

Chapter 14
Statistics and Probability

Exercise No. 14.1

Multiple Choice Questions:

Write the correct answer in each of the following:

1. The class mark of the class 90-120 is:

- (A) 90
- (B) 105
- (C) 115
- (D) 120

Solution:

According to the question,

Upper limit = 120 and lower limit = 90,

Now, the class mark will be calculated as follows:

$$\begin{aligned}\text{Class mark} &= \frac{\text{Upper class} + \text{Lower class}}{2} \\ &= \frac{120 + 90}{2} \\ &= \frac{210}{2} \\ &= 105\end{aligned}$$

Hence, the correct option is (B).

2. The range of the data:

25, 18, 20, 22, 16, 6, 17, 15, 12, 30, 32, 10, 19, 8, 11, 20 is

- (A) 10
- (B) 15
- (C) 18
- (D) 26

Solution:

Consider the data set:

25, 18, 20, 22, 16, 6, 17, 15, 12, 30, 32, 10, 19, 8, 11, 20

Now, in the given data:

Minimum value of data set = 6

Maximum value of data set = 32

As, Range = Maximum value of data set - Minimum value of data set

$$\text{Range} = 32 - 6$$

$$\text{Range} = 26$$

Hence, the correct option is (D).

3. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is:

- (A) 6
- (B) 7
- (C) 8
- (D) 12

Solution:

Given: Mid-value = 10

Range = 6

Let upper class and lower class of the interval are x and y respectively.

Now,

$$\text{Mid-value} = \frac{\text{Upper class} + \text{Lower class}}{2}$$

$$10 = \frac{x + y}{2}$$

$$x + y = 20 \quad \dots \text{(I)}$$

As, Range = Maximum value of data set - Minimum value of data set

$$6 = x - y$$

$$x - y = 6 \quad \dots \text{(II)}$$

Adding equation (I) and (II), get:

$$x + y + (x - y) + 20 + 6$$

$$2x = 26$$

$$x = \frac{26}{2}$$

$$x = 13$$

Putting the value of x in equation (I), get:

$$13 + y = 20$$

$$y = 20 - 13$$

$$y = 7$$

Hence, the correct option is (B).

4. The width of each of five continuous classes in a frequency distribution is 5 and the lower class-limit of the lowest class is 10. The upper class-limit of the highest class is:

- (A) 15
- (B) 25
- (C) 35
- (D) 40

Solution:

Given: Lower class limit = 10

Width = 5

As, we know that, width = Upper class limit - Lower class limit

So, Upper class limit = 10 + 5

Upper class limit = 15

Since, the five continuous classes are:

10-15, 15-20, 20-25, 25-30, and 30-35

Therefore, the upper class limit of the highest class is 35.

Hence, the correct option is (C).

5. Let m be the mid-point and l be the upper class limit of a class in a continuous frequency distribution. The lower class limit of the class is :

(A) $2m + l$

(B) $2m - l$

(C) $m - l$

(D) $m - 2l$

Solution:

Given:

Mid-point = m

Upper class limit = l

As, we know that, mid-point = $\frac{\text{Upper class limit} + \text{Lower class limit}}{2}$

So,

$$m = \frac{l + \text{Lower class limit}}{2}$$

$$l + \text{Lower class limit} = 2m$$

$$\text{Lower class limit} = 2m - l$$

Hence, the correct option is (B).

6. The class marks of a frequency distribution are given as follows:

15, 20, 25, ...

The class corresponding to the class mark 20 is:

(A) 12.5 – 17.5

(B) 17.5 – 22.5

(C) 18.5 – 21.5

(D) 19.5 – 20.5

Solution:

Given: the class marks are: 15, 20, 25...

The size of the each class interval is: $25 - 20 = 20 - 15 = 5$

Therefore, the class interval corresponding to the class mark 20 is:
 $20 - 2.5 - (20+2.5)$ that is $17.5 - 22.5$
Hence, the correct option is (B).

- 7. In the class intervals 10-20, 20-30, the number 20 is included in:**
(A) 10-20
(B) 20-30
(C) Both the intervals
(D) None of these intervals

Solution:

In the class intervals 10-20, 20-30, the number 20 is included in 20 – 30.
Hence, the correct option is (B).

- 8. A grouped frequency table with class intervals of equal sizes using 250-270 (270 not included in this interval) as one of the class interval is constructed for the following data:**

268, 220, 368, 258, 242, 310, 272, 342,
310, 290, 300, 320, 319, 304, 402, 318,
406, 292, 354, 278, 210, 240, 330, 316,
406, 215, 258, 236.

The frequency of the class 310-330 is:

- (A) 4**
(B) 5
(C) 6
(D) 7

Solution:

Given data set is,
268, 220, 368, 258, 242, 310, 272, 342,
310, 290, 300, 320, 319, 304, 402, 318,
406, 292, 354, 278, 210, 240, 330, 316,
406, 215, 258, 236

The observation corresponding to class 310 – 330 are class 310, 310, 320, 319, 318, 316 that is 6 observations.

So, the frequency of the class 310 – 330 is 6.

Hence, the correct option is (B).

- 9. A grouped frequency distribution table with classes of equal sizes using 63-72 (72 included) as one of the class is constructed for the following data:**
30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88,

40, 14, 20, 15, 35, 44, 66, 75, 84, 95, 96,
102, 110, 88, 74, 112, 14, 34, 44.

The number of classes in the distribution will be:

- (A) 9
- (B) 10
- (C) 11
- (D) 12

Solution:

Consider the data set:

30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88,
40, 14, 20, 15, 35, 44, 66, 75, 84, 95, 96,
102, 110, 88, 74, 112, 14, 34, 44.

So, in the above given data set:

Minimum value = 14

Maximum value = 112

Since, the class are:

13 -22, 23, 32, 33-42, 43-52, 53-62, 63-72, 73-82, 83-92, 93-102 and 103 -112.

Since, the number of class in the distribution is 10.

10. To draw a histogram to represent the following frequency distribution:

Class interval	5-10	10-15	15-25	25-45	45-75
Frequency	6	12	10	8	15

the adjusted frequency for the class 25-45 is :

- (A) 6
- (B) 5
- (C) 3
- (D) 2

Solution:

The adjusted frequency for the class 25 – 45 will be:

$$= \frac{\text{Frequency of the class}}{\text{Class width}} \times \text{Minimum width}$$

$$= \frac{8}{20} \times 5$$

$$= 2$$

Hence, the correct option is (D).

11. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is:

- (A) 28

(B) 30

(C) 35

(D) 38

Solution:

Given: The mean of five numbers is 30.

Mean = 28

Sum of the five numbers = $30 \times 5 = 150$

Suppose that the excluded number be a . So,

Sum of four data point = $150 - a$

The formula of the mean is: $\text{Mean} = \frac{\text{Sum of all data points}}{\text{Number of data points}}$

Now,

$$28 = \frac{150 - a}{4}$$

$$150 - a = 28 \times 4$$

$$150 - a = 112$$

$$a = 150 - 112$$

$$a = 38$$

Hence, the correct option is (D).

12. If the mean of the observations:

$x, x + 3, x + 5, x + 7, x + 10$

is 9, the mean of the last three observations is

(A) $10\frac{1}{3}$

(B) $10\frac{2}{3}$

(C) $11\frac{1}{3}$

(D) $11\frac{2}{3}$

Solution:

Consider the data set:

$x, x + 3, x + 5, x + 7, x + 10$

Mean = 9

The formula of the mean is: $\text{Mean} = \frac{\text{Sum of all data points}}{\text{Number of data points}}$

$$9 = \frac{x+x+3+x+5+x+7+x+10}{5}$$

$$5x + 25 = 45$$

$$5x = 45 - 25$$

$$5x = 20$$

$$x = 4$$

So, last three observation at $x=4$ are: $4+5=9$, $4+7=11$, and $4+10=14$.

Now, the mean of last three observation is:

$$\text{Mean} = \frac{9+11+14}{3}$$

$$= \frac{34}{3}$$

$$= 11\frac{1}{3}$$

Hence, the correct option is (C).

13. If \bar{x} represents the mean of n observations x_1, x_2, \dots, x_n , then value of

$\sum_{i=1}^n (x_i - \bar{x})$ is:

(A) -1

(B) 0

(C) 1

(D) $n - 1$

Solution:

The formula of the mean (\bar{x}) is:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\sum_{i=1}^n x_i = n\bar{x} \quad \dots \text{(I)}$$

Here, n is total number of observation.

The value of $\sum_{i=1}^n (x_i - \bar{x})$ is calculated as follows:

$$\sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n x_i - \sum_{i=1}^n \bar{x}$$

Now, from equation (I), get:

$$\begin{aligned}
\sum_{i=1}^n (x_i - \bar{x}) &= n\bar{x} - \sum_{i=1}^n \bar{x} \\
&= n\bar{x} - \bar{x} \sum_{i=1}^n 1 \\
&= n\bar{x} - n\bar{x} \\
&= 0
\end{aligned}$$

Hence, the correct option is (B).

14. If each observation of the data is increased by 5, then their mean

- (A) remains the same
- (B) becomes 5 times the original mean
- (C) is decreased by 5
- (D) is increased by 5

Solution:

The formula of the mean (\bar{x}) is:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Now, according to the question: let new mean is \bar{x}_{New} .

$$\begin{aligned}
\bar{x}_{\text{New}} &= \frac{(x_1 + 5) + (x_2 + 5) + \dots + (x_n + 5)}{n} \\
&= \frac{x_1 + x_2 + \dots + x_n + 5n}{n} \\
&= \frac{x_1 + x_2 + \dots + x_n}{n} + \frac{5n}{n}
\end{aligned}$$

$$\bar{x}_{\text{New}} = \bar{x} + 5$$

Therefore, the new mean is increased by 5.

Hence, the correct option is (D).

15. Let \bar{x} be the mean of x_1, x_2, \dots, x_n and \bar{y} the mean of y_1, y_2, \dots, y_n . If \bar{z} is the mean of $x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n$, then \bar{z} is equal to

- (A) $\bar{x} + \bar{y}$
- (B) $\frac{\bar{x} + \bar{y}}{2}$
- (C) $\frac{\bar{x} + \bar{y}}{n}$
- (D) $\frac{\bar{x} + \bar{y}}{2n}$

Solution:

The formula of the mean (\bar{x}) is:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Now, according to the question:

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad \dots(\text{I})$$

And :

$$\bar{y} = \frac{y_1 + y_2 + \dots + y_n}{n} \quad \dots(\text{II})$$

Then,

$$\bar{z} = \frac{x_1 + x_2 + \dots + x_n + y_1 + y_2 + \dots + y_n}{2n}$$

$$\bar{z} = \frac{1}{2} \left(\frac{x_1 + x_2 + \dots + x_n}{n} + \frac{y_1 + y_2 + \dots + y_n}{n} \right)$$

Now, by equations (I) and (II):

$$\bar{z} = \frac{\bar{x} + \bar{y}}{2}$$

Hence, the correct option is (D).

16. If \bar{x} is the mean of x_1, x_2, \dots, x_n , then for $a \neq 0$, the mean of

$ax_1, ax_2, \dots, ax_n, \frac{x_1}{a}, \frac{x_2}{a}, \dots, \frac{x_n}{a}$ is

(A) $\left(a + \frac{1}{a}\right)\bar{x}$

(B) $\left(a + \frac{1}{a}\right)\frac{\bar{x}}{2}$

(C) $\left(a + \frac{1}{a}\right)\frac{\bar{x}}{n}$

(D) $\frac{\left(a + \frac{1}{a}\right)\bar{x}}{2n}$

Solution:

Given: \bar{x} is the mean of x_1, x_2, \dots, x_n .

Then,

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

Let the mean of the data set $ax_1, ax_2, \dots, ax_n, \frac{x_1}{a}, \frac{x_2}{a}, \dots, \frac{x_n}{a}$ is \bar{y} . So,

$$\bar{y} = \frac{ax_1 + ax_2 + \dots + ax_n + \frac{x_1}{a} + \frac{x_2}{a} + \dots + \frac{x_n}{a}}{2n}$$

$$\bar{y} = \frac{a(x_1 + x_2 + \dots + x_n) + \frac{1}{a}(x_1 + x_2 + \dots + x_n)}{2n}$$

$$\bar{y} = \frac{\left(a + \frac{1}{a}\right)(x_1 + x_2 + \dots + x_n)}{2n}$$

From equation (I):

$$\bar{y} = \frac{\left(a + \frac{1}{a}\right)\bar{x}}{2}$$

Hence, the correct option is (B).

17. If $\bar{x}_1, \bar{x}_2, \bar{x}_3, \dots, \bar{x}_n$ are the means of n groups with n_1, n_2, \dots, n_i number of observations respectively, then the mean \bar{x} of all the groups taken together is given by:

- (A) $\sum_{i=1}^n n_i \bar{x}_i$
- (B) $\frac{\sum_{i=1}^n n_i \bar{x}_i}{n^2}$
- (C) $\frac{\sum_{i=1}^n n_i \bar{x}_i}{\sum_{i=1}^n n_i}$
- (D) $\frac{\sum_{i=1}^n n_i \bar{x}_i}{2n}$

Solution:

The formula of the mean (\bar{x}) is:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Now, according to the question:

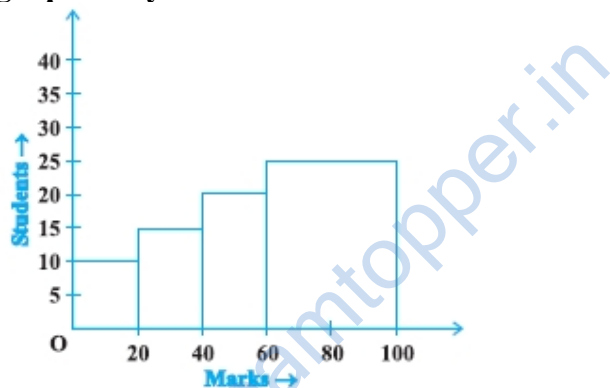
Exercise No. 14.2

Short Answer Questions with Reasoning:

1. The frequency distribution:

Marks	0-20	20-40	40-60	60-100
Number of students	10	15	20	25

has been represented graphically as follows:



Do you think this representation is correct? Why?

Solution:

The class of the frequency distribution table is not uniform width. Hence, this representation is not correct.

2. In a diagnostic test in mathematics given to students, the following marks (out of 100) are recorded:

46, 52, 48, 11, 41, 62, 54, 53, 96, 40, 98, 44

Which 'average' will be a good representative of the above data and why?

Solution:

Consider the data set:

46, 52, 48, 11, 41, 62, 54, 53, 96, 40, 98, 44

Median will be a good representative of the given data. Because:

1. Each value occurs once.
2. The data is influenced by extreme values.

3. A child says that the median of 3, 14, 18, 20, 5 is 18. What doesn't the child understand about finding the median?

Solution:

When we have to find out the median of the data set, then first we have to arrange them in ascending order or descending order.

**4. A football player scored the following number of goals in the 10 matches:
1, 3, 2, 5, 8, 6, 1, 4, 7, 9**

Since the number of matches is 10 (an even number), therefore, the median

$$= \frac{5^{\text{th}} \text{ observation} + 6^{\text{th}} \text{ observation}}{2}$$

$$= \frac{8+6}{2} = 7$$

Is it the correct answer and why?

Solution:

The calculated answer is not correct.

Before finding the median, first, data have to arrange in ascending or descending order.

5. Is it correct to say that in a histogram, the area of each rectangle is proportional to the class size of the corresponding class interval? If not, correct the statement.

Solution:

As we know that in a histogram, the area of each rectangle is proportional to the corresponding frequency of its class.

Hence, the given statement is not correct.

6. The class marks of a continuous distribution are:

1.04, 1.14, 1.24, 1.34, 1.44, 1.54 and 1.64

Is it correct to say that the last interval will be 1.55 - 1.73? Justify your answer.

Solution:

The difference between two consecutive class marks should be equal to the class size. Since, difference between two consecutive marks is 0.1 and class size of 1.55-1.73 is 0.18, which are not equal.

Hence, the given statement is not correct.

7. 30 children were asked about the number of hours they watched TV programmes last week. The results are recorded as under:

Number of hours	0-5	5-10	10-15	15-20
Frequency	8	16	4	2

Can we say that the number of children who watched TV for 10 or more hours a week is 22? Justify your answer.

Solution:

The number of children who watched TV for 10 or more hour in week is = $4 + 2$ that is 6.
Hence, the given statement is false.

8. Can the experimental probability of an event be a negative number? If not, why?

Solution:

The number of trials in which the event can happen cannot be negative and the total number of trials is always positive.

Hence, the given statement is false.

9. Can the experimental probability of an event be greater than 1? Justify your answer.

Solution:

The number of trials in which the event can happen cannot be greater than the total number of trials.

Hence, the given statement is false.

10. As the number of tosses of a coin increases, the ratio of the number of heads to the total number of tosses will be $\frac{1}{2}$. Is it correct? If not, write the correct one.

Solution:

The number of coin increases, the ratio of the number of heads to the total number of tosses will be nearer to $\frac{1}{2}$ but not exactly $\frac{1}{2}$.

Hence, the given statement is false.

Exercise No. 14.3

Short Answer Questions:

1. The blood groups of 30 students are recorded as follows:

A, B, O, A, AB, O, A, O, B, A, O, B, A, AB, B, A, AB, B,
A, A, O, A, AB, B, A, O, B, A, B, A

Prepare a frequency distribution table for the data.

Solution:

A frequency distribution table for the given data is given below:

Blood group	Tally marks	Number of students (Frequency)
A		12
B		8
AB		4
O		6
Total		30

2. The value of π upto 35 decimal places is given below:

3. 14159265358979323846264338327950288

Make a frequency distribution of the digits 0 to 9 after the decimal point.

Solution:

A frequency distribution table for the given data is given below:

Digit	0	1	2	3	4	5	6	7	8	9	Total
Tally marks											
Frequency	1	2	5	6	3	4	3	2	5	4	35

3. The scores (out of 100) obtained by 33 students in a mathematics test are as follows:

69, 48, 84, 58, 48, 73, 83, 48, 66, 58, 84, 000

66, 64, 71, 64, 66, 69, 66, 83, 66, 69, 71

81, 71, 73, 69, 66, 66, 64, 58, 64, 69, 69

Represent this data in the form of a frequency distribution.

Solution:

A frequency distribution table for the given data is given below:

Scores	Tally marks	Frequency
48		3
58		3
64		4
66		7
69		6
71		3
73		2
81		1
83		2
84		2
Total		33

4. Prepare a continuous grouped frequency distribution from the following data:

Mid-point	Frequency
5	4
15	8
25	13
35	12
45	6

Also find the size of class intervals.

Solution:

The common difference of mid-point = $15 - 5 = 10$.

Let lower limit is a . So, upper limit will be $= a + 10$

As we know that:

$$\text{Mid value} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

$$5 = \frac{a + a + 10}{2}$$

$$2a + 10 = 10$$

$$2a = 10 - 10$$

$$a = 0$$

Therefore, the first interval is 0-10.

Now, a frequency distribution table for the given data is given below:

Mid-point	Class interval	Frequency
5	0 - 10	4
15	10 - 20	8
25	20 - 30	13
35	30 - 40	12
45	40 - 50	6

5. Convert the given frequency distribution into a continuous grouped frequency distribution:

Class interval	Frequency
150-153	7
154-157	7
158-161	15
162-165	10
166-169	5
170-173	6

In which intervals would 153.5 and 157.5 be included?

Solution:

See the given table in the question:

Upper limit of class interval 150-153 is 153. And lower limit of class interval 154-157 is 154.

Now, difference between upper and lower limit will be = $154 - 153 = 1$

So, half of the difference = 0.5

Therefore, we have to subtracted 0.5 from each lower limit and add 0.5 to each upper limit.

Hence, the table for continuous grouped frequency distribution is given below:

Class interval	Frequency
149.5 - 153.5	7
153.5 - 157.5	7
157.5 - 161.5	15
161.5 - 165.5	10
165.5 - 169.5	5
169.5 - 173.5	6

Thus, 153.5 and 157.5 would use in the class intervals 153.5-157.5 and 157.5-161.5, respectively.

6. The expenditure of a family on different heads in a month is given below:

Head	Food	Education	Clothing	House rent	Others	Savings
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Expenditure (in Rs.)	4000	2500	1000	3500	2500	1500
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Draw a bar graph to represent the data above.

Solution:

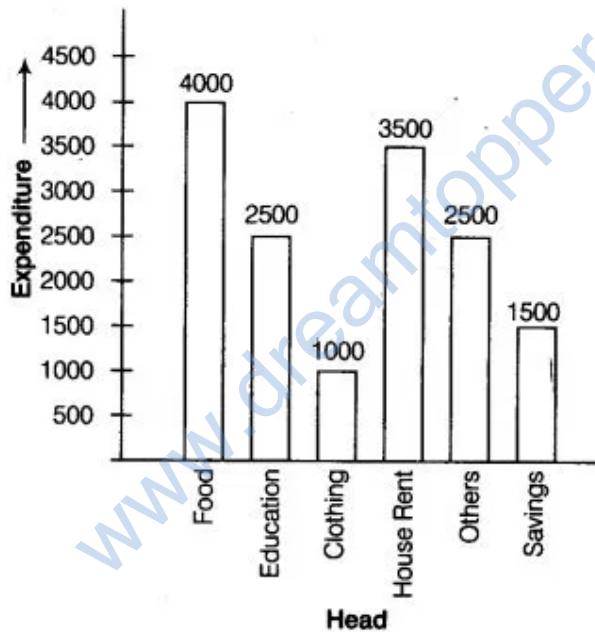
We represent the head variable on the x-axis and expenditure on the y-axis.

Let choose the scale as 1 unit = Rs. 500

To represent our first head that is, food, we draw a rectangular bar with width 1 unit and height 8 units.

Similarly, other heads are represented by leaving a gap of $\frac{1}{2}$ unit in between two consecutive bars.

The bar graph for given data is shown below:



7. Expenditure on Education of a country during a five year period (2002-2006), in crores of rupees, is given below:

Elementary education	240
Secondary education	120
University education	190
Teacher's training	20
Social education	10
Other educational programmes	115
Cultural programmes	25
Technical education	125

Represent the information above by a bar graph.

Solution:

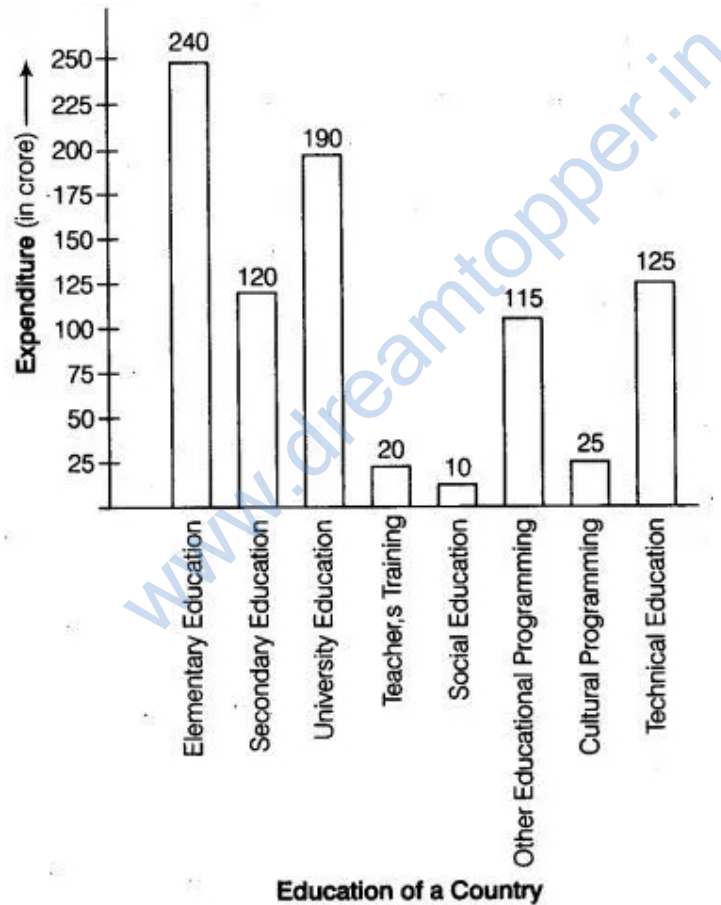
We represent the education of a country as a variable on the x-axis and expenditure on the y-axis.

The maximum expenditure is Rs. 240crore. So, we can choose the scale as 1 unit = Rs. 25 crore.

To represent our first education of a country that is, elementary education, we draw a rectangular bar with width 1 unit and height 9.6 units.

Similarly, other heads are represented by leaving gap of $\frac{1}{2}$ unit in between two consecutive bars.

The bar graph for given data is shown below:



8. The following table gives the frequency of most commonly used letters *a*, *e*, *i*, *o*, *r*, *t*, *u* from a page of a book:

Letters	<i>a</i>	<i>e</i>	<i>i</i>	<i>o</i>	<i>r</i>	<i>t</i>	<i>u</i>
Frequency	75	125	80	70	80	95	75

Represent the information above by a bar graph.

Solution:

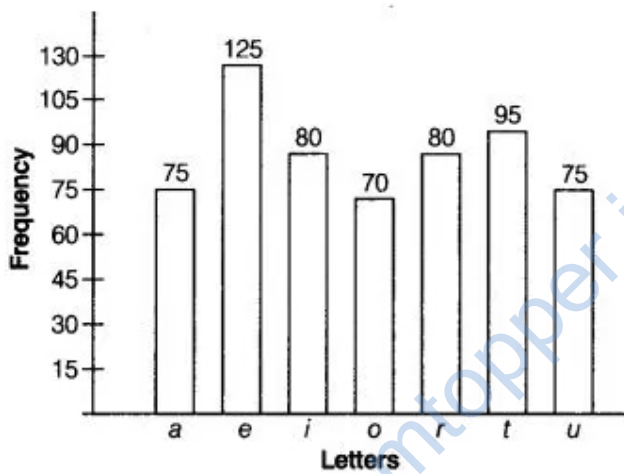
We represent the letter as a variable on the x-axis and frequency on the y-axis.

According to the given table, we can choose the scale as 1 unit = 15 frequency.

To represent our first letter that is, a, we draw a rectangular bar with width 1 unit and height 5 units.

Similarly, other heads are represented by leaving a gap of $\frac{1}{2}$ unit in between two consecutive bars.

The bar graph for given data is shown below



9. If the mean of the following data is 20.2, find the value of p :

x	10	15	20	25	30
f	6	8	p	10	6

Solution:

Consider the data:

x	10	15	20	25	30
f	6	8	p	10	6

As we know that:

$$\text{Mean of the data } (\bar{x}) = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

So,

$$20.2 = \frac{f_1x_1 + f_2x_2 + f_3x_3 + f_4x_4 + f_5x_5}{f_1 + f_2 + f_3 + f_4 + f_5}$$

$$20.2 = \frac{6 \times 10 + 8 \times 15 + p \times 20 + 10 \times 25 + 6 \times 30}{6 + 8 + p + 10 + 6}$$

$$20.2 = \frac{60 + 120 + 20p + 250 + 180}{30 + p}$$

$$20p + 610 = 606 + 20.2p$$

$$610 - 606 = 0.2p$$

$$\frac{2p}{10} = 4$$

$$p = 10 \times 2$$

$$p = 20$$

Therefore, the value of p is 20.

10. Obtain the mean of the following distribution:

Frequency	Variable
4	4
8	6
14	8
11	10
3	12

Solution:

As we know that:

$$\text{Mean of the data } (\bar{x}) = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

So,

$$= \frac{4 \times 4 + 8 \times 6 + 14 \times 8 + 11 \times 10 + 3 \times 12}{4 + 8 + 14 + 11 + 3}$$

$$= \frac{16 + 48 + 112 + 110 + 36}{40}$$

$$= \frac{322}{40}$$

$$= 8.05$$

Hence, the mean of the given data set is 8.05.

11. A class consists of 50 students out of which 30 are girls. The mean of marks scored by girls in a test is 73 (out of 100) and that of boys is 71. Determine the mean score of the whole class.

Solution:

Given: In a class consists of 50 students out of which 30 are girls.

So, number of boys in the class = $50 - 30 = 20$

Now, the marks obtained by 30 girls = $30 \times 73 = 2190$

And the marks obtained by 20 boys = $20 \times 71 = 1420$

Then, mean score of the whole class will be calculated as follows:

$$\begin{aligned}\text{Mean} &= \frac{\text{Marks obtained by 30 girls} + \text{Marks obtained by 20 girls}}{\text{Number of girls} + \text{Number of boys}} \\ &= \frac{2190 + 1420}{50} \\ &= \frac{3610}{50} \\ &= 72.2\end{aligned}$$

Hence, mean of the whole class is 72.2.

12. Mean of 50 observations was found to be 80.4. But later on, it was discovered that 96 was misread as 69 at one place. Find the correct mean.

Solution:

Given, the mean of 50 observations = 80.4

The formula of the mean (\bar{x}) is:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

So, sum of 50 observations = $80.4 \times 50 = 4020$

Now, according to the question, 96 was misread as 69 at one place.

Therefore, the correct mean will be = $\frac{(\text{Sum of 50 observations}) - 69 + 96}{50}$

$$\begin{aligned}&= \frac{4020 - 69 + 96}{50} \\ &= \frac{4047}{50} \\ &= 80.94\end{aligned}$$

Hence, the correct mean is 80.94.

13. Ten observations 6, 14, 15, 17, $x + 1$, $2x - 13$, 30, 32, 34, 43 are written in an ascending order. The median of the data is 24. Find the value of x .

Solution:

Consider the data set:

6, 14, 15, 17, $x + 1$, $2x - 13$, 30, 32, 34, 43

Since, n is even. So, median will be calculated as follows:

$$\begin{aligned} \text{Median} &= \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ observation}}{2} \\ &= \frac{\left(\frac{10}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{10}{2} + 1\right)^{\text{th}} \text{ observation}}{2} \\ &= \frac{(5)^{\text{th}} \text{ observation} + (6)^{\text{th}} \text{ observation}}{2} \\ 24 &= \frac{x + 1 + 2x - 13}{2} \end{aligned}$$

$$3x - 12 = 48$$

$$3x = 60$$

$$x = 20$$

Hence, the value of x is 20.

14. The points scored by a basketball team in a series of matches are as follows:

17, 2, 7, 27, 25, 5, 14, 18, 10, 24, 48, 10, 8, 7, 10, 28

Find the median and mode for the data.

Solution:

Consider the data set:

17, 2, 7, 27, 25, 5, 14, 18, 10, 24, 48, 10, 8, 7, 10, 28

Let arrange the given points scored in ascending order, get:

2, 5, 7, 7, 8, 10, 10, 10, 14, 17, 18, 24, 25, 27, 28, 48

Number of given observation in this data set is 16 that is even. So, median will be calculated as follows:

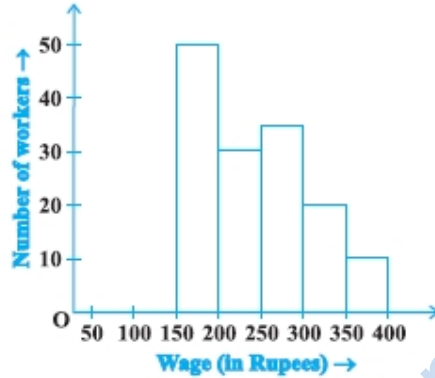
$$\begin{aligned} \text{Median} &= \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ observation}}{2} \\ &= \frac{\left(\frac{16}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{16}{2} + 1\right)^{\text{th}} \text{ observation}}{2} \\ &= \frac{(8)^{\text{th}} \text{ observation} + (9)^{\text{th}} \text{ observation}}{2} \\ &= \frac{10 + 14}{2} \\ &= \frac{24}{2} \\ &= 12 \end{aligned}$$

As we know that, mode is the observation which is repeated maximum number of times that is 10.

Hence, median and mode for the given data are 12 and 10, respectively.

15. In Fig. 14.2, there is a histogram depicting daily wages of workers in a factory.

Construct the frequency distribution table.



Solution:

See the above histogram that class interval start from 150-200, 200-250, having width. The frequency distribution table is given below:

Wage (₹)	Number of workers
150-200	50
200-250	30
250-300	35
300-350	20
350-400	10
Total	145

16. A company selected 4000 households at random and surveyed them to find out a relationship between income level and the number of television sets in a home. The information so obtained is listed in the following table:

Monthly income (in Rs.)	Number of televisions/household			
	0	1	2	Above 2
<10000	20	80	10	0
10000-14999	10	240	60	0
15000-19999	0	380	120	30
20000-24999	0	520	370	80
25000 and	0	1100	760	220

Exercise No. 14.4

Long Answer Questions:

1. The following are the marks (out of 100) of 60 students in mathematics.
16, 13, 5, 80, 86, 7, 51, 48, 24, 56, 70, 19, 61, 17, 16, 36, 34, 42, 34, 35, 72, 55, 75, 31, 52, 28, 72, 97, 74, 45, 62, 68, 86, 35, 85, 36, 81, 75, 55, 26, 95, 31, 7, 78, 92, 62, 52, 56, 15, 63, 25, 36, 54, 44, 47, 27, 72, 17, 4, 30.
Construct a grouped frequency distribution table with width 10 of each class starting from 0 - 9.

Solution:

Arranging the given data in ascending order, get:

4, 5, 7, 7, 13, 15, 16, 16, 17, 17, 19, 24, 25, 26, 27, 28, 30, 31, 31, 34, 34, 35, 35, 36, 36, 36, 42, 44, 45, 47, 48, 51, 52, 52, 54, 55, 55, 56, 56, 61, 62, 62, 63, 68, 70, 72, 72, 72, 74, 75, 75, 78, 80, 81, 85, 86, 86, 92, 95, 97

Now, arrange the given data set into class interval like 0-9, 10-19, ... So, the class width in each case is 10.

The frequency distribution of the given data is given below:

Class interval	Tally marks	Frequency
0-9		4
10-19		7
20-29		5
30-39		10
40-49		5
50-59		8
60-69		5
70-79		8
80-89		5
90-99		3

2. Refer to Q1 above. Construct a grouped frequency distribution table with width 10 of each class, in such a way that one of the classes is 10 - 20 (20 not included).

Solution:

Arrange the given data into class interval 0-10, 10-20, ... The class width in each case is 10.

The frequency distribution of the given data is given below:

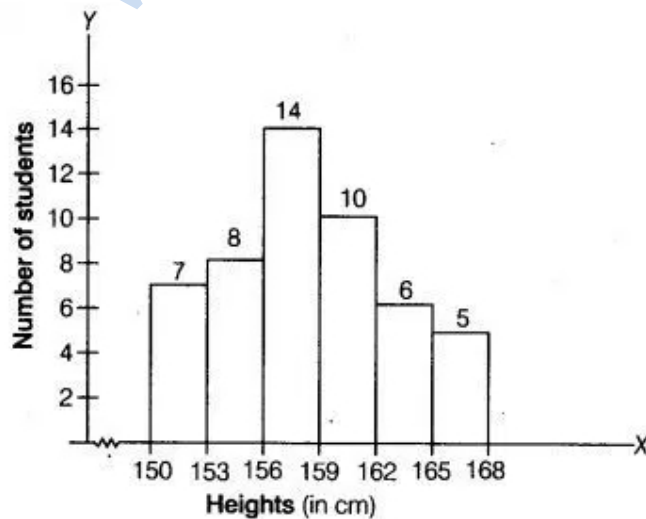
Class interval	Tally marks	Frequency
0-10		4
10-20		7
20-30		5
30-40		10
40-50		5
50-60		8
60-70		5
70-80		8
80-90		5
90-100		3

3. Draw a histogram of the following distribution:

Heights (in cm)	Number of students
150-153	7
153-156	8
156-159	14
159-162	10
162-165	6
165-168	5

Solution:

Histogram which represents the given frequency distribution is shown below:



4. Draw a histogram to represent the following grouped frequency distribution:

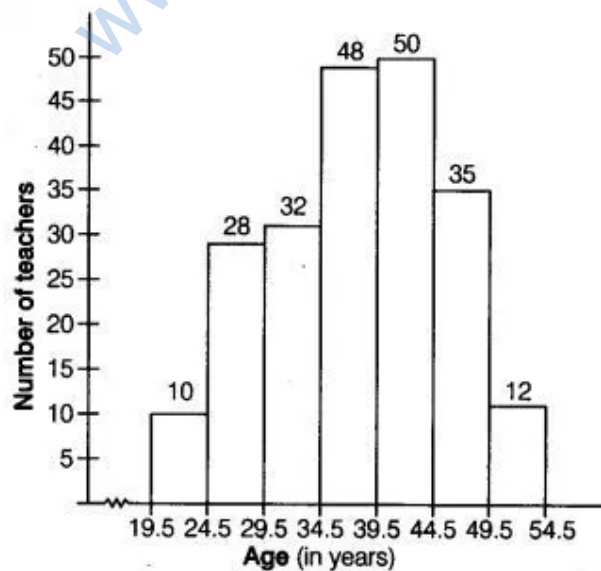
Ages (in years)	Number of teachers
20-24	10
25-29	28
30-34	32
35-39	48
40-44	50
45-49	35
50-54	12

Solution:

The given table is in exclusive form. So, first convert it into an exclusive form, as given below:

Age (in years)	Number of teachers
19.5-24.5	10
24.5-29.5	28
29.5-34.5	32
34.5-39.5	48
39.5-44.5	50
44.5-49.5	35
49.5-54.5	12

A histogram for this table is shown in the figure given below:



5. The lengths of 62 leaves of a plant are measured in millimetres and the data is represented in the following table:

Length (in mm)	Number of leaves
118-126	8
127-135	10
136-144	12
145-153	17
154-162	7
163-171	5
172-180	3

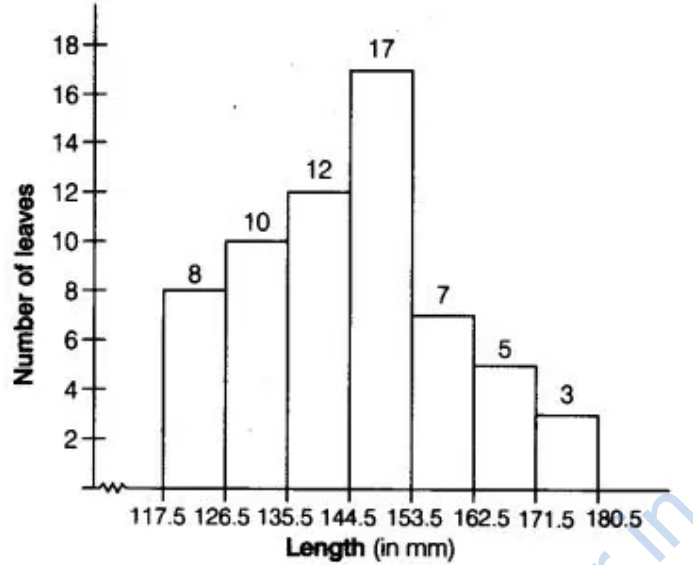
Draw a histogram to represent the data above.

Solution:

The given table is in exclusive form. So, first convert it into an exclusive form, as given below:

Length (in mm)	Number of leaves
117.5-126.5	8
126.5-135.5	10
135.5-144.5	12
144.5-153.5	17
153.5-162.5	7
162.5-171.5	5
171.5-180.5	3

A histogram for this table is shown in the figure given below:



6. The marks obtained (out of 100) by a class of 80 students are given below:

Marks	Number of students
10-20	6
20-30	17
30-50	15
50-70	16
70-100	26

Construct a histogram to represent the data above.

Solution:

See the given table, the class intervals are of unequal width. So, we calculate the adjusted frequency for each class.

Now, minimum size = 20 - 10 = 10

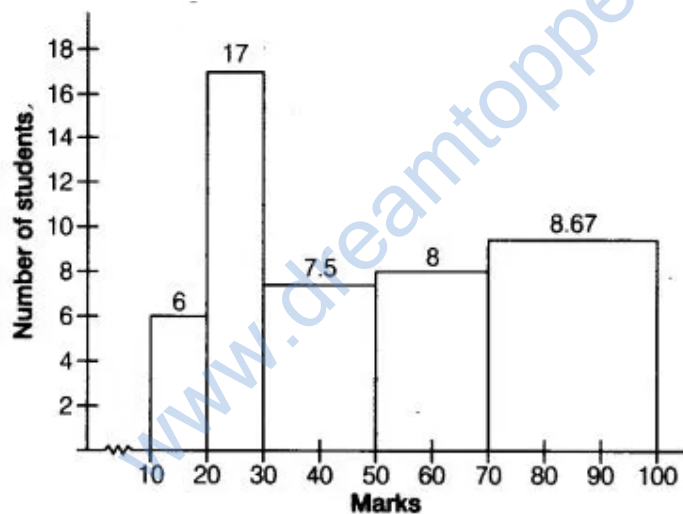
The formula of adjusted frequencies are:

$$\text{Adjusted frequency} = \frac{\text{Minimum class size}}{\text{Class size}} \times \text{Frequency of the class}$$

Now, the modified table for frequency distribution is given by:

Marks	Number of students (Frequency)	Adjusted frequency
10-20	6	$\frac{10}{10} \times 6 = 6$
20-30	17	$\frac{10}{10} \times 17 = 17$
30-50	15	$\frac{10}{20} \times 15 = \frac{15}{2} = 7.5$
50-70	16	$\frac{10}{20} \times 16 = \frac{16}{2} = 8$
70-100	26	$\frac{10}{30} \times 26 = \frac{26}{3} = 8.67$

Now, let's construct rectangles with class-limits as base and respective adjusted frequencies as heights.



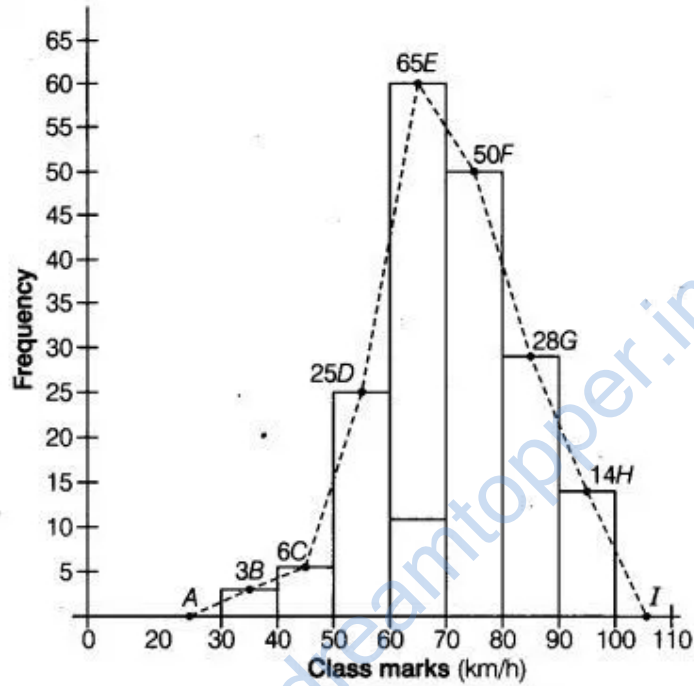
7. Following table shows a frequency distribution for the speed of cars passing through at a particular spot on a high way:

Class interval (km/h)	Frequency
30-40	3
40-50	6
50-60	25
60-70	65
70-80	50
80-90	28
90-100	14

Draw a histogram and frequency polygon representing the data above.

Solution:

In the given figure below, a histogram and a frequency polygon are drawn on the same scale. The class interval has represented on the x-axis and the frequency has represented on the y-axis.

**8. Refer to Q. 7:**

Draw the frequency polygon representing the above data without drawing the histogram.

Solution:

First we obtain in the class marks (mid-marks) of the given table as:

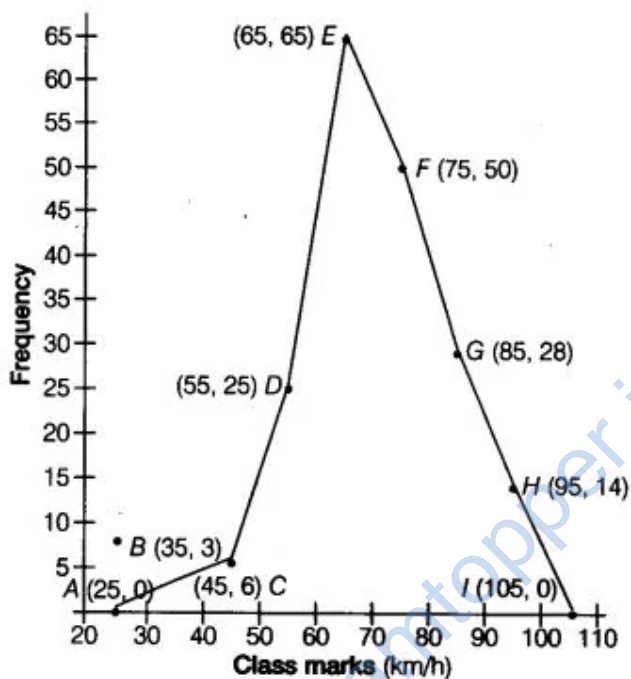
$$\text{Class-marks} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

Since, the new table is shown below:

Class interval (km/h)	Class marks	Frequency
30-40	35	3
40-50	45	6
50-60	55	25
60-70	65	65
70-80	75	50
80-90	85	28
90-100	95	14

Now, let's draw a frequency polygon by plotting the class marks along the x-axis and the frequency along y-axis.

Also, plotting all the points as B(35,3), C(45, 6), D(55, 25), E(65, 65), F(75, 50), G(85, 28) and H(95, 14). Then, join all these point line segment, shown below:



9. Following table gives the distribution of students of sections A and B of a class according to the marks obtained by them.

Section A		Section B	
Marks	Frequency	Marks	Frequency
0-15	5	0-15	3
15-30	12	15-30	16
30-45	28	30-45	25
45-60	30	45-60	27
60-75	35	60-75	40
75-90	13	75-90	10

Represent the marks of the students of both the sections on the same graph by two frequency polygons. What do you observe?

Solution:

First we obtain in the mid-marks of the given section A and B as:

$$\text{Mid-marks} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

Since, the new table for section A and B is shown below:

Section A			Section B		
Marks	Mid marks	Frequency	Marks	Mid marks	Frequency
0-15	7.5	5	0-15	7.5	3
15-30	22.5	12	15-30	22.5	16
30-45	37.5	28	30-45	37.5	25
45-60	52.5	30	45-60	52.5	27
60-75	67.5	35	60-75	67.5	40
75-90	82.5	13	75-90	82.5	10

Now, we represent the mid-marks on x-axis and the frequency on y-axis.

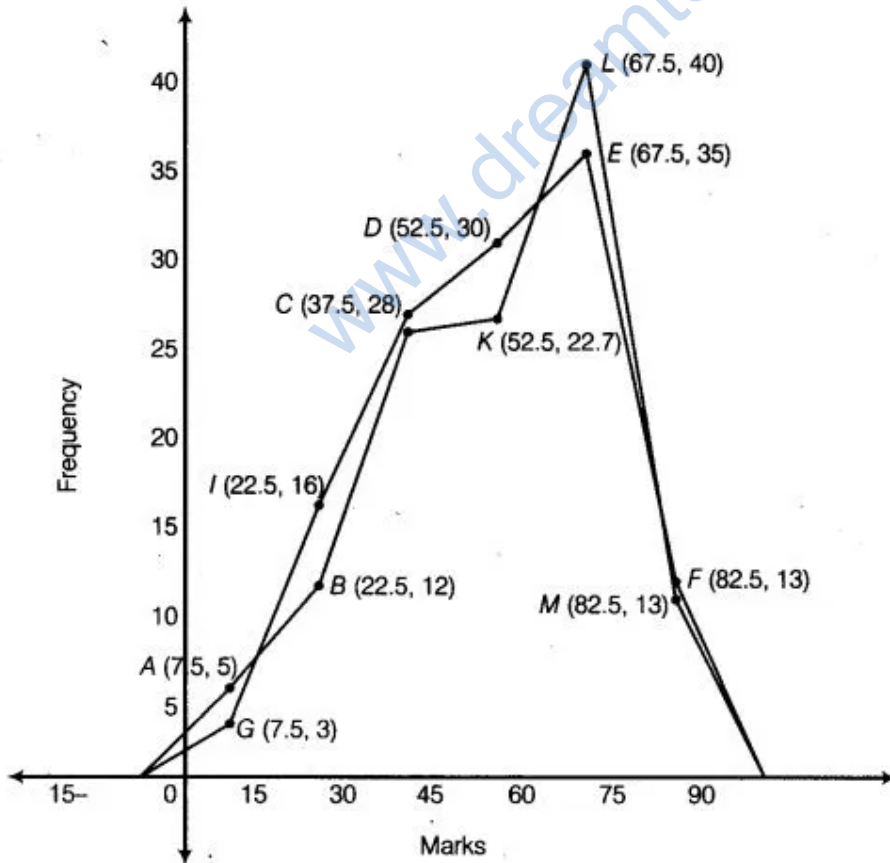
So, plotting all the points for section A as:

A(7.5, 5), B(22.5, 12), C(37.5, 28), D(52.5, 30), E(67.5, 35), F(82.5, 13)

Similarly, plotting all the points for section B as:

H(7.5, 3), I(22.5, 16), J(37.5, 25), K(52.5, 27), L(67.5, 40), M(82.5, 10)

Now, the two frequency polygon are shown in the figure given below:



10. The mean of the following distribution is 50.

x	f
10	17
30	$5a+3$
50	32
70	$7a-11$
90	19

Find the value of a and hence the frequencies of 30 and 70.

Solution:

The formula of the mean (\bar{x}) is:

$$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

So, the value of a will be calculated as follows:

$$50 = \frac{17 \times 10 + (5a + 3) \times 30 + 32 \times 50 + (7a - 11) \times 70 + 19 \times 90}{17 + 5a + 3 + 32 + 7a - 11 + 19}$$

$$50 = \frac{170 + 150a + 90 + 1600 + 490a - 770 + 1710}{12a + 60}$$

$$50 = \frac{3570 - 770 + 640a}{12a + 60}$$

$$2800 + 640a = 600a + 3000$$

$$40a = 200$$

$$a = 5$$

So, frequency of 30 will be = $5a + 3 = 5 \times 5 + 3 = 25 + 3 = 28$

Similarly, frequency of 70 will be = $7a - 11 = 7 \times 5 - 11 = 35 - 11 = 24$

Therefore, the frequency of 30 and 70 are 28 and 24 respectively.

11. The mean marks (out of 100) of boys and girls in an examination are 70 and 73, respectively. If the mean marks of all the students in that examination is 71, find the ratio of the number of boys to the number of girls.

Solution:

Let the number of boys be n_1 and number of girls be n_2 .

According to the question,

Mean marks of boys (\bar{x}_1) = 70

Mean marks of girls (\bar{x}_2) = 73

The formula of mean (\bar{x}) will be: