Chapter – 3

Coordinate Geometry

Introduction to Coordinate Geometry

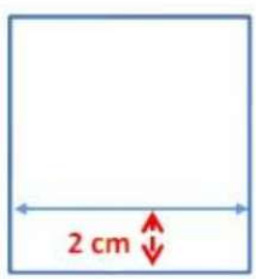
Introduction

Suppose I put a small dot on a sheet of paper with a pen. Can you locate this dot on the paper if I tell you that the dot is at the lower right corner of the paper?

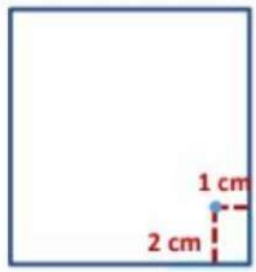
Now, you are able to see the dot but, can you tell me the exact position of the dot?

You will see that the information given above is not sufficient to fix the position of the dot.

Now, if I tell you that the point is nearly 2 cm away from the bottom line of the paper then this will give some idea but still is not sufficient because this would mean that the point could be anywhere, which is 2 cm away from the bottom line.



Therefore, to fix the position of the dot we have to specify its distance from two fixed lines, the right edge and the bottom line of the paper. Therefore, if I say that the dot is also 1 cm away from the right edge of the paper, then we can easily fix the position of the dot.



We see that position of any object lying in a plane can be represented with the help of two perpendicular lines.

Coordinate geometry is the branch of mathematics where we study the position of an object on a plane with reference to two mutually perpendicular lines in the same plane.

> **Coordinate geometry was initially developed by the French philosopher and Mathematician Rene Descartes. In his honour, the system used for describing the position of a point in a plane is also known as the Cartesian System.**

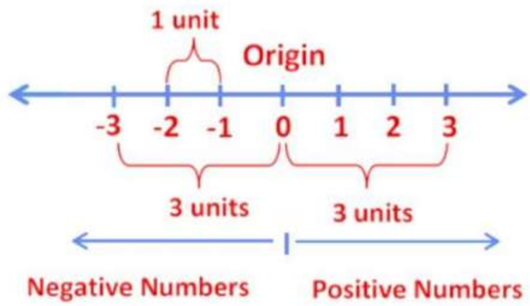
Cartesian System

Number Line

The number line is used to represent the numbers by marking points on a line at equal distances.

On a number line distances from the fixed point are marked in equal units positively in one direction and negatively in the other.

This fixed point from which the distances are marked is called the origin. In the figure O denotes the origin.



3 on number line is located at a distance of 3 units on the right side of origin O. Similarly, -3 is located at the same distance from origin but on its left side.

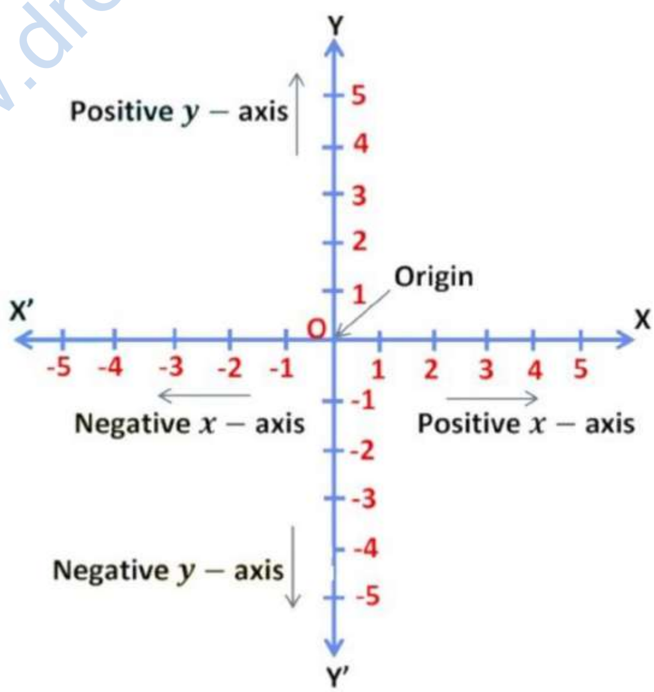
In Cartesian system, two perpendicular lines are used, one of them is horizontal (XX’) and the other is vertical (YY’).

The horizontal line X'X is called the x – axis and the vertical line Y'Y is called the y – axis.

The point where X'X and Y'Y intersect is called the origin (denoted by O).

Directions OX and OY are the positive directions of X - axis and Y - axis, respectively.

Similarly, directions OX' and OY' are the negative directions of X - axis and Y - axis, respectively.

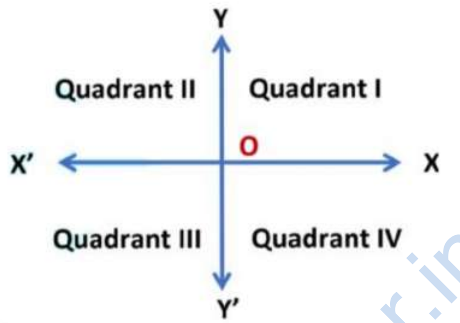


Quadrant

The axes (plural of the word 'axis') divide the plane into four parts. These four parts are called the quadrants ($\frac{1}{4}$ th part), numbered I, II, III and IV anticlockwise from OX.

III and IV anticlockwise from OX.

XOY	I Quadrant
X'OY	II Quadrant
X'OY'	III Quadrant
XOY'	IV Quadrant



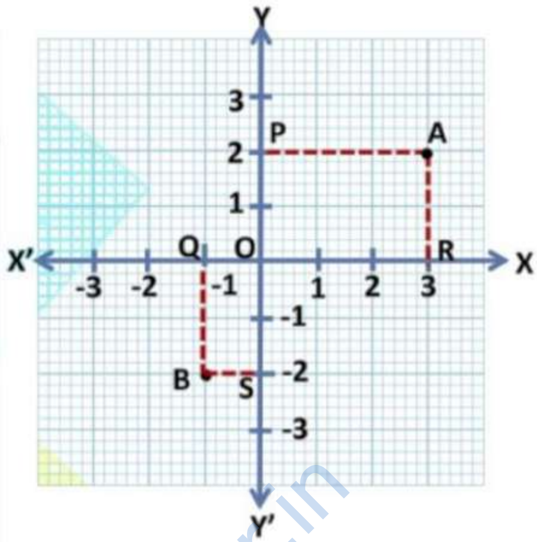
The plane consists of the axes and the four quadrants. We call the plane, the Cartesian plane, or the coordinate plane, or the xy-plane. The axes are called the coordinate axes.

A plane is a flat surface that goes infinitely in both directions.

Coordinates of a Point in Cartesian Plane

Coordinates of a Point in Cartesian Plane

Perpendicular distance of point A from the $y-$axis along positive direction of $x-$axis	AP = OR = 3 units
Perpendicular distance of point A from the $x-$axis along positive direction of $y-$axis	AR = OP = 2 units
Perpendicular distance of point B from the $y-$axis along negative direction of $x-$axis	BS = OQ = 1 unit
Perpendicular distance of point B from the $x-$axis along negative direction of $y-$axis	BQ = OS = 2 units

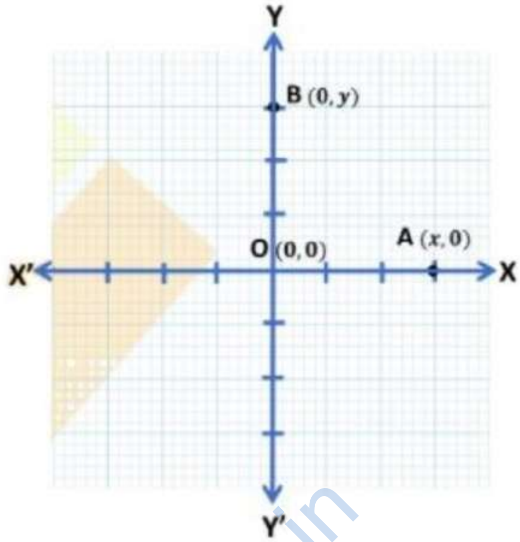


The perpendicular distance of a point from the y axis measured along the x axis is called its $x-$coordinate, or abscissa. For the point A it is +3 and for B, it is -1.

The perpendicular distance of a point from the x axis measured along the y axis is called its $y-$coordinate, or ordinate. For the point A it is +2 and for B, it is -2.

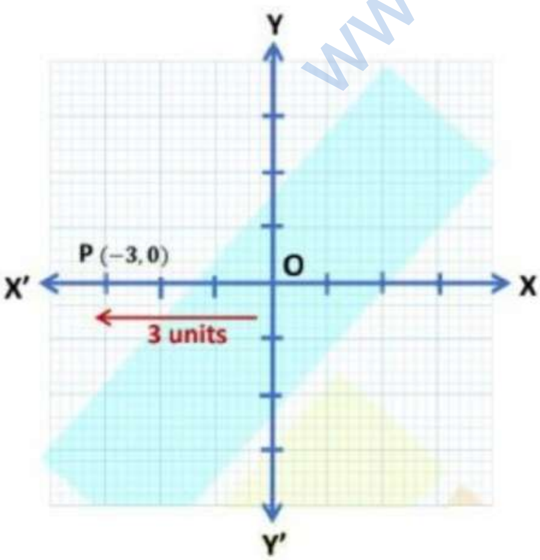
In stating the coordinates of a point in the coordinate plane, the x - coordinate comes first, and then the y - coordinate. We place the coordinates in brackets. Therefore, coordinates of A are (3,2) and B are (-1, -2)

1) Coordinates of a point on the x-axis are of the form $(x, 0)$ as every point on the x-axis has zero perpendicular distance from the x-axis.
2) Coordinates of a point on the y-axis are of the form $(0, y)$ as every point on the y-axis has zero perpendicular distance from the y-axis.
3) The coordinate of origin is $(0, 0)$ because it has zero distance from both the axes.
4) If we interchange the coordinates x and y then the position of (y, x) will be different from the position of (x, y). For example the ordered pair $(2, 5) \neq (5, 2)$



Example: Point P is on the x-axis and is at a distance of 3 units from the y-axis to its left. Write the coordinates of point P.

Point P is at a distance of 3 units towards left, from the y-axis.



Coordinates of point P are (-3, 0).

Example: Find distances of points C (-3, -2) and D (2, 1) from x-axis and y-axis.

C (-3, -2)

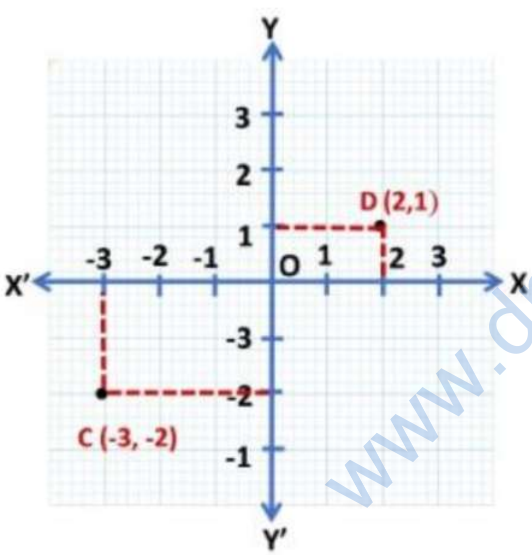
Distance from $x - axis = 2$ units

Distance from $y - axis = 3$ units

D (2, 1)

Distance from $x - axis = 1$ units

Distance from y-axis = 2 unit



Example: Locate and write the coordinates of a point:

a) lying on the x-axis to the left of origin at a distance of 4 units. b) above x-axis lying on the y-axis at a distance of 4 units from the origin.

b) above x- axis lying on y- axis at a distance of 4 units from origin.

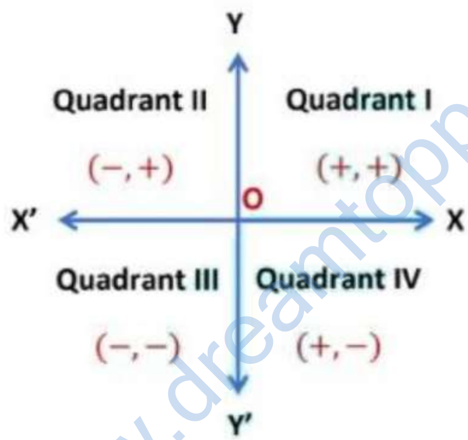
a) The given point is at a distance of 4 units towards left from the y-axis and at a zero distance from the x-axis. Therefore, the $x -$ coordinate of the point is -4 and the $y -$ coordinate is 0.

Hence, the coordinates of the given point are (-4, 0). Coordinates of a point on the x-axis are of the form (x, 0) as every point on the x-axis has zero perpendicular distance from the x-axis.

b) The given point is at a zero distance from the y-axis at a distance of 4 units from the x-axis. Therefore, the x − coordinate of the point is 0 and the y − coordinate is 4. Hence,

the coordinates of the given point are (0, -4). Coordinates of a point on the y-axis are of the form (0, y) as every point on the y-axis has zero perpendicular distance from the y-axis.

Signs of Coordinates in different Quadrants



Quadrant	Nature of x and y coordinates	
I Quadrant	$x > 0, y > 0$	Points in Quadrant I will be of the form $(+,+)$, as I quadrant is enclosed by the positive X – axis and the positive Y - axis
II Quadrant	$x < 0, y > 0$	Points in Quadrant II will be of the form $(-,+)$, as II quadrant is enclosed by the negative X – axis and the positive Y - axis
III Quadrant	$x < 0, y < 0$	Points in Quadrant III will be of the form $(-,-)$, as III quadrant is enclosed by the negative X – axis and the negative Y - axis
IV Quadrant	$x > 0, y < 0$	Points in Quadrant IV will be of the form $(+,-)$, as IV quadrant is enclosed by the positive X – axis and the negative Y - axis

Example: Write the quadrant in which each of the following points lie:

i) (-2, -4)

ii) (1, -4)

iii) (-3, 2)

i) (-2, -4)

Here, x coordinate = -2 and y coordinate = -4

As x coordinate and y coordinate both are negative ($x < 0$, $y < 0$) ,the given point lies in III quadrant.

ii) (1, -4)

Here, x coordinate = 1 and y coordinate = -4

As x coordinate is positive and y coordinate is negative ($x > 0$, $y < 0$) the given point lies in IV quadrant.

iii) (-3, 2)

Here, x coordinate = -3 and y coordinate = 2

As x coordinate is negative and y coordinate is positive $(x < 0, y < 0)$ the given point lies in II quadrant.

Example: If the coordinates of a point M are (-2,9) which can also be expressed as $(1+x, y^2)$ and $y > 0$, then find in which quadrant do the following points lie: $P(y, x)$, $Q(2, x)$, $R(x^2, y - 1)$, $S(2x, -3y)$

We know,

$(-2, 9) = (1 + x, y^2)$

$\therefore -2 = 1 + x \Rightarrow x = -2 - 1$

$x = -3$

$9 = y^2 \Rightarrow y = \pm 3$

Now, it is given that $y > 0$, so we choose the positive value of y.

So, $y = 3$

Therefore, $x = -3$ and $y = 3$

i) P (y, x)

$P(y, x) = P(3, -3)$ ($\because y = 3$ and $x = -3$)

As x coordinate is positive and y coordinate is negative $(x > 0, y < 0)$ the given point lies in IV quadrant.

ii) Q (2, x)

$Q(2, x) = Q(2, -3)$ ($\because x = -3$)

The x coordinate is positive and y coordinate is negative $(x > 0, y < 0)$ so the given point lies in IV quadrant.

iii) $R(x^2, y - 1)$

$x^2 = (-3)^2 = 9$; $y - 1 = 3 - 1 = 2$

$R(x^2, y-1) = (9, 2)$

As x coordinate and y coordinate both are positive $(x > 0, y > 0)$,the given point lies in I quadrant.

iv) $S(2x, -3y)$

$2x = 2\times(-3) = -6$; $-3y = -3\times3 = -9$

$S(2x, -3y) = S(-6, -9)$

As x coordinate and y coordinate both are negative $(x < 0, y < 0)$,the given point lies in III quadrant.

Plotting Coordinate

Plotting a point in a plane if its coordinates are given

In order to plot the points in a plane we follow the steps given below:

1) We first draw two mutually perpendicular lines on the graph paper, one horizontal (XX') and the other is vertical (YY').

2) Mark their point of intersection as O (origin). The line XOX' is the x-axis and the line YOY' is the y-axis.

3) Next, we get the coordinates of the point which is to be plotted. Let the point be A (5, 3).

4) To plot this point, we start from origin, and move 5 units in the positive direction along the x-axis and mark the corresponding point as P.

5) Now, starting from P we move 3 units in the positive direction of the y-axis.

6) The point where we arrive finally is the required point.

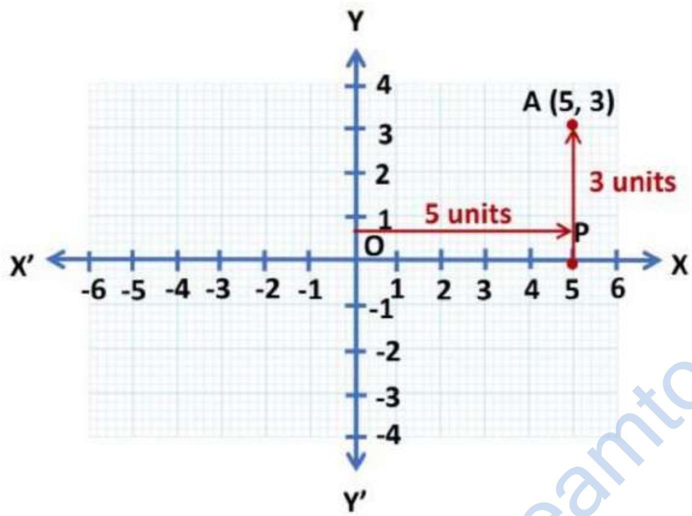
We first draw two mutually perpendicular lines on the graph paper, one horizontal (XX') and the other vertical (YY').

Mark their point of intersection as O (origin).

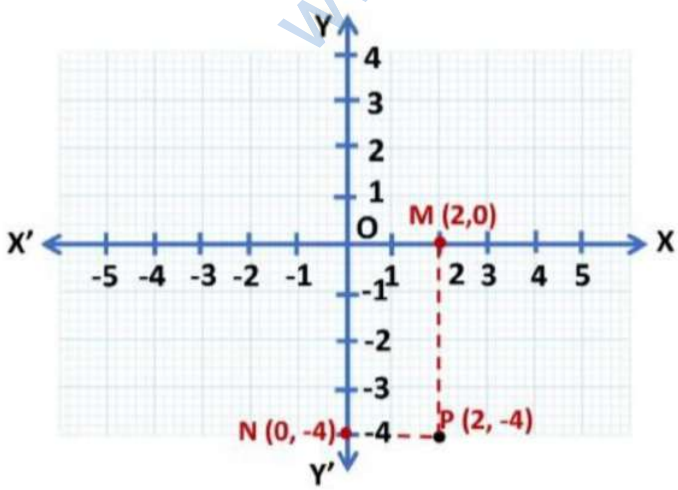
To plot point (2,-4) we start from origin, and move 2 units along the x-axis in the positive direction and then we move 4 units in the negative direction of the y-axis.

We reach the point P(2,-4). From point P draw PM and PN perpendiculars to x-axis and y-axis respectively.

Coordinate of M and N are (2,0) and (0,-4) respectively.



Example: Plot the point P (2, -4) on a graph paper and from it draw PM and PN perpendiculars to the x-axis and y-axis respectively. Write the coordinates of the point M and N.



Example: Plot the points A (0, 4), B (5, 4), C (4, 0) and D (-1, 0) on the graph paper. Identify the figure ABCD and find whether the point (2, 2) lies inside the figure or not?

We first draw two mutually perpendicular lines on the graph paper, one horizontal (XX') and the other vertical (YY').

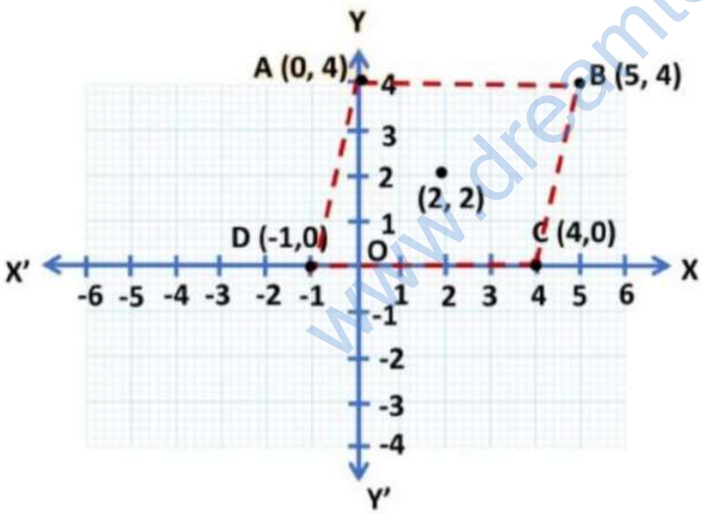
Mark their point of intersection as O (origin).

To plot point A(0,4) we move 4 units in the positive direction of the y-axis.

To plot point B(5,4), we start from origin, and move 5 units along the x-axis in the positive direction and then we move 4 units in the positive direction of the y-axis.

To plot point C(4,0) we move 4 units in the positive direction of the x-axis.

To plot point D(-1,0) we move 1 unit in the negative direction of x-axis Next we join the points A and B, B and C, C and D, D and A.



The figure ABCD is a parallelogram. We see that (2, 2) will lie inside the figure.

A parallelogram is a quadrilateral with opposite sides parallel and equal in length.