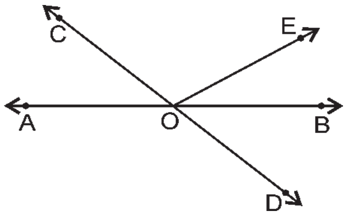
6 LINES AND ANGLES

EXERCISE 6.1

Q.1. *In the figure lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.*

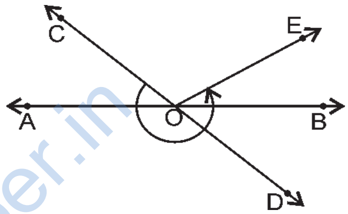


Sol. Lines AB and CD intersect at O.

$\angle AOC + \angle BOE = 70°$ (Given) ...(1)

$\angle BOD = 40°$ (Given) ...(2)

Since, $\angle AOC = \angle BOD$ (Vertically opposite angles)



Therefore, $\angle AOC = 40°$ [From (2)]

and $40° + \angle BOE = 70°$ [From (1)]

$\Rightarrow$ $\angle BOE = 70° - 40° = 30°$

Also, $\angle AOC + \angle BOE + \angle COE = 180°$ ($\because$ AOB is a straight line)

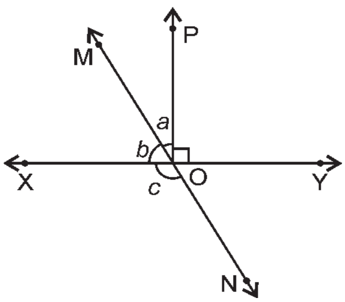
$\Rightarrow$ $70° + \angle COE = 180°$ [Form (1)]

$\Rightarrow$ $\angle COE = 180° - 70° = 110°$

Now, reflex $\angle COE = 360° - 110° = 250°$

Hence, **$\angle BOE = 30°$ and reflex $\angle COE = 250°$ Ans.**

Q.2. *In the figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, find c.*



Sol. In the figure, lines XY and MN intersect at O and $\angle POY = 90°$.

Also, given $a : b = 2 : 3$

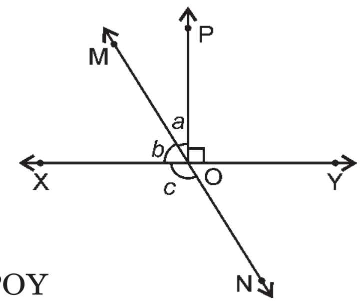
Let $a = 2x$ and $b = 3x$.

Since, $\angle XOM + \angle POM + \angle POY = 180°$ (Linear pair axiom)

$\Rightarrow$ $3x + 2x + 90° = 180°$

$\Rightarrow$ $5x = 180° - 90°$

$\Rightarrow$ $x = \dfrac{90°}{5} = 18°$



$\therefore$ $\angle XOM = b = 3x = 3 \times 18° = 54°$

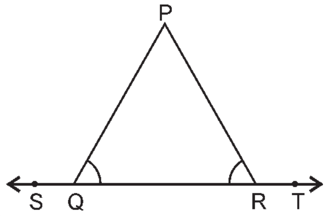
and $\angle POM = a = 2x = 2 \times 18° = 36°$

Now, $\angle XON = c = \angle MOY = \angle POM + \angle POY$ (Vertically opposite angles)

$= 36° + 90° = 126°$

Hence, **$c = 126°$ Ans.**

Q.3. *In the figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.*



Sol. $\angle PQS + \angle PQR = 180°$ ...(1)

(Linear pair axiom)

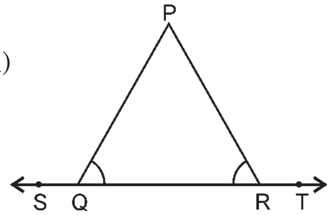
$\angle PRQ + \angle PRT = 180°$ ...(2)

(Linear pair axiom)

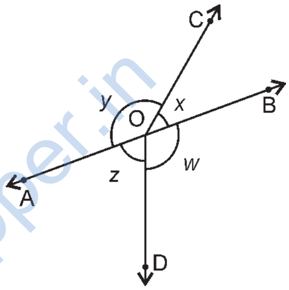
But, $\angle PQR = \angle PRQ$ (Given)

$\therefore$ From (1) and (2)

$\angle PQS = \angle PRT$ **Proved.**



Q.4. *In the figure, if $x + y = w + z$, then prove that AOB is a line.*



Sol. Assume AOB is a line.

Therefore, $x + y = 180°$ ...(1)

[Linear pair axiom]

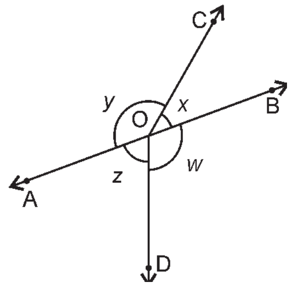
$w + z = 180°$ ...(2)

[Linear pair axiom]

Now, from (1) and (2)

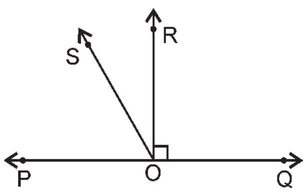
$x + y = w + z$

Hence, our assumption is correct, AOB is a line **Proved.**



Q.5. *In the figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that*

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$



Sol. $\angle ROS = \angle ROP - \angle POS$ ...(1)

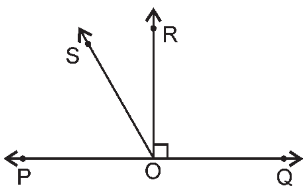
and $\angle ROS = \angle QOS - \angle QOR$ ...(2)

Adding (1) and (2),

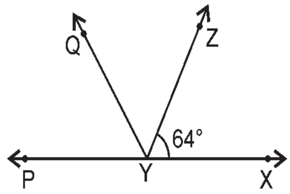
$\angle ROS + \angle ROS = \angle QOS - \angle QOR + \angle ROP - \angle POS$

$\Rightarrow$ $2\angle ROS = \angle QOS - \angle POS$ ($\because$ $\angle QOR = \angle ROP = 90°$)

$\Rightarrow$ $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$ **Proved.**



Q.6. *It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.*



Sol. From figure,

$\angle XYZ = 64°$ (Given)

Now, $\angle ZYP + \angle XYZ = 180°$ (Linear pair axiom)

$\Rightarrow \angle ZYP + 64° = 180°$

$\Rightarrow \angle ZYP = 180° - 64° - 116°$

Also, given that ray YQ bisects $\angle ZYP$.

But, $\angle ZYP = \angle QYP \ \angle QYZ = 116°$

Therefore, $\angle QYP = 58°$ and $\angle QYZ = 58°$

Also, $\angle XYQ = \angle XYZ + \angle QYZ$

$\Rightarrow \angle XYQ = 64° + 58° = 122°$

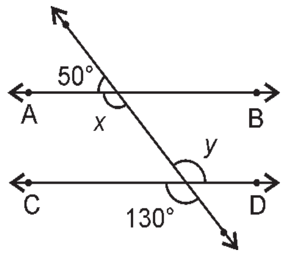
and reflex $\angle QYP = 360° - \angle QYP = 360° - 58° = 302°$ $(\because \angle QYP = 58°)$

Hence, $\mathbf{\angle XYQ = 122°}$ **and reflex** $\mathbf{\angle QYP = 302°}$ **Ans.**

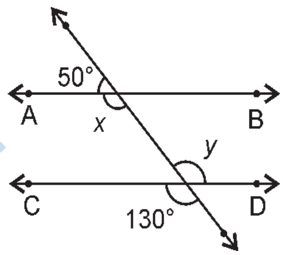
6 LINES AND ANGLES

EXERCISE 6.2

Q.1. *In the figure, find the values of x and y and then show that $AB \parallel CD$.*



Sol. In the given figure, a transversal intersects two lines AB and CD such that



$x + 50° = 180°$ (Linear pair axiom)

$\Rightarrow \quad x = 180° - 50°$

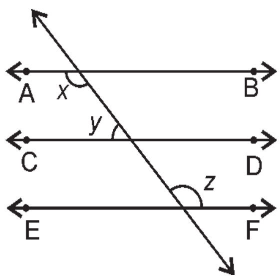
$= 130°$

$y = 130°$ (Vertically opposite angles)

Therefore, $\angle x = \angle y = 130°$ (Alternate angles)

$\therefore$ AB || CD (Converse of alternate angles axiom) **Proved.**

Q.2. *In the figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x.*



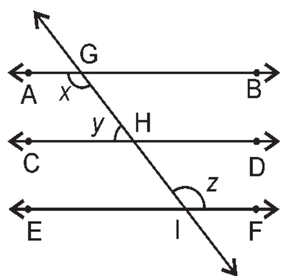
Sol. In the given figure, AB || CD, CD || EF and $y : z = 3 : 7$.

Let $y = 3a$ and $z = 7a$

$\angle DHI = y$ (vertically opposite angles)

$\angle DHI + \angle FIH = 180°$

(Interior angles on the same side of the transversal)



$\Rightarrow \quad y + z = 180°$

$\Rightarrow \quad 3a + 7a = 180°$

$\Rightarrow \quad 10a = 180° \quad \Rightarrow \quad a = 18°$

$\therefore y = 3 \times 18° = 54°$ and $z = 18° \times 7 = 126°$

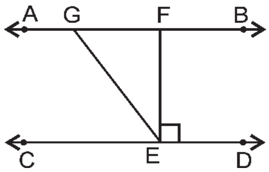
Also, $\quad x + y = 180°$

$\Rightarrow \quad x + 54° = 180°$

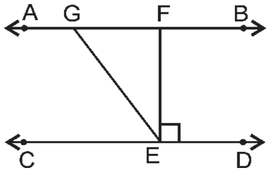
$\therefore \quad x = 180° - 54° = 126°$

Hence, $\boldsymbol{x = 126°}$ **Ans.**

Q.3. *In the figure, if AB || CD, EF ⊥ CD and ∠GED = 126°. Find ∠AGE, ∠GEF and ∠FGE.*



Sol. In the given figure, AB || CD, EF ⊥ CD and ∠GED = 126°



∠AGE = ∠LGE (Alternate angle)

∴ ∠AGE = 126°

Now, ∠GEF = ∠GED – ∠DEF

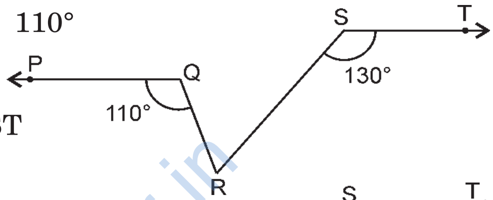
= 126° – 90° = 36° (∵ ∠DEF = 90°)

Also, ∠AGE + ∠FGE = 180° (Linear pair axiom)

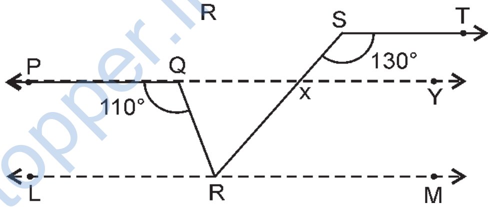
⇒ 126° + FGE = 180°

⇒ ∠FGE = 180° – 126° = 54°

Q.4. *In the figure, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.*



Sol. Extend PQ to Y and draw LM || ST through R.



∠TSX = ∠QXS

[Alternate angles]

⇒ ∠QXS = 130°

∠QXS + ∠RXQ = 180°

[Linear pair axiom]

⇒ ∠RXQ = 180° – 130° = 50° ...(1)

∠PQR = ∠QRM [Alternate angles]

⇒ ∠QRM = 110° ...(2)

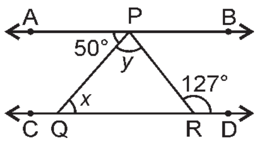
∠RXQ = ∠XRM [Alternate angles]

⇒ ∠XRM = 50° [By (1)]

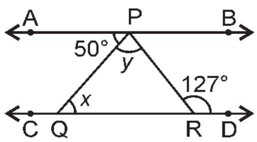
∠QRS = ∠QRM – ∠XRM

= 110° – 50° = **60°** **Ans.**

Q.5. *In the figure, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find* x and y.



Sol. In the given figure, AB || CD, ∠APQ = 50° and ∠PRD = 127°



∠APQ + ∠PQC = 180°

[Pair of consecutive interior angles are supplementary]

⇒ 50° + ∠PQC = 180°

⇒ ∠PQC = 180° – 50° = 130°

Now, ∠PQC + ∠PQR = 180° [Linear pair axiom]

⇒ 130° + x = 180°

⇒ x = 180° – 130° = 50°

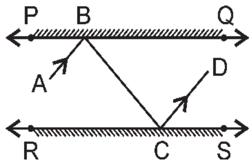
Also, $x + y$ = 127° [Exterior angle of a triangle is equal to the sum of the two interior opposite angles]

⇒ 50° + y = 127°

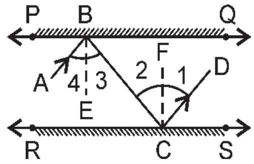
⇒ y = 127° – 50° = 77°

Hence, $\boldsymbol{x}$ **= 50° and** $\boldsymbol{y}$ **= 77°** **Ans.**

Q.6. *In the figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.*



Sol. At point B, draw BE ⊥ PQ and at point C, draw CF ⊥ RS.



$\angle 1 = \angle 2$ ...(i)

(Angle of incidence is equal to angle of reflection)

$\angle 3 = \angle 4$ ...(ii) [Same reason]

Also, $\angle 2 = \angle 3$ ... (iii) [Alternate angles]

$\Rightarrow \angle 1 = \angle 4$ [From (i), (ii), and (iii)]

$\Rightarrow 2\angle 1 = 2\angle 4$

$\Rightarrow \angle 1 + \angle 1 = \angle 4 + \angle 4$

$\Rightarrow \angle 1 + \angle 2 = \angle 3 + \angle 4$ [From (i) and (ii)]

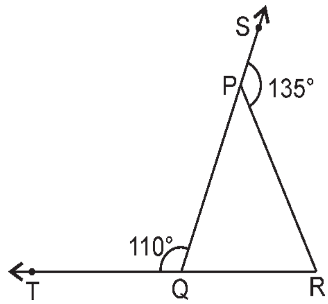
$\Rightarrow \angle BCD = \angle ABC$

Hence, AB || CD. [Alternate angles are equal] **Proved.**

6 LINES AND ANGLES

EXERCISE 6.3

Q.1. *In the figure, sides QP and RQ of ∆ PQR are produced to points S and T respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.*



Sol. In the given figure, ∠SPR = 135° and ∠PQT = 110°.

∠PQT + ∠PQR = 180° [Linear pair axiom]

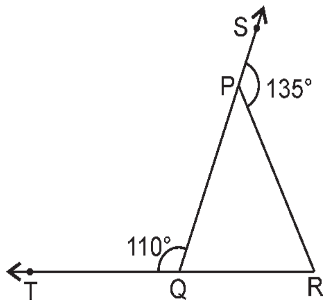
⇒ 110° + ∠PQR = 180°

⇒ ∠PQR = 180° – 110° = 70°

Also, ∠SPR + ∠QPR = 180° [Linear pair axiom]

⇒ 135° + ∠QPR = 180°

⇒ ∠QPS = 180° – 135° = 45°



Now, in the triangle PQR

∠PQR + ∠PRQ + ∠QPR = 180°

[Angle sum property of a triangle]

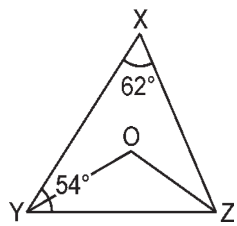
⇒ 70° + ∠PRQ + 45° = 180°

⇒ ∠PRQ + 115° = 180°

⇒ ∠PRQ = 180° – 115° = 65°

Hence, **∠PRQ = 65° Ans.**

Q.2. *In the figure, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of ∆ XYZ, find ∠OZY and ∠YOZ.*

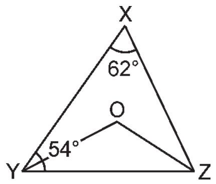


Sol. In the given figure,

∠X = 62° and ∠XYZ = 54°.

$\angle XYZ + \angle XZY + \angle YXZ = 180°$ ...(i)

[Angle sum property of a triangle]



$\Rightarrow$ $54° + \angle XZY + 62° = 180°$

$\Rightarrow$ $\angle XZY + 116° = 180°$

$\Rightarrow$ $\angle XZY = 180° - 116° = 64°$

Now, $\angle OZY = \frac{1}{2} \times \angle XZY$ [$\because$ ZO is bisector of $\angle XZY$]

$= \frac{1}{2} \times 64° = 32°$

Similarly, $\angle OYZ = \frac{1}{2} \times 54° = 27°$

Now, in ΔOYZ, we have

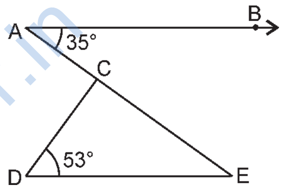
$\angle OYZ + \angle OZY + \angle YOZ = 180°$ Angle sum property of a triangle]

$\Rightarrow 27° + 32° + \angle YOZ = 180°$

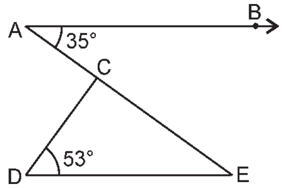
$\Rightarrow \angle YOZ = 180° - 59° = 121°$

Hence, **$\angle$OZY = 32° and $\angle$YOZ = 121° Ans.**

Q.3. *In the figure, if AB || DE, $\angle$BAC = 35° and $\angle$CDE = 53°, find $\angle$DCE.*



Sol. In the given figure



$\angle BAC = \angle CED$

[Alternate angles]

$\Rightarrow$ $\angle CED = 35°$

In ΔCDE,

$\angle CDE + \angle DCE + \angle CED = 180°$ [Angle sum property of a triangle]

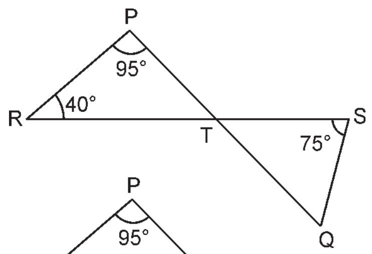
$\Rightarrow$ $53° + \angle DCE + 35° = 180°$

$\Rightarrow$ $\angle DCE + 88° = 180°$

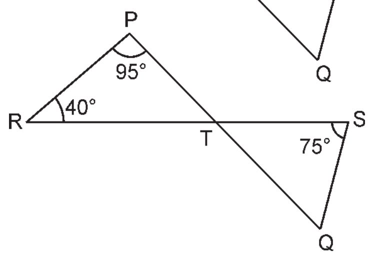
$\Rightarrow$ $\angle DCE = 180° - 88° = 92°$

Hence, **$\angle$DCE = 92° Ans.**

Q.4. *In the figure, if lines PQ and RS intersect at point T, such that $\angle$PRT = 40°, $\angle$RPT = 95° and $\angle$TSQ = 75°, find $\angle$SQT.*



Sol. In the given figure, lines PQ and RS intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$.



In ΔPRT

$\angle PRT + \angle RPT + \angle PTR = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $40° + 95° + \angle PTR = 180°$

$\Rightarrow$ $135° + \angle PTR = 180°$

$\Rightarrow$ $\angle PTR = 180° - 135° = 45°$

Also, $\angle PTR = \angle STQ$ [Vertical opposite angles]

$\therefore$ $\angle STQ = 45°$

Now, in ΔSTQ,

$$\angle STQ + \angle TSQ + \angle SQT = 180° \quad \text{[Angle sum property of a triangle]}$$

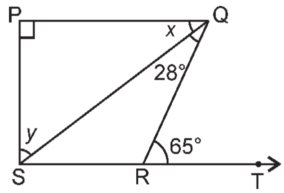
$$\Rightarrow \quad 45° + 75° + \angle SQT = 180°$$

$$\Rightarrow \quad 120° + \angle SQT = 180°$$

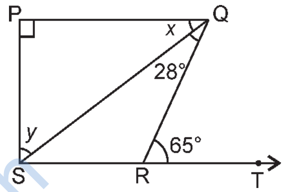
$$\Rightarrow \quad \angle SQT = 180° - 120° = 60°$$

Hence, **$\angle SQT = 60°$ Ans.**

Q.5. *In the figure, if $PT \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$, then find the values of x and y.*



Sol. In the given figure, lines $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$



$$\angle PQR = \angle QRT \quad \text{[Alternate angles]}$$

$$\Rightarrow \quad x + 28° = 65°$$

$$\Rightarrow \quad x = 65° - 28° = 37°$$

In ΔPQS,

$$\angle SPQ + \angle PQS + \angle QSP = 180° \quad \text{[Angle sum property of a triangle]}$$

$$\Rightarrow \quad 90° + 37° + y = 180°$$

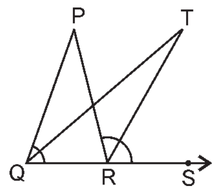
[$\because PQ \perp PS$, $\angle PQS = x = 37°$ and $\angle QSP = y$)

$$\Rightarrow \quad 127° + y = 180°$$

$$\Rightarrow \quad y = 180° - 127° = 53°$$

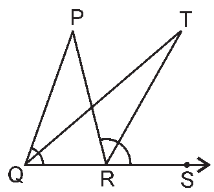
Hence, **$x = 37°$ and $y = 53°$ Ans.**

Q.6. *In the figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.*



Sol. Exterior $\angle PRS = \angle PQR + \angle QPR$

[Exterior angle property]



Therefore, $\frac{1}{2}\angle PRS = \frac{1}{2}\angle PQR + \frac{1}{2}\angle QPR$

$$\Rightarrow \quad \angle TRS = \angle TQR + \frac{1}{2}\angle QPR \quad \text{...(i)}$$

But in ΔQTR,

Exterior $\angle TRS = \angle TQR + \angle QTR$ ...(ii)

[Exterior angles property]

Therefore, from (i) and (ii)

$$\angle TQR + \angle QTR = \angle TQR + \frac{1}{2}\angle QPR$$

$$\Rightarrow \quad \angle QTR = \frac{1}{2}\angle QPR$$

Proved.