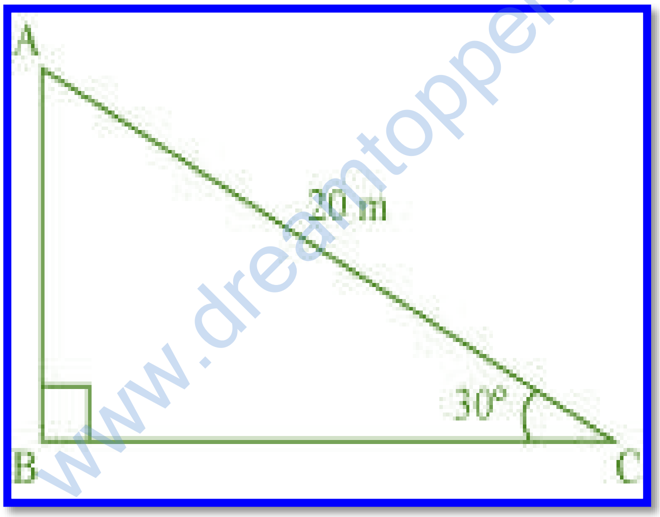
Mathematics

(Chapter – 9) (Some Applications of Trigonometry)

(Class – X)

Exercise 9.1

Question 1:
A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30 °.



Answer 1:
It can be observed from the figure that AB is the pole.

In ΔABC,

$$\frac{AB}{AC} = \sin 30°$$

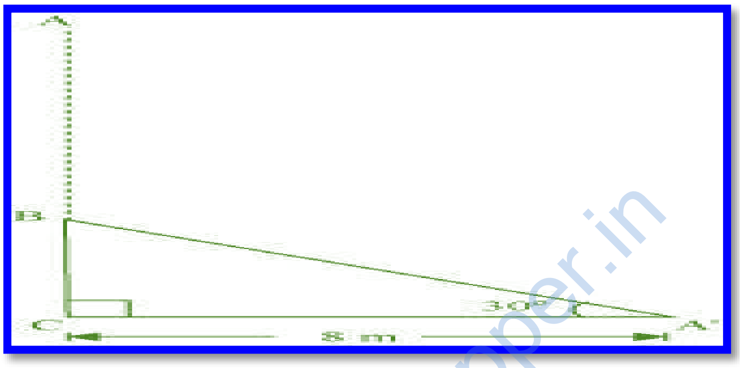
$$\frac{AB}{20} = \frac{1}{2}$$

$$AB = \frac{20}{2} = 10$$

Therefore, the height of the pole is 10 m.

Question 2:
A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30 ° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Answer 2:



Let AC was the original tree. Due to storm, it was broken into two parts. The broken part $A'B$ is making 30° with the ground.

In $\Delta A'BC$,

$$\frac{BC}{A'C} = \tan 30°$$

$$\frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$BC = \left(\frac{8}{\sqrt{3}}\right) m$$

$$\frac{A'C}{A'B} = \cos 30°$$

$$\frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$A'B = \left(\frac{16}{\sqrt{3}}\right) m$$

Height of the tree = A'B + BC

$$= \left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) m = \frac{24}{\sqrt{3}} \text{ m}$$

$$= 8\sqrt{3} \text{ m}$$

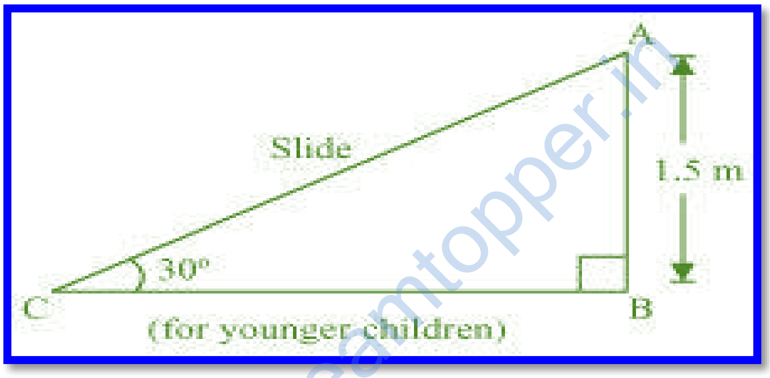
Hence, the height of the tree is $8\sqrt{3}$ m.

Question 3:

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30 ° to the ground, where as for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of 60 ° to the ground. What should be the length of the slide in each case?

Answer 3:

It can be observed that AC and PR are the slides for younger and elder children respectively.

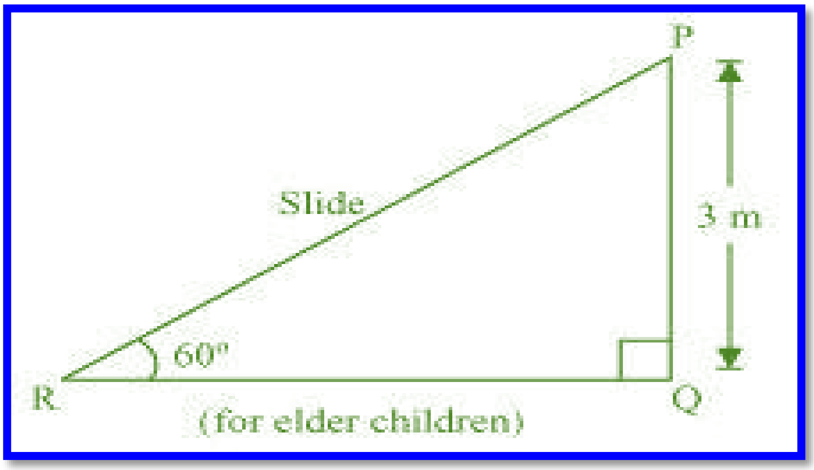


In ΔABC,

$$\frac{AB}{AC} = \sin 30°$$

$$\frac{1.5}{AC} = \frac{1}{2}$$

$$AC = 3 \text{ m}$$



In ΔPQR,

$$\frac{PQ}{PR} = \sin 60$$

$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

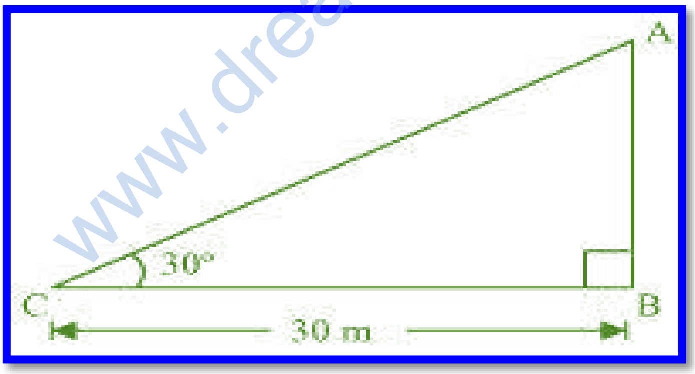
$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3}\ \text{m}$$

Therefore, the lengths of these slides are 3 m and $2\sqrt{3}$ m.

Question 4:

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. Find the height of the tower.

Answer 4:



Let AB be the tower and the angle of elevation from point C (on ground) is 30°.
In ΔABC,

$$\frac{AB}{BC} = \tan 30^\circ$$

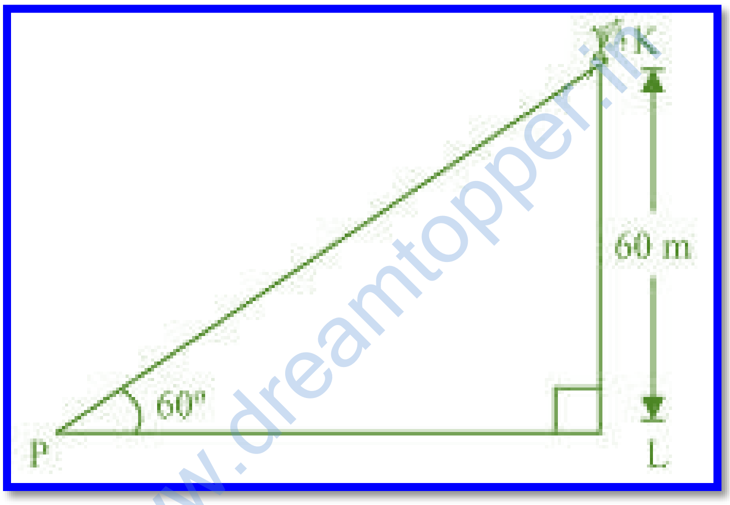
$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}\ \text{m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m.

Question 5:
A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Answer 5:



Let K be the kite and the string is tied to point P on the ground.

In ΔKLP,

$$\frac{KL}{KP} = \sin 60°$$

$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$

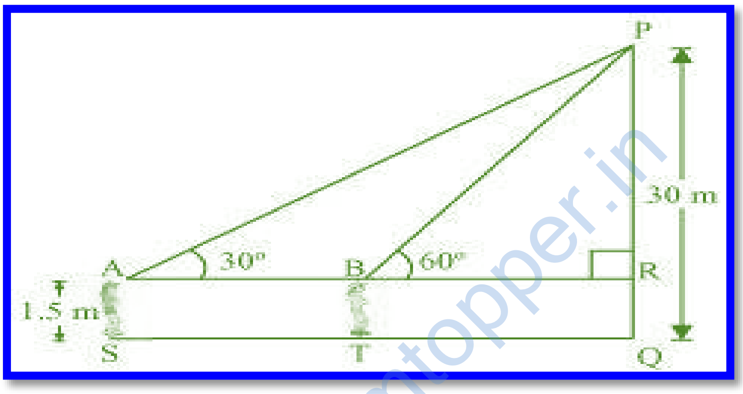
$$KP = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

Hence, the length of the string is $40\sqrt{3}$ m.

Question 6:

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer 6:



Let the boy was standing at point S initially. He walked towards the building and reached at point T. It can be observed that

PR = PQ − RQ

= (30 − 1.5) m = 28.5 m = $\frac{57}{2}$ m

In ΔPAR,

$$\frac{PR}{AR} = \tan 30^\circ$$

$$\frac{57}{2AR} = \frac{1}{\sqrt{3}}$$

$$AR = \left(\frac{57}{2}\sqrt{3}\right) \text{m}$$

In ΔPRB,

$$\frac{PR}{BR} = \tan 60^\circ$$

$$\frac{57}{2\,BR} = \sqrt{3}$$

$$BR = \frac{57}{2\sqrt{3}} = \left(\frac{19\sqrt{3}}{2}\right) \text{m}$$

ST = AB

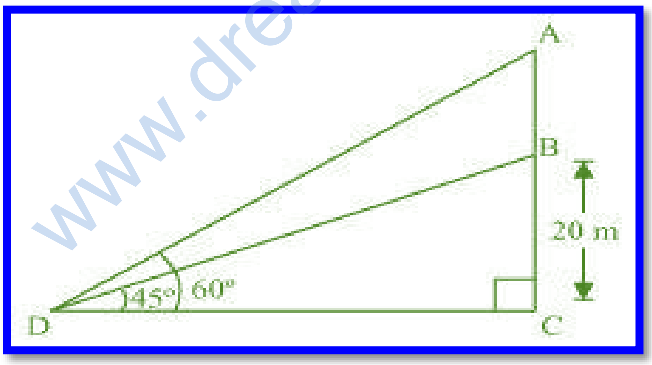
$$= AR - BR = \left(\frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2}\right) m$$

$$= \left(\frac{38\sqrt{3}}{2}\right) m = 19\sqrt{3} \text{ m}$$

Hence, he walked $19\sqrt{3}$ m towards the building.

Question 7:
From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Answer 7:



Let BC be the building, AB be the transmission tower, and D be the point on the ground from where the elevation angles are to be measured.
In ΔBCD,

$$\frac{BC}{CD} = \tan 45°$$

$$\frac{20}{CD} = 1$$

$$CD = 20 \text{ m}$$

In ΔACD,

$$\frac{AC}{CD} = \tan 60°$$

$$\frac{AB + BC}{CD} = \sqrt{3}$$

$$\frac{AB + 20}{20} = \sqrt{3}$$

$$AB = \left(20\sqrt{3} - 20\right) m$$

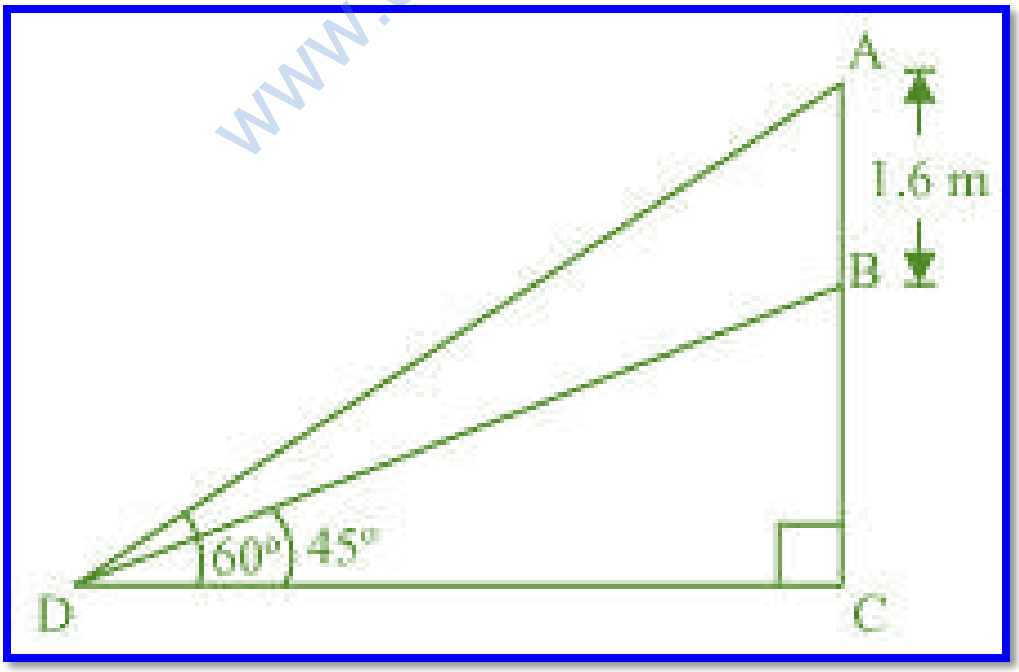
$$= 20\left(\sqrt{3} - 1\right) m$$

Therefore, the height of the transmission tower is $20(\sqrt{3} - 1)$ m.

Question 8:

A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45 °. Find the height of the pedestal.

Answer 8:



Let AB be the statue, BC be the pedestal, and D be the point on the ground from where the elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD} = \tan 45°$$
$$\frac{BC}{CD} = 1$$
$$BC = CD$$

In ΔACD,

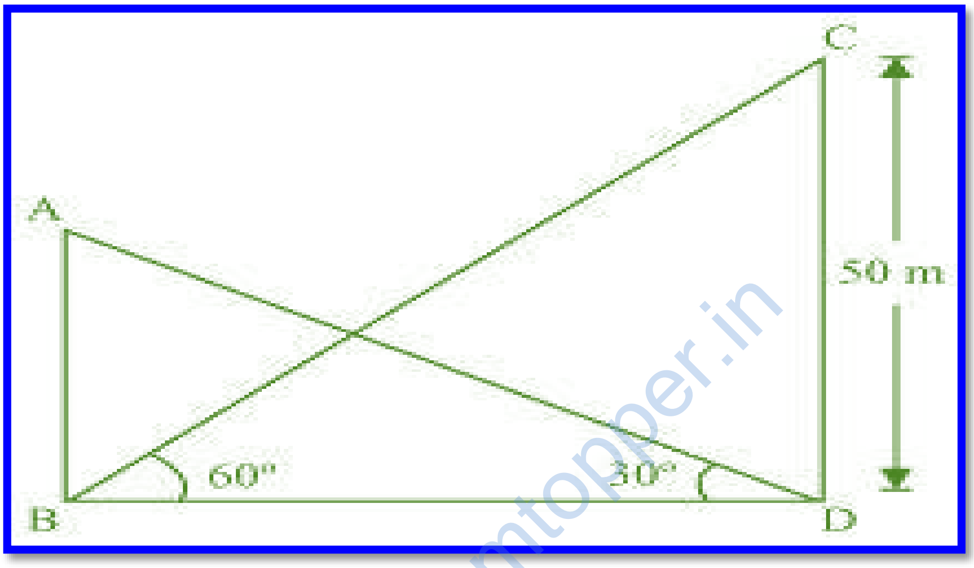
$$\frac{AB + BC}{CD} = \tan 60°$$
$$\frac{AB + BC}{BC} = \sqrt{3}$$
$$1.6 + BC = BC\sqrt{3}$$
$$BC\left(\sqrt{3} - 1\right) = 1.6$$
$$BC = \frac{(1.6)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$
$$= \frac{1.6\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2 - (1)^2}$$
$$= \frac{1.6\left(\sqrt{3}+1\right)}{2} = 0.8\left(\sqrt{3}+1\right)$$

Therefore, the height of the pedestal is $0.8\left(\sqrt{3}+1\right)$ m.

Question 9:

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Answer 9:



Let AB be the building and CD be the tower.

In ΔCDB,

$$\frac{CD}{BD} = \tan 60^\circ$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$

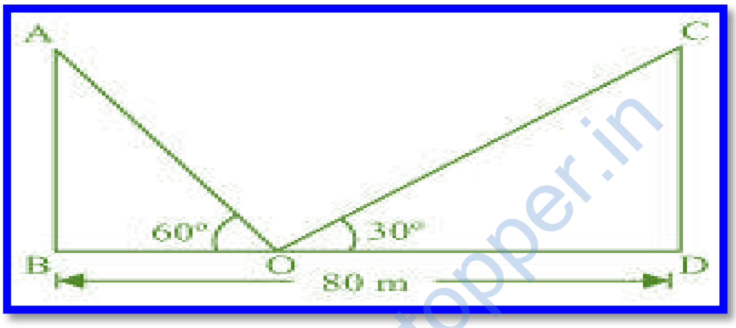
$$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$$

Therefore, the height of the building is $16\frac{2}{3}$ m.

Question 10:

Two poles of equal heights are standing opposite each other and either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30º, respectively. Find the height of poles and the distance of the point from the poles.

Answer 10:



Let AB and CD be the poles and O is the point from where the elevation angles are measured.

In ΔABO,

$$\frac{AB}{BO} = \tan 60°$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}}$$

In ΔCDO,

$$\frac{CD}{DO} = \tan 30°$$

$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$

$$CD\sqrt{3} = 80 - BO$$

$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$$

$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

Since the poles are of equal heights,

CD = AB

$$CD\left[\sqrt{3}+\frac{1}{\sqrt{3}}\right]=80$$

$$CD\left(\frac{3+1}{\sqrt{3}}\right)=80$$

$$CD=20\sqrt{3}\text{ m}$$

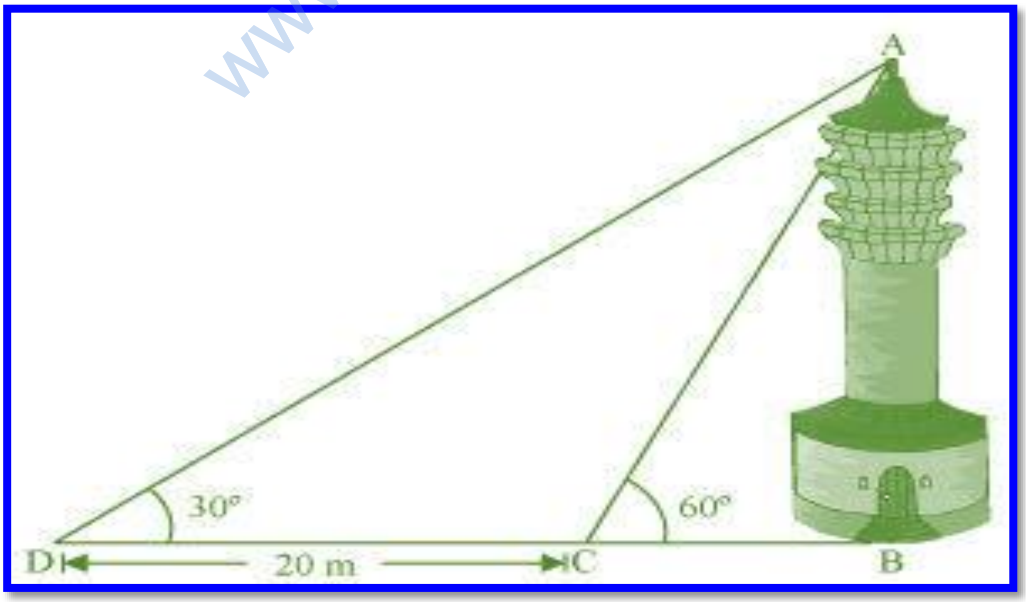
$$BO=\frac{AB}{\sqrt{3}}=\frac{CD}{\sqrt{3}}=\left(\frac{20\sqrt{3}}{\sqrt{3}}\right)\text{m}=20\text{ m}$$

DO = BD − BO = (80 − 20) m = 60 m

Therefore, the height of poles is $20\sqrt{3}$ m and the point is 20 m and 60 m far from these poles.

Question 11:

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.



Answer 11:

In ΔABC,

$$\frac{AB}{BC} = \tan 60°$$

$$\frac{AB}{BC} = \sqrt{3}$$

$$BC = \frac{AB}{\sqrt{3}}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

$$\frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}}$$

$$\frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$

$$2AB = 20\sqrt{3}$$

$$AB = 10\sqrt{3} \text{ m}$$

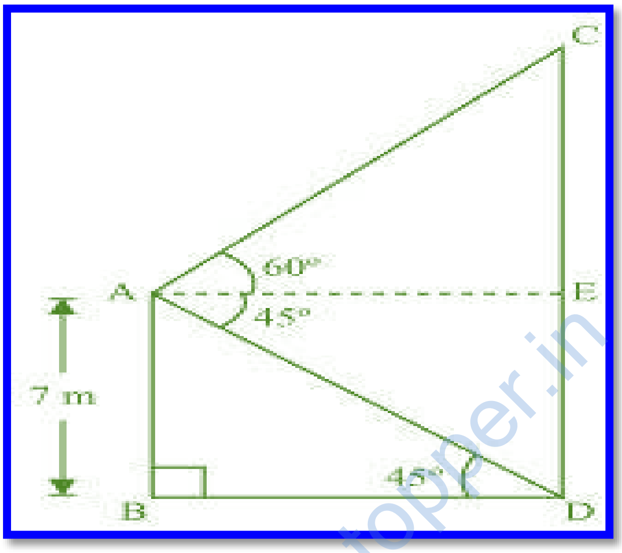
$$BC = \frac{AB}{\sqrt{3}} = \left(\frac{10\sqrt{3}}{\sqrt{3}}\right) \text{m} = 10 \text{ m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

Question 12:

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Answer 12:



Let AB be a building and CD be a cable tower.

In ΔABD,

$$\frac{AB}{BD} = \tan 45°$$

$$\frac{7}{BD} = 1$$

$$BD = 7 \text{ m}$$

In ΔACE,

AC = BD = 7 m

$$\frac{CE}{AE} = \tan 60°$$

$$\frac{CE}{7} = \sqrt{3}$$

$$CE = 7\sqrt{3} \text{ m}$$

$$CD = CE + ED = \left(7\sqrt{3} + 7\right)\text{m}$$

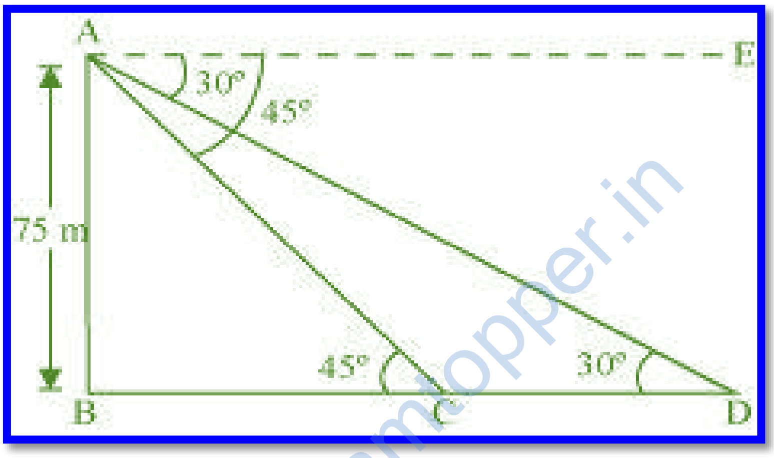
$$= 7\left(\sqrt{3} + 1\right)\text{m}$$

Therefore, the height of the cable tower is $7\left(\sqrt{3}+1\right)$ m.

Question 13:

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer 13:



Let AB be the lighthouse and the two ships be at point C and D respectively.

In ΔABC,

$$\frac{AB}{BC} = \tan 45°$$

$$\frac{75}{BC} = 1$$

$$BC = 75 \text{ m}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

$$\frac{75}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{75}{75 + CD} = \frac{1}{\sqrt{3}}$$

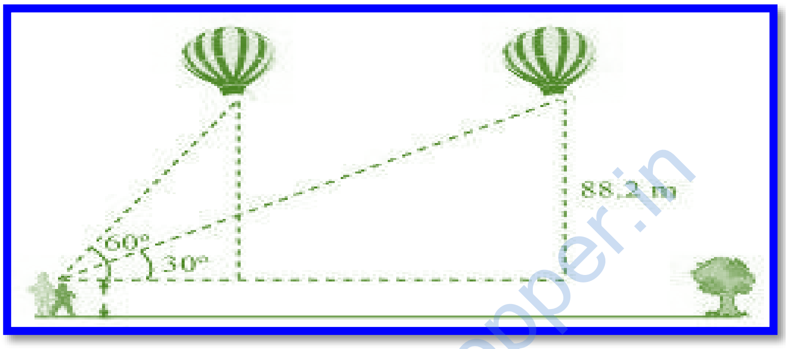
$$75\sqrt{3} = 75 + CD$$

$$75\left(\sqrt{3} - 1\right)\text{m} = CD$$

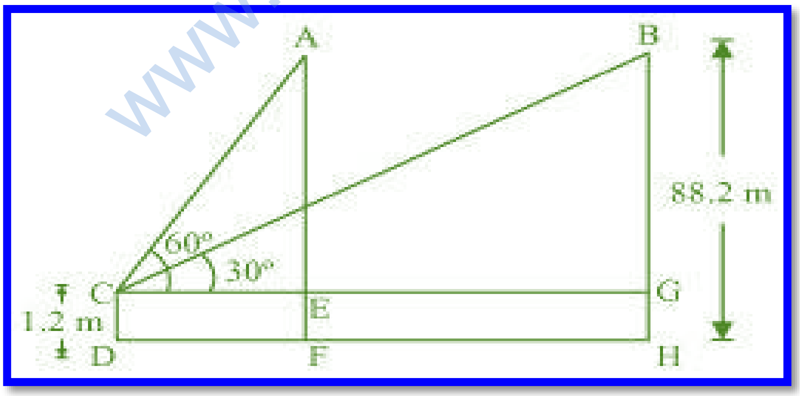
Therefore, the distance between the two ships is $75\left(\sqrt{3}-1\right)$ m.

Question 14:

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



Answer 14:



Let the initial position A of balloon change to B after some time and CD be the girl.

In ΔACE,

$$\frac{AE}{CE} = \tan 60°$$

$$\frac{AF - EF}{CE} = \tan 60°$$

$$\frac{88.2 - 1.2}{CE} = \sqrt{3}$$

$$\frac{87}{CE} = \sqrt{3}$$

$$CE = \frac{87}{\sqrt{3}} = 29\sqrt{3} \text{ m}$$

In ΔBCG,

$$\frac{BG}{CG} = \tan 30°$$

$$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$$

$$87\sqrt{3} \text{ m} = CG$$

Distance travelled by balloon = EG = CG − CE

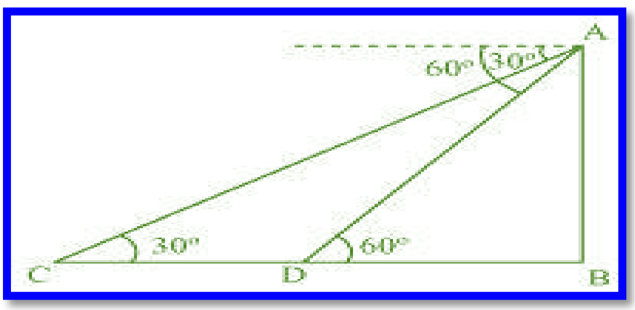
$$= \left(87\sqrt{3} - 29\sqrt{3}\right)\text{m}$$

$$= 58\sqrt{3} \text{ m}$$

Question 15:

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Answer 15:



Let AB be the tower.

Initial position of the car is C, which changes to D after six seconds.

In ΔADB,

$$\frac{AB}{DB} = \tan 60^\circ$$

$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}}$$

In ΔABC,

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

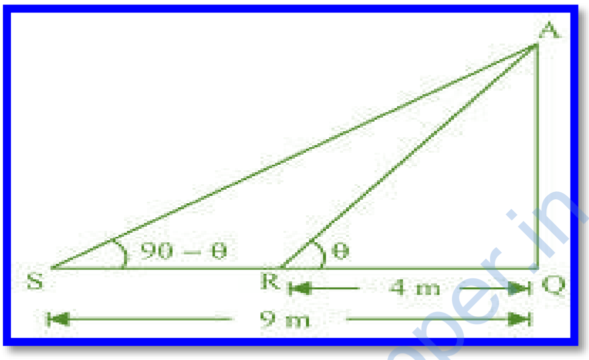
$$= \frac{2AB}{\sqrt{3}}$$

Time taken by the car to travel distance DC= $\left(\text{i.e., } \frac{2AB}{\sqrt{3}}\right)$ 6 seconds Time taken by the car to travel distance DB $\left(\text{i.e., } \frac{AB}{\sqrt{3}}\right) = \frac{6}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}}$

$$= \frac{6}{2} = 3 \text{ seconds}$$

Question 16:
The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m. from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Answer 16:



Let AQ be the tower and R, S are the points 4m, 9m away from the base of the tower respectively.
The angles are complementary. Therefore, if one angle is θ, the other will be 90 − θ.

In ΔAQR,

$$\frac{AQ}{QR} = \tan\theta$$

$$\frac{AQ}{4} = \tan\theta \qquad ...(i)$$

In ΔAQS,

$$\frac{AQ}{SQ} = \tan(90 - \theta)$$

$$\frac{AQ}{9} = \cot\theta \qquad ...(ii)$$

On multiplying equations (*i*) and (*ii*), we obtain

$$\left(\frac{AQ}{4}\right)\left(\frac{AQ}{9}\right) = (\tan\theta).(\cot\theta)$$

$$\frac{AQ^2}{36} = 1$$

$$AQ^2 = 36$$

$$AQ = \sqrt{36} = \pm 6$$

However, height cannot be negative.
Therefore, the height of the tower is 6 m.