Chapter - 9

Some Applications of Trigonometry

Height and Distance

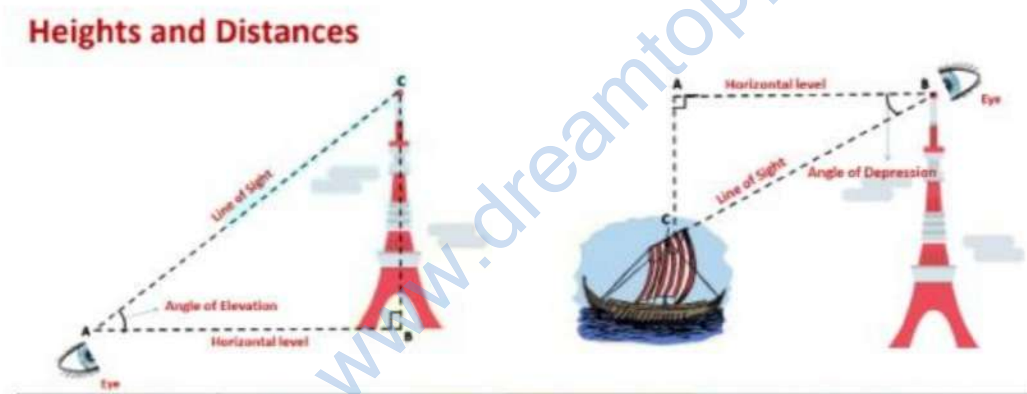
Introduction

Astronomers have used trigonometry to calculate distances from the earth to the planets and stars.

Trigonometry is used to measure the height of a building or a mountain.

Trigonometry is also used to construct maps and to determine the position of an island in relation to the longitudes and latitudes.

Heights and Distances



Horizontal Level	The horizontal level is the line parallel to the ground from the eye level of the observer.
Line of Sight	The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer
Angle of Elevation	The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, that is, when the head is raised to look at objects.
Angle of Depression	The angle of depression of a point being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, that is when the head is lowered to look at the object.

If we have to find the height of the tower without actually measuring it, then we need the following informations,

- The distance AB, of the observer from the foot of the tower.
- The angle of elevation, ∠BAC

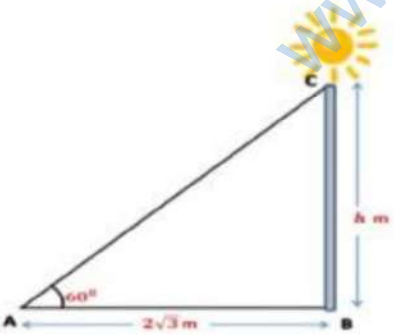
Now we will use trigonometric ratios to find the value of BC (Height of the tower)

• So, we see that when we use trigonometric ratios, tan or cot we are using the two values that we know, that is, AB and ∠BAC.

• tan A = $\frac{BC}{AB}$ or cot A = $\frac{AB}{BC}$, which on solving give us BC.

Example: A wall casts a shadow of length 2√3 m on the ground, when the sun's elevation is 60°

Find the height of the wall.



Let BC = h m be the height of the wall and AB = $2\sqrt{3}$ m be the length of the shadow.

∠BAC = 60° is the Angle of elevation.

In right-angled triangle Δ ABC

$\tan 60° = \frac{BC}{AB} \Rightarrow \sqrt{3} = \frac{BC}{2\sqrt{3}}$

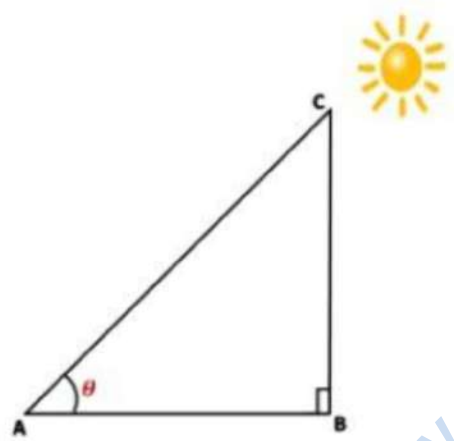
$BC = \sqrt{3} \times 2\sqrt{3} = 2\times3 = 6$ m

Example: If the height and the length of the shadow of a man are the same, then find the angle of elevation of the Sun.

Let BC be the height of the man and AB be the length of the shadow of the man.

Let the angle of elevation of the sun be θ

We know that the height and the length of the shadow of a man are equal.



$\therefore AB = BC$

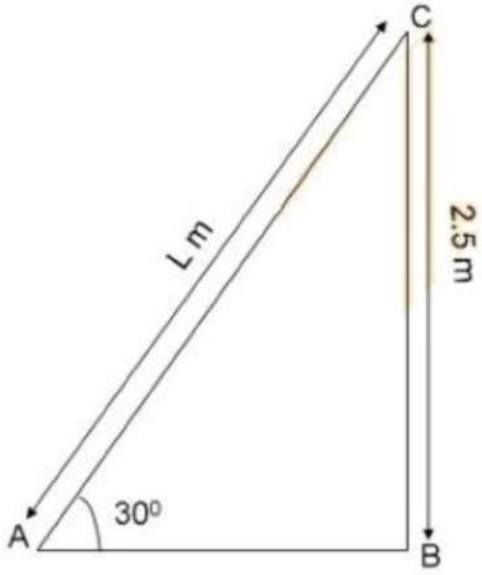
$\tan\theta = \frac{BC}{AB} \Rightarrow \tan\theta = \frac{BC}{BC}$ $(\because AB = BC)$

$\tan\theta = 1 \Rightarrow \tan\theta = \tan 45^{\circ}$

$\theta = 45^{\circ}$

Example: A ladder, leaning against a wall, makes an angle of 30° with the horizontal. If the height of the wall is 2.5 m, then find the length of the ladder.

Let AC = l m be the length of the ladder.



Angle

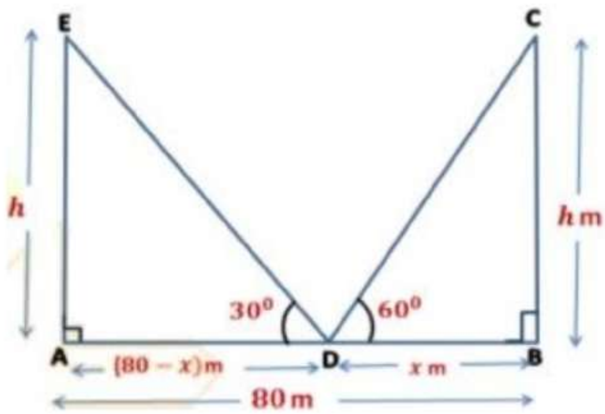
In right-angled Δ ABC,

$\sin 30^\circ = \frac{BC}{AC} \Rightarrow \frac{1}{2} = \frac{2.5}{l}$

l = 5 m

Example: Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

(REFERENCE: NCERT)



Here,

Width of the road = 80 m

Let the height of the two poles be h m.

Let D be the point on AB such that angle of elevations from point D are ∠ADE = 30° and ∠BDE = 60°

Let BD = x m

Then, AD = AB – BD = (80 - x) m

In right angled Δ DAE, tan θ = $\frac{AE}{AD} = \frac{h}{80-x} \Rightarrow tan30 = \frac{h}{(80-x)}$

tan 30° = $\frac{h}{(80-x)} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{(80-x)}$ (tan 30° = $\frac{1}{\sqrt{3}}$)

⇒ $\sqrt{3}h = 80 - x$

⇒ $\sqrt{3}h + x = 80 \longrightarrow$ Eq 1

In right-angled Δ DBC, tan θ = $\frac{BC}{BD} = \frac{h}{x} \Rightarrow tan60 = \frac{h}{x}$

tan60° = $\frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$ (tan 60°=$\sqrt{3}$)

$h = \sqrt{3}x \longrightarrow$ Eq 2

On putting h = $\sqrt{3}$x in Eq 1, we get

$\sqrt{3}(\sqrt{3}x) + x = 80 \Rightarrow 3x + x = 80 \Rightarrow 4x = 80 \Rightarrow x = 20$ m

On putting x = 20 m in Eq 2 we get,

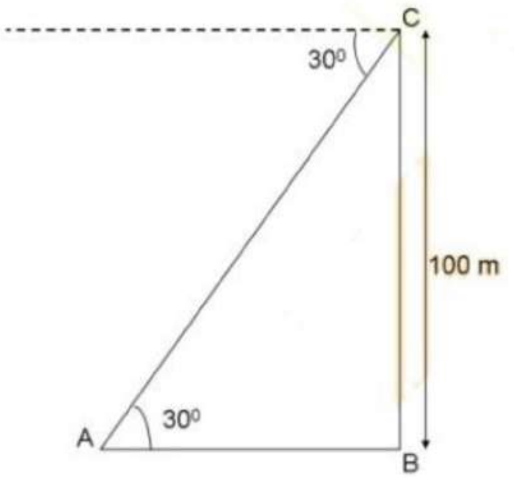
h = 20$\sqrt{3}$ m

Now, AD = (80 − 20) = 60 m

Height of the poles is $20\sqrt{3}$ m and the distances of the point D from the poles are 60 m and 20 m.

Example: The angle of depression of a car standing on the ground, from the top of a 100 m high tower, is 30^{o}.

Find the distance of the car from the base of the tower.



Let AB be the distance of the car from the base of the tower.

The height of the tower, BC is 100 m.

Angle of Depression, $\angle DCA = 30^{o}$

Now, $\angle DCA = \angle CAB = 30^{o}$ (Alternate Angles)

Now, in right-angled Δ ABC,

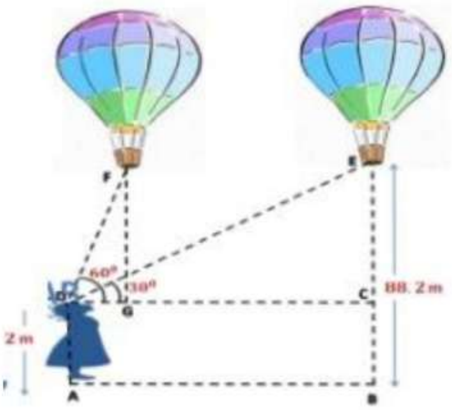
$\tan 30o = \frac{BC}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{AB}$ $(\tan 30^{o} = \frac{1}{\sqrt{3}})$

$\Rightarrow AB = 100\sqrt{3}$m

Therefore, the distance between the car and the base of the tower is $100\sqrt{3}$ m

Example: A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60^{o}. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

(REFERENCE: NCERT)



Here,

Height of the girl, AD = 1.2 m

FH = EB = 88.2 m is the height of the balloon from AB.

The angles of elevation at the eye of the girl are,

∠FDC = 60° and ∠EDC = 30°

Now, FG = EC = EB – BC

FG = 88.2 – 1.2 = 87 m

Let DG = xm and GC = x m be the distance travelled by the balloon In right angled Δ FGD

$$\tan 60° = \frac{FG}{DG} \Rightarrow \sqrt{3} = \frac{87}{x}$$

$$x = \frac{87}{\sqrt{3}} m \rightarrow Eq1$$

In right-angled Δ ECD

tan 30° = $\frac{EC}{CD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{x+y}$ (tan 30° = and CD = CG +GD)

$x + y = 87\sqrt{3} \rightarrow Eq2$

On putting x = $\frac{87}{\sqrt{3}}$in Eq 2

$\frac{87}{\sqrt{3}} + y = 87\sqrt{3} - \frac{87}{\sqrt{3}} = \frac{87X3 - 87}{\sqrt{3}} = \frac{87(3-1)}{\sqrt{3}}$

$= \frac{87X2}{\sqrt{3}}$

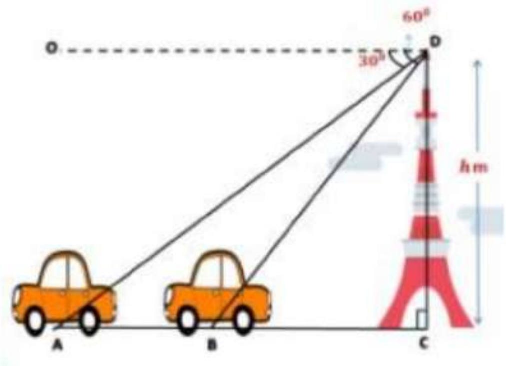
$= \frac{174}{\sqrt{3}} X \frac{\sqrt{3}}{\sqrt{3}} = \frac{174X\sqrt{3}}{3} = 58\sqrt{3}m$

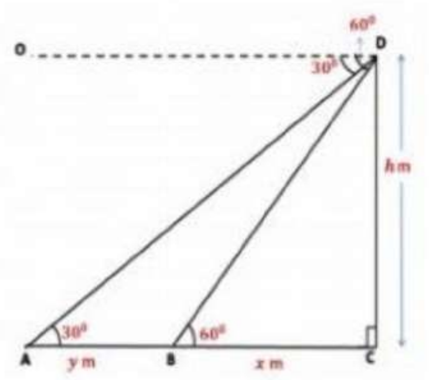
Therefore, the distance traveled by the balloon during the interval is58$\sqrt{3}$ m.

Example: A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30° , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

(REFERENCE: NCERT)

Let h be the height of the tower. The observer is standing at point D of the tower and observes the car at an angle of depression of 30°. After 6 seconds, the angle of depression of the car is 60°





Here, ∠ ODA = 30° and ∠ ODB = 60°

Now, ∠ ODA = ∠ DAC = 30°

(Alternate Angles)

∠ ODB = ∠ DBC = 60° (Alternate Angles)

Let AB = y m and BC = x m

In Δ ACD

tan 30° $= \frac{DC}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y} \Rightarrow x + y = \sqrt{3h} \rightarrow Eq1$

In Δ BCD

tan60° = $\frac{DC}{BC} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h \Rightarrow \sqrt{3x} \rightarrow Eq2$

Putting the value of h in Eq 1 we get,

$x + y = \sqrt{3}X\sqrt{3x} \Rightarrow x + y = 3x \Rightarrow 2x = y \rightarrow Eq2$

Let the speed of the car be v km/s. Now the car moves from A to B in 6 seconds.

∴ Time = $\frac{distance}{speed} = \frac{y}{v} = 6 = \frac{y}{v} = y = 6v$

On putting y = 6v in Eq 1 we get,

$2x = 6v \Rightarrow x = 3v$

$\therefore$ Time $= \frac{x}{v} = \frac{3v}{v} = 3s$

Therefore, the car moves from point B to C in 3 s.