Chapter 4

Quadrilateral Equations

Exercise No. 4.1

Choose the correct answer from the given four options in the following questions:

1.

Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$ **(B) $-2x^2 = (5 - x)(2x-(2/5))$**

(C) $(k + 1) x^2 + (3/2) x = 7$, where $k = -1$ **(D) $x^3 - x^2 = (x - 1)^3$**

Solution:

Correct answer is (D) $x^3 - x^2 = (x - 1)^3$

Justification:
We have quadratic equation:
$ax^2 + bx + c = 0$,

(A)
$x^2 + 2x + 1 = (4 - x)^2 + 3$
$x^2 + 2x + 1 = 16 - 8x + x^2 + 3$
$10x - 18 = 0$
This is linear equation.

(B)
$-2x^2 = (5 - x)(2x - 2/5)$
$-2x^2 = 10x - 2x^2 - 2 + 2/5x$
$52x - 10 = 0$
This is also a linear equation.

(C)
$(k + 1) x^2 + 3/2 x = 7$,
As $k = -1$
$(-1 + 1) x^2 + 3/2 x = 7$
$3x - 14 = 0$
This is a linear equation.

(D)
$x^3 - x^2 = (x - 1)^3$
$x^3 - x^2 = x^3 - 3x^2 + 3x - 1$
$2x^2 - 3x + 1 = 0$
Above equation represents a quadratic equation.

2.

Which of the following is not a quadratic equation?

(A) $2(x - 1)^2 = 4x^2 - 2x + 1$ (B) $2x - x^2 = x^2 + 5$

(C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Solution:

Correct answer is (D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Equation to be quadratic, it should be in the form,

$ax^2 + bx + c = 0, a \neq 0$

(A)

$2(x - 1)^2 = 4x^2 - 2x + 1$

$2(x^2 - 2x + 1) = 4x^2 - 2x + 1$

$2x^2 + 2x - 1 = 0$

This equation represents a quadratic equation.

(B)

$2x - x^2 = x^2 + 5$

$2x^2 - 2x + 5 = 0$

This equation represents a quadratic equation.

(C)

$(\sqrt{2}x + \sqrt{3})^2 = 3x^2 - 5x$

$2x^2 + 2\sqrt{6}x + 3 = 3x^2 - 5x$

$x^2 - (5 + 2\sqrt{6})x - 3 = 0$

This equation represents a quadratic equation.

(D)

$(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

$x^4 + 4x^3 + 4x^2 = x^4 + 3 + 4x^2$

$4x^3 - 3 = 0$

This equation represents a cubic equation.

3.

Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$ (B) $x^2 + 3x - 12 = 0$

(C) $2x^2 - 7x + 6 = 0$ (D) $3x^2 - 6x - 2 = 0$

Solution:

Correct answer is (C) $2x^2 - 7x + 6 = 0$

As 2 is a root then putting value 2 in place of x, we should get zero.

(A)

$x^2 - 4x + 5 = 0$

$(2)^2 - 4(2) + 5 = 1$

$1 \neq 0$

Therefore, x = 2 is not a root of $x^2 - 4x + 5 = 0$

(B)

$x^2 + 3x - 12 = 0$

$(2)^2 + 3(2) - 12 = -2 \neq 0$

Therefore, x = 2 is not a root of $x^2 + 3x - 12 = 0$

(C)

$2x^2 - 7x + 6 = 0$

$2(2)^2 - 7(2) + 6 = 0$

So, x = 2 is a root of $2x^2 - 7x + 6 = 0$

(D)

$3x^2 - 6x - 2 = 0$

$3(2)^2 - 6(2) - 2 = -2$

$-2 \neq 0$

Therefore, x = 2 is not a root of $3x^2 - 6x - 2 = 0$

4.

If ½ is a root of the equation $x^2 + kx - 5/4 = 0$, then the value of k is

(A) 2 (B) – 2

(C) ¼ (D) ½

Solution:

Correct answer is (A) 2.

As, ½ is a root of the equation $x^2 + kx - 5/4 = 0$.

Putting the value of ½ in place of x gives us the value of k.

As,

x = ½

$(½)^2 + k(½) - (5/4) = 0$

$(k/2) = (5/4) - ¼$

So, k = 2

5.

Which of the following equations has the sum of its roots as 3?

(A) $2x^2 - 3x + 6 = 0$ (B) $-x^2 + 3x - 3 = 0$

(C) $\sqrt{2}x^2 - 3/\sqrt{2}x + 1 = 0$ (D) $3x^2 - 3x + 3 = 0$

Solution:

Correct answer is (B) $-x^2 + 3x - 3 = 0$.

The sum of the roots of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by,

Coefficient of x / coefficient of $x^2 = -(b/a)$

(A) We have,

$2x^2 - 3x + 6 = 0$

Sum of the roots = – b/a

= - (-3/2)

Sum of the roots = 3/2

(B) We have,

$-x^2 + 3x - 3 = 0$

Sum of the roots = – b/a

= -(3/-1)

= 3

(C) We have,

$\sqrt{2}x^2 - 3/\sqrt{2}x+1=0$

$2x^2 - 3x + \sqrt{2} = 0$

Sum of the roots = – b/a

= -(-3/2)

= 3/2

(D) We have,

$3x^2 - 3x + 3 = 0$

Sum of the roots = – b/a

= -(-3/3)

= 1

6.

Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(a) 0 only
(b) 4
(c) 8 only
(d) 0, 8

Solution:

(d)
The condition for equal roots of quadratic equation $ax^2 + bx + c= 0$ is
$b^2 - 4ac = 0$.

We have:
$2x^2 - kx + k = 0$

Condition for equal roots,
$b^2 - 4ac = 0$

$(-k)^2 - 4(2)(k) = 0$ (As, a = 2, b = –k, c = +k)
$k^2 - 8k = 0$
$k(k - 8) = 0$
$k = 0$
Or
$k - 8 = 0$
$k = 8$
As, the values of k are 0 and 8.
The answer is (d).

7.

Which constant must be added and subtracted to solve the quadratic equation
$$9x^2 + \frac{3}{4}x - \sqrt{2} = 0$$
by the method of completing the square?
(a) $\frac{1}{8}$
(b) $\frac{1}{64}$
(c) $\frac{1}{4}$
(d) $\frac{9}{64}$

Solution:
(b) $\frac{1}{64}$

The given equation is
$$9x^2 + \frac{3}{4}x - \sqrt{2} = 0$$

So, to make the expression a complete square, we have to subtract $\frac{1}{64}$.
$$9x^2 + \frac{3}{4}x + \frac{1}{64} - \sqrt{2} - \frac{1}{64} = 0$$

$$\left(3x+\frac{1}{8}\right)^2=\sqrt{2}+\frac{1}{64}$$

8.

The quadratic equation has:

$$2x^2-\sqrt{5}x+1=0$$

(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than two real roots

Solution:

(c)no real roots

We have,

$$2x^2-\sqrt{5}x+1=0$$

Now,
D = b^2– 4ac,

Checking the following conditions:

(i) for no real rootsD < 0
(ii) for two equal roots D = 0
(iii) for two distinct roots D > 0 and any quadratic equation must have only roots.

The equation is:

$$2x^2-\sqrt{5}x+1=0$$

So,
D = b^2– 4ac

Where,
a = 2,
b = $-\sqrt{5}$
c = 1

D= 5 – 8
D = –3

As D < 0 so, the given equations has no real roots.

9.

Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

(b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$

(d) $5x^2 - 3x + 1 = 0$

Solution:

Correct answer is (b)$x^2 + x - 5 = 0$

We have,
For real distinct roots $D > 0$

(a) Equation is $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

$D = b^2 - 4ac$
$D = 9 \times 2 - 18$
$D = 0$

For,$D = 0$, the given equation has two real equal roots.

(b) Equation is $x^2 + x - 5 = 0$
$D = b^2 - 4ac$
$D = (1)^2 - 4(1)(-5)$ (where, $a = 1$, $b = 1$, $c = -5$)
$D = 1 + 20$
$D = 21$

For, $D > 0$, the given equation has two distinct real roots.

(c) Equation is $x^2 + 3x + 2\sqrt{2} = 0$
$D = b^2 - 4ac$
$D = (3)2 - 4(1)2\sqrt{2}$ (Where, $a = 1$, $b = 3$, $c = 2\sqrt{2}$)
$D = 9 - 11.312$
$D = -2.312$
As $D < 0$, so the given equation has no real roots.

(d) Equation is $5x^2 - 3x + 1 = 0$
$D = b2 - 4ac$
$D = (-3)2 - 4(5)(1)$ (Where, $a = 5$, $b = -3$, $c = 1$)
$D = 9 - 20$
$D = -11$
As $D < 0$, so the given equation has no real roots.

10.

Which of the following equations has no real roots.

(a) $x^2 - 4x + 3\sqrt{2} = 0$
(b) $x^2 + 4x - 3\sqrt{2} = 0$
(c) $x^2 - 4x - 3\sqrt{2} = 0$
(d) $3x^2 + 4\sqrt{3}x + 4 = 0$

Solution:

Correct answer is **(**a) $x^2 - 4x + 3\sqrt{2} = 0$.

Given equation is $x^2 - 4x + 3\sqrt{2} = 0$
D = $b^2 - 4ac$

D= $(-4)^2 - 4(1)(3\sqrt{2})$ (a = 1, b = –4, c = $3\sqrt{2}$)
D= 16 – 12 × 1.414
D= 16 – 16.968
D = –0.968
As D < 0, so the given equation has no real roots.

(b)
D = $b^2 - 4ac$
D= $(4)^2 - 4(1)3\sqrt{2}$ (a = 1, b = 4, c = $3\sqrt{2}$)
D = 16 +$12\sqrt{2}$
Here,
D > 0
Hence, the given equation has two distinct real roots.

(c)
D = $b^2 - 4ac$
D = $(-4)^2 - 4(1)(-3\sqrt{2})$ (where, a = 1, b = –4, c = $-3\sqrt{2}$)
D = 16 +$12\sqrt{2}$
Here,

D > 0
So, the given equation has two real distinct roots.

(d)
D = $b^2 - 4ac$
D= $12\sqrt{2} - 4(3)(4)$(where, a = 3, b =$12\sqrt{2}$, c = 4)
D= 16 × 3 – 48 = 48 – 48
D = 0
So, the given equation has two real and equal roots.

11.

$(x^2 + 1)^2 - x^2 = 0$ has

(a) four real roots
(b) two real roots
(c) no real roots
(d) one real root

Solution:
(c)no real roots

We have,
$(x^2+1)^2 - x^2 = 0$
$(x^2)^2 + (1)^2 + 2(x^2)(1) - x^2 = 0$
$(x^2)^2 + 1x^2 + 1 = 0$

Taking,
$x^2 = y$so,$y^2 + 1y + 1 = 0$

Now,
$D = b^2 - 4ac$

$D = (1)^2 - 4(1)(1)$
$D = 1 - 4$ (Where, $a = 1, b = 1, c = 1$)
$D = -3$
$D < 0$
So, the given equation $y^2 + y + 1 = 0$ has no values of y in equation $y^2 + 1y + 1 = 0$ or if y is not real then x^2will not be real so no values of x are real or the given equation has no real roots.

Exercise No. 4.2

1.

State whether the following quadratic equations have two distinct real roots. Justify your answer.

- **i.** $\mathbf{-3x + 4 = 0}$
- **ii.** $\mathbf{2x^2 + x - 1 = 0}$
- **iii.** $\mathbf{x^2\ \ 2x^2 - 6x + 9/2 = 0}$
- **iv.** $\mathbf{3x^2 - 4x + 1 = 0}$
- **v.** $\mathbf{(x + 4)^2 - 8x = 0}$
- **vi.** $\mathbf{(x - \sqrt{2})^2 - 2(x + 1) = 0}$
- **vii.** $\mathbf{\sqrt{2}\,x^2 - (3/\sqrt{2})x + 1/\sqrt{2} = 0}$
- **viii.** $\mathbf{x(1 - x) - 2 = 0}$
- **ix.** $\mathbf{(x - 1)(x + 2) + 2 = 0}$
- **x.** $\mathbf{(x + 1)(x - 2) + x = 0}$

Solution:

(i)

Equation $x^2 - 3x + 4 = 0$ has no real roots.

$D = b^2 - 4ac$

$D = (-3)^2 - 4(1)(4)$

$D = 9 - 16 < 0$

So, the roots are imaginary.

(ii)

Equation $2x^2 + x - 1 = 0$ has two real and distinct roots.

$D = b^2 - 4ac$

$D = 1^2 - 4(2)(-1)$

$D = 1 + 8 > 0$

So, the roots are real and distinct.

(iii)

Equation $2x^2 - 6x + (9/2) = 0$ has real and equal roots.

$D = b^2 - 4ac$

$D = (-6)^2 - 4(2)(9/2)$

$D = 36 – 36 = 0$

So, the roots are real and equal.

(iv)

Equation $3x^2 – 4x + 1 = 0$ has two real and distinct roots.

$D = b^2 – 4ac$

$D = (-4)^2 – 4(3)(1)$

$D = 16 – 12 > 0$

So, the roots are real and distinct.

(v)

Equation $(x + 4)^2 – 8x = 0$ has no real roots.

Simplifying the above equation,

$x^2 + 8x + 16 – 8x = 0$

$x^2 + 16 = 0$

$D = b^2 – 4ac$

$D = (0) – 4(1) (16) < 0$

So, the roots are imaginary.

(vi)

Equation $(x – \sqrt{2})^2 – \sqrt{2}(x+1)=0$ has two distinct and real roots.

Simplifying the above equation,

$x^2 – 2\sqrt{2}x + 2 – \sqrt{2}x – \sqrt{2} = 0$

$x^2 – \sqrt{2}(2+1)x + (2 – \sqrt{2}) = 0$

$x^2 – 3\sqrt{2}x + (2 – \sqrt{2}) = 0$

$D = b^2 – 4ac$

$D = (– 3\sqrt{2})^2 – 4(1)(2 – \sqrt{2})$

$D = 18 – 8 + 4\sqrt{2} > 0$

So, the roots are real and distinct.

(vii)

Equation $\sqrt{2}x^2 - 3x/\sqrt{2} + ½ = 0$ has two real and distinct roots.

$D = b^2 - 4ac$

$D = (-3/\sqrt{2})^2 - 4(\sqrt{2})(½)$

$D = (9/2) - 2\sqrt{2} > 0$

So, the roots are real and distinct.

(viii)

The equation $x(1 - x) - 2 = 0$ has no real roots.

Solving the above equation,

$x^2 - x + 2 = 0$

$D = b^2 - 4ac$

$D = (-1)^2 - 4(1)(2)$

$D = 1 - 8 < 0$

So, the roots are imaginary.

(ix)

Equation $(x - 1)(x + 2) + 2 = 0$ has two real and distinct roots.

Solving the above equation,

$x^2 + x = 0$

$D = b^2 - 4ac$

$D = 1^2 - 4(1)(0)$

$D = 1 - 0 > 0$

So, the roots are real and distinct.

(x)

Equation $(x + 1)(x - 2) + x = 0$ has two real and distinct roots.

Solving the above equation,

$x^2 + x - 2x - 2 + x = 0$

$x^2 - 2 = 0$

$D = b^2 - 4ac$

$D = (0)^2 - 4(1)(-2)$

D= 0 + 8 > 0

So, the roots are real and distinct.

2.

Write whether the following statements are true or false. Justify your answers.

i. **Every quadratic equation has exactly one root.**
ii. **Every quadratic equation has at least one real root.**
iii. **Every quadratic equation has at least two roots.**
iv. **Every quadratic equations has at most two roots.**
v. **If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.**
vi. **If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.**

Solution:

(i) False.

For example, a quadratic equation $x^2 – 16 = 0$ has two distinct roots $– 4$ and 4.

(ii) False.

For example, equation $x^2 + 6 = 0$ has no real root.

(iii) False.

For example, a quadratic equation $x^2 – 4x + 4 = 0$ has only one root which is 2.

(iv) True,

Because every quadratic polynomial has almost two roots.

(v) True,

Because in this case discriminant is always positive.

For example, in $ax^2+ bx + c = 0$, as a and c have opposite sign, $ac < 0$

Discriminant = $b^2 – 4ac > 0$.

(vi) True,

Because in this case discriminant is always negative.

For example, in $ax^2 + bx + c = 0$, as $b = 0$, and a and c have same sign then $ac > 0$

Discriminant = $b^2 - 4ac = -4ac < 0$

3.

A quadratic equation with integral coefficient has integral roots. Justify your answer.

Solution:

No, a quadratic equation with integral coefficients may or may not have integral roots.

Explanation:

Consider the following equation,

$16x^2 - 4x - 2 = 0$

The roots of the given equation are ½ and – ¼ which are not integers.

Therefore, a quadratic equation with integral coefficient can or cannot have integral roots.

4.

Does there exist a quadratic equation, whose coefficients are rational both of its roots are irrational? Justify your answer.

Solution:

Yes, a quadratic equation having coefficients as rational number, has irrational roots.
Taking example,
$2x^2 - 3x - 15 = 0$ has rational coefficients.

Now, calculating,
$D = b^2 - 4ac$ $\qquad$ $(a = 2, b = -3, c = -15)$
$D = (-3)^2 - 4(2)(-15)$
$D = 9 + 120$
$D = 129$

Roots of the equation can be given as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{129}}{2 \times 2}$$

$$x = \frac{3 \pm \sqrt{129}}{4}$$

So, we can say that roots are irrational as $\sqrt{129}$ is irrational.

5.

Does there exist a quadratic equation whose coefficient are all distinct irrationals but both the roots are rational? Why?

Solution:

Yes, there may be a quadratic equation whose coefficients are all distinct irrationals, but both the roots are rational.

For example, consider a quadratic equation having distinct irrational coefficients.

In this equation, the roots are real.

$\sqrt{3}x^2 - 7\sqrt{3}x + 12\sqrt{3} = 0$

Now,

$b^2 - 4ac = 147 - 144$

$= 3$

Roots of the equation can be given as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7\sqrt{3} \pm \sqrt{3}}{2\sqrt{3}}$$

$$x = \frac{7 \pm 1}{2}$$

$$x = 4 \, or \, 3$$

Hence, the roots are rational while coefficients a, b, c were irrational.

6.

Is 0.2 a root of equation $x^2 – 0.4 = 0$? Justify your answer.

Solution:

If we consider 0.2 as a root of equation $x^2 – 0.4 = 0$, then 0.2 must satisfy the given equation.

$x^2 – 0.4 = 0$ [Given]

$(0.2)^2 – 0.4 = 0$

$0.04 - 0.4 = 0$

$–0.36 \neq 0$

Therefore, 0.2 is not a root of the given equation.

7.

If b = 0, c < 0, is it true that the roots of x^2+ bx + c = 0 are numerically equal and opposite in sign? Justify your answer.

Solution:
Equation is $x^2 + bx + c = 0$
b = 0 [Given]
So, $x^2 + c = 0$
$x^2 = -c$
$x = \sqrt{-c}$

Since, c is negative so –c becomes positive or $\sqrt{-c}$ is real.

So, the roots of the given equation are
$x = +\sqrt{-c}$
and
$x = -\sqrt{-c}$ [where (–c) is positive]
Therefore, the roots of the given equation are real, equal and opposite in sign.

Exercise No. 4.3

1.

Find the roots of the quadratic equations by using the quadratic formula in each of the following:

i. $2x^2 - 3x - 5 = 0$
ii. $5x^2 + 13x + 8 = 0$
iii. $-3x^2 + 5x + 12 = 0$
iv. $-x^2 + 7x - 10 = 0$
v. $x^2 + 2\sqrt{2}x - 6 = 0$
vi. $x^2 - 3\sqrt{5}x + 10 = 0$
vii. $(1/2)x^2 - \sqrt{11}x + 1 = 0$

Solution:

The quadratic formula to find the roots of quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) $2x^2 - 3x - 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{49}}{4}$$

$$x = \frac{3 \pm 7}{4}$$

So,

$$x_1 = \frac{5}{2}$$

$$x_2 = \frac{-1}{1}$$

(ii) $5x^2 + 13x + 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-13 \pm \sqrt{9}}{10}$$

$$x = \frac{-13 \pm 3}{10}$$

So,

$$x_1 = -1$$

$$x_2 = \frac{-8}{5}$$

(iii) $-3x^2 + 5x + 12 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{169}}{-6}$$

$$x = \frac{5 \pm 13}{6}$$

So,

$$x_1 = 3$$

$$x_2 = \frac{-4}{3}$$

(iv) $-x^2 + 7x - 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{9}}{-2}$$

$$x = \frac{7 \pm 3}{2}$$

So,

$$x_1 = 5$$

$$x_2 = 2$$

(v) $x^2 + 2\sqrt{2}x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2\sqrt{2} \pm \sqrt{32}}{2}$$

$$x = \frac{-2\sqrt{2} \pm 4\sqrt{2}}{2}$$

So,

$$x_1 = \sqrt{2}$$

$$x_2 = -3\sqrt{2}$$

(vi) $x^2 - 3\sqrt{5}x + 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3\sqrt{5} \pm \sqrt{5}}{2}$$

So,

$$x_1 = \frac{3\sqrt{5} + \sqrt{5}}{2}$$

$$= 2\sqrt{5}$$

$$x_2 = \frac{3\sqrt{5} - \sqrt{5}}{2}$$

$$= \sqrt{5}$$

(vii) $(\frac{1}{2})x^2 - \sqrt{11}x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{\sqrt{11} \pm \sqrt{9}}{1}$$

$$x = \frac{\sqrt{11} \pm 3}{1}$$

So,

$$x_1 = \sqrt{11} + 3$$

$$x_2 = \sqrt{11} - 3$$

2.

Find the roots of the following quadratic equations by the factorization method:

$(i)\ 2x^2 + \frac{5}{3}x - 2 = 0$

$(ii)\ \frac{2}{5}x^2 - x - \frac{3}{5} = 0$

$(iii)\ 3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$

$(iv)\ 3x^2 + 5\sqrt{5}x - 10 = 0$

$(v)\ 21x^2 - 2x + \frac{1}{21} = 0$

Solution:

(i)

$2x^2 + \frac{5}{3}x - 2 = 0$

$6x^2 + 5x - 6 = 0$

$6x^2 + 9x - 4x - 6 = 0$

$3x(2x + 3) - 2(2x + 3) = 0$

$(2x + 3)(3x - 2) = 0$

$2x + 3 = 0$ or $3x - 2 = 0$

$2x = -3$ or $3x = 2$

So, the roots of the given quadratic equation are -3/2 and 2/3.

(ii)

$\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

$2x^2 - 5x - 3 = 0$

$2x^2 - 6x + 1x - 3 = 0$

$2x(x - 3) + 1(x - 3) = 0$

$(x - 3)(2x + 1) = 0$

$x - 3 = 0$

and,

$2x + 1 = 0$

$x = 3$

or $2x = -1$

So, the roots of the quadratic equation are 3 and -1/2.

(iii)

$$3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$$
$$3\sqrt{2}x^2 - 6x + 1x - \sqrt{2} = 0$$
$$3\sqrt{2}x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$$
$$(x - \sqrt{2})(3\sqrt{2}x + 1) = 0$$
$$\text{x} = \sqrt{2}$$
or,
$$\text{x} = \frac{-1}{3\sqrt{2}}$$
$$= \frac{-\sqrt{2}}{6}$$

Above are the roots of the given equation.

(iv)
$$3x^2 + 5\sqrt{5}x - 10 = 0$$
$$3x^2 + 5\sqrt{5}x - 10 = 0$$
$$3x^2 + 6\sqrt{5}x - \sqrt{5} - 10 = 0$$
$$3x(x + 2\sqrt{5}) - \sqrt{5}(x + 2\sqrt{5}) = 0$$
$$(x + 2\sqrt{5})(3x - \sqrt{5}) = 0$$
$$x = \frac{\sqrt{5}}{3}$$
or
$$x = -2\sqrt{5}$$
So, these are the roots of the given quadratic equation.

(v)
$$21x^2 - 2x + \frac{1}{21} = 0$$
$441x^2 – 42x + 1 = 0$
$441x^2 – 21x – 21x + 1 = 0$
$21x(21x – 1) – 1(21x – 1) = 0$
$(21x – 1) (21x – 1) = 0$
$(21x – 1) = 0$ or $(21x – 1) = 0$
$21x = 1$ or $21x = 1$
$$x = \frac{1}{21}, \frac{1}{21}$$
So, the roots of the given equation are $x = \frac{1}{21}, \frac{1}{21}$

Exercise No. 4.4

1.

Find whether the following equations have real roots. If real roots exist, find them.

(i) $8x^2 + 2x - 3 = 0$

(ii) $-2x^2 + 3x + 2 = 0$

(iii) $5x^2 - 2x - 10 = 0$

(iv) $\frac{1}{2x-3} + \frac{1}{x-5} = 1$

(v) $x^2 + 5\sqrt{5}x - 70 = 0$

Solution:

For real roots of quadratic equation $ax^2 + bx + c = 0$,

Also, $b^2 - 4ac > 0$

(i)

The given equation is $8x^2 + 2x - 3 = 0$

Discriminant (D) = $b^2 - 4ac$

$D = (2)^2 - 4(8)(-3)$ (where, a = 8, b = 2, c = –3)

$D = 4 + 96$

$D = 100$

As D > 0, so, roots are real.

Now,

Discriminant = 100

So, roots are

$$x = \frac{-b \pm \sqrt{D}}{2a}$$
$$x = \frac{-2 \pm 10}{2 \times 8}$$
$$x = \frac{-2 \pm 10}{16}$$
So,
$$x_1 = \frac{8}{16}$$
$$= \frac{1}{2}$$
$$x_2 = \frac{-12}{16}$$
$$= \frac{-3}{4}$$

So, the roots of the given equation are 1/2 and -3/4.

(ii)

$-2x^2 + 3x + 2 = 0$

Discriminant $D = b^2 - 4ac$

$D = (3)^2 - 4(-2)(2)$ (where, $a = -2$, $b = 3$, $c = 2$)

$D = 9 + 16$

$D = 25 > 0$

So, the given equation has real and distinct roots.

Now,

$D = 25$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$
$$x = \frac{-3 \pm 5}{2 \times -2}$$
$$x = \frac{-2 \pm 10}{-4}$$

So,

$x_1 = \frac{2}{-4}$

$= \frac{1}{-2}$

$x_2 = \frac{-8}{-4}$

$= \frac{2}{1}$

So, the roots of the given equation are 2 and -1/2.

(iii)

$5x^2 - 2x - 10 = 0$

Discriminant $D = b^2 - 4ac$

$D = (-2)^2 - 4(5)(-10)$ (where, $a = 5$, $b = -2$, $c = -10$)

$D = 4 + 200$

$D = 204 > 0$

So, the roots of the given equation are real and distinct.

Now,

$x = \frac{-b \pm \sqrt{D}}{2a}$

$x = \frac{2 \pm 2\sqrt{51}}{2 \times 5}$

$x = \frac{1 \pm \sqrt{51}}{5}$

So,

$x_1 = \frac{1 + \sqrt{51}}{5}$

$x_2 = \frac{1 - \sqrt{51}}{5}$

So, the roots of the given equation are, $\frac{1 + \sqrt{51}}{5}$ and $\frac{1 - \sqrt{51}}{5}$.

(iv)

$$\frac{1}{2x-3}+\frac{1}{x-5}=1$$

$2x^2 - 10x - 3x + 15 = x - 5 + 2x - 3$

$2x^2 - 13x + 15 = 3x - 8$

$2x^2 - 13x + 15 - 3x + 8 = 0$

$2x^2 - 16x + 23 = 0$

Now,

$D = b^2 - 4ac$

$D = (-16)^2 - 4(2)(23)$ (where, $a = 2$, $b = -16$, $c = 23$)

$D = 256 - 184$

$D = 72 > 0$

So, the roots are,

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$x=\frac{16\pm6\sqrt{2}}{2\times2}$$

$$x=\frac{16}{4}\pm\frac{6\sqrt{2}}{4}$$

So,

$$x_1=4+\frac{3\sqrt{2}}{2}$$

$$x_2=4-\frac{3\sqrt{2}}{2}$$

So, these are the roots of the given quadratic equation.

(v)

$$x^2+5\sqrt{5}x-70=0$$

$D = b^2 - 4ac$

$D = 25 \times 5 + 280$

$D = 125 + 280$

$D = 405 > 0$

So, the roots of the given equation are real and distinct.

For roots

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-5\sqrt{5} \pm 9\sqrt{5}}{2 \times 1}$$

$$x = \frac{-5\sqrt{5} \pm 9\sqrt{5}}{2}$$

So,

$$x_1 = \frac{-5\sqrt{5} + 9\sqrt{5}}{2}$$

$$= 2\sqrt{5}$$

$$x_2 = \frac{-5\sqrt{5} - 9\sqrt{5}}{2}$$

$$= -7\sqrt{5}$$

Above are the roots of the given quadratic equation.

2.

Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

Solution:

Let us take the natural number = 'x'.

Now from the question,

We have the equation,

$x^2 - 84 = 3(x+8)$

$x^2 - 84 = 3x + 24$

$x^2 - 3x - 84 - 24 = 0$

$x^2 - 3x - 108 = 0$

$x^2 – 12x + 9x – 108 = 0$

$x(x – 12) + 9(x – 12) = 0$

$(x + 9) (x – 12)$

$x = -9$ and $x = 12$

As, natural numbers cannot be negative.

The number is 12.

3.

A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

Solution:

Let us take the natural number = x

When the number increased by 12 = x + 12

Reciprocal of the number = 1/x

From the question,

x + 12 = 160 times of reciprocal of x

$x + 12 = 160/ x$

$x(x + 12) = 160$

$x^2 + 12x – 160 = 0$

$x^2 + 20x – 8x – 160 = 0$

$x(x + 20) – 8(x + 20)= 0$

$(x + 20) (x – 8) = 0$

$x + 20 = 0$ or $x – 8 = 0$

$x = – 20$ or $x = 8$

As, natural numbers cannot be negative.

The required number = x = 8

4.

A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

Solution:

Let us take the original speed of train = x km/h

We have,

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

From the question, we have,

Time taken by train = 360/x hour

And, Time taken by train its speed increase 5 km/h = 360/(x + 5)

It is given that,

Time taken by train in first – time taken by train in 2nd case = 48 min

= 48/60 hour

$$\frac{360}{x} + \frac{360}{x+5} = \frac{48}{60}$$

$$360\left(\frac{1}{x} + \frac{1}{x+5}\right) = \frac{4}{5}$$

$450 \times 5 = x^2 + 5x$

$x^2 + 5x - 2250 = 0$

On solving,

x = -50, 45

But,

$x \neq -50$ as speed cannot be negative

Therefore,

x = 45 km/h

So, original speed of train = 45 km/h

5.

If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

Solution:

Let us take Zeba's age = x

From the question, we have,

$(x-5)^2 = 11+5x$

$x^2+25-10x = 11+5x$

$x^2-15x+14 = 0$

$x^2-14x-x+14 = 0$

$x(x-14)-1(x-14) = 0$

$x = 1$ or $x = 14$

We neglect 1 as 5 years younger than 1 is not possible.

So, Zeba's present age = 14 years.

6.

At present Asha's age (in years) is 2 more than the square of her daughter Nisha'sage. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.

Solution:

Taking the present age of Asha = x years

The present age of her daughter Nisha = y years

At Present, Asha's age, $x = (y^2) + 2$ .... (i)

Age of Nisha will be equal to age of her mother (x) after no of years = Age of Mother – Age of Daughter

$= (x – y)$

$= (y^2+ 2 – y)$

$= (y^2– y + 2)$ years

∴ Age of (Nisha) daughter after $(y^2– y + 2)$ years $= y^2– y + 2 + y$

$= (y^2+ 2)$ years

Age of Asha (mother) after $(y^2– y + 2)$ years

$= x + y^2– y + 2$

$= y^2 + 2 + y^2 - y + 2$ [From (i)]

$= 2y^2 - y + 4$ years

After $(y^2 - y - 2)$ years, age of Asha $= 2y^2 - y + 4 = 10y - 1$

$2y^2 - y - 10y + 5 = 0$

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - 1y + 5 = 0$

$2y(y - 5) - 1(y - 5) = 0$

$(y - 5)(2y - 1) = 0$

$y - 5 = 0$ or $2y - 1 = 0$

$y = 5$ or $y =$ years

From (i), we have

$x = y^2 + 2$

Putting $y = 5$, we have

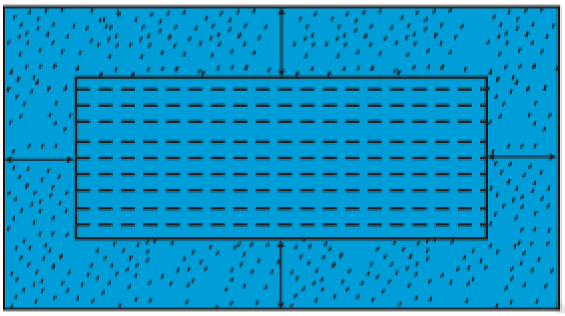
$x = (5)^2 + 2 = 25 + 2 = 27$

Mother's age can never be years, so it is not taken.

So, the ages of Asha and Nisha are 27 years and 5 years respectively.

7.

In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2 [see Fig]. Find the length and breadth of the pond.



Solution:

In the figure both Pond and lawn are rectangular.

Pond is inside the lawn.

Let the length of pond = (50 – 2x) m

The breadth of pond = (40 – 2x) m

Also,

Area of grass around the pond = 1184 m^2

Area of Lawn – Area of Pond = 1184

$50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

$2000 - (2000 - 100x - 80x + 4x^2) - 1184 = 0$

$2000 - (2000 - 180x + 4x^2) - 1184 = 0$

$2000 - 2000 + 180x - 4x^2 - 1184 = 0$

$4x^2 - 180x + 1184 = 0$

$x^2 - 45x + 296 = 0$

$x^2 - 37x - 8x + 296 = 0$

$x(x - 37) - 8(x - 37) = 0$

$(x - 37)(x - 8) = 0$

$x - 37 = 0$

$x = 37$

or

x – 8 = 0

x = 8

When x = 37, then

The length of pond = 50 – 2 × 37

= 50 – 74

= –24 m

Length cannot be negative. So, x = 37 is not taken.

When x = 8, then

The length of pond = 50 – 2x

= 50 – 2x × 8

= 50 – 16

= 34 m

And the breadth of the pond = 40 – 2x

= 40 – 2 × 8

= 40 – 16

= 24 m

Hence, the length and breadth of the pond are 34 m and 24 m respectively.

8.

At t minutes past 2 p.m., the time needed by the minute hand of a clock to show 3 p.m. was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.

Solution:

It is given that total time taken by min. hand from 2 p.m. to 3 p.m. is 60 min.

After t min past 2 p.m. the time needed by min. hand of a clock to show 3 p.m. is given by 3 min less than $\frac{t^2}{4}$ min.

$t + (\frac{t^2}{4} - 3) = 60$

$4t + t^2 - 12 = 240$

$t^2 + 4t - 252 = 0$

$t^2 + 18t - 14t - 252 = 0$

$t(t + 18) - 14(t + 18) = 0$

$(t + 18)(t - 14) = 0$

$t + 18 = 0$ or $t - 14 = 0$

$t = -18$ or $t = 14$ min.

Being, negative value, $t = -18$ is not taken.

So, $t = 14$ min.