Chapter 5

Introduction to Euclid's Geometry

Exercise No. 5.1

Multiple Choice Questions:

1. The three steps from solids to points are:
(A) Solids - surfaces - lines - points
(B) Solids - lines - surfaces - points
(C) Lines - points - surfaces - solids
(D) Lines - surfaces - points - solids

Solution:
The three steps from solids to point are solids-surface-lines-points.
Hence, the correct option is (A).

2. The number of dimensions, a solid has:
(A) 1
(B) 2
(C) 3
(D) 0

Solution:
A solid has shape, position, and size and can be move from one place to another. So, a solid has three dimensions. For example: cuboid, cube, cylinder, cone etc.
Hence, the correct option is (C).

3. The number of dimensions, a surface has:
(A) 1
(B) 2
(C) 3
(D) 0

Solution:
A surface has 2 dimensions.
Hence, the correct option is (B).

4. The number of dimension, a point has:
(A) 0
(B) 1
(C) 2
(D) 3

Solution:
According to Euclid, a point is that which has no part i.e., length, no breath, no height that why it has no dimension.
Hence, the correct option is (A).

5. Euclid divided his famous treatise "The Elements" into:
(A) 13 chapters
(B) 12 chapters
(C) 11 chapters
(D) 9 chapters

Solution:
Euclid divided his famous treatise "the element" into 13 chapters.

6. The total number of propositions in the Elements are:
(A) 465
(B) 460
(C) 13
(D) 55

Solution:
The total number of propositions in the Elements are 465.
Hence, the correct option is (A).

7. Boundaries of solids are:
(A) surfaces
(B) curves
(C) lines
(D) points

Solution:
Boundaries of solids are surface.
Hence, the correct option is (A).

8. Boundaries of surfaces are:
(A) surfaces
(B) curves
(C) lines
(D) points

Solution:
Boundaries of surfaces are curves.
Hence, the correct option is (B).

9. In Indus Valley Civilisation (about 300 B.C.), the bricks used for construction work were having dimensions in the ratio

(A) 1 : 3 : 4

(B) 4 : 2 : 1

(C) 4 : 4 : 1

(D) 4 : 3 : 2

Solution:

The dimension of the bricks used for construction work where having dimensions in the ratio are 4 : 2 : 1.

Hence, the correct option is (B).

10. A pyramid is a solid figure, the base of which is

(A) only a triangle

(B) only a square

(C) only a rectangle

(D) any polygon

Solution:

A pyramid is a solid figure, the base of which is any polygon.

Hence, the correct option is (D).

11. The side faces of a pyramid are:

(A) Triangles

(B) Squares

(C) Polygons

(D) Trapeziums

Solution:

The side faces of a pyramid are triangles.

Hence, the correct option is (A).

12. It is known that if $x + y = 10$ then $x + y + z = 10 + z$. The Euclid's axiom that illustrates this statement is:

(A) First Axiom

(B) Second Axiom

(C) Third Axiom

(D) Fourth Axiom

Solution:

It is known that if $x + y = 10$ then $x + y + z = 10 + z$. The Euclid's axiom that illustrates this statement is axiom second according to which. If equals are added to equals, the wholes are equal.
Hence, the correct option is (B).

13. In ancient India, the shapes of altars used for house hold rituals were:
(A) Squares and circles
(B) Triangles and rectangles
(C) Trapeziums and pyramids
(D) Rectangles and squares

Solution:
In ancient India, the shapes of altars used for house hold rituals were squares and circles.
Hence, the correct option is (A).

14. The number of interwoven isosceles triangles in Sriyantra (in the Atharvaveda) is:
(A) Seven
(B) Eight
(C) Nine
(D) Eleven

Solution:
The number of interwoven isosceles triangles in Sriyantra (in the Atharvaveda) is nine.
Hence, the correct option is (C).

15. Greek's emphasised on:
(A) Inductive reasoning
(B) Deductive reasoning
(C) Both A and B
(D) Practical use of geometry

Solution:
Greek's emphasised on deductive reasoning.
Hence, the correct option is (B).

16. In Ancient India, Altars with combination of shapes like rectangles, triangles and trapeziums were used for:
(A) Public worship
(B) Household rituals
(C) Both A and B
(D) None of A, B, C

Solution:

In Ancient India, Altars with combination of shapes like rectangles, triangles and trapeziums were used for public worship.
Hence, the correct option is (A).

17. Euclid belongs to the country:
(A) Babylonia
(B) Egypt
(C) Greece
(D) India

Solution:
Euclid belongs to the country Greece.
Hence, the correct option is (C).

18. Thales belongs to the country:
(A) Babylonia
(B) Egypt
(C) Greece
(D) Rome

Solution:
Thales belongs to the country Greece.
Hence, the correct option is (C).

19. Pythagoras was a student of:
(A) Thales
(B) Euclid
(C) Both A and B
(D) Archimedes

Solution:
Pythagoras was a student of Thales.
Hence, the correct option is (A).

20. Which of the following needs a proof?
(A) Theorem
(B) Axiom
(C) Definition
(D) Postulate

Solution:
Theorem
Hence, the correct option is (A).

21. Euclid stated that all right angles are equal to each other in the form of
(A) an axiom
(B) a definition
(C) a postulate
(D) a proof

Solution:
Euclid stated that all right angles are equal to each other in the form of a postulate.
Hence, the correct option is (C).

22. 'Lines are parallel if they do not intersect' is stated in the form of
(A) an axiom
(B) a definition
(C) a postulate
(D) a proof

Solution:
Lines are parallel if they do not intersect' is stated in the form of a definition.
Hence, the correct option is (B).

Exercise No. 5.2

Short Answer Questions with Reasoning:

Write whether the following statements are True or False? Justify your answer:

1. Euclidean geometry is valid only for curved surfaces.

Solution:
We know that Euclidean geometry is valid only for the figure in the plane. Hence, the given statement is false.

2. The boundaries of the solids are curves.

Solution:
The boundaries of the solids are surfaces.
Hence, the given statement is false.

3. The edges of a surface are curves.

Solution:
The edges of a surface are line.
Hence, the given statement is false.

4. The things which are double of the same thing are equal to one another.

Solution:
The given statement is true because, it is one of the Euclid's axioms.

5. If a quantity B is a part of another quantity A, then A can be written as the sum of B and some third quantity C.

Solution:
The Euclid's axiom statement is:
If a quantity B is a part of another quantity A, then A can be written as the sum of B and some third quantity C
Hence, the given statement is true.

6. The statements that are proved are called axioms.

Solution:

The given statement is false because the statement that are proved are called theorems.

7. "For every line *l* and for every point P not lying on a given line *l*, there exists a unique line *m* passing through P and parallel to *l* " is known as Playfair's axiom.

Solution:
The given statement is an equivalent version of Euclid's fifth postulate and it is known as Playfair's axiom.
Hence, the given statement is true.

8. Two distinct intersecting lines cannot be parallel to the same line.

Solution:
The given statement is an equivalent version of Euclid's fifth postulate. Hence, it is true.

9. Attempts to prove Euclid's fifth postulate using the other postulates and axioms led to the discovery of several other geometries.

Solution:
These geometries are different from Euclidean geometry called non-Euclidean geometry.
Hence, the given statement is true.

Exercise No. 5.3

Short Answer Questions:

Solve each of the following question using appropriate Euclid's axiom:
1. Two salesmen make equal sales during the month of August. In September, each salesman doubles his sale of the month of August. Compare their sales in September.

Solution:
Let the sales of 2 salesmen in the month of August are x and y.
Now, according to the question:
Two salesmen make equal sales during the month of August. So,
$x = y$

In September, each salesman doubles his sale of the month of August. So,
$2x = 2y$

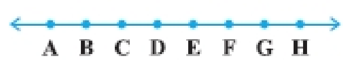
Therefore, by Euclid's axiom, thing which are double of the same things are equal to one another.
Hence, in the month of September also, two salesmen make equal sales.

2. It is known that $x + y = 10$ and that $x = z$. Show that $z + y = 10$?

Solution:
It is known that $x + y = 10$ and that $x = z$
Therefore, $x + y = z + y$ [by the Euclid's axiom 2]

Now, $10 = y + z$ [by $x + y = 10$]
Hence, $z + y = 10$.

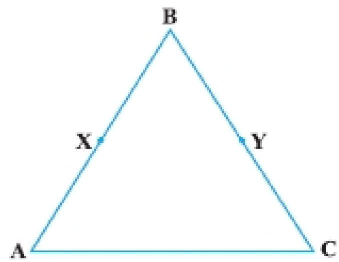
3. Look at the Fig. Show that length AH > sum of lengths of AB + BC + CD.



Solution:
Given in the question, AB, BC and CD are parts of line.
Then, AB + BC + CD = AD …(I)
And AD is the part of line AH.

Now, By Euclid's axiom 5, the whole is greater than the part.
So, AH > AD
That is length AH > sum of length of AB + BC +CD [by using (1)]

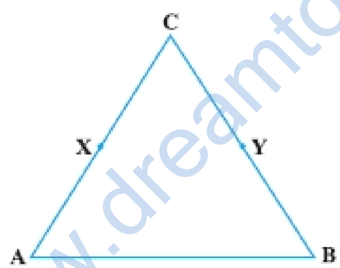
4. In the Fig., we have AB = BC, BX = BY. Show that AX = CY.



Solution:
Given: AB = BC …(I)
And BX = BY …(II)

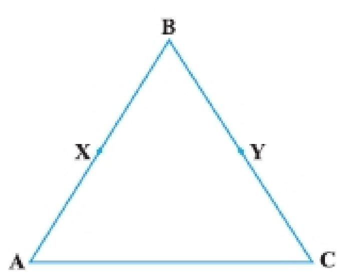
Subtracting (II) from (I), get:
AB – BX = BC – BY
Hence, AX = CY [By Euclid's axiom 3]

5. In the Fig., we have X and Y are the mid-points of AB and BC and AX = CY. Show that AB = BC.



Solution:
Given: AX = CY
Now, 2AX = 2CY [By Euclid's axiom 6]
Hence, AC = BC [X and Y are the mid-points of AC and BC]

6. In the Fig., we have $BX = \frac{1}{2}AB$**,** $BY = \frac{1}{2}BC$ **and** $AB = BC$**. Show that BX = BY.**

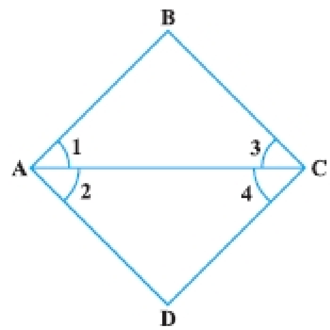


Solution:
Given: AB = BC

Now, $\frac{1}{2}AB = \frac{1}{2}BC$ [By Euclid's axiom 7]

Hence, BX = BY. [Given: $BX = \frac{1}{2}AB$ and $BY = \frac{1}{2}BC$]

7. In the Fig., we have $\angle 1 = \angle 2, \angle 2 = \angle 3$. Show that $\angle 1 = \angle 3$.



Solution:

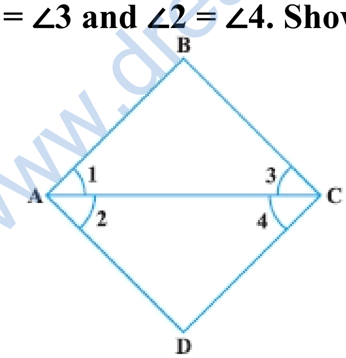
Given:

$\angle 1 = \angle 2$

$\angle 2 = \angle 3$

Hence, $\angle 1 = \angle 3$ [By Euclid's axiom 1]

8. In the Fig., we have ∠1 = ∠3 and ∠2 = ∠4. Show that ∠A = ∠C.



Solution:

Given:

$\angle 1 = \angle 3$ …(I)

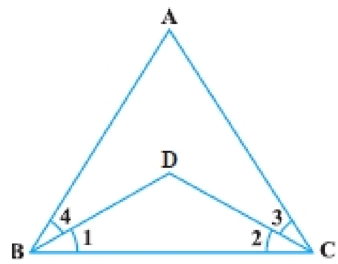
$\angle 2 = \angle 4$ …(II)

Adding (I) and (II), get:

$\angle 1 + \angle 2 = \angle 3 + \angle 4$ [By Euclid's axiom 2]

Hence, $\angle A = \angle C$.

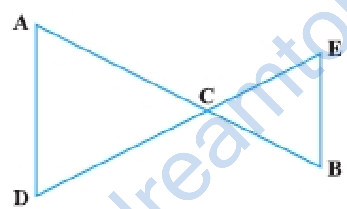
9. In the Fig., we have ∠ABC = ∠ACB, ∠3 = ∠4. Show that ∠1 = ∠2.



Solution:
Given:
$\angle ABC = \angle ACB \ldots$(I)
And $\angle 4 = \angle 3 \ \ldots$(II)

Now, subtracting (II) from (I), get:
$\angle ABC - \angle 4 = \angle ACB - \angle 3$ [By Euclid's axiom 3]
Hence, $\angle 1 = \angle 2$.

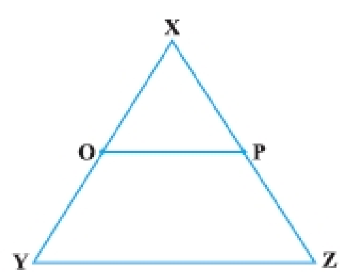
10. In the Fig., we have AC = DC, CB = CE. Show that AB = DE.



Solution:
Given:
$AC = DC \ldots$(I)
$CB = CE \ldots$(II)

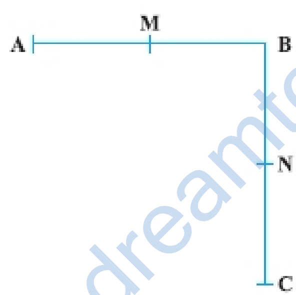
Adding (I) and (II), get:
AC + CB = DC + CE [By axiom 2]
Hence, AB = DE.

11. In the Fig., if $\mathrm{OX} = \frac{1}{2}\mathrm{XY}$, $\mathrm{PX} = \frac{1}{2}\mathrm{XZ}$ and OX = PX, show that XY = XZ.



Solution:
Given:
$OX = PX$
Now, $OX = \frac{1}{2}XY$ and $PX = \frac{1}{2}XY$
$\frac{1}{2}XY = \frac{1}{2}XZ$ [By Euclid's axiom 7]
Therefore, XY = XZ [By Euclid's axiom 6].

12. In the Fig.:
(i) AB = BC, M is the mid-point of AB and N is the mid- point of BC. Show that
AM = NC.
(ii) BM = BN, M is the mid-point of AB and N is the mid-point of BC. Show that
AB = BC.



Solution:
(i) Given:
AB = BC …(I)
The point M are lies in between A and B. So,
AM + MB = AB …(II)
Similarly, BN + NC = BC …(III)
By equation (I), (II) and (III), get:
AM + MB = BN + NC
Now, M is the midpoint of AB and N is the midpoint of BC, therefore:
2AM = 2NC
Hence, AM = NC [using axiom 6]

(ii) Given:
BM = BN …(I)
As M is the midpoint of AB, So,
BM = AM …(II)
And N is the midpoint of BC,
BN = NC …(III)
Then, from equation (I), (II) and (III) and Euclid's axiom 1, get:

AM = NC …(IV)

Adding (IV) and (I), get:
AM + BM = NC + BN
Hence, AB = BC. [By axiom 2].

Exercise No. 5.4

Long Answer Questions:

1. Read the following statement:
An equilateral triangle is a polygon made up of three line segments out of which two line segments are equal to the third one and all its angles are 60° each.
Define the terms used in this definition which you feel necessary. Are there any undefined terms in this? Can you justify that all sides and all angles are equal in an equilateral triangle.

Solution:
The terms need to be defined are:
(i) Polygon: A closed figure bounded by three or more line segments.
(ii) Line segment: Part of a line with two end points.
(iii) Line: Undefined term.
(iv) Point: Undefined term.
(v) Angle: A figure formed by two rays with one common initial point.
(vi) Acute angle: Angle whose measure is between 0° to 90°.
Now, undefined terms are line and point.
Given in the question, All the angles of equilateral triangle are 60° each. Since, all the sides of an equilateral triangle are equal.
Hence, all the sides and all angles are equal in an equilateral triangle.

2. Study the following statement:
“Two intersecting lines cannot be perpendicular to the same line”.
Check whether it is an equivalent version to the Euclid’s fifth postulate.
[Hint: Identify the two intersecting lines *l* and *m* and the line *n* in the above statement.]

Solution:
Two equivalent versions of Euclid’s fifth postulate are:
(i) For every line l and for every point P not lying on Z, there exists a unique line m passing through P and parallel to Z.
(ii) Two distinct intersecting lines cannot be parallel to the same line. According to the above statement it is clear that given statement is not an equivalent version to the Euclid’s fifth postulate.

3. Read the following statements which are taken as axioms:
(i) If a transversal intersects two parallel lines, then corresponding angles are not necessarily equal.

(ii) If a transversal intersect two parallel lines, then alternate interior angles are equal.
Is this system of axioms consistent? Justify your answer.

Solution:
This system of axioms is not consistent because if a transversal intersects two parallel lines, and if corresponding angles are not equal then alternative interior angle cannot be equal.

4. Read the following two statements which are taken as axioms:
(i) If two lines intersect each other, then the vertically opposite angles are not equal.
(ii) If a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180°.
Is this system of axioms consistent? Justify your answer.

Solution:
The system of axioms are not consistence because If a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180° and then for two line which intersect each other, the vertically opposite angle become equal.

5. Read the following axioms:
(i) Things which are equal to the same thing are equal to one another.
(ii) If equals are added to equals, the wholes are equal.
(iii) Things which are double of the same thing are equal to one another.
Check whether the given system of axioms is consistent or inconsistent.

Solution:
The given three axioms are consistent because (i), (ii) and (iii) are Euclid's axiom.