Chapter – 10

Gravitation

Gravitation

The concept of 'Gravitation':

Ever wondered why an object dropped from a height reaches the Earth's surface?

Ever felt curious why the planets revolve around the sun?

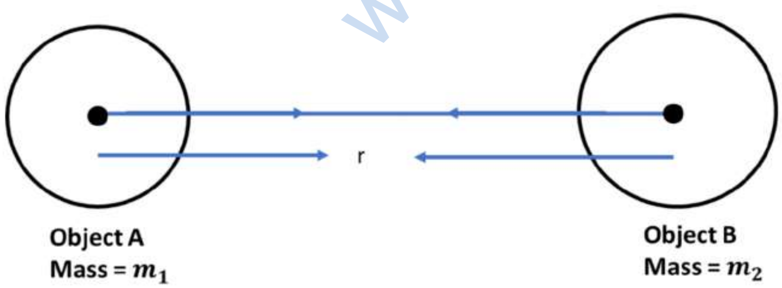
Ever felt inquisitive why the moon revolves around the Earth?

Well, the answer is gravitation. All the above-mentioned phenomenon are happening due to gravitation!

Gravitation (Scientific approach):

If two objects of masses m_1 and m_2 are separated by a distance 'r', then the gravitational force between both the bodies is directly proportional to the product of the masses of the objects and inversely proportional to the square of the distance between them.

i.e. $F \propto \frac{m_1 m_2}{r^2}$



On removing the proportionality sign, we get-

$$F = \frac{G m_1 m_2}{r^2}$$

Where m_1 = mass of the first object, m_2 = mass of the second object, r = distance of separation between both the objects, G = universal gravitational constant.

$G = 6.67\times 10^{-11}\ Nm^2/kg^2$

Example: Two objects of masses 2 kg and 3 kg are separated by a distance of 3 m. Calculate the gravitational force exerted by one object on the other. (Take $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$)

Solution: Mass of first object, $m_1 = 2$ kg

Mass of second object, $m_2 = 3$ kg

Distance of separation between the objects, r = 3 m

Here, we have to find the gravitational force exerted by one object on the other.

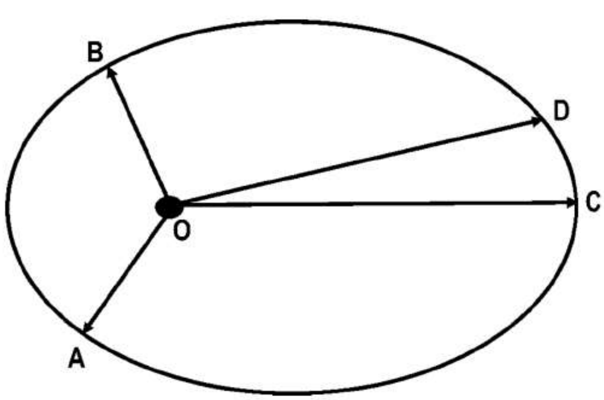
We know that, $F = \frac{Gm_1m_2}{r^2}$

$$F = \frac{6.67 \times 10^{-11} \times 2 \times 3}{(3)^2}$$

$$F = 4.44 \times 10^{-11}\ N$$

__Kepler's laws__: Kepler gave three laws that govern the motion of the planets.

1) The orbit of a planet is an ellipse with the Sun at one of the foci. (According to the image, O is the position of the Sun.)



2) The line joining the planet and the Sun sweep equal areas in equal intervals of time. Thus, if the time of travel from A to B is the same as that from C to D, then area OAB and OCD are equal.

3) The cube of the mean distance of a planet from the Sun is proportional to the square of its orbital period T. In other words, $\frac{r^3}{T^2} \propto Constant$

Free Fall and Acceleration Due to Gravity

Acceleration due to gravity: When an object falls towards the Earth, then, it experiences an acceleration due to Earth's gravitational field. This acceleration experienced by the object is called as acceleration due to gravity. It is represented by 'g' and its value is 9.8 m/s^2.

Scientific approach towards acceleration due to gravity: From the second law of motion, we know that F = mg

where F = force experienced by the object, m = mass of the object, g = acceleration due to gravity.

From the universal law of gravitation, we know that-

$$F = \frac{GmM}{r^2}$$

Where M = mass of the earth. m = mass of the second object, G = universal gravitational constant, r = distance of separation between both the objects (here, this distance is nearly equal to the radius of the Earth, R)

Now, as they both are forces due to gravity, therefore, they are equal.

Therefore $mg = \frac{GmM}{r^2}$

This implies $g = \frac{GM}{r^2}$

But here r = radius of Earth(R),

So. $g = \frac{GM}{R^2}$

Example: A hollow cylinder and a solid cylinder of different mass are dropped from a 20m high building. Which of them will experience a greater value of acceleration due to gravity?

Solution: Acceleration due to gravity is given by -

$$g = \frac{GM}{R^2}$$

where g = acceleration due to gravity, G = universal gravitational constant, R = radius of Earth.

This acceleration due to gravity experienced by any object is independent of its mass. So, both the objects i.e. hollow cylinder and solid cylinder will experience the same value of 'g' and will fall at the same rate.

Mass and Weight

Mass: Mass is the amount of matter contained in a body. It remains the same throughout the universe and doesn't vary with different surroundings. The mass of an object is a measure of its inertia.

Example: Mass of an object on Earth is 200kg. What will be the mass of the same object on the moon?

Solution: Mass of an object remains the same throughout the universe. So, the mass of the object on the moon will be the same as that on Earth i.e. 200kg.

Weight: The force by which an object is attracted towards the Earth is known as weight.

Weight depends on the mass of the body and the acceleration due to gravity.

$$W = mg$$

As the weight of an object is the force by which it is attracted towards the Earth, therefore, its unit is the same as that of force i.e. newtons (N).

Example: An object of mass 200g is kept on the surface of the Earth. Calculate its weight.

Solution: Given- Mass of the object, m = 200 g = 0.2 kg

Weight of the object = ?

The force by which an object is attracted towards the Earth is known as weight.

Acceleration due to gravity on the surface of the earth is given by-

$g = 9.8\ m/s^2$

Therefore, weight of the object, W = mg = 0.2 x 9.8 N= 19.6 N

Weight on an object on the moon:

$$\frac{Weight\ of\ object\ on\ the\ Moon}{Weight\ of\ object\ on\ the\ Earth} = \frac{1}{6}$$

i.e.

$$Weight\ of\ object\ on\ the\ Moon = \frac{Weight\ of\ object\ on\ the\ Earth}{6}$$

INFERENCE- As we know that mass of the object remains the same everywhere, therefore, we can infer that the weight of the object is less on the moon due to less value of acceleration due to gravity.

This can be formulated as-

$$Weight\ of\ object\ on\ the\ Moon = \frac{1}{6} \times Weight\ of\ object\ on\ the\ Earth$$
$$Mass\ of\ the\ object \times g_{moon} = \frac{1}{6} \times Mass\ of\ the\ object \times g_{earth}$$

Where, g_{moon} = acceleration due to gravity on the moon and g_{earth}= acceleration due to gravity on the Earth

So, $g_{moon} = \frac{1}{6} \times g_{earth}$

$$g_{moon} = \frac{1}{6} \times 9.8\ ms^{-2}$$

$$g_{moon} = 1.63\ ms^{-2}$$

Example: An object weighs 60 N on Earth. What will be the weight of the object on the moon?

Solution: Given- Weight of the object on Earth = 60 N

To find- Weight on the object on the moon

Solution- As we know that $\frac{Weight\ of\ object\ on\ the\ Moon}{Weight\ of\ object\ on\ the\ Earth} = \frac{1}{6}$

Putting values in the above formula, we get-

$$\frac{Weight\ of\ object\ on\ the\ Moon}{60\ N} = \frac{1}{6}$$

Therefore, the weight of the object on the moon = 10 N

Thrust and Pressure

Thrust: The force acting on an object perpendicular to the surface is known as thrust. Its unit is Newton (N).

Pressure: Pressure is defined as the thrust applied per unit area.

$$Pressure = \frac{Thrust}{Area}$$

Its S.I unit is Pascal (Pa) or N/m^2.

INFERENCE- 1. It is clearly evident from the above formula that pressure is directly proportional to the thrust. Therefore, increasing the thrust on the same area will increase the pressure too.

2. It is clearly evident from the above formula that pressure is inversely proportional to the area on which the thrust is applied. So, if the area on which the thrust is applied is increased (keeping the thrust constant), pressure will decrease.

In other words, if the same thrust is applied on a smaller area, larger pressure is exerted and vice-versa.

This principle is utilised in sharp objects like knives, blades etc. As these sharp objects have very less area on which the thrust is being applied, so high pressure is involved there. That's why we can cut things easily.

Example: A cube of mass 2 kg and side 10 m is kept on a table. Taking g= $10 m/s^2$, calculate the force exerted by the cube on the table.

Solution: Given- Mass of the cube = 2kg, Side of cube = 10 m and g = $10m/s^2$

As all the sides of a cube are equal, therefore, the area of the base will be given by-

Area of base = $(side)^2 = (10\ m)^2 = 100\ m^2$

As the cube is kept on the table, so, its weight will be acting downwards, and that is the perpendicular force (thrust) which the cube will be exerting on the table.

Therefore, thrust exerted by the cube = mg = 2 x10 N = 20 N

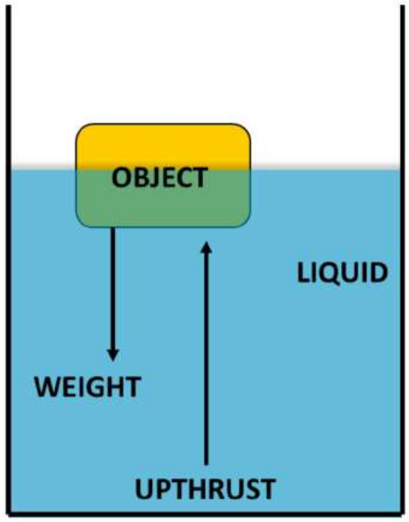
Now we know that $Pressure = \frac{Thrust}{Area}$

Therefore, putting values in the formula, we get-

$$Pressure = \frac{20}{100} = \frac{1}{5} = 0.2\ Pa$$

Therefore, the cube exerts a pressure of 0.2 Pa on the table.

Buoyant force: Whenever an object is immersed in a liquid, its weight is acting downwards and the liquid exerts an upward force on the object. This upward force exerted by the liquid is known as buoyant force.



- Object floats on the surface of the liquid when the buoyant force exerted by the liquid is greater than the weight of the object.
- Object sinks in the liquid when the weight of the object is greater than the buoyant force exerted by the liquid.

Density: Density is defined as the mass per unit volume.

$$Density = \frac{Mass}{Volume}$$

Its unit is kg/m^3 or g/cm^3.

INFERENCE-

- It is clearly evident from the formula that the density of an object is directly proportional to the mass of the object. This implies that the density of the object increases correspondingly if the mass of the object is increased.
- It is clearly evident from the formula that the density of an object is inversely proportional to the volume of the object. So, if the volume of the object is decreased, its density will be increased correspondingly.

Tip: An object floats on the surface of a liquid only when its density is lesser than the density of the liquid.

Tip: An object sinks in the liquid only when its density is greater than the density of the liquid.

Example: A cube of side 2 m and mass 16 kg is suspended in a liquid of density 3 kg/m^3. Calculate its density. Also, find out if the cube will sink/float in the liquid.

Solution: Given- Mass of the cube = 16 kg

Side of the cube = 2 m

Density of the liquid = 3 kg/m^3

To find- 1) Density of the cube.

2) If the cube will sink/ float in the liquid.

Solution- Volume of the cube = $(side)^3 = (2)^3 = 8\ m^3$

As we know that density is defined as mass per unit volume i.e.

$$Density\ of\ the\ cube = \frac{Mass}{Volume}$$

Therefore, putting values in the formula, we get-

$$Density\ of\ the\ cube = \frac{16}{8}\ kg/m^3 = 2\ kg/m^3$$

Density of the liquid = 3 kg/m^3

As the density of the cube is lesser than the density of the liquid, therefore, it will float.

Archimedes Principle

Archimedes Principle: When a body is immersed fully or partially in a fluid, it experiences an upward force that is equal to the weight of the fluid displaced by it.

This principle has multiple applications.

1) It is used in designing ships and submarines.

2) It is used in lactometers, which are used to determine the purity of a sample of milk.

3) It is used in hydrometers used for determining the density of liquids.

Example: The volume of an object is 250 cm^3. If the object sinks in water, calculate the mass of the water displaced by the object. (Density of water is $1gm/cm^3$)

Solution: Given- Volume of the object = 250 cm^3

Density of water = $1gm/cm^3$

According to Archimedes principle, "When a body is immersed fully or partially in a fluid, it experiences an upward force which is equal to the weight of the fluid displaced by it."

So, applying Archimedes principle here,

Mass of the water displaced = volume of the object x density of water

Therefore, mass of the water displaced = 250 cm^3 x 1 gm/cm^3 = 250 g

Relative density

Relative density: Relative density is defined as the ratio of the density of the substance to the density of water. As it's a ratio of two densities, therefore, it is unitless.

$$Relative\ density = \frac{Density\ of\ the\ substance}{Density\ of\ water}$$

Cases arising with relative density-

- Relative density < 1 - Here, as the density of the object is less than the density of water, therefore, the object will float.

- Relative density > 1 - Here, as the density of the object, is greater than the density of water, therefore, the object will sink in water.

- Relative density $= 1$ - Here, the density of the object is equal to the density of water, therefore, the object will remain fully submerged in water at the same depth.

Example: Calculate the relative density, if the density of the substance is 15 kg/m^3 and the density of water is 1kg/m^3.

Solution: Given- Density of the substance = 15 kg/m^3

Density of water = 1kg/m^3

To find- Relative density.

Solution- As we know that,

$$Relative\ density = \frac{Density\ of\ the\ substance}{Density\ of\ water}$$

Therefore, putting values in the formula, we get-

$$Relative\ density = \frac{15}{1} = 15$$