Chapter - 11
Mensuration

Exercise

In questions 1 to 28, there are four options out of which one is correct. Write the correct answer.

1. A cube of side 5 cm is painted on all its faces. If it is sliced into 1 cubic centimetre cubes, how many 1 cubic centimetre cubes will have exactly one of their faces painted?
(a) 27 (b) 42 (c) 54 (d) 142

Solution:
(c) 54

Given:
The cube side = 5 cm
The side of cube 5cm is cut into 5 equal parts, in which each of 1 cm
Therefore, the total number of cubes of side 1 cm = 25 + 25 + 25 + 25 + 25
= 125
In one face of cube, there are total of 9 small cubes painted.
We know that, there are 6 faces in cube.
Thus, total of 9 x 6 faces will have one face painted.
So,
9 x 6 = 54

2. A cube of side 4 cm is cut into 1 cm cubes. What is the ratio of the surface areas of the original cubes and cut-out cubes?
(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 6

Solution:

(c) 1: 4
Given: The cube side is 4cm
The side of cube 4cm is cut into small cubes, in which each of 1 cm
Therefore, the total number of cubes = 4 x 16
= 64 cubes
Thus, the number of cut-out cubes = $\frac{64}{1}$
Now, the surface area of the cut-out cubes = 64 x 6 x $(1)^2$ cm^2
The surface area of the original cube = 6 x 4^2
Hence, the required ratio obtained is:
$= \frac{6 \times 4^2}{64 \times 6}$

= 1: 4

3. A circle of maximum possible size is cut from a square sheet of board. Subsequently, a square of maximum possible size is cut from the resultant circle. What will be the area of the final square?

(a) $\frac{3}{4}$ of original square. (b) $\frac{1}{2}$ of original square.

(c) $\frac{1}{4}$ of original square. (d) $\frac{2}{3}$ of original square.

Solution:

(b) 1/2 of the original square

Let "a" be the side of the square sheet

Thus, the area of the bigger square sheet = a^2 …. (1)

Now, the circle of maximum possible size from it is given as:

The radius of the circle = $\frac{a}{2}$ … (2)

Then the diameter = a

We know that, any square in the circle of maximum size should have the length of the diagonal which is equal to the diameter of the circle.

It means that, the diagonal of a square formed inside a circle is "a"

Hence, the square side = $\frac{a}{\sqrt{2}}$

Thus, the area of square = $\frac{a^2}{2}$

By equating the equations (1) and (2), we will get:

Area of the resultant square is $\frac{1}{2}$ of the original square.

4. What is the area of the largest triangle that can be fitted into a rectangle of length l units and width w units?

(a) lw/2 (b) lw/3 (c) lw/6 (d) lw/4

Solution:

(a) $\frac{lw}{2}$

We know that,

Area of a triangle is ($\frac{1}{2}$) x base x height

Let ABCD be a triangle with length "l" and width "w".

Here, we have to construct a triangle of maximum area inside the rectangle in all possible ways.
Now, the maximum base length is "l"
Maximum height is "w".

Therefore, the area of a largest triangle is $(\frac{1}{2})$ x l x w.

5. If the height of a cylinder becomes $\frac{1}{4}$ of the original height and the radius is doubled, then which of the following will be true?
(a) Volume of the cylinder will be doubled.
(b) Volume of the cylinder will remain unchanged.
(c) Volume of the cylinder will be halved.
(d) Volume of the cylinder will be $\frac{1}{4}$ of the original volume.

Solution:
The correct answer is option (b)
Volume of the cylinder will remain unchanged.

We know that, the volume of a cylinder is $\pi \times r^2 \times h$
Also,
base radius and height of the cylinder is "r" and "h" respectively.
Now, height "h" becomes $(\frac{1}{4})$h and "r" becomes "2r", then the volume of the cylinder is

$V = \pi \times 4r^2 \times (\frac{1}{4})\ h$
$\quad = \pi r^2 h = v$
Therefore, the volume of new cylinder = the volume of original cylinder.

6. If the height of a cylinder becomes $\frac{1}{4}$ of the original height and the radius is doubled, then which of the following will be true?
(a) Curved surface area of the cylinder will be doubled.
(b) Curved surface area of the cylinder will remain unchanged.
(c) Curved surface area of the cylinder will be halved.
(d) Curved surface area will be $\frac{1}{4}$ of the original curved surface.

Solution:
(c) Curved surface area of the cylinder will be halved.

We know that the curved surface area of a cylinder with radius "r" and height "h" is given as
The curved surface area of a cylinder = $2\pi rh$ … (1)

Now, the new curved surface area of cylinder with radius 2r and height $(\frac{1}{4})$h, then the new curved surface area is

$= 2\pi(2r)(\frac{1}{4})h$

$= \pi rh$

Now, multiply an divide the new curved surface area by 2, we will get

$= (\frac{1}{2})\ (2)\ \pi rh$ …. (2)

Now, by comparing (1) and (2),

The new curved surface area of a cylinder is $\frac{1}{2}$ times of the original curved surface area of a cylinder.

7. If the height of a cylinder becomes $\frac{1}{4}$ of the original height and the radius is doubled, then which of the following will be true?
(a) Total surface area of the cylinder will be doubled.
(b) Total surface area of the cylinder will remain unchanged.
(c) Total surface of the cylinder will be halved.
(d) None of the above.

Solution:

(d) None of the above.

We know that, the total surface area of a cylinder is $2\pi r(h + r)$, when the radius is "r" and height is "h".

If the radius is 2r and the height is $(\frac{1}{4})$h, then the total surface area becomes,

$= 2\pi\ (2r)\ ((\frac{1}{4})h + 2r)$

$= 4\ \pi r\ \frac{h+8r}{4}$

$= \pi r\ (h+8r)$

8. The surface area of the three coterminous faces of a cuboid are 6, 15 and 10 cm^2 respectively. The volume of the cuboid is
(a) 30 cm^3 (b) 40 cm^3 (c) 20 cm^3 (d) 35 cm^3

Solution:

(a) 30 cm^3

It is given that, the coterminous faces of a cuboid is given as:

$l \times b = 6$
$l \times h = 15$
$b \times h = 10$
The formula for volume of a cuboid is $l \times b \times h$
So,
$l^2 \times b^2 \times h^2 = 6 \times 15 \times 10$
$(lbh)^2 = (900)$
$lbh = 30$

9. A regular hexagon is inscribed in a circle of radius r. The perimeter of the regular hexagon is
(a) 3r (b) 6r (c) 9r (d) 12r

Solution:
(b) 6r

We know that a hexagon contains six equilateral triangles, where one of the vertices of each equilateral triangles meet at the centre of the hexagon.

The radius of the smallest which is inscribing the hexagon is equal to the sides of the equilateral triangle.

Therefore, the perimeter of a regular hexagon is 6r, as each side of the hexagon is equal to the radius of the hexagon.

10. The dimensions of a go down are 40 m, 25 m and 10 m. If it is filled with cuboidal boxes each of dimensions 2 m × 1.25 m × 1 m, then the number of boxes will be
(a) 1800 (b) 2000 (c) 4000 (d) 8000

Solution:
(c) 4000
Given that, the dimensions of the godown are 40 m, 25 m and 10 m

Volume = 40 m × 25 m × 10 m
= 10000 m^3
Given that,
Volume of each cuboidal box = 2 m × 1.25 m × 1 m
= 2.5 m^3
Therefore,
Total number of boxes to be filled in the godown is = $\frac{10000}{2.5}$
= 4000

11. The volume of a cube is 64 cm^3. Its surface area is

(a) 16 cm^2 (b) 64 cm^2 (c) 96 cm^2 (d) 128 cm^2

Solution:

(c) 96 cm^2

Let a be the side of the cube,
Given that,
Volume of cube = 64 cm^3
It means that $a^3 = 64$ cm^3
Hence,
a = 4 cm
Therefore,
Surface area of a cube = 6×4^2
= 6×16
= 96

12. If the radius of a cylinder is tripled but its curved surface area is unchanged, then its height will be
(a) tripled (b) constant (c) one sixth (d) one third

Solution:
(d) one third

We know that the curved surface area of a cylinder is $2\pi rh$, when the radius is "r" and height is "h".
Let "H" be the new height.
When the radius of a cylinder is tripled, then the CSA of a cylinder becomes,
CSA = 2π (3r) H
CSA = $6\pi r$. H
Now, compare the CSA of the cylinder to find the height
$6\pi rH = 2\pi rh$
$H = \frac{2\pi rh}{6\pi r}$
$H = (\frac{1}{3})h$
Hence, the new height of the cylinder is one-third of the original height.

13. How many small cubes with edge of 20 cm each can be just accommodated in a cubical box of 2 m edge?
(a) 10 (b) 100 (c) 1000 (d) 10000

Solution:
(c) 1000

We know that,
Volume of cube = $(\text{side})^3$
Therefore,
Volume of each small cube = $(20)^3$
= $8000\ cm^3$
When it is converted into m^3, we get
$V = 0.008\ m^3$
It is given that,
Volume of the cuboidal box is $2^3 = 8\ m^3$
Now,
Number of small cubes that can be accommodated in the cuboidal box is = $\frac{8}{0.008}$
= 1000

14. The volume of a cylinder whose radius r is equal to its height is
(a) $1/4\ \pi r^3$ (b) $\pi r^3/32$ (c) πr^3 (d) $\pi r^3/8$

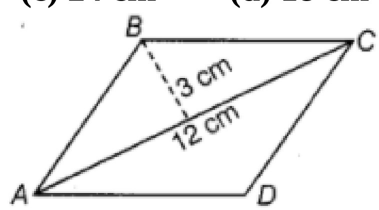
Solution:
(c) πr^3

Volume of cylinder = $\pi r^2 h$
Given,
$r = h$
Then,
Volume of cylinder = $\pi r^2 (r)$
$V = \pi r^3$

15. The volume of a cube whose edge is 3x is
(a) $27x^3$ (b) $9x^3$ (c) $6x^3$ (d) $3x^3$

Solution:
(a) $27x^3$
The volume of a cube = $(\text{side})^3$
$V = (3x)^3$
$V = 27x^3$

16. The figure ABCD is a quadrilateral in which AB = CD and BC = AD. Its area is
(a) $72\ cm^2$ (b) $36\ cm^2$ (c) $24\ cm^2$ (d) $18\ cm^2$



Solution:

(B) 36 cm^2

From the given figure, it is clear that, a quadrilateral ABCD is a parallelogram.

Here, the diagonal AC divides the parallelogram into two equal triangles.

Hence, the area of a triangle ABC = $(\frac{1}{2})$ bh

Here,

b = 12 and h = 3

Area = $(\frac{1}{2})$ (12)(3)

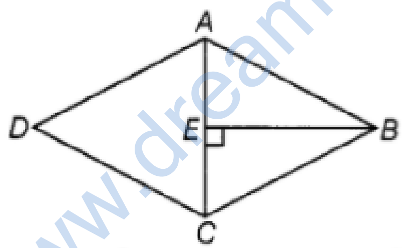
= 18

Therefore, the area of a parallelogram ABCD = 2 (18)

= 36 cm^2

17. What is the area of the rhombus ABCD below if AC = 6 cm, and BE = 4cm?

(a) 36 cm^2 (b) 16 cm^2 (c) 24 cm^2 (d) 13 cm^2



Solution:

(c) 24 cm^2

From figure, the diagonal AC divides the rhombus into two triangles of equal area.

Therefore, the area of a triangle ABC = $(\frac{1}{2})$ bh

= $(\frac{1}{2})$ (4) (6)

=12 cm^2

Therefore, the area of a rhombus ABCD = 2(12)

= 24

18. The area of a parallelogram is 60 cm^2 and one of its altitude is 5 cm. The length of its corresponding side is

(a) 12 cm (b) 6 cm (c) 4 cm (d) 2 cm

Solution:
(a) 12 cm

The area of a parallelogram = base x altitude
b. h = A
b (5) = 60
b = $\frac{60}{5}$
b = 12cm

19. The perimeter of a trapezium is 52 cm and its each non-parallel side is equal to 10 cm with its height 8 cm. Its area is
(a) 124 cm^2 (b) 118 cm^2 (c) 128 cm^2 (d) 112 cm^2

Solution:
(c) 128 cm^2
Given:
The perimeter of a trapezium = 52 cm
The sum of its parallel sides = 52 – (10+10)
= 32 cm
We know that, the area of a trapezium = $(\frac{1}{2})$ (a+b) h

A = $(\frac{1}{2})$ (32) (8)
A = 128 cm^2

20. Area of a quadrilateral ABCD is 20 cm^2 and perpendiculars on BD from opposite vertices are 1 cm and 1.5 cm. The length of BD is
(a) 4 cm (b) 15 cm (c) 16 cm (d) 18 cm

Solution:
(c) 16 cm
Given that, the area of a quadrilateral = 20 cm^2
We know that,
Area of a quadrilateral = $(\frac{1}{2})$ (diagonal) (sum of the altitudes)

20 = $(\frac{1}{2})$ (1+1.5) BD

20 = $(\frac{1}{2})$ (2.5) BD
20×2 = 2.5 BD
40 = 2.5 BD
BD = 16 cm

21. A metal sheet 27 cm long, 8 cm broad and 1 cm thick is melted into a cube. The side of the cube is
(a) 6 cm (b) 8 cm (c) 12 cm (d) 24 cm

Solution:

(a) 6 cm

Given that,
The metal sheet dimension is 27 cm long, 8 cm broad and 1 cm thick.
Thus,
Volume of the sheet = (27)(8)(1) = 216 cm^3
Given that, the metal sheet is melted to make a cube
Let the edge be a,
Hence,
$a^3 = 216\ cm^3$
a = 6 cm

22. Three cubes of metal whose edges are 6 cm, 8 cm and 10 cm respectively are melted to form a single cube. The edge of the new cube is
(a) 12 cm (b) 24 cm (c) 18 cm (d) 20 cm

Solution:
(a) 12 cm

Given that,
Sum of the volume of the three metal cubes = 63 + 83 +103
V = 216+ 512+ 1000
V = 1728 cm^3
Let the side of the new cube be "a"
Therefore,
Volume of the new cube = sum of the volume of the three cubes
$a^3 = 1728$
Hence,
a = 12 cm

23. A covered wooden box has the inner measures as 115 cm, 75 cm and 35 cm and thickness of wood as 2.5 cm. The volume of the wood is
(a) 85,000 cm^3 (b) 80,000 cm^3 (c) 82,125 cm^3 (d) 84,000 cm^3

Solution:
(c) 82,125 cm^3

The thickness of the wooden box is 2.5 cm

So,
The outer measure of the wooden box = 115+5, 75+5, 35+5
Therefore,
Outer volume be = (120)(80)(40)
Outer volume = 384000 cm^3
Given that,
Inner volume = (115)(80)(40)
Inner volume = 301875 cm^3

Hence, the volume of a wood = Outer volume – Inner volume
V = 384000 – 301875 cm^3
V= 82125 cm^3

24. The ratio of radii of two cylinders is 1 : 2 and heights are in the ratio 2 : 3. The ratio of their volumes is
(a) 1 : 6 (b) 1 : 9 (c) 1 : 3 (d) 2 : 9

Solution:
(a) 1:6

Assuming that r and R be the radii of the two cylinders and h and H be the height of the two cylinders
We have,
$$\frac{r}{R}=\frac{1}{2}$$
$$\frac{h}{H}=\frac{2}{3}$$
Also,
Volume of a cylinder $= \pi r^2 h$
$$\frac{v}{V}=\frac{\pi r^2 h}{\pi R^2 h}$$
$$\frac{v}{V}=\frac{r^2 h}{R^2 h}$$
$$\frac{v}{V}=\left(\frac{1}{2}\right)^2\left(\frac{2}{3}\right)$$
$$\frac{v}{V}=\frac{1}{6}$$

Therefore, the ratio of their volume is 1/6

25. Two cubes have volumes in the ratio 1 : 64. The ratio of the area of a face of first cube to that of the other is

(a) 1 : 4 (b) 1 : 8 (c) 1 : 16 (d) 1 : 32

Solution:

(c) 1:16

Let a and b be two cubes

It is given that,

$$\frac{a^3}{b^3} = \frac{1}{64}$$

So,

$$\frac{a}{b} = \frac{1}{4}$$

The ratio of the areas are:

$$\left(\frac{a}{b}\right)^2 = \left(\frac{1}{4}\right)^2$$

$$= \frac{1}{16}$$

26. The surface areas of the six faces of a rectangular solid are 16, 16, 32, 32, 72 and 72 square centimetres. The volume of the solid, in cubic centimetres, is

(a) 192 (b) 384 (c) 480 (d) 2592

Solution:

(a) 192

We have, the solid has a rectangular faces,

lb =16 …(1)

bh = 32 ….(2)

lh = 72 …(3)

Multiply the equations (1), (2), (3);

$(l)^2(b)^2 (h)^2 = (16)(32)(72)$

$= 36864$

lbh = 192

Therefore, the volume of a solid is 192 cubic centimetre.

27. Ramesh has three containers.

(a) Cylindrical container A having radius r and height h,

(b) Cylindrical container B having radius 2r and height 1/2 h, and

(c) Cuboidal container C having dimensions r × r × h

The arrangement of the containers in the increasing order of their volumes is

(a) A, B, C (b) B, C, A (c) C, A, B (d) cannot be arranged

Solution:

(c) C, A, B

(i) If the cylinder have radius r and height h, then the volume will be $\pi r^2 h$

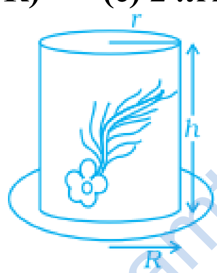
(ii) If the cylinder have radius 2r and height $(\frac{1}{2})$h, then the volume will be $2\pi r^2 h$

(ii) The volume of the cuboidal container with dimensions is r^2 h

Then, the arrangement of the containers in the increasing order of their volumes is C, A, B

28. If R is the radius of the base of the hat, then the total outer surface area of the hat is

(a) $\pi r (2h + R)$ (b) $2\pi r (h + R)$ (c) $2 \pi rh + \pi R^2$ (d) None of these



Solution:

(c) $2\pi rh + \pi R^2$

The total surface area of a hat = CSA + TSA + Base Surface Area

$= 2\pi rh + \pi r^2 + \pi (R^2 - r^2)$

$= 2\pi rh + \pi R^2$

In questions 29 to 52, fill in the blanks to make the statements true.

29. A cube of side 4 cm is painted on all its sides. If it is sliced in 1 cubic cm cubes, then number of such cubes that will have exactly two of their faces painted is __________.

Solution:

24

Given that,

Cube side is 4 cm, then the volume of cube is 43 = 64 cm^3

When it is sliced into 1 cubic cm, we will get 64 small cubes

In each side of the larger cube, the smaller cubes in the edges should have more than one face painted. Therefore, the cube which are located at the corner of the larger cube, have three faces painted.

So, in each edge two small cubes are left, in which two faces painted.

It is known that the total numbers of edges in a cubes = 12.

Hence, the number of small cubes with two faces painted = 12×2
= 24 small cubes.

30. A cube of side 5 cm is cut into 1 cm cubes. The percentage increase in volume after such cutting is __________.

Solution:

No change
Volume of cube = 5^3
= 125
Now, when the cube is cut into 1 cubic cm, we will get 125 small cubes

Therefore,
Volume of the big cube = volume of 125 cm with 1 cubic cm.
It means that, there is no change in the volume.

31. The surface area of a cuboid formed by joining two cubes of side a face to face is __________.

Solution:

$10a^2$

Let "a" be the side of two cubes.
When the two cubes are joined face to face, the figure obtained should be a cuboid having the same breadth and height. As the combined cube has a length twice of the length of a cube.

It means that l = 2a, b = a and h = a
Hence, the total surface area of cuboid = 2(lb + bh + hl)
$= 2(2a \times a + a \times a + a \times 2a)$
$= 2[2a^2 + a^2 + 2a^2]$
$= 10a^2$

32. If the diagonals of a rhombus get doubled, then the area of the rhombus becomes __________ its original area.

Solution:

4 times

Let p and q be the two diagonals of the rhombus

We know that area of a rhombus = $\frac{pq}{2}$

If the diagonals are doubled, we will get

$A = \frac{2p.2q}{2}$

Take 4 outside,

$A = \frac{4pq}{2}$

33. If a cube fits exactly in a cylinder with height h, then the volume of the cube is __________ and surface area of the cube is __________.

Solution:

Volume is h^3 and surface area is $6h^2$

Each side of a cube = h

So,

Volume of cube = h^3

Surface area of a cube = $6 (h^3)$

34. The volume of a cylinder becomes __________ the original volume if its radius becomes half of the original radius.

Solution:

$\frac{1}{4}$ times

Volume of cylinder = $\pi r^2 h$ (when radius is r and height is h)

When the radius is halved, then it becomes

$V = \pi \left(\frac{r}{2}\right)^2 h$

$V = \frac{1}{4} (\pi r^2 h)$

35. The curved surface area of a cylinder is reduced by ___________ per cent if the height is half of the original height.

Solution:

50%

The CSA of cylinder with radius "r" and height "h" is $2\pi rh$

When the height is halved, then new CSA is $2\pi r \left(\frac{h}{2}\right) = \pi rh$

Therefore, the percentage reduction in CSA = $\frac{2\pi rh - \pi rh}{2\pi rh} \times 100$

= 50%

36. The volume of a cylinder which exactly fits in a cube of side a is __________.

Solution:

$\frac{\pi a^3}{4}$

When the cylinder exactly fits in the cube of side "a", the height equals to the edges of the cube and the radius equal to half the edges of a cube.

It means that,
h = a, and
r = $\frac{a}{2}$
Then the volume of a cylinder be = $\pi r^2 h$

$= \pi(\frac{a}{2})^2(a)$

$= \frac{\pi a^3}{4}$

37. The surface area of a cylinder which exactly fits in a cube of side b is __________.

Solution:
πb^2
When the cylinder exactly fits in the cube of side "b", the height equals to the edges of the cube and the radius equal to half the edges of a cube.

It means that,
h = b, and
r = $\frac{b}{2}$

Then the CSA of a cylinder be = $2\pi rh$

$= 2\pi (\frac{b}{2})(b)$

$= \pi b^2$

38. If the diagonal d of a quadrilateral is doubled and the heights h_1 and h_2 falling on d are halved, then the area of quadrilateral is __________.

Solution:

$\frac{1}{2}(h_1 + h_2)\, d$

Assume that ABCD be a quadrilateral, h_1 and h_2 are the heights on the diagonal BD = d, then, the area of a quadrilateral be

$= (\frac{1}{2})(h_1 + h_2)\, BD$

Since the diagonal is doubled and the heights are halved, we will get

$= (\frac{1}{2})\,[\,(\frac{h_1}{2}) + (\frac{h_2}{2})\,]\, 2d$

$= \frac{1}{2}(h_1 + h_2)\, d$

39. The perimeter of a rectangle becomes __________ times its original perimeter, if its length and breadth are doubled.

Solution:

Two times

We know that the perimeter of a rectangle is 2(l+b)
When the length and breadth of the perimeter are doubled, we will get
P = 2(2l +2b)
Now take 2 outside,
P = 2 [2(l+b)]

40. A trapezium with 3 equal sides and one side double the equal side can be divided into __________ equilateral triangles of _______ area.

Solution:

3, equal areas

By using SSS congruency rule of triangle, we can show that a trapezium can be divided into three equilateral triangle with equal areas.

41. All six faces of a cuboid are __________ in shape and of ______ area.

Solution:

Rectangular shape, different

It is known that, a cuboid is made up of 6 rectangular face which different lengths and breadths. Hence, it has different area.

42. Opposite faces of a cuboid are _________ in area.

Solution:

Equal
A cuboid is made up of 6 rectangular faces, but the opposite sides have equal length and breadth. Hence, the opposite areas are equal.

43. Curved surface area of a cylinder of radius h and height r is _______.

Solution:

$2\pi rh$
The CSA of a cylinder with radius "r" and height "h" is
CSA = $2\pi(r)(h)$

44. Total surface area of a cylinder of radius h and height r is _________

Solution:

$2\pi h(r + h)$

Given radius = r and height = h
TSA of cylinder = CSA of cylinder + Area of top surface + Base area
TSA = $2\pi rh + \pi r^2 + \pi r^2$

45. Volume of a cylinder with radius h and height r is __________.

Solution:

$\pi r^2 h$ cubic units

46. Area of a rhombus = $\frac{1}{2}$ product of _________.

Solution:

Diagonals
We know that the area of a rhombus = $\frac{pq}{2}$
Where p and q are diagonals.

47. Two cylinders A and B are formed by folding a rectangular sheet of dimensions 20 cm × 10 cm along its length and also along its breadth respectively. Then volume of A is ________ of volume of B.

Solution:

Twice

Rectangular sheet dimension is 20 cm × 10 cm
When a cylinder is folded along its length, which is 20 cm, then the resultant cylinder is with height 10 cm.

Again, if a cylinder is folded along its breadth, which is 10 cm, then the resultant cylinder is with height 20 cm

When the above conditions are applied in the volume of cylinder formula,
We get,
v = 2V

48. In the above question, curved surface area of A is ________ curved surface area of B.

Solution:

Same

For cylinder A, h= 10 cm and
$r = \frac{10}{\pi}$
Thus,
CSA of cylinder A = $2\pi rh$
= 200
For cylinder B, h= 12 cm and
$r = \frac{5}{\pi}$
So,
CSA of cylinder B = $2\pi rh$
= 200

49. __________ of a solid is the measurement of the space occupied by it.

Solution:

Volume

The space occupied by any solids or three dimensional shaped are always measured in terms of volume.

50. __________ surface area of room = area of 4 walls.

Solution:

Lateral

We know that, the rooms are in the cuboid shape.
The walls are considered as the lateral faces of the cuboid shaped room.

51. Two cylinders of equal volume have heights in the ratio 1:9. The ratio of their radii is __________.

Solution-

3 : 1

52. Two cylinders of same volume have their radii in the ratio 1:6, then ratio of their heights is __________.

Solution-

$h_1 : h_2 = 36 : 1$

In question 53 to 61, state whether the statements are true (T) or false (F).

53. The areas of any two faces of a cube are equal.

Solution-

The given statement is true.

54. The areas of any two faces of a cuboid are equal.

Solution-

The given statement is False.

A cuboid has rectangular face with different lengths and breadths.
Only opposite faces of cuboid have the same length and breadth.
Therefore, areas of only opposite faces of a cuboid are equal.

55. The surface area of a cuboid formed by joining face to face 3 cubes of side x is 3 times the surface area of a cube of side x.

Solution-

The given statement is False.
Three cubes having side x are joined face-to-face, then the cuoid so formed has the same height and breadth as the cubes but its length will be thrice that of the cubes.
Hence, the length, breadth and height of the cuboid so formed are 3x, x and x respectively**.**
Then,
Surface area = 2 (lb + bh + hl)
$= 2(3x \times x + x \times x + x \times 3x)$
$= 2(3x^2 + x^2 + 3x^2)$
$= 2(7x^2)$
$= 14x^2$
As the surface area of the cube of side x = $6a^2$
$= 6x^2$

56. Two cuboids with equal volumes will always have equal surface areas.

Solution-

The given statement is False.

57. The area of a trapezium become 4 times if its height gets doubled.

Solution-

The given statement is False.

58. A cube of side 3 cm painted on all its faces, when sliced into 1 cubic centimetre cubes, will have exactly 1 cube with none of its faces painted.

Solution-

The given statement is True.

59. Two cylinders with equal volume will always have equal surface areas.

Solution-

The given statement is False.
Consider two cylinders with the following measures-

r_1 = 2 cm,
h = 9cm,

r = 3 cm,
h = 4 cm for the first cylinder,
Volume = $\pi r^2 h$
$= \pi \times 2^2 \times 9$
$= 36\pi$ cm^3

Again for the second cylinder,
Volume = $\pi r^2 h$
$= \pi \times 3^2 \times 4$
$= 36\pi$ cm^3

Thus, the volumes are equal.
Now, surface area of first cylinder = $2\pi rh$
$= 2\pi \times 2 \times 9$
$= 36\pi$ cm^2.
Surface area of second cylinder = $2\pi rh$
$= 2\pi \times 3 \times 4$
$= 24\pi$ cm^2
Therefore, the surface areas are not equal.

60. The surface area of a cube formed by cutting a cuboid of dimensions 2 × 1 × 1 in 2 equal parts is 2 sq. units.

Solution-

The given statement is False.

The dimensions of the given cuboid are 2 x 1 x 1. It is sliced into two equal parts, which are cubes.
Then, the dimensions of the cube, so formed 1 x 1 x 1.

Therefore, the surface area of the cube so formed = $6(a)^2$,
$= 6\ (1)^2$
= 6 sq units.

61. Ratio of area of a circle to the area of a square whose side equals radius of circle is 1 : π.

Solution-

The given statement is False.
Given, side of a square equals radius of a circle.
Then,
area of the square = r^2
And
area of the circle = πr^2,

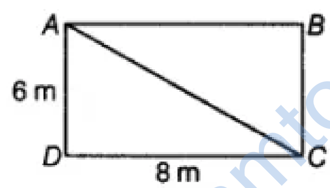
where r is a radius of the circle.
Now,
the ratio of area of the circle to area of the square = $\pi r^2 : r^2$
$= \pi : 1$.

Solve the following:

62. The area of a rectangular field is 48 m^2 and one of its sides is 6m. How long will a lady take to cross the field diagonally at the rate of 20 m/minute?

Solution-

Given,
Area of a rectangular field is 48 m^2 and one side of the rectangle is = 6 m.



Therefore, area of a rectangle = length x breadth
48 = 6 x breadth
Breadth = 8 m
In triangle ACD,
$\angle D = 90^o$

Use Pythagoras theorem;

$$AC^2 = AD^2 + DC^2$$
$$AC^2 = 6^2 + 8^2$$
$$AC^2 = 100$$
$$AC = 10m$$

Time taken at rate of 20m/min= distance/ speed

$$= \frac{10}{20}$$
$$= \frac{1}{2}\text{min}$$
$$= 30\,\text{sec}$$

63. The circumference of the front wheel of a cart is 3 m long and that of the back wheel is 4 m long. What is the distance travelled by the cart, when the front wheel makes five more revolutions than the rear wheel?

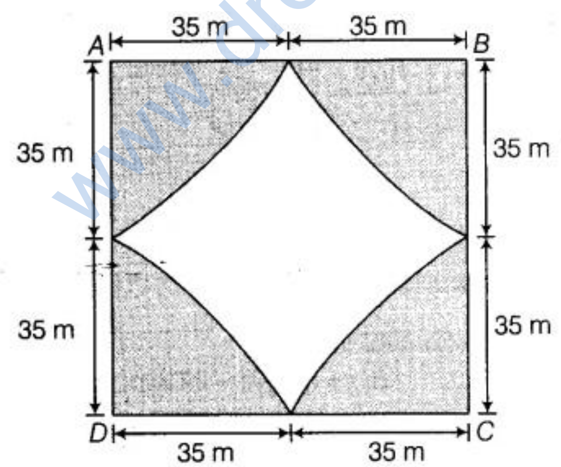
Solution-

Given,
Circumference of front wheel = 3m.
Now,
Distance covered by front wheel of the cart in 1 revolution = circumference of front wheel.
Therefore, the distance covered by front wheel in 5 revolutions = 3 x 5
= 15 m.

64. Four horses are tethered with equal ropes at 4 corners of a square field of side 70 metres so that they just can reach one another. Find the area left ungrazed by the horses.

Solution-

Side of a square= 70m
Also, four horses are tethered with equal ropes at 4 corners of the square field.
Hence, each horse can graze upto 35 m of distance along the side



Therefore,
Area of the square field = side x side
= 70 x 70
= 4900 m^2

The grazed area is making a complete circle by taking all the four grazed parts
So,

Area of grazed part = πr^2

$= \frac{22}{7} \times 35 \times 35$

$= 3850\ m^2$.

Ungrazed area left= area of square field – area of grazed part

$= 4900 - 3850$

$= 1050\ m^2$

65. The walls and ceiling of a room are to be plastered. The length, breadth and height of the room are 4.5 m, 3 m, and 350 cm respectively. Find the cost of plastering at the rate of Rs 8 per m^2.

Solution-

Given,

length of the room (l) = 4.5 m

Breadth of the room (b) = 3m

Height of the room (h) = 350 cm

= 3.5 m

And the cost of plastering = Rs 8 per m^2

Therefore, the area of the walls = 2h (l + b)

$= 2 \times 3.5 \times (4.5 + 3)$

$= 7 \times (7.5)$

$= 52.5\ m^2$

Area of the ceiling = lb

$= 4.5 \times 3$

$= 13.5\ m^2$

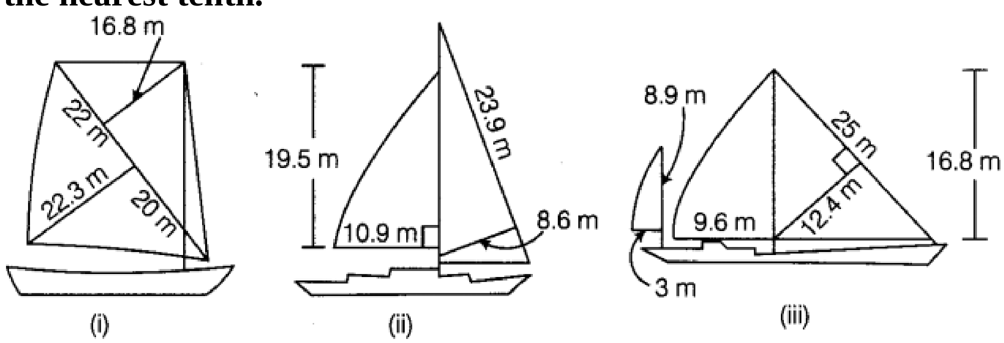
Area of the room = 52.5 + 13.5

$= 66\ m^2$

Hence, the cost of plastering = 66 x 8

= Rs 528

66. Most of the sailboats have two sails, the jib and the mainsail. Assume that the sails are triangles. Find the total area of each sail of the sail boats to the nearest tenth.



Solution-

In the sailboat (i),

Area of a triangle = $\frac{1}{2}$ x base x height

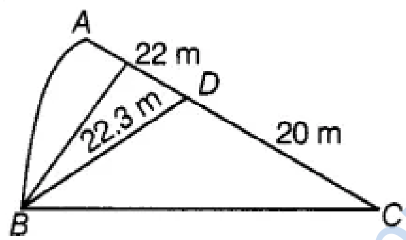
In triangle ABC,

AC = base

= 22 + 20

= 42 m.

BD = height = 22.3 m



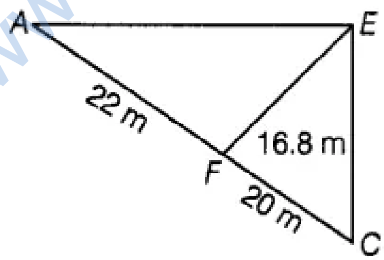
Therefore,

area of triangle ABC = $\frac{1}{2}$ x 42 x 22.3

= $\frac{936.6}{2}$

= 468.3 m^2

In another triangular part,



In ACE,

EF = height = 16.8 m

AC = Base = 22 + 20

= 42 m

Therefore area of ACE = $\frac{1}{2}$ x 42 x 16.8

= $\frac{705.6}{2}$

= 352.8m^2

Therefore, area of sailboat (i) = 468.3 + 352.8

$= 821.1\ m^2$

In sailboat (ii),

Area of a triangle = $\frac{1}{2}$ x base x height

In,ABC , B = 90^o

(BC) = 10.9 m and

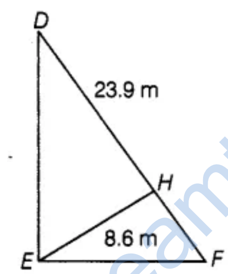
height (AB) = 19.5m

Area of ABC = $\frac{1}{2}$ x 10.9 x 19.5

$= \frac{212.55}{2}$

$= 106.275\ m^2$

In another triangular part,



Area of DEF = $\frac{1}{2}$ x DF x EH

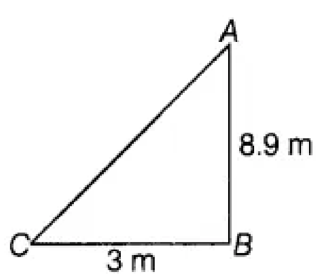
$= \frac{1}{2}$ x 23.9 x 8.6

$= 102.77\ m^2$

Therefore, the area of sailboat (ii) = 106.275 + 102.77

$=209.045\ m^2$

In sailboat (iii),



ar(ABC) = $\frac{1}{2}$ x BC x AB

$= \frac{1}{2} \times 8.9 \times 3$

$= 13.35 \text{ m}^2$

ar(DEF) = $\frac{1}{2}$ x DE x EH

$= \frac{1}{2} \times 9.6 \times 16.8$

$= 80.64 \text{ m}^2$

Therefore, area of sailboat (iii) = 155 + 80.64
= 235.64 m^2

67. The area of a trapezium with equal non-parallel sides is 168 m^2. If the lengths of the parallel sides are 36 m and 20 m, find the length of the non-parallel sides.

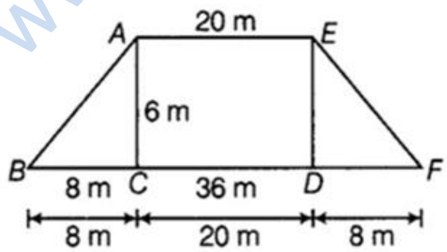
Solution-

Length of the parallel sides is 36 m and 20 m.
Area of trapezium given= 168 m^2
Since,

Area of a trapezium = $\frac{1}{2}$ x (sum of parallel sides) x height

$168 = \frac{1}{2} \times (36+20) \times h$

h = 6m



Now,
In ACB , use Pythagoras theorem,

$$AB^2 = BC^2 + AC^2$$
$$= 8^2 + 6^2$$
$$= 64 + 36$$
$$= 100$$
$$AB = 10m$$

68. Mukesh walks around a circular track of radius 14 m with a speed of 4 km/hr. If he takes 20 rounds of the track, for how long does he walk?

Solution-

Radius of the circular track = 14m
Circumference of the circular track = $2\pi r$

$$= 2 \times \frac{22}{7} \times 14$$
$$= 44 \times 2$$
$$= 88 \text{ m}$$

Total distance cover in 20 rounds = 88 x 20
= 1760 m

Speed of Mukesh on the circular track = 4km/h

$$= \frac{200}{3} \text{ m/min}$$

Time take by mukesh = $\dfrac{1760}{\frac{200}{3}}$

$$= \frac{1760 \times 3}{200}$$
$$= 26.4 \text{ min}$$

26.4 min = 26 min and 24 sec.

69. The areas of two circles are in the ratio 49:64. Find the ratio of their circumferences.

Solution-

Ratio of the area of two circles given= 49 : 64

Area of circle= πr^2
And the area of the first and second circle = πr_1^2 and πr_1^2

Now,

$$\frac{49}{64} = \frac{\pi r_1^2}{\pi r_2^2}$$

$$\left(\frac{7}{8}\right)^2 = \frac{r_1^2}{r_2^2}$$

$$\left(\frac{7}{8}\right)^2 = \left(\frac{r_1}{r_2}\right)^2$$

So,

$$r_1 = 7$$

$$r_2 = 8$$

$$\text{Ratio of circumference} = \frac{2\pi r_1}{2\pi r_2}$$

$$= \frac{r_1}{r_2}$$

$$= \frac{7}{8}$$

70. There is a circular pond and a footpath runs along its boundary. A person walks around it, exactly once keeping close to the edge. If his step is 66 cm long and he takes exactly 400 steps to go around the pond, find the diameter of the pond.

Solution-

Let the radius of the pond be r. Then, diameter of the pond d= 2 x r

Since, a person takes exactly 400 steps with 66 cm long each step to go round the pond.

Hence, the circumference of the pond = 66 x 400

= 26400 cm

$= \frac{26400}{100}$

= 264 m

As the circumference of a circle is 2πr

2πr = 264

$r = \frac{264}{2} \times \frac{7}{22}$

= 42m

Hence, the radius of the pond = 42 m

So, diameter of the pond = 2 x 42

= 84 m.

71. A running track has 2 semi-circular ends of radius 63 m and two straight lengths. The perimeter of the track is 1000 m. Find each straight length.

Solution-

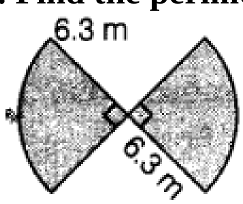
Radius of semi-circular track = 63 m
Perimeter of 2 semi-circles = Perimeter of 1 circle
Perimeter of a circular track = $2\pi r$

Perimeter of circular track = $2 \text{ x } \frac{22}{7} \text{ x } 63$
= 2 x 22 x 9
= 44x 9
= 396 m

Since, the perimeter of the total track is 1000 m
Therefore, the length of two straight lengths track = 1000 – 396
= 604 m

Length of 1 straight length track = $\frac{604}{2}$
= 302 m.

72. Find the perimeter of the given figure.



Solution-

Radius of the given figure = 6.3 m
Two sections in figure form of a semi-circle.

Perimeter of semi-circle figure= $\frac{2\pi r}{2} + 2r$
$= \pi r + 2r$
$= \frac{22}{7} \text{ x } 6.3 + 2 \text{ x } 6.3$
= 22 x 0.9 + 2 x 6.3
= 19.8 + 12.6
= 32.4 m

73. A bicycle wheel makes 500 revolutions in moving 1 km. Find the diameter of the wheel.

Solution-

A bicycle wheel makes 500 revolutions in moving 1 km.
In 1 revolution, the bicycle wheel covers

$= \frac{1}{500}$ km

$= \frac{1}{500}$ m

= 2m

1 revolution distance = circumference of the wheel

$2\pi r = 2$

$2 \times \frac{22}{7} r = 2$

$r = \frac{2 \times 7}{2 \times 22}$

$= \frac{7}{22}$

Therefore,
diameter (d) = 2r

$= 2 \text{ x } \frac{7}{22}$

$= \frac{7}{11}$

= 0.636 m.

74. A boy is cycling such that the wheels of the cycle are making 140 revolutions per hour. If the diameter of the wheel is 60 cm, calculate the speed in km/h with which the boy is cycling.

Solution-

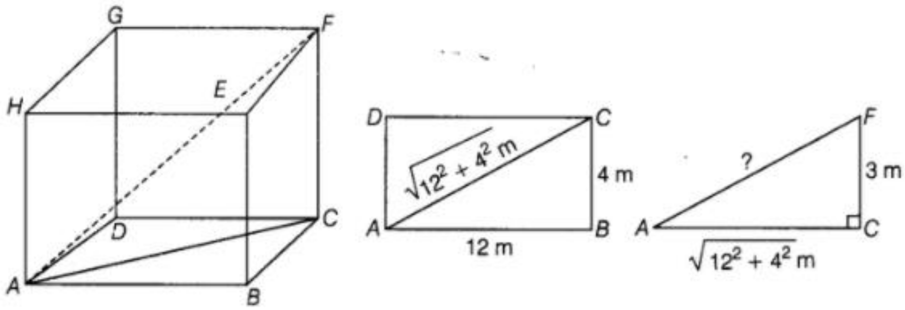
Circumference of a circle = $2\pi r$
= 2 x 22/7 x 30
= 188.57 cm
Distance cover in 140 revolutions = 140 x 188.57
= 26400 cm
Therefore,

Speed= $\frac{Distance}{time}$

$= \frac{26400}{100000}$ km/h

= 0.264 km/h.

75. Find the length of the largest pole that can be placed in a room of dimensions 12 m × 4 m × 3 m.



Find the area of the following fields. All dimensions are in metres.

Solution-

We have,

$In\,\Delta ACF,$

$\angle C = 90^o$

$CF = 3m$

$AC = \sqrt{12^2 + 4^2}$

The length of the largest pole = length of diagonal of cuboid $(in\ shape\ of\ room)$

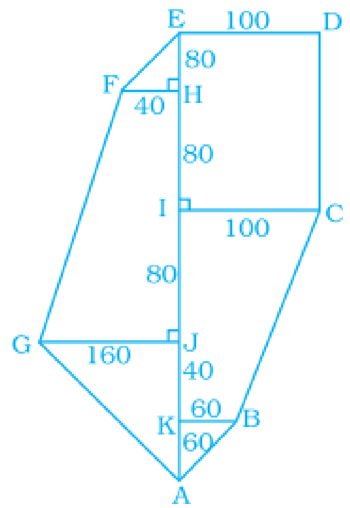
$(AF)^2 = (AC)^2 + (CF)^2$

$(AF)^2 = (12)^2 + (4)^2 + (3)^2$

$AF = \sqrt{169}$

$AF = 13m$

76.



Solution-

Area of the given figure = Area of EFH + Area of rectangle EDCI + Area of Trapezium FHJG + Area of trapezium ICBK + Area of GJA+ Area of KBA

According to question,
We find the individual areas and then add all the areas of figures.
So,

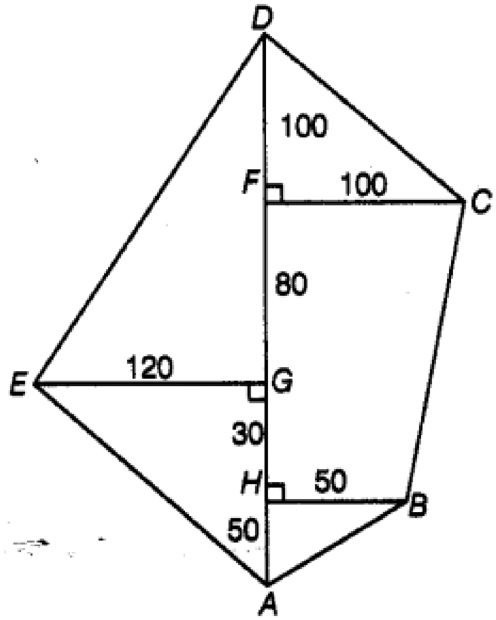
$$\text{Area}\left(EFH\right)=\frac{1}{2}\times 40\times 80$$
$$=1600m^2$$
$$\text{Area}\left(EDCI\right)=l\times b$$
$$=100\times 160$$
$$=1600m^2$$
$$\text{Area of trapezium } ICBK=\frac{1}{2}\times\left(60+100\right)\times 120$$
$$=9600m^2$$
$$ar(AJG)=\frac{1}{2}\times 160\times 100$$
$$=8000m^2$$
$$ar(\text{KBA})=\frac{1}{2}\times 60\times 60$$
$$=1800$$

Thus, the area of the complete figure –

$= 1600 + 16000 + 16000 + 9600 + 8000 + 1800$

$= 53000\ m^2$

77.



Find the area of the shaded portion in the following figures.

Solution-

Area of the given figure = Area of DCF + Area of EGD + Area of trapezium FCBH + Area of AHB + Area of EGA

Therefore,

$$\text{Area}\left(DCF\right)=\frac{1}{2}\times 100\times 100$$
$$=5000m^2$$

$$\text{Area}\left(EGD\right)=\frac{1}{2}\times 120\times 180$$
$$=10800m^2$$

$$\text{Area of trapezium}=\frac{1}{2}\times\left(100+50\right)\times 110$$
$$=8250m^2$$

$$ar(\text{EGA})=\frac{1}{2}\times 120\times 80$$
$$=4800m^2$$

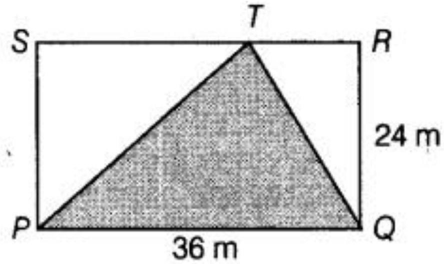
$$ar(\text{AHB})=\frac{1}{2}\times 50\times 50$$
$$=1250$$

Thus, the complete area of the given figure is

$= 5000 + 10800 + 8250 + 4800 + 1250$

$= 30100\ m^2$.

78.

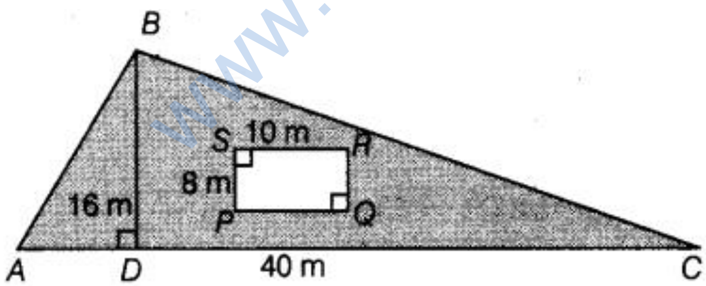


Solution-

Area of the shaded portion= area of PTQ
RQ = height
So,

Area (PTQ) = $\frac{1}{2}$ x x 36 x 24
= 18 x 24
= 432 m^2

79.



Solution-

Area of shaded triangle = Area ABC - Area of rectangle PQRS
Now,

Ar(ABC) = $\frac{1}{2}$ x 40 x 16 m^2
= 20 x 16
= 320 m^2

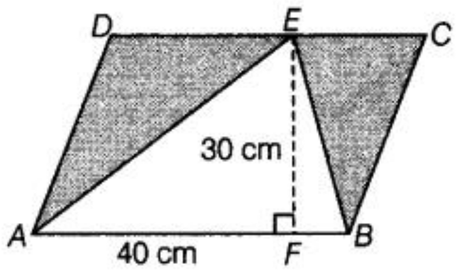
Area of rectangle = length x breadth
Area of rectangle = 10 x 8

$= 80 \text{ m}^2$

Area of shaded region = 320 – 80

$= 240 \text{ m}^2$

80.



Solution-

Area of shaded region = area of the parallelogram ABCD – Area of ABE

So,

Area of a parallelogram = Side x height

Area of the parallelogram ABCD = 40 x 30

$= 1200 \text{ cm}^2$

Area of ABE = $\frac{1}{2}$ x AB x EF

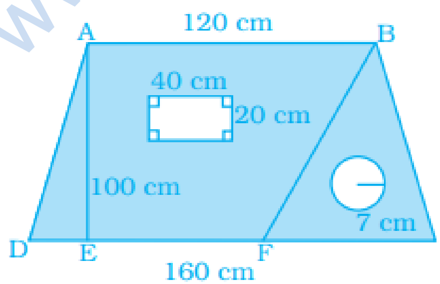
$= \frac{1}{2}$ x 40 x 30

$= 600 \text{ cm}^2$

Thus, the area of shaded region = 1200 – 600

$= 600 \text{ cm}^2$

81.



Solution-

Area of shaded portion = Area of trapezium – area of rectangle – area of circle

Now,

Area of trapezium= $\frac{1}{2}$ x (sum of parallel side) x height

$= \frac{1}{2}$ x (120 + 160) x 100

$= \frac{1}{2} \text{ x } 280 \text{ x } 100$

$= \frac{28000}{2}$

$= 14000 \text{ cm}^2$

Area of rectangle = length x breadth

= 40 x 20

= 800 cm^2

Area of circle = πr^2

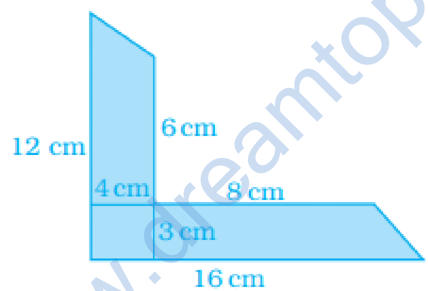
$= \frac{22}{7} \text{ x } 7 \text{ x } 7$

= 154 cm^2

Therefore, area of shaded portion = 14000 – 800 – 154

= 13046 cm^2

82.



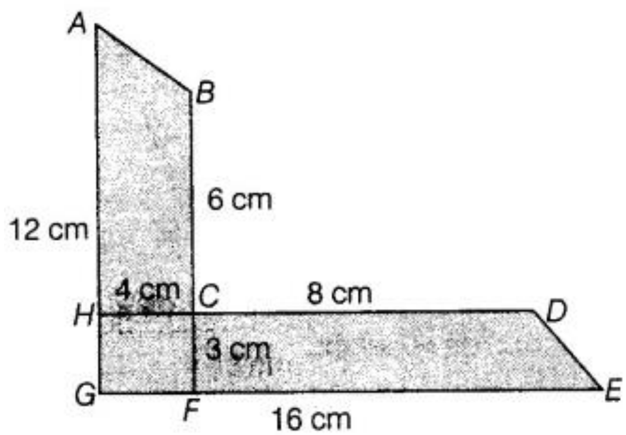
Solution-

Area of shaded portion = area of trapezium ABCH + area of trapezium CDEF

Area of trapezium = $\frac{1}{2}$ x (sum of parallel sides) x height

Area of trapezium ABCH = $\frac{1}{2}$ x (12 + 6) x 4

= 36 cm^2



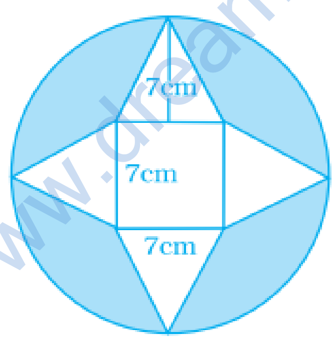
Area of trapezium CDEF = $\frac{1}{2}$ x (8 +16) x 3

= 36cm^2

Area of shaded portion = 36 + 36

= 72 cm^2

83.



Solution-

Area of shaded portion = area of the circle – area of four triangles – area of a square

Area of four triangles = 4 x $\frac{1}{2}$ x base x height

= 4 x $\frac{1}{2}$ x 7 x 7

= 98cm^2

Area of a square= (side)^2

= (7)^2

= 49 cm^2

Area of the circle = πr^2

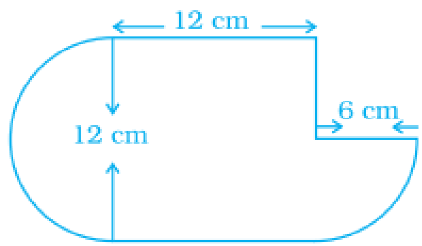
r = $\frac{21}{2}$

= 346.5cm^2

Area of shaded region = (346.5 – 98 – 49)

= 199.5 cm^2

84.



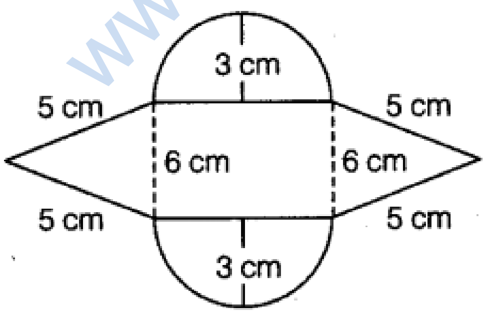
Solution-

Area of the given figure = Area of section with 6 cm+ area of square with side measure 12 cm + area of semi circle with radius 6 cm

$$= \frac{22\times6\times6}{7\times4} + (12)^2 + \frac{22\times6\times6}{7\times2}$$

$$= 228.85cm^2$$

85.



Solution-

Area of the given figure= area of two semi-circles + area of two triangles + area of a square

Using Heron's formula,

Therefore, area of triangle= $\sqrt{s(s-a)(s-b)(s-c)}$

$$s = \frac{a+b+c}{2}$$

$$= \frac{5+5+6}{2}$$

$$= 8cm$$

Therefore, area of a triangle $= \sqrt{8(8-5)(8-5)(8-6)}$

$$= \sqrt{144}$$

$$= 12cm^2$$

Area of 2 triangles $= 2 \times 12$

$$= 24\ cm^2$$

Area of 2 semi-circles = area of 1 circle

Area of a circle $= \pi r^2$

$r = 3,$

Area of a circle $= 28.28cm^2$

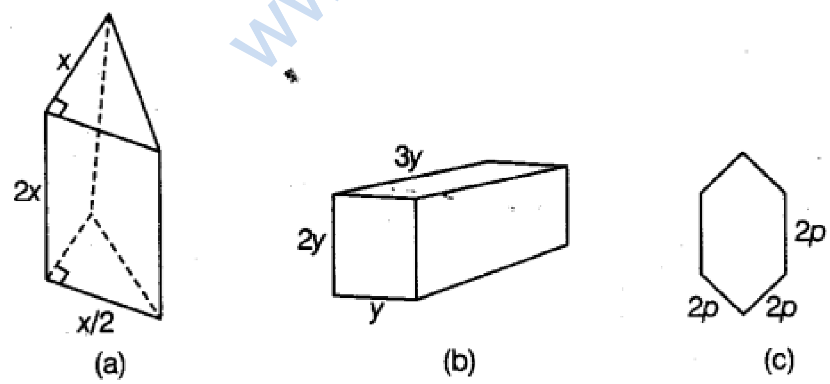
Area of a square $= 6 \times 6$

$$= 36\ cm^2$$

Thus, the area of given figure $= (24 + 28.28 + 36)$

$$= 88.28\ cm^2$$

86. Find the volume of each of the given figure if volume = base area × height.



Solution-

Volume of each of the given figure= base area x height

In fig (a), base is rectangle

So, area of rectangle = $2x \times \frac{x}{2} = x^2$

Therefore, height = $\frac{x}{2}$

So, the volume of the figure= $x^2 \times \frac{x}{2} = \frac{x^3}{2}$

In Fig (b), base is rectangle

So, the area of rectangle = y x 3 y

= $3\,y^2$

Height = 2 y

Therefore, the volume of the figure= $3\,y^2$ x 2 y

= $6\,y^3$

In fig (c), base is rectangle,

So, the area of rectangle = 2p x 2p

= $4p^2$

Height= 2p

Volume of the figure= $4p^2$ x 2p

= $8p^3$

87. A cube of side 5 cm is cut into as many 1 cm cubes as possible. What is the ratio of the surface area of the original cube to that of the sum of the surface areas of the smaller cubes?

Solution-

Surface area of a cube = $6a^2$, where a is side of a cube.

Therefore,

Side of cube = 5cm

Surface area of the cube = 6 x $(5)^2$

= 6 x 25

= 150 cm^2

Now, surface area of the cube with side 1 cm = 6 x $(1)^2$

= $6cm^2$

Therefore, surface area of 5 cubes with side 1 cm = 5 x 6

= 30 cm^2

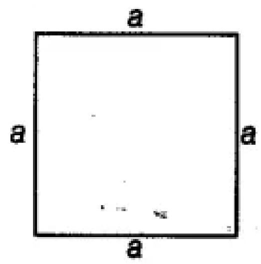
Ratio of the surface area of the original cube to that of the sum of the surface area of the smaller cubes = 30/150

= 1:5

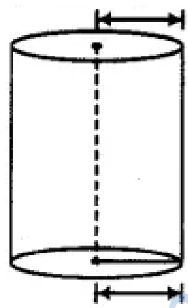
88. A square sheet of paper is converted into a cylinder by rolling it along its side. What is the ratio of the base radius to the side of the square?

Solution-

Let the sides of a square paper be a



A cylinder is formed by rolling the paper along its side.



Therefore, base of a cylinder is circle so the circumference of the circle is equal to the length of each side of the square sheet.

We have, $2\pi r = a$

$r = \dfrac{a}{2\pi}$

Therefore, ratio $= \dfrac{a}{2\pi} : a$

$= 1 : 2\pi$

Ratio to the side of square $= 1 : 2\pi$

89. How many cubic metres of earth must be dug to construct a well 7 m deep and of diameter 2.8 m?

Solution-

A well is in the form of cylindrical form
Earth must be dug to construct a well 7 m deep and diameter
2.8 m is equal to the volume of a cylinder with 7 m height and diameter 2.8 m

Volume of a cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{2.8}{2} \times \frac{2.8}{2} \times 7$$
$$= 11 \times 2.8 \times 1.4$$
$$= 43.12 m^3$$

90. The radius and height of a cylinder are in the ratio 3:2 and its volume is 19,404 cm^3. Find its radius and height.

Solution-
The radius and height of a cylinder are in the ratio 3:2

Let, the radius be 3x and height be 2x.
As the vol. of cylinder = $\pi r^2 h$
Vol. of cylinder = 19404 cm^3

$$19404 = \frac{22}{7} \times (3x)^2 \times (2x)$$
$$19404 = \frac{396x^3}{7}$$
$$x^3 = 343$$
$$x^3 = (7)^3$$
$$x = 7cm$$

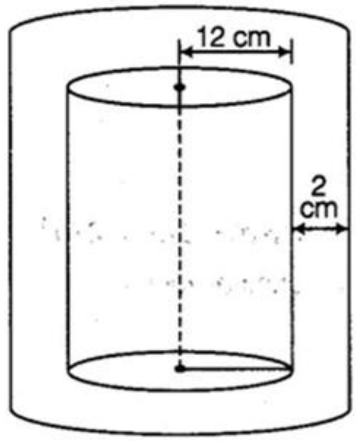
Hence, radius of the cylinder $= 3 \times 7$
$= 21\ cm$

And height of the cylinder $= 2 \times 7$
$= 14\ cm$

91. The thickness of a hollow metallic cylinder is 2 cm. It is 70 cm long with outer radius of 14 cm. Find the volume of the metal used in making the cylinder, assuming that it is open at both the ends. Also find its weight if the metal weighs 8 g per cm^3.

Solution-

Thickness of the hollow metallic cylinder = 2 cm Height of the cylinder = 70 cm
Outer radius = 14 cm
Inner radius = 14 – 2
= 12 cm



Volume of the metal used in making the cylinder = volume of the hollow cylinder

$= \pi(R^2 - r^2) \times h$

$= \frac{22}{7}((14)^2 - (12)^2) \times 70$

$= 22 \times [196 - 144] \times 10$

$= 22 \times 52 \times 10$

$= 11440 cm^3$

$Weight\ of\ 11440\ cm^3,\ if\ metal\ is\ 8g\ per\ cm^3 = 11440 \times 8$

$= 91520\ g$

92. Radius of a cylinder is r and the height is h. Find the change in the volume if the
(a) height is doubled.
(b) height is doubled and the radius is halved.

Solution-

Volume of a cylinder = $\pi r^2 h$
r is radius of base of the cylinder,
h is height

(i) If height is doubled ,
h = 2 x h
= 2h
So, its volume = $\pi r^2 .2h$
Therefore, volume became double of original volume.

(ii) If height is doubled and the radius is halved, Volume = $\dfrac{\pi r^2 h}{2}$

93. If the length of each edge of a cube is tripled, what will be the change in its volume?

Solution-

Let the edge of a cube be a
If edge of the cube became tripled = a = 3 x a = 3a
Since, the volume of the cube= a^3
Therefore, volume of the cube with edge tripled = $(3a)^3$
= $27a^3$.
Hence, volume is 27 times of the original volume.

94. A carpenter makes a box which has a volume of 13,400 cm^3. The base has an area of 670 cm^2. What is the height of the box?

Solution-

Let the height of the box be h,
Volume of the box = 13400 cm^2
Area of base of the box = 670 cm^2
Since,
Volume of the box = area of base x height
Therefore,
13400 = 670 x h
h = 20cm

95. A cuboidal tin box opened at the top has dimensions 20 cm × 16 cm × 14 cm. What is the total area of metal sheet required to make 10 such boxes?

Solution-

Given,
Dimensions of cuboidal tin box = 20 cm x 16 cm x 14 cm
Surface area of cuboid = 2 (lb + bh + hl)
Therefore,
Area of metal sheet for 1 box = Surface area of cuboid
= 2 (20 x 16 + 16 x 14 + 14 x 20)
= 2 (320 + 224 + 280)
= 1648 cm^2
So,
Area of metal sheet required to make 10 such boxes = 10 x 1648
= 16460 cm^2

96. Find the capacity of water tank, in litres, whose dimensions are 4.2 m, 3 m and 1.8 m?

Solution-

Dimensions of the water tank are 4.2m, 3m and 1.8m
Capacity of water tank = length x breadth x height
= 4.2 x 3 x 1.8
= 22.68 m^3

97. How many cubes each of side 0.5 cm are required to build a cube of volume 8 cm^3 ?

Solution-

Volume of a cube = $(side)^3$
Since side of cube = 0.5 cm
Therefore, the volume of the cube = $(0.5)^3$
= 0.125 cm^3

The number of cubes required to make volume of 8 cm^3 cube = $\frac{8}{0.125}$
= 64 cubes

98. A wooden box (including the lid) has external dimensions 40 cm by 34 cm by 30 cm. If the wood is 1 cm thick, how many cm^3 of wood is used in it?

Solution-

External dimension of wooden box are 40cm x 34 cm x 30 cm
Since, the wood is 1 cm thick,
Internal dimensions will be = (40 – 2) x (34 – 2) x (30 – 2)
= 38cm x 32cm x 28cm

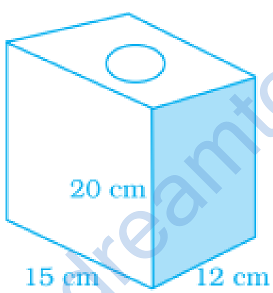
Therefore, wood used for the box-
= Vol. of the wooden box with external dimensions – vol. of the wooden box with internal dimensions
= 40 x 34 x 30 – 38 x 32 x 28
= 40800 – 34048
= 6752 cm^3

99. A river 2 m deep and 45 m wide is flowing at the rate of 3 km per hour. Find the amount of water in cubic metres that runs into the sea per minute.

Solution-

Depth of the river = 2m
Width of the river = 45 m
Flowing rate of the water = 3km/h
= 50 m/min
The amount of water into sea per minute = depth x width x length of water of 1 min
= 2 x 45 x 50
= 4500 m^3/min

100. Find the area to be painted in the following block with a cylindrical hole. Given that length is 15 cm, width 12 cm, height 20 cm and radius of the hole 2.8 cm.



Solution-

length of the given figure = 15 cm
Width of the given figure = 12 cm
Height of the given figure = 20 cm
Radius of the hole = 2.8 cm
Therefore,
Area to be painted = surface area of the figure – 2 x area of the circular hole

$= 2\,(lb + bh + hl) - 2\pi r^2$

$= 2\,(15 \times 12 + 12 \times 20 + 15 \times 20) - 2 \times \frac{22}{7} \times 2.8 \times 2.8$

$= 2 \times 720 - 49.28$

$= 1440 - 49.28$

$= 1390.72\ cm^2$

101. A truck carrying 7.8 m^3 concrete arrives at a job site. A platform of width 5 m and height 2 m is being contructed at the site. Find the length of the platform, constructed from the amount of concrete on the truck?

Solution-

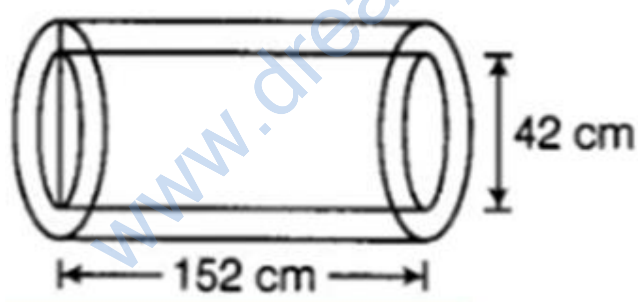
Total volume of concrete = 7.8 m^3
Width of the platform = 5m
Height of the platform = 2m
Let the length of the platform = x m

According to the question,
Volume of concrete = Volume used to make platform
Therefore,
7.8 = 5 x 2 x x
7.8= 10 x
10x = 7.8
x = 0.78 m

102. A hollow garden roller of 42 cm diameter and length 152 cm is made of cast iron 2 cm thick. Find the volume of iron used in the roller.

Solution-

Diameter of the hollow garden roller = 42 cm



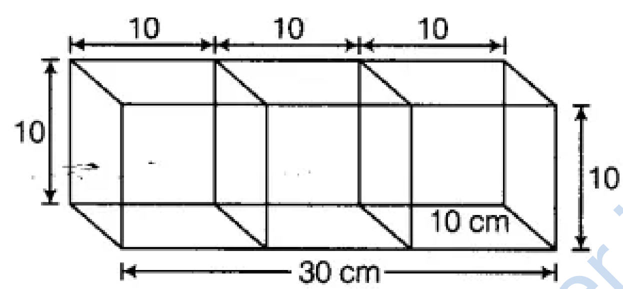
Therefore, inner radius = $\frac{42}{2}$
= 21 cm
Thickness of cast iron = 2cm
Outer radius = 21 + 2
= 23 cm

$$\begin{aligned} Vol.\ of\ \ hollow\ cylinder\ &= \pi(R^2 - r^2)\times h \\ &= \frac{22}{7}(23^2 - 21^2)\times 152 \\ &= 42038.85 cm^3 \end{aligned}$$

103. Three cubes each of side 10 cm are joined end to end. Find the surface area of the resultant figure.

Solution-

If three cubes each of side 10 cm are joined, then a cuboid will be formed of dimensions 30cm x 10cm x 10cm



Therefore,

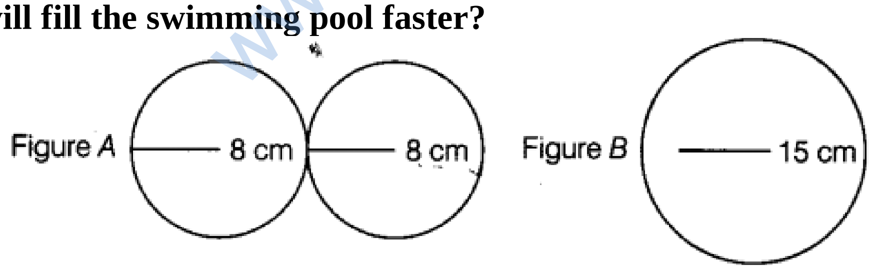
Surface area of the cuboid $= 2\,(lb + bh + hl)$

$= 2\,(30 \times 10 + 10 \times 10 + 30 \times 10)$

$= 2\,(300 + 100 + 300)$

$= 1400\ cm^2$

104. Below are the drawings of cross sections of two different pipes used to fill swimming pools. Figure A is a combination of 2 pipes each having a radius of 8 cm. Figure B is a pipe having a radius of 15 cm. If the force of the flow of water coming out of the pipes is the same in both the cases, which will fill the swimming pool faster?



Solution-

In fig. A,

2 pipes each having a radius of 8 cm

Since, area of a circle $= \pi r^2$

Area of one pipe $= \dfrac{22}{7} \times 8 \times 8$

$= \dfrac{1408}{7}$

Area of 2 pipes = 2 x $\frac{1408}{7}$

= 402.28cm^2

In fig B, a pipe having radius of 15 cm,
Therefore,
Area of the pipe = πr^2

$$\pi r^2 = \frac{22}{7} \times 15 \times 15$$
$$= 707.14 cm^2$$

The surface area of pipe B is greater so pipe B fills the swimming pool faster.

105. A swimming pool is 200 m by 50 m and has an average depth of 2 m. By the end of a summer day, the water level drops by 2 cm. How many cubic metres of water is lost on the day?

Solution-

Dimensions of swimming pool are 200 m x 50 m
Average depth of the swimming pool = 2m
At the end of summer day the water level drops by 2 cm
Therefore,
Volume of water in swimming pool = length x breadth x depth
= 200 x 50 x 2
= 20000 m^3

If water level drops by 2 cm, it means new level of water $= 2 - \frac{2}{100}$
= 1.98 m

Volume of water after summer day = 200 x 50 x 1.98
= 19800m^3

So,
Water in cubic metres was lost on that day = Initial volume – volume after summer day
= 20000 – 19800
= 200m^3

106. A housing society consisting of 5,500 people needs 100 L of water per person per day. The cylindrical supply tank is 7 m high and has a diameter 10 m. For how many days will the water in the tank last for the society?

Solution-

Total no. of peoples = 5500

Water required per person per day = 100 L
Total requirement of water by 5500 people = 100 x 5500
= 550000 L
Height of the cylindrical tank = 7m
Diameter of the cylindrical tank = 10 m
Therefore,
Radius= 5 m

$$\begin{aligned}\text{Volume of cylinder } &= \pi r^2 h \\ &= \frac{22}{7}\times 5\times 5\times 7 \\ &= 550 m^3 \\ &= 550\times 1000 \\ &= 550000 L \qquad [As\ 1\ m = \ 1000\ L]\end{aligned}$$

Hence, for 1 day the water in the tank lost for the society and in one day society needs 550000 L of water.

107. Metallic discs of radius 0.75 cm and thickness 0.2 cm are melted to obtain 508.68 cm^3 of metal. Find the number of discs melted (use π = 3.14).

Solution-

Radius of metallic disc = 0.75 cm
Thickness of disc = 0.2 cm
Total volume of material which will be used in forming of disc = 508.68 cm^3
Therefore, material required for one disc = volume of cylinder

$$\begin{aligned}\pi r^2 h &= \frac{22}{7}\times 0.75\times 0.75\times 0.2 \\ &= 0.35325 cm^3\end{aligned}$$

$$\begin{aligned}\text{Number of discs can be melted } &= \frac{\text{total vol. of metal obtained after melting}}{\text{vol of one disc}} \\ &= \frac{508.68}{0.35325} \\ &= 1440\, discs\end{aligned}$$

108. The ratio of the radius and height of a cylinder is 2:3. If its volume is 12,936 cm^3, find the total surface area of the cylinder.

Solution-

The ratio of the radius and height of a cylinder = 2:3
Let the radius of the cylinder be 2x and
the height of the cylinder be 3x

Volume of the cylinder = $\pi r^2 h$
= 12936 cm^3
Therefore,

$$12936 = \pi r^2 h$$
$$12936 = \frac{22}{7} \times 2x \times 2x \times 3x$$
$$12936 = \frac{264}{7} x^3$$
$$x = 7$$

So, radius $= 2x$
$= 2 \times 7$
$= 14\ cm$

height $= 3x$
$= 3 \times 7$
$= 21 cm$

The total surface area of the cylinder $= 2\pi r(r+h)$
$= 2 \times \frac{22}{7} \times 14(14+21)$
$= 3080 cm^2$

109. External dimensions of a closed wooden box are in the ratio 5:4:3. If the cost of painting its outer surface at the rate of Rs 5 per dm^2 is Rs 11,750, find the dimensions of the box.

Solution-

External dimensions of a close wooden box are in the ratio 5 : 4 : 3

Let the external dimensions of the closes wooden box be 5x, 4x and 3x
The cost of painting = Rs 5 per dm^2
Total cost of painting = Rs 11750

Therefore,
Total surface area = Total cost of painting / cost of painting per dm^2
$= \frac{11750}{5}$

= 2350 dm^2

Total surface area of a cuboid =2 (lb + bh + hl)

= 2 (5x x 4x + 4x x 3x + 3x x 5x)

= $2(20x^2+12x^2+15x^2)$

= 2 x $47x^2$

= 94 x^2

Since,

Total surface area = 2350 dm^2

94 x^2 = 2350

x = 5

Hence, dimensions of the box are:

5x= 5 x 5 = 25 dm

4x = 4 x 5 = 20 dm

3x = 3 x 5 = 15 dm

110. The capacity of a closed cylindrical vessel of height 1 m is 15.4 L. How many square metres of metal sheet would be needed to make it?

Solution-

Height of cylindrical vessel = 1m

Capacity of the cylindrical vessel = 15.4 L

In metre cube = $\frac{15.4}{1000}$

= 0.154 m^3

Volume of a cylinder $= \pi r^2 h$

$$0.0154 = \frac{22}{7} \times r^2 \times 1$$

$$\frac{0.0154}{3.14} = r^2$$

$$r = 0.07m$$

Therefore, metal of sheet required $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 0.07 \times 1$$

$$= 0.44m^3$$

111. What will happen to the volume of the cube, if its edge is (a) tripled (b) reduced to one-fourth?

Solution-

Let each side of cube be a, then its volume = a^3

a) If side became triple, then volume will be = $(3a)^3$

$= 27a^3$

Hence, new volume of the cube will be 27 times of original volume of the cube.

b) If side is reduced to one fourth = a x ¼ = a/4

$$Now,\ its\ volume\ = \left(\frac{a}{4}\right)^3$$

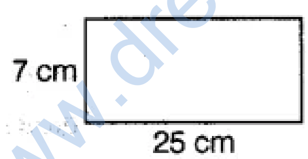
$$= \frac{a^3}{64}$$

Hence, new volume is 1/64 times of original volume.

112. A rectangular sheet of dimensions 25 cm × 7 cm is rotated about its longer side. Find the volume and the whole surface area of the solid thus generated.

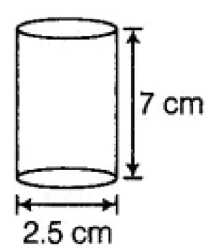
Solution-

A rectangular sheet of dimensions 25 cm x 7 cm is rotated about its longer side which makes a cylinder with base 25 cm and height 7 cm.



Therefore,
Surface area of a base = $2\pi r$
So, $2\pi r = 25$ cm

$$r = \frac{177}{44} cm$$



Volume of a cylinder = $\pi r^2 h$

$$\pi r^2 h = \frac{22}{7} \times \frac{177}{44} \times \frac{177}{44} \times 7$$
$$= \frac{30625}{88}$$
$$= 348.011 cm^3$$
$$Surface\ area = 2\pi rh$$
$$= 2 \times \frac{22}{7} \times \frac{177}{44} \times 7$$
$$= 175 cm^2$$

113. From a pipe of inner radius 0.75 cm, water flows at the rate of 7 m per second. Find the volume in litres of water delivered by the pipe in 1 hour.

Solution-

Radius of pipe = 0.75 cm
= 0.0075 m
The rate of water flow = 7 m/s
Therefore, the length of water in 1 sec = 7 m
So the volume of water flow in 1 hour =60 x 60 x 22/7 x 0.0075 x 0.0075 x 7
= 3600 x 3.14 x 0.00005625 x 7
= 79128 x 0.00005625
= 4.45095 m^3
= 4.45095 m^3
= 4450000cm^3
= 4450 L

114. Four times the area of the curved surface of a cylinder is equal to 6 times the sum of the areas of its bases. If its height is 12 cm, find its curved surface area.

Solution:

Let the radius and height of the cylinder be r and h, respectively.

Curved surface area of cylinder = 2 πrh
Area of base = πr^2
Sum of areas of bases = 2 πr^2
4 x curved surface area = 6 x sum of areas of bases
4 x 2πrh = 6 x 2πr^2
8 πrh = 12 πr^2

$r = \frac{2}{3}h$

$r = \frac{2}{3} \times 12$

$r = 8cm$

$Therefore,\ curved\ surface\ area\ of\ the\ cylinder\ = 2\pi rh$

$= 2\pi \times 8 \times 12$

$= 603.428 cm^2$

115. A cylindrical tank has a radius of 154 cm. It is filled with water to a height of 3 m. If water to a height of 4.5 m is poured into it, what will be the increase in the volume of water in kl?

Solution-

Radius of cylindrical tank = 154 cm
Initial height of water tank = 3m
= 3 x 100 cm
Therefore,
volume of water = $\pi r^2 h$
= $\frac{22}{7}$ x154 x154x 3x10
= 22360800cm^3
If height of water 4.5 m is poured into it, then
volume of water = $\frac{22}{7}$ x 154 x 154 x 4.5 x 100
= 33541200 cm^3
Increases in volume = 33541200 – 22360800
= 11180400 cm^3
= 11180.4L
= 11.1804kL

116. The length, breadth and height of a cuboidal reservoir is 7 m, 6 m and 15 m respectively. 8400 L of water is pumped out from the reservoir. Find the fall in the water level in the reservoir.

Solution-

Length of cuboid reservoir = 7m
Breadth of a cuboidal reservoir = 6m
Height of a cuboidal reservoir = 15 m
Capacity of cuboidal reservoir = l x b x h
= 7 x 6 x 15

$= 630m^3$

If 8400 L of water is pumped out

In meter cubic = $\frac{8400}{1000}$

$= 8.4m^3$

Fall in water level = 630 – 8.4

$= 6216\ m^3$

= 621.6 x 1000

= 621600 L

117. How many bricks of size 22 cm × 10 cm × 7 cm are required to construct a wall 11m long, 3.5 m high and 40 cm thick, if the cement and sand used in the construction occupy $(1/10)^{th}$ part of the wall?

Solution-

Volume of each brick = 22cm x 10cm x 7cm

$= 1540\ cm^3$

$= 0.00154\ m^3$

Vol. of wall = l x b x h

= 11m x 3.5m x 40/100 m

= 11 x 3.5 x 0.4

$= 15.4\ m^3$

If 1/10th part of the wall used in cement and sand, then part of wall used by remaining

cement and sand = $\frac{15.4}{10} m^3$

$= 1.54m^3$

Remaining part = 15.4 – 1.54

$= 13.86\ m^3$

No. of bricks = $\frac{\text{vol. of wall to be constructed}}{\text{vol. of each brick}}$

$= \frac{13.86}{0.00154}$

= 9000

118. A rectangular examination hall having seats for 500 candidates has to be built so as to allow 4 cubic metres of air and 0.5 square metres of floor area per candidate. If the length of hall be 25 m, find the height and breadth of the hall.

Solution-

Total no. of seats in rectangular examination hall= 500

Length of the hall = 25m

Cubic meter of air for per candidates = $4m^3$
Square metres of floor area for per candidate = $0.5m^2$

Therefore,
Height of the hall =

$$\frac{\text{vol.of air per candidate}}{\text{Sqmtr of floor for 1 candidate}} = \frac{4}{0.5}$$

$$= 8m$$

$$\text{Total capacity of the hall} = 500 \times 4$$

$$= 2000\ m^3$$

$$\text{Therefore, breadth of the hall} = \frac{2000}{25 \times 8}$$

$$= 10m$$

119. The ratio between the curved surface area and the total surface area of a right circular cylinder is 1:2. Find the ratio between the height and radius of the cylinder.

Solution-

Curved surface area of cylinder = 2πrh
Total surface area of a cylinder = 2πr (r + h)
The ratio between the curved surface area and the total surface area of a cylinder is 1:2

Therefore**,**

$$\frac{1}{2} = \frac{2\pi rh}{2\pi r(h+r)}$$

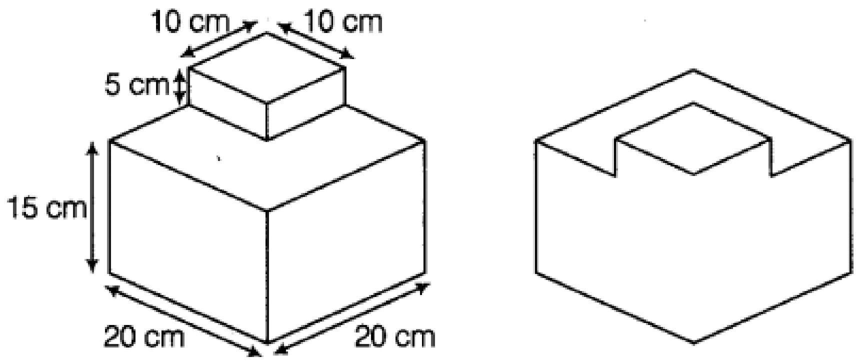
$$2h = (r+h)$$

$$2h - h = r$$

$$h = r$$

$$\textit{Therefore, the ratio of height and radius} = 1:1$$

120. A birthday cake has two tiers as shown in the figure below. Find the volume of the cake.

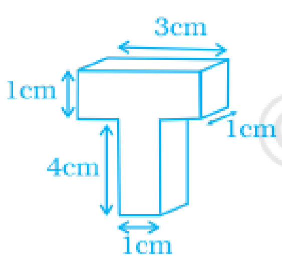


Solution-

Volume of the cake = Volume of lower cuboid + volume of upper cuboid
= 20 x 20 x 15 x 10 x 10 x 5
= 6000 + 500
= 6500 cm^3

Work out the surface area of following shapes in questions 121 to 124 (use π = 3.14).

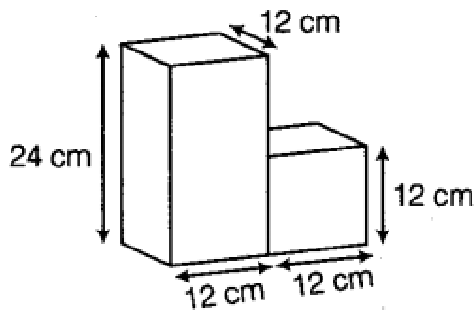
121.



Solution-

Surface area of figure = 2 (3 x 1 + 1 x 1 + 3 x 1) + 2 (4 x 1 + 1 x 1 + 4 x 1) – (1 x 1 + 1 x 1)
= 2 (3+1+3) + 2 (4 + 1 +4) – (1+1)
= 14 + 18 – 2
=32 – 2
= $30cm^2$

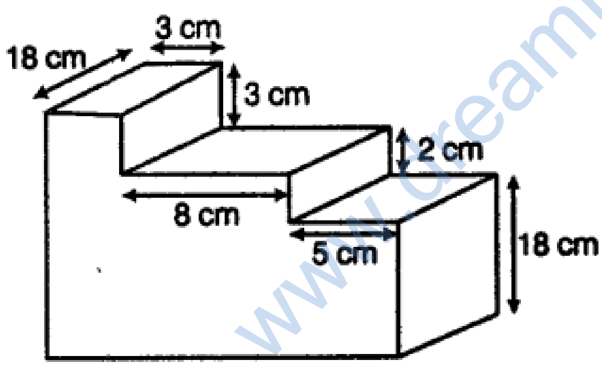
122.



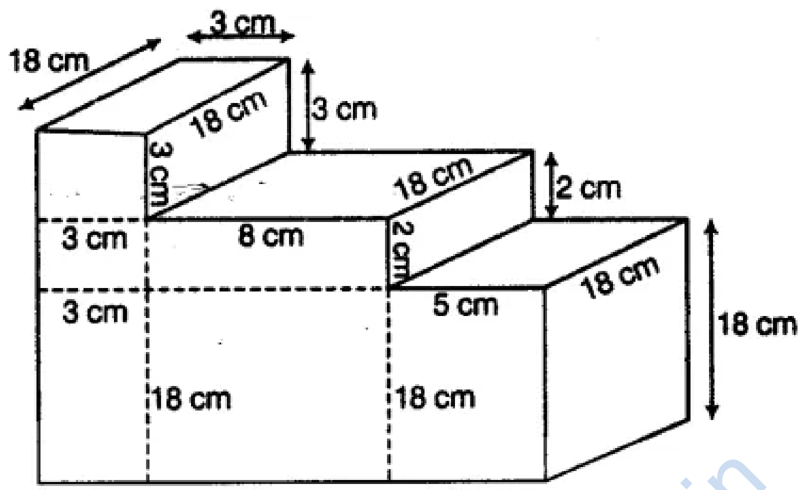
Solution-

In the given figure, there are two cuboides joined
Therefore,
Total surface area of the given solid = 2 (24 x 12 + 12 x 12 + 12 x 24) + 2 (12 x 12 + 12 x 12 + 12 x 12) – 2(12 x 12)
= 1440 + 864 – 288
= 2016 cm^2

123.

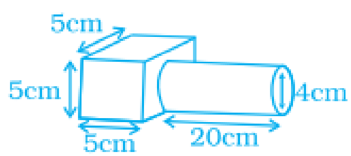


Solution-



Therefore, upper surface area = 18 x 3 + 8 x 18 + 5 x 18
= 54 + 144 + 90
= 288 cm^2
Lower surface area = 288 cm^2
Surface area of those faces which are flat from right = 18 x 18 + 2 x 18 + 3 x 18
= 324 + 36 + 54
= 414 cm^2
Also,
Surface area of that face which are flat from right = 414cm^2
Surface area of from face = 18 x 5 + 2 x 8 + 8 x 18 + 3 x 2 + 3 x 18 + 3 x 3
= 90 + 16 + 144 + 6 + 54 + 9
= 319 cm^2
Surface area of the back face = 319 cm^2
Therefore, total surface area = 288 + 288 + 414 + 414 + 319 + 319
= 288 + 1754
= 2042 cm^2

124.



Solution-

In the given figure,
There is a cube of side 5 cm and a cylinder of height 20 cm and radius is 2 cm
So,
total surface area = surface area of the cube ($6a^2$) + curved surface area of cylinder ($2\pi rh$)
= $6(5)^2 + 2\pi\ (2) \times 20$

$= 150 + 80\pi$
Total surface area $= 150 + 251.2$
$= 401.2 \text{ cm}^2$

125. Water flows from a tank with a rectangular base measuring 80 cm by 70 cm into another tank with a square base of side 60 cm. If the water in the first tank is 45 cm deep, how deep will it be in the second tank?

Solution-

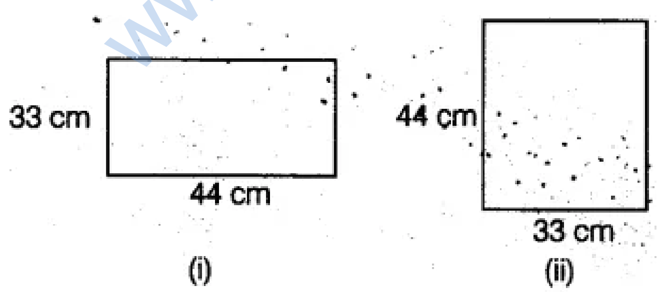
Dimensions of rectangular base tank are 80 cm x 70cm
Height of rectangle base tank = 45 cm
Each side of square base tank = 60 cm

Let h be the height of square base tank.
Volume of rectangular tank = volume of square tank
80 x 70 x 45 = 60 x 60 x h
h = 70 cm

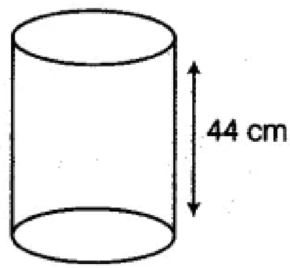
Hence, water in second tank will be 70 cm deep.

126. A rectangular sheet of paper is rolled in two different ways to form two different cylinders. Find the volume of cylinders in each case if the sheet measures 44 cm × 33 cm.

Solution:

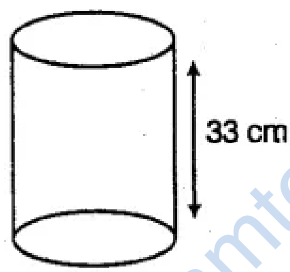


If sheet is rolled along 44cm,



$2\pi r = 44cm$
So,
$r = 7cm$

In 2^{nd} case, if sheet is rolled along 33cm,



$2\pi r = 33cm$
So,
$r = \frac{21}{4}$ cm