of side a is given by
$V = a \times a \times a = a^3 = (\text{side})^3$.

Where V and a are respectively the volume and side of a cube.

Example 1: A matchbox measure 4 cm × 2 cm × 1 cm. What will be the volume of a packet containing 15 such boxes?




Solution: The matchbox is in the form of a Cuboid. So, first, we will find the volume of one matchbox. Then, we will find the volume of the packet containing 15 such matchboxes.

Here, length, breadth, and height of matchbox are 4 cm, 2 cm, and 1 cm respectively.

Volume of Cuboid = length × breadth × height = $l \times b \times h$.

Therefore, the volume of 1 matchbox = $l \times b \times h$

= 4 cm × 2 cm × 1 cm.

= 8 $cm^3$.

The volume of 15 matchboxes = 15 × Volume of 1 matchbox Hence, volume of 15 matchboxes = 15 × 8 $cm^3$= 120 $cm^3$.

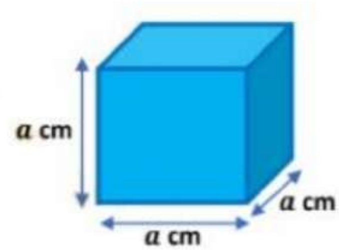
Example 2: The volume of an ice cube is 125 $cm^3$. Find its total surface area.

Solution: Clearly, the ice cube is in the form of a Cube.

Suppose, side of the ice cube = a cm

Volume of cube = a × a × a = $a^3$.

So, the volume of ice cube = a cm × a cm × a cm.




⇒ 125 $cm^3$ = $a^3$ $cm^3$.

⇒ a = $\sqrt[3]{5X5X5}$

⇒ a = 5.

So, Each edge of the ice cube is 5 cm.

Now, the surface area of ice cube = $6a^2$.

= 6 (5 cm)$^2$.
= 6 × 5 × 5 $cm^2$.

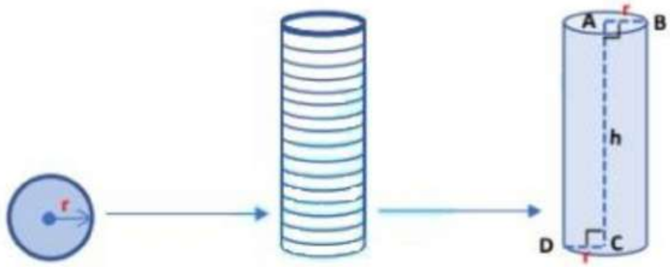
= 150 $cm^2$.

Hence, surface area of ice cube is 150 $cm^2$.

## Volume of a Right Circular Cylinder

The volume of a Right Circular Cylinder

Suppose that we take circular sheets of radius r and stack up vertically as shown in the figure below to form a right circular cylinder of height h.




Therefore, Volume of the cylinder = Measure of the space occupied by the cylinder.

The volume of Cylinder = The area of each circular sheet × height.

The volume of right circular Cylinder = Area of the base × height.

$$= \pi r^2 \times h$$

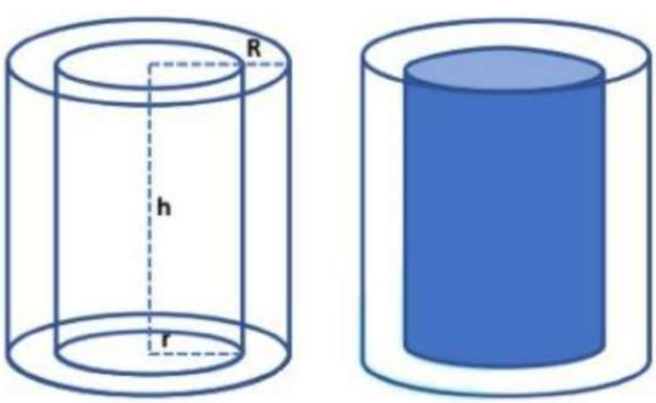
$$= \pi r^2 h.$$

Volume of a right circular Cylinder = $\pi r^2 h$.

Where r and h are the radius of the base and height of the right circular cylinder respectively.

Volume of a hollow Cylinder

Suppose that R and r be the external and internal radii of a hollow cylinder (shaded in blue) & h be its height as shown in the figure below.



The volume of the hollow cylinder = Exterior volume – Interior volume

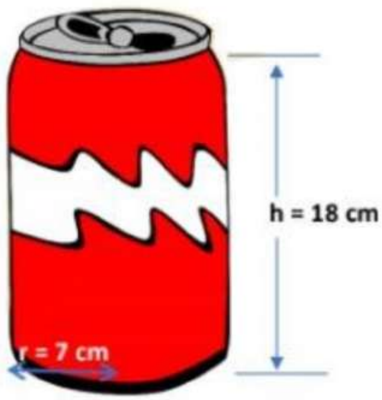
$= \pi R^2h - \pi r^2h$

$= \pi h\ (R^2 - r^2)$

The volume of the hollow cylinder = $\pi h\ (R^2 - r^2)$

Where h is the height of the hollow cylinder.

Example 1: Find the volume of a soda can, if the radius of its base and height is 7 cm and 18 cm respectively.






Solution: Clearly, soda cane is in the form of a right circular cylinder.

Suppose that, height and radius of soda cane are 18 cm and 7 cm respectively.

Volume of a cylinder = $\pi r^2h$.

So, Volume of the soda can = $\pi r^2h$.

$$= \frac{22}{7} \times (7\text{ cm})^2 \times 18\text{ cm}.$$

$$= \frac{22}{7} \times 7\text{ cm} \times 7\text{ cm} \times 18\text{ cm}.$$

$$= 22 \times 1\text{ cm} \times 7\text{ cm} \times 18\text{ cm}.$$

$$= 2772\text{ cm}^3.$$

Hence, the volume of a soda can is = 2772 $cm^3$.

Example 2: The thickness of a hollow wooden cylinder is 2 cm. It is 35 cm long and its inner radius is 5 cm. Find the volume of the wood required to make the cylinder.


Solution: We have,

r = radius of the inner cylinder = 5 cm.

The thickness of the cylinder = 2 cm.

∴ R = Outer radius of the cylinder = (5 cm + 2 cm) = 7 cm.

h = height of the cylinder = 35 cm.

∴ The volume of the wood = $\pi h (R^2 - r^2)$.

$= \frac{22}{7} \times 35 \text{ cm } [(7 \text{ cm})^2 - (5 \text{ cm})^2]$

$= \frac{22}{7} \times 35 \text{ cm } [(7 \text{ cm})^2 - (5 \text{ cm})^2]$.

$= \frac{22}{7} \times 35 \text{ cm } [(7 \text{ cm} + 5 \text{ cm}) \times (7 \text{ cm} - 5 \text{ cm})]$.

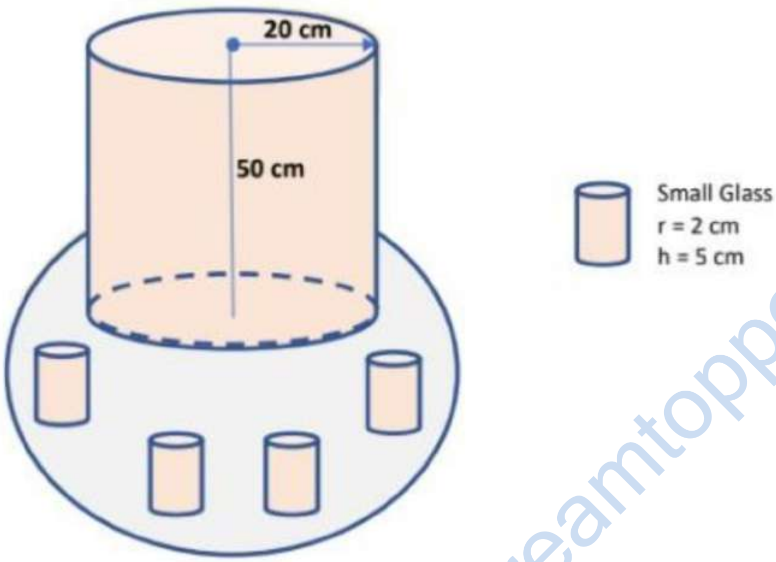
$= \frac{22}{7} \times 35 \text{ cm} \times 12 \text{ cm} \times 2 \text{ cm}$.

$= 22 \times 5 \text{ cm} \times 12 \text{ cm} \times 2 \text{ cm}$.

$= 2640 \text{ cm}^3$.

Hence, the volume of the wood is 2640 $cm^3$.

Example 3: In a Ramzan month, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 20 cm filled up to a height of 50 cm with orange juice. The juice is filled in small cylindrical glasses (as shown in figures below) of radius 2 cm up and a height of 5 cm, which is sold for ₹ 10 each. How much money does the stall keeper earn by selling the whole juice?






Solution: The volume of juice in the vessel = volume of the cylindrical vessel = $\pi R^2 H$

Where R = 20 cm and H = 50 cm are the radius and height respectively of the vessel.0

Similarly, the volume of juice each glass can hold = $\pi r^2 h$.

Where r = 2 cm and h = 5 cm are taken as the radius and height respectively of a small glass.

So, the number of glasses of juice that are sold = $\frac{volume\ of\ the\ vessel}{volume\ of\ a\ small\ glass}$

$= \frac{\pi R^2 H}{\pi r^2 h}$

$$= \frac{\pi\,(20cm)^2\,50cm}{\pi\,(2cm)^2\,5cm}.$$

$$= \frac{\pi \times 20cm \times 20cm \times 50cm}{\pi \times 2cm \times 2cm \times 5cm}.$$

$= 10 \times 10 \times 10 = 1000.$
Therefore, money earned by the stall keeper

= Price of 1 glass × Total number of glasses sold.

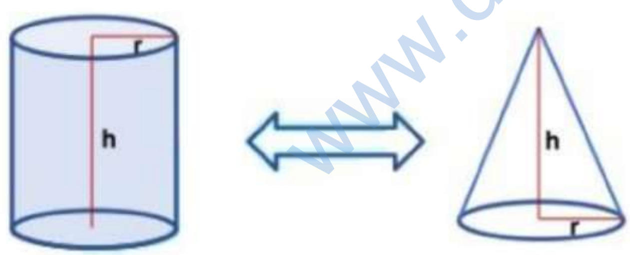
Therefore, money earned by the stall keeper = ₹ 10 × 1000.
= ₹ 10000.

## Volume of a right circular cone

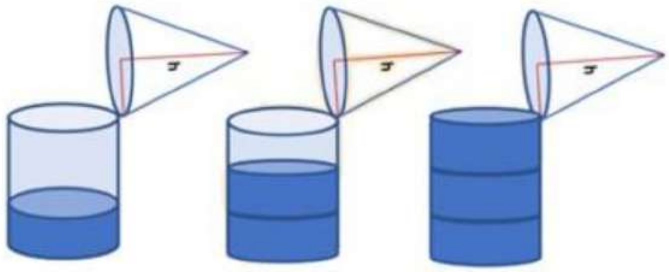
Volume of a right circular cone

To find the volume of a right circular cone, let us consider the following experiment.

We take a conical cup of radius r and height h. Also, take a cylindrical jar of radius r and height h.




Fill the conical cup with water to the brim and transfer the water to the cylindrical jar. Repeat the process two times more. We will find that 3 conical cups will fill the cylindrical jar completely.

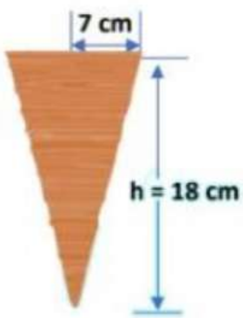


So, we conclude that 3(Volume of a cone of radius r and height h) = (Volume of a cylinder of radius r and height h) = $\pi r^2 h$ cubic units.

∴ The volume of a cone of radius r and height h = $\frac{1}{3}\pi r^2 h$ cubic units.

Thus, Volume of a cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}$ (Area of the base) × (height).

Example: Find the volume of a waffle cone 18 cm high, if the radius of its base is 7 cm.




Solution: The waffle cone is in the form of a right circular cone.

Suppose, the height and the radius of the waffle cone are 18 cm and 7 cm respectively.

We know that,

Volume of a cone = $\frac{1}{3}\pi r^2 h$.

So, Volume of a waffle cone = 13πr2h.

$$= \frac{1}{3} \times \frac{22}{7} \times (7\ \text{cm})2 \times 18\ \text{cm}.$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7\ \text{cm} \times 7\ \text{cm} \times 18\ \text{cm}.$$

$$= 22 \times 1\ \text{cm} \times 7\ \text{cm} \times 6\ \text{cm}.$$

$$= 924\ \text{cm}^3.$$

Hence, the volume of the waffle cone = 924 $\text{cm}^3$

## Volume of a Sphere

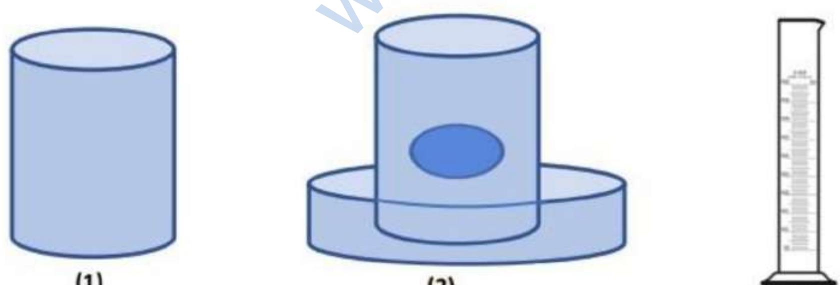
Volume of a Sphere

To find the volume of a sphere, let us consider the following experiment.

We take two spheres of different radii, and a container big enough to be able to put each of the spheres into it, one at a time. Also, take a large trough in which we can place the container. Then, fill the container up to the brim with water as shown in figure (1) below. Now, carefully place one of the spheres in the container. Some of the water from the container will overflow into the trough in which it is kept as shown in figure (2) below.

Carefully pour out the water from the trough into a measuring cylinder (graduated cylindrical jar) an measure the water overflowed as shown in figure (3).





(1) (2) (3)

Suppose the radius of the immersed sphere is r, then the volume of overflow water is equal to $\frac{4}{3}\pi r^3$.

Once again repeat the procedure done just now, with a different size of the sphere. Find the radius R of this sphere and then calculate $\frac{4}{3}\pi r^3$

Once again, this value is nearly equal to the measure of the volume of the water displaced (overflowed) by the sphere.

Therefore, the volume of the sphere is equal to $\frac{4}{3}\pi$ times the cube of its radius.

This gives us the idea that Volume of a sphere $= \frac{4}{3}\pi r^3$.

Where r is the radius of the sphere.

Since a hemisphere is half of a sphere, then

Volume of a hemisphere $= \frac{2}{3}\pi r^3$.

Where r is the radius of the sphere.

Example 1: Find the volume of a football of radius 7 cm.




Solution: Clearly, football is in the form of a sphere which radius is 7 cm.

We know that,

Volume of a sphere $= \frac{4}{3}\pi r^3$.

So, Volume of the football $= \frac{4}{3}\pi r^3$.

$$= \frac{4}{3} \times \frac{22}{7} \times (7\ cm)^3.$$

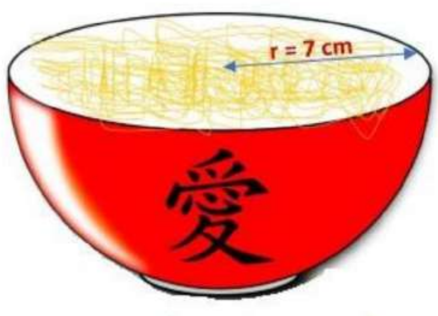
$$= \frac{4}{3} \times \frac{22}{7} \times 7\ cm \times 7\ cm \times 7\ cm.$$

$$= \frac{4}{3} \times 22 \times 1\ cm \times 7\ cm \times 7\ cm.$$

$$= 1437.33\ cm^3.$$

Hence, volume of a football is = 1437.33 $cm^3$.

Example 2: Find the volume of a bowl of radius 7 cm.




Solution: The bowl is in the shape of a hemisphere. Hemisphere has a radius of 7 cm.

We know that,

Volume of a hemisphere = $\frac{2}{3} \pi r^3$.

So, Volume of a hemisphere = $\frac{2}{3} \pi r^3$.

$$= \frac{2}{3} \times \frac{22}{7} \times (7\ cm)^3.$$

$$= \frac{2}{3} \times \frac{22}{7} \times 7\ cm \times 7\ cm \times 7\ cm.$$

$$= \frac{2}{3} \times 22 \times 1\ cm \times 7\ cm \times 7\ cm.$$

$= 718.66 \text{ cm}^3.$

Hence, the volume of a hemisphere is $718.66 \text{ cm}^3$.