Chapter - 7
Coordinate Geometry
Exercise No. 7.1

Multiple Choice Questions:

Choose the correct answer from the given four options:

1. The distance of the point P (2, 3) from the x-axis is
(A) 2
(B) 3
(C) 1
(D) 5

Solution:
We have,
(x, y) is a point on the Cartesian plane in first quadrant.

So,
x = Perpendicular distance from Y – axis and
y = Perpendicular distance from X – axis

Hence, the perpendicular distance from X-axis = y coordinate is 3.

2. The distance between the points A (0, 6) and B (0, –2) is
(A) 6
(B) 8
(C) 4
(D) 2

Solution:

We have,
Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

As given in the question,
$x_1 = 0$,
$x_2 = 0$

$y_1 = 6$,
$y_2 = -2$

So,
$d^2 = (0 - 0)^2 + (-2 - 6)^2$
$d = \sqrt{(0)^2 + (-8)^2}$

$d = \sqrt{64}$
$d = 8$ units

Hence, the distance between A (0, 6) and B (0, 2) is 8.

3. The distance of the point P (–6, 8) from the origin is
(A) 8
(B) $2\sqrt{7}$
(C) 10
(D) 6

Solution:
Distance formula: $d^2 = (x_2 – x_1)^2 + (y_2 – y_1)^2$

As given in the question,
$x_1 = – 6$,
$x_2 = 0$

$y_2 = 8$,
$y_2 = 0$

Now,
$d^2 = [0 – (– 6)]^2 + [0 – 8]^2$
$d= \sqrt{((0-(-6))^2+ (0-8)^2}$
$d= \sqrt{((6)^2+ (-8)^2)}$
$d = \sqrt{(36 + 64)}$
$d = \sqrt{100}$
$d = 10$

Hence, the distance between P (– 6, 8) and origin O (0, 0) is 10

4. The distance between the points (0, 5) and (–5, 0) is
(A) 5
(B) $5\sqrt{2}$
(C) $2\sqrt{5}$
(D) 10

Solution:
Distance formula: $d^2 = (x_2 – x_1)^2 + (y_2 – y_1)^2$

As given in the question,
$x_1 = 0$, $x_2 = – 5$
$y_1 = 5$, $y_2 = 0$

$d^2 = ((– 5) – 0)^2 + (0 – 5)^2$
$d = \sqrt{(-5-0)^2+ (0-5)^2}$

$d = \sqrt{((-5)^2 + (-5)^2)}$
$d = \sqrt{(25 + 25)}$
$d = \sqrt{50}$
$= 5\sqrt{2}$

Hence, the distance between (0, 5) and (– 5, 0) = $5\sqrt{2}$

5. AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is
(A) 5
(B) 3
(C) $\sqrt{34}$
(D) 4

Solution:
The three vertices are:
A = (0, 3),
O = (0, 0),
B = (5, 0)

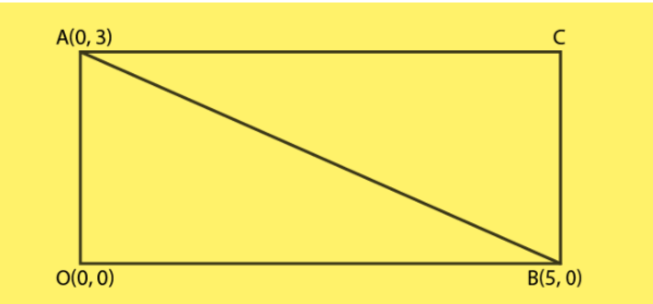
We know that, the diagonals of a rectangle are of equal length,

Length of the diagonal AB = Distance between the points A and B

Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

As given in the question,

We have;



$x_1 = 0$,
$x_2 = 5$

$y_1 = 3$,
$y_2 = 0$

$d^2 = (5 - 0)^2 + (0 - 3)^2$

$d = \sqrt{((5-0)^2 + (0-3)^2)}$
$d = \sqrt{(25 + 9)}$
$= \sqrt{34}$

Distance between A (0, 3) and B (5, 0) is $\sqrt{34}$.

So, the length of its diagonal is $\sqrt{34}$.

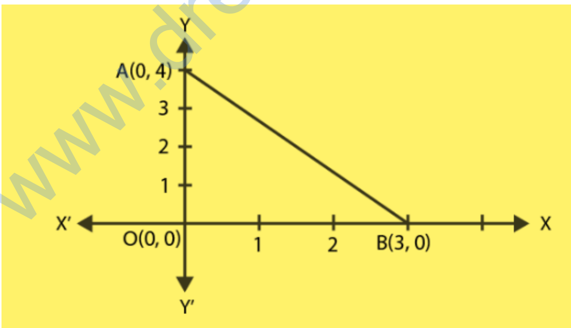
6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is
(A) 5
(B) 12
(C) 11
(D) $7+\sqrt{5}$

Solution:
The vertices of a triangle are (0, 4), (0, 0) and (3, 0).

Now, perimeter of ΔAOB = Sum of the length of all its sides:
= distance between (OA+OB+AB)

Distance between the points (x_1,y_1) and (x_2, y_2) is given by,

$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$



To find:
Distance between A(0, 4) and O(0, 0) + Distance between O(0, 0) and B(3, 0) + Distance between A(0, 4) and B(3, 0).
$= \sqrt{((0-0)^2 + (0-4)^2)} + \sqrt{((3-0)^2 + (0-0)^2)} + \sqrt{((3-0)^2 + (0-4)^2)}$
= 12

Hence, the required perimeter of triangle is 12

7. The area of a triangle with vertices A (3, 0), B (7, 0) and C (8, 4) is
(A) 14
(B) 28
(C) 8

(D) 6

Solution:

Vertices of the triangle are,
A $(x_1, y_1) = (3, 0)$
B $(x_2, y_2) = (7, 0)$
C $(x_3, y_3) = (8, 4)$

$$\begin{aligned} Area\ of\ triangle &= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2}[3(0-4) + 7(4-0) + 8(0-0)] \\ &= 8 \end{aligned}$$

Therefore, the area of ΔABC is 8.

8. The points (–4, 0), (4, 0), (0, 3) are the vertices of a
(A) right triangle
(B) isosceles triangle
(C) equilateral triangle
(D) scalene triangle

Solution:

(b)

Let A(- 4, 0), B(4, 0), C(0, 3) are the given vertices.

So, distance between A (-4, 0) and B (4, 0),
$d^2 = (4 - (-4))^2 + (0 - 0)^2$
d= 8

Now, distance between B (4, 0) and C (0, 3),
$d^2 = (0 - (4))^2 + (3 - 0)^2$
d = 5

Now, distance between A (-4, 0) and C (0, 3)
$d^2 = (0 - (-4))^2 + (3 - 0)^2$
d = 5

As, BC=AC

ΔABC is an isosceles triangle because an isosceles triangle has two sides equal.

9. The point which divides the line segment joining the points (7, –6) and (3, 4) in ratio 1 : 2 internally lies in the

(A) I quadrant
(B) II quadrant
(C) III quadrant
(D) IV quadrant

Solution:

(d)
If P(x, y) divides the line segment joining A(x_1,y_2) and B(x_2, y_2) internally in the ratio,
m : n

$$x = \frac{mx_2 + nx_1}{m+n},$$

$$y = \frac{my_2 + ny_1}{m+n}$$

Here,

$$x_1 = 7$$

$$x_2 = 3$$

$$y_1 = -6$$

$$y_2 = 4$$

$$m = 1$$

$$n = 2$$

$$x, y = \frac{1\times 3 + 2\times 7}{1+2}, \frac{1\times 4 + 2\times -6}{1+2}$$

$$x, y = \frac{17}{3}, \frac{-8}{3}$$

It lies in 4rth quadrant.

10. The point which lies on the perpendicular bisector of the line segment joining the points A (–2, –5) and B (2, 5) is
(A) (0, 0)
(B) (0, 2)
(C) (2, 0)
(D) (–2, 0)

Solution:
(a)
We know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid-point of the line segment.

Mid-point of the line segment joining the points A (-2, -5) and S(2, 5), is given by :

$$x = \frac{-2+2}{2},$$
$$y = \frac{-5+5}{2}$$
$$(x, y) = (0, 0)$$

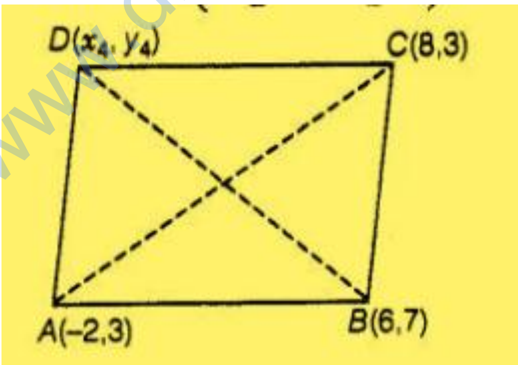
So, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

11. The fourth vertex D of a parallelogram ABCD whose three vertices are A (–2, 3), B (6, 7) and C (8, 3) is
(A) (0, 1)
(B) (0, –1)
(C) (–1, 0)
(D) (1, 0)

Solution:
(b)

Let the fourth vertex of parallelogram,
D= (x_4, y_4)

L, M be the middle points of AC and BD, respectively,



Therefore,
$$L = \left(\frac{-2+8}{2}, \frac{3+3}{2}\right)$$
$$= (3,3)$$
$$M = \left(\frac{6+x_4}{2}, \frac{7+y_4}{2}\right)$$

Since, ABCD is a parallelogram, therefore diagonals AC and BD will bisect each other.
Hence, L and M are the same points.

So,

$$\frac{6+x_4}{2}=3$$

$$x_4=0$$

and

$$\frac{7+y_4}{2}=3$$

$$y_4=-1$$

So, the fourth vertex of parallelogram is D = (x_4, y_4) s (0,-1).

12. If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4),then

(A) $AP=\frac{1}{3}AB$

(B) $AP=PB$

(C) $PB=\frac{1}{3}AB$

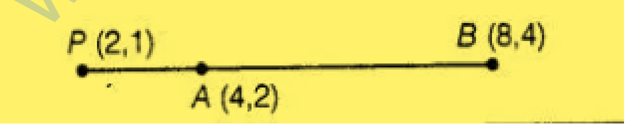
(D) $AP=\frac{1}{2}AB$

Solution:

(d)

Given,

The point P(2,1) lies on the line segment joining the points A(4,2) and 8(8, 4),



$$Dis\tan ce\ AP=\sqrt{(2-4)^2+(1-2)^2}$$

$$=\sqrt{5}$$

$$Dis\tan ce\ AB=\sqrt{(8-4)^2+(4-2)^2}$$

$$=2\sqrt{5}$$

$$Dis\tan ce\ BP=\sqrt{(8-2)^2+(4-1)^2}$$

$$=3\sqrt{5}$$

So,

$$AB=2\sqrt{5}$$

$$=2AP$$

$or,$

$$AP = \frac{AB}{2}$$

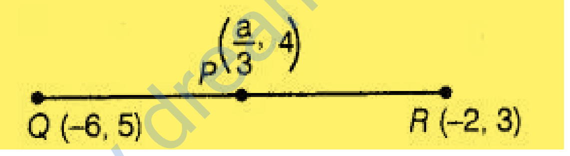
So, required condition is $AP = \frac{AB}{2}$.

13. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q $(-6, 5)$ and $R(-2, 3)$, then the value of *a* is

(A) – 4
(B) – 12
(C) 12
(D) – 6

Solution:
(b)

Given that, $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q(-6, 5) and R (-2, 3), which shows in the figure ,



$$\begin{aligned} Midpoint\ of\ AB &= P \\ &= \left(\frac{-6-2}{2}, \frac{5+3}{2}\right) \\ &= (-4, 4) \end{aligned}$$

$But\ midpoint\ P\left(\frac{a}{3}, 4\right) is\ given,$

$So,$

$$\left(\frac{a}{3}, 4\right) = (-4, 4)$$

On comparing,

$$\frac{a}{3} = -4$$

$$a = -12$$

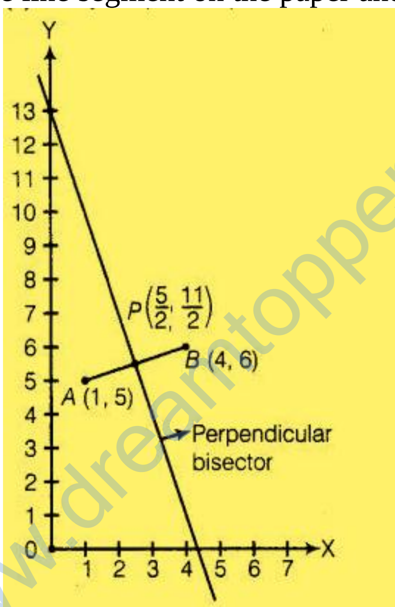
So, the required value of a is -12.

14. The perpendicular bisector of the line segment joining the points $A(1,5)$ and $B(4,6)$ cuts the *y*-axis at

(A) (0, 13)
(B) (0, –13)
(C) (0, 12)
(D) (13, 0)

Solution:
(a)

At first, we plot the points of the line segment on the paper and join them.



We know that, the perpendicular bisector of the line segment AB bisect the segment AB, i.e., perpendicular bisector of line segment AB passes through the mid-point of AB.

Midpoint of (A,B) = P

$$= \left(\frac{1+4}{2}, \frac{5+6}{2}\right)$$

$$= \left(\frac{5}{2}, \frac{11}{2}\right)$$

Now, we draw a straight line on paper passes through the mid-point P. We see that the perpendicular bisector cuts the Y-axis at the point (0, 13).

So, the required point is (0, 13).

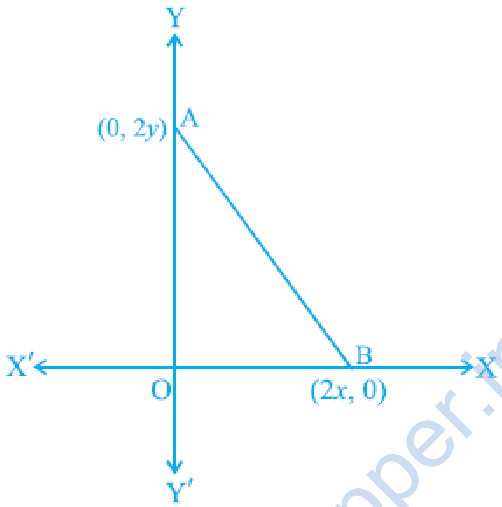
15. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the fig. is

(A) (x, y)

(B) (y, x)

(C) $\frac{x}{2}, \frac{y}{2}$

(D) $\frac{y}{2}, \frac{x}{2}$



Solution:

(a)

Let the coordinate of the point which is equidistant from the three vertices 0(0, 0), A(0,2y) and B(2x, 0) is P(h,k).

Then, $\quad PO = PA = PB$

$$(PO)^2 = (PA)^2 = (PB)^2 \quad \dots \text{(i)}$$

By distance formula,

$$\left(\sqrt{(h-0)^2+(k-0)^2}\right)^2 = \left(\sqrt{(h-0)^2+(k-2y)^2}\right)^2 = \left(\sqrt{(h-2x)^2+(k-0)^2}\right)^2$$

$$h^2+k^2 = h^2+(k-2y)^2 = (h-2x)^2+k^2$$

Taking 1st two equations,

$h^2+k^2 = h^2+(k-2y)^2$

$4y(y-k)=0$

$y=k$

Taking first and last equations,

$h^2+k^2 =(h-2x)^2+k^2$

$4x(x-h)=0$

$x=h$

Required points = (h,k)

= (x,y)

16. A circle drawn with origin as the center passes through $\left(\frac{13}{2},0\right)$. The point which does not lie in the interior of the circle is

(A) $\frac{-3}{4},1$

(B) $2,\frac{7}{3}$

(C) $5,\frac{-1}{2}$

(D) $\left(-6,\frac{5}{2}\right)$

Solution:

(d)
Given,

A circle drawn with origin as the center passes through $\left(\frac{13}{2},0\right)$.

Radius of the circle = distance between (0,0) and $\left(\frac{13}{2},0\right)$

$$=\sqrt{\left(\frac{13}{2}-0\right)^2+(0-0)^2}$$

$$=\frac{13}{2}$$

$= 6.5$ (using distance formula)

Now we calculate the distance of each point from the origin.
If the distance is less than 6.5 than the point lies inside the circle otherwise not.

So, point $\left(-6,\frac{5}{2}\right)$ lies on the circle and other point lies inside the circle.

17. A line intersects the *y*-axis and *x*-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively

(A) (0, – 5) and (2, 0)

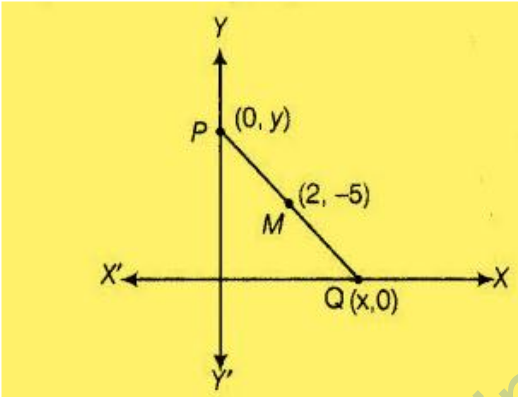
(B) (0, 10) and (– 4, 0)

(C) (0, 4) and (– 10, 0)

(D) (0, – 10) and (4, 0)

Solution:
(d)

Let the coordinates of P and 0 (0, y) and (x, 0), respectively.



Now,

Midpoint of PQ is M,

M=(2,-5)

$$\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = M$$

$$\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = (2, -5)$$

$$\frac{0+x}{2} = 2 \quad and$$

$$\frac{y+0}{2} = -5$$

On solving,
x = 4
y= -10

So, the coordinates of P and Q are (0, -10) and (4, 0).

18. The area of a triangle with vertices (*a*, *b* + *c*), (*b*, *c* + *a*) and (*c*, *a* + *b*) is
(A) $(a + b + c)^2$
(B) 0
(C) ***a*+ *b* + *c***
(D) ***abc***

Solution:

(b)

Let the vertices of a triangle are,

A = (x_1, y_1)
  = (a, b + c)

B = (x_2, y_2)
  = (b,c + a) and

C = (x_3, y_3)
  =(c, a + b)

$$\begin{aligned} Area\,of\;\; ABC &= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 + y_2) \\ &= \frac{1}{2}[a(\mathrm{c} + a - a - b) + b(\mathrm{a} + \mathrm{b} - \mathrm{b} - \mathrm{c}) + c(b + c - c - a) \\ &= \frac{1}{2}(0) \\ &= 0 \end{aligned}$$

Hence, the required area of triangle is 0.

19. If the distance between the points (4, *p*) and (1, 0) is 5, then the value of *p* is

(A) 4 only

(B) ± 4

(C) – 4 only

(D) 0

Solution:

(b)

As given in the question,

The distance between the points (4, p) and (1, 0) = 5

$$\sqrt{(1-4)^2 + (0-p)^2} = 5$$
$$\sqrt{9 + p^2} = 5$$

Squaring both sides,

$$\begin{aligned} 9 + p^2 &= 25 \\ p^2 &= 16 \\ p &= \pm 4 \end{aligned}$$

Hence, the required value of p is ± 4,

20. If the points A (1, 2), O (0, 0) and C (*a, b*) are collinear, then

(A) *a*= *b*

(B) *a*= 2*b*

(C) 2*a* = *b*
(D) *a*= –*b*

Solution:
(c)
Let the given points are

A = (x_1,y_1)
= $(1,2)$,

B = (x_2,y_2)
= $(0,0)$

C = (x_3,y_3)
= (a, b).

$$Area\ of\ ABC = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1+y_2)$$
$$= \frac{1}{2}[1(0-b)+0(b-2)+a(2+0)$$
$$= \frac{1}{2}(2a-b)$$

As the points are collinear,
ar(ABC)=0

$$\frac{1}{2}(2a-b)=0$$
$$2a=b$$

So, 2a=b is required relation.

Hence, the required relation is 2a = b.

Exercise No. 7.2

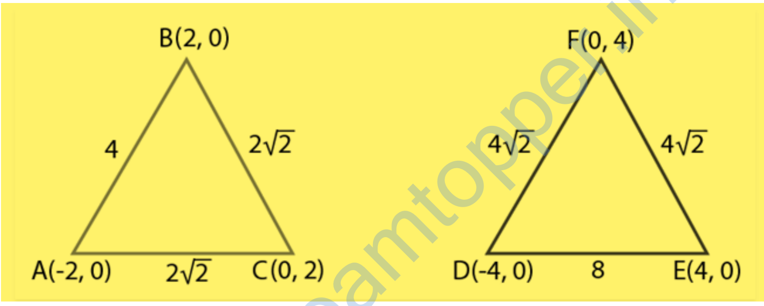
Short Answer Questions with Reasoning:

State whether the following statements are true or false. Justify your answer.

1. ΔABC **with vertices A (–2, 0), B (2, 0) and C (0, 2) is similar to** ΔDEF **with vertices D (–4, 0) E (4, 0) and F (0, 4).**

Solution:
True.

Explanation:



By distance formula,
$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

We can find,

$AB=\sqrt{(2+2)^2+(0)^2}$

$AB=4$

$BC=\sqrt{(0-2)^2+(2-0)^2}$

$BC=2\sqrt{2}$

$CA=\sqrt{(-2-0)^2+(0-2)^2}$

$CA=2\sqrt{2}$

Also,

$DE=\sqrt{(-4-4)^2+(6-2)^2}$

$DE=8$

$EF=\sqrt{(-4-4)^2+(6-2)^2}$

$EF=4\sqrt{2}$

$FD=\sqrt{(-4-4)^2+(6-2)^2}$

$FD=4\sqrt{2}$

So,

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}=\frac{1}{2}$$

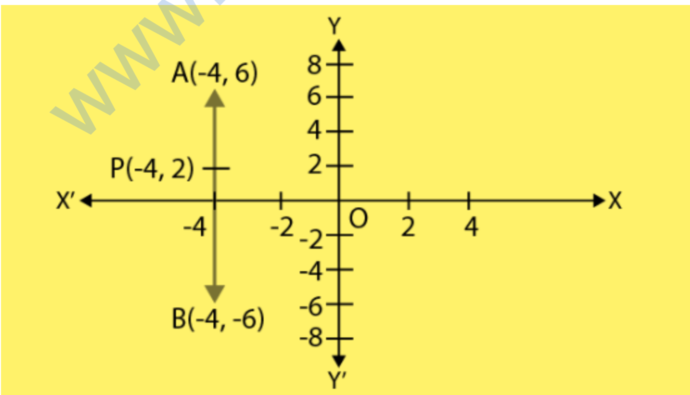
Hence, triangle ABC and DEF are similar.

2. Point P (– 4, 2) lies on the line segment joining the points A (– 4, 6) and B (– 4, – 6).

Solution:

True.

Explanation:
We will plot the points P(- 4, 2), A (- 4, 6) and B (- 4, – 6) on a graph paper and connect the points .



So, from the graph it is clear that, point P (– 4, 2) lies on the line segment joining the points A (– 4, 6) and B (– 4, – 6).

3. The points (0, 5), (0, –9) and (3, 6) are collinear.

Solution:

False

Explanation,
The points are collinear if area of a triangle formed by its points is equals to the zero.

Given,
$x_1 = 0$,
$x_2 = 0$,
$x_3 = 3$ and

$y_1 = 5$,
$y_2 = -9$,
$y_3 = 6$

Area of triangle,

$$= \frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$$

$$= \frac{1}{2}(0(-9-6) - 0(6-5) + 4(5+9))$$

$$= \frac{1}{2}(42)$$

$$= \frac{42}{2} \neq 0$$

So, the points are not collinear.

From the above equation, it is clear that the points are not collinear.

4. Point P (0, 2) is the point of intersection of *y*–axis and perpendicular bisector of line segment joining the points A (–1, 1) and B (3, 3).

Solution:

False

Explanation:
The points lying on perpendicular bisector of the line segment joining the two points is equidistant from the two points.

So,
PA should be equals to the PB.

Using distance formula,
$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

$PA=\sqrt{(-4-4)^2+(6-2)^2}$

PA=4

$PB=\sqrt{(-4-4)^2+(-6-2)^2}$

PB=8

So, PA is not equal to PB.

5. Points A (3, 1), B (12, –2) and C (0, 2) cannot be the vertices of a triangle.

Solution:

True.

Explanation:

Coordinates of A = (x_1, y_1)
= (3, 1)

Coordinates of B = (x_2, y_2)
= (12, – 2)

Coordinates of C = (x_3, y_3)
= (0, 2)

Area of ΔABC = $\Delta = 1/2\ [x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)]$

$\Delta = ½ [3 – (2 – 2) + 12(2 – 1) + 0\{1 – (- 2)\}]$

$\Delta = ½ [3(- 4) + 12(1) + 0]$

$\Delta = ½ (- 12 + 12)$

$= 0$

Area of ΔABC = 0

As, the points A (3, 1), B (12, – 2) and C (0, 2) are collinear.

So, the points A (3, 1), B (12, – 2) and C (0, 2) can't be the vertices of a triangle.

6. Points A (4, 3), B (6, 4), C (5, –6) and D (–3, 5) are the vertices of a parallelogram.

Solution:

False

Explanation:

The given points are A (4, 3), B (6, 4), C (5, –6) and D (–3, 5).

We will find the distances AB, BC, CD and AD respectively by distance formula.

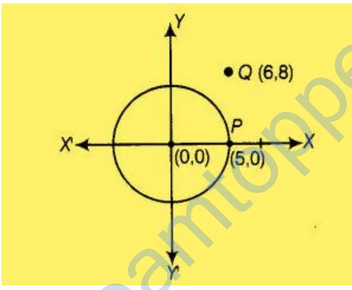
We will see that,
Every distance is different from one another.

As the distances are different, we can conclude that the points are not the vertices of a parallelogram.

7. A circle has its center at the origin and a point P (5, 0) lies on it. The point Q (6, 8) lies outside the circle.

Solution:
True

At first, we draw a circle and a point,



Also, if the distance of any point from the centre is less than/equal to/ more than the radius, then the point is inside/on/outside the circle, respectively.

$$OP=\sqrt{(5-0)^2+(0-0)^2}$$
$$OP=\sqrt{25}$$
$$OP=5\,units$$
$$which\ is\ radius\ of\ the\ circle$$
$$and,$$
$$OQ=\sqrt{(6-0)^2+(8-0)^2}$$
$$OQ=\sqrt{100}$$
$$OQ=10\,units$$

We see that, OQ > OP

So, it is true that point Q (6, 8), lies outside the circle.

8. The point A (2, 7) lies on the perpendicular bisector of line segment joining the points P (6, 5) and Q (0, – 4).

Solution:

False

If A (2,7) lies on perpendicular bisector of P(6,5) and Q (0, – 4), then AP=AQ

$$\text{AP}=\sqrt{(6-2)^2+(5-7)^2}$$

$$\text{AP}=\sqrt{16+4}$$

$$AP=\sqrt{20}\,units$$

$$and,$$

$$\text{AQ}=\sqrt{(0-2)^2+(-4-7)^2}$$

$$AQ=\sqrt{125}\,units$$

Therefore, A does not lies on the perpendicular bisector of PQ.

9. Point P (5, –3) is one of the two points of trisection of the line segment joining the points A (7, – 2) and B (1, – 5).

Solution:

Let the point P (5,-3) divides the line segment joining the points A (7,-2) and B (1 ,-5) in the ratio k: 1 internally.
By section formula, the coordinate of point P will be,

$$\left(\frac{k(1)+1(7)}{k+1},\frac{k(-5)+1(-2)}{k+1}\right)=(5,-3)$$

$$\left(\frac{k+7}{k+1},\frac{-5\text{k}-2}{k+1}\right)=(5,-3)$$

$$So,$$

$$\frac{k+7}{k+1}=5$$

$$k+7=5(k+1)$$

$$-4k=-2$$

$$k=\frac{1}{2}$$

The point P divides the line segment AB in ratio 1: 2.

Therefore, point P in the point of trisection of AB.

10. Points A (–6, 10), B (–4, 6) and C (3, –8) are collinear such that $AB = \frac{2}{9}AC$.

Solution:

True

If the area of triangle formed by the points (x_1,y_2), (x_2, y_2) and (x_3, y_3) is zero, then the points are collinear,

$$ar(ABC) = \frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)) = 0$$

$$= \frac{1}{2}(-6(6-(-8)) - 4(-8-10) + 3(10-6)) = 0$$

$$= -84 + 84 = 0$$

So, it is true.

And the points are collinear.

Also,

$$AC = \sqrt{(3+6)^2 + (-8-10)^2}$$

$$AC = 9\sqrt{5}\,units$$

and,

$$AB = \sqrt{(-4-(-6))^2 + (6-10)^2}$$

$$AB = 2\sqrt{5}\,units$$

Now,

$$\frac{AB}{AC} = \frac{2\sqrt{5}}{9\sqrt{5}}$$

$$AB = \frac{2}{9}AC$$

Which is the required relation.

11. The point P (–2, 4) lies on a circle of radius 6 and centre C (3, 5).

Solution:

False

If the distance between the centre and any point is equal to the radius, then we say that point lie on the circle.

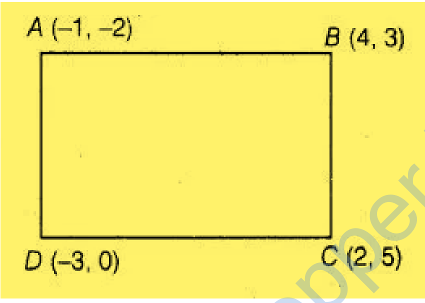
So, distance between P (-2,4) and centre (3, 5)

$$\text{Distance}=\sqrt{(3+2)^2+(5-4)^2}$$
$$=\sqrt{26}$$

It is not equal to the radius of the circle.
So, the point P(-2, 4) does not lies on the circle.

12. The points A (–1, –2), B (4, 3), C (2, 5) and D (–3, 0) in that order form a rectangle.

Solution:
True



Distance between A(-1,-2) and B(4,3) is :

$$AB=\sqrt{(4+1)^2+(3+2)^2}$$
$$AB=5\sqrt{2}$$
Also,
$$CD=\sqrt{(-3-2)^2+(0-5)^2}$$
$$CD=5\sqrt{2}$$
$$AD=\sqrt{(-3+1)^2+(0+2)^2}$$
$$AD=2\sqrt{2}$$
$$BC=\sqrt{(4-2)^2+(3-5)^2}$$
$$BC=2\sqrt{2}$$
Also,
$$\text{Diagonal } AC=\sqrt{(2+1)^2+(5+2)^2}$$
$$AC=\sqrt{58}$$
$$\text{Diagonal } BD=\sqrt{(4+3)^2+(3-0)^2}$$
$$BD=\sqrt{58}$$

As, opposite sides and diagonals AC and BD are equal.
Hence, the points A (-1, – 2), B (4, 3), C (2, 5)and D (- 3 0) form a rectangle.

Exercise No. 7.3

Short Answer Questions:

1. Name the type of triangle formed by the points A (–5, 6), B (–4, –2) and C (7, 5).

Solution:
Given points are:
A (–5, 6), B (–4, –2) and C (7, 5)

Using distance formula,
$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

$AB = \sqrt{((-4+5)^2 + (-2-6)^2)}$
$= \sqrt{1+64}$
$= \sqrt{65}$

$BC = \sqrt{((7+4)^2 + (5+2)^2)}$
$= \sqrt{121 + 49}$
$= \sqrt{170}$

$AC = \sqrt{((7+5)^2 + (5-6)^2)}$
$= \sqrt{144 + 1}$
$= \sqrt{145}$

As all sides are of different length, ABC is a scalene triangle.

2. Find the points on the *x*–axis which are at a distance of $2\sqrt{5}$ from the point (7, –4). How many such points are there?

Solution:

Let coordinates of the point=(x, 0) (as the point lies on x axis)
$x_1=7$.
$y_1=-4$

$x_2=x$.
$y_2=0$

Distance $=\sqrt{(x_2-x_1)^2+ (y_2-y_1)^2}$

As given in the question,
$2\sqrt{5}=\sqrt{(x-7)^2+ (0-(-4))^2}$

Squaring L.H.S and R.H.S

$20= x^2+49-14x+16$
$20= x^2+65-14x$
$0= x^2-14x+45$
$0= x^2-9x-5x+45$
$0= x(x-9)-5(x-9)$
$0= (x-9)(x-5)$

So,
$x-9 =0$.
and
$x-5= 0$

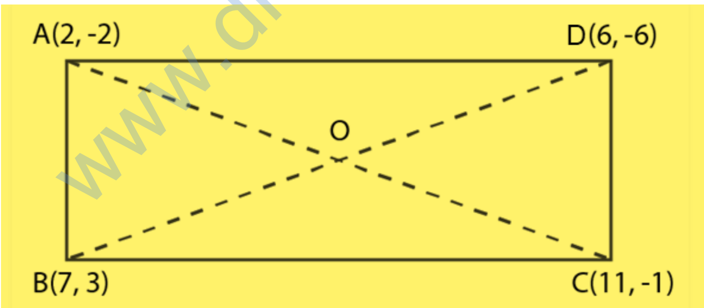
Therefore,
$x = 9$ or $x = 5$

So, coordinates of points…..(9,0) or (5,0).

3. What type of a quadrilateral do the points A (2, –2), B (7, 3), C (11, –1) and D (6, –6) taken in that order, form?

Solution:

The points are A (2, –2), B (7, 3), C (11, –1) and D (6, –6)



Using distance formula,
$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

AB= $5\sqrt{2}$
BC=$4\sqrt{2}$
CD=$5\sqrt{2}$
DA=$4\sqrt{2}$

Finding diagonals AC and BD, we get,
AC= $\sqrt{82}$
BD=$\sqrt{82}$

So, it is a rectangle.

4. Find the value of *a*, if the distance between the points A (–3, –14) and B (*a*, –5) is 9 units.

Solution:

Distance between two points (x_1,y_1) (x_2,y_2) is :

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance between A (–3, –14) and B (a, –5) $=\sqrt{[(a+3)^2+(-5+14)^2]}$
$=9$

Squaring on L.H.S and R.H.S.

$(a+3)^2+81 = 81$

$(a+3)^2 = 0$

$(a+3)(a+3) = 0$

$a+3 = 0$

$a= -3$

5. Find a point which is equidistant from the points A (–5, 4) and B (–1, 6)? How many such points are there?

Solution:

Taking the point be P.

As given in the question,

P is equidistant from A (–5, 4) and B (–1, 6)

So,

Point P $= ((x_1+x_2)/2, (y_1+y_2)/2)$ (using midpoint theorem)

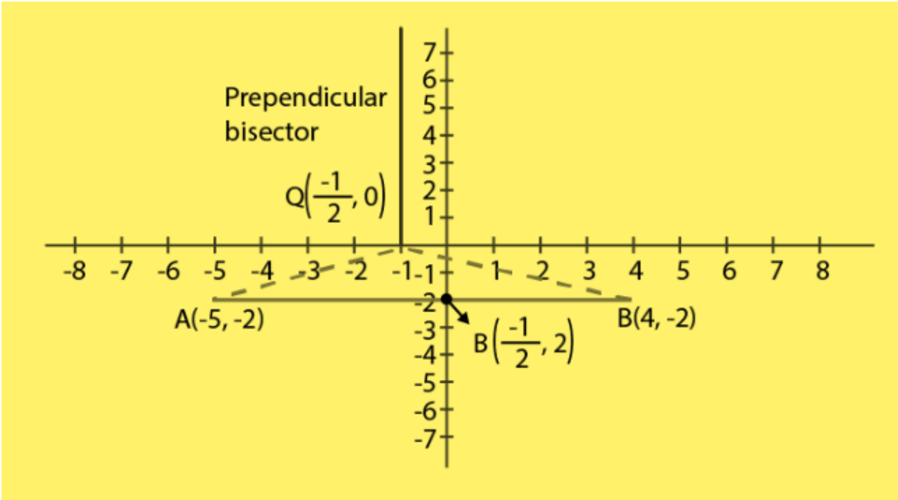
$= ((-5-1)/2, (6+4)/2)$

$= (-3 , 5)$

6. Find the coordinates of the point Q on the x–axis which lies on the perpendicular bisector of the line segment joining the points A (–5, –2) and B(4, –2). Name the type of triangle formed by the points Q, A and B.

Solution:

Point Q is the midpoint of AB as the point P lies on the perpendicular bisector of AB.



By mid-point formula:

$(x_1 + x_2)/2 = (-5+4)/2$

$= -½$

$x = -½$

It is given that, P lies on x axis,

$y = 0$

$P(x,y) = (-½ , 0)$

Therefore, It is an isosceles triangle.

7. Find the value of m if the points (5, 1), (–2, –3) and (8, 2m) are collinear.

Solution:

The points A(5, 1), B(–2, –3) and C(8, 2m) are collinear.

Area of ΔABC = 0

$$\frac{1}{2}\,[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0$$

$$\frac{1}{2}\,[5(-3 - 2m) + (- 2)(2m - 1) + 8(1 - (- 3))] = 0$$

$$\frac{1}{2}\,(-15 - 10m - 4m + 2 + 32) = 0$$

$$\frac{1}{2}\,(-14m + 19) = 0$$

$$m = 19/14$$

8. If the point A (2, – 4) is equidistant from P (3, 8) and Q (–10, y), find the values of y. Also find distance PQ.

Solution:

The points are,
A (2, – 4)

P (3, 8)

Q (–10, y)

As given in the question,

PA=QA

$$\sqrt{(-2-3)^2+(-4-8)^2}=\sqrt{(2-10)^2+(-4-y)^2}$$
$$\sqrt{145}=\sqrt{160+8y+y^2}$$
Squaring both sides,
$$145=160+8y+y^2$$
$$160+8y+y^2-145=0$$
$$8y+y^2+15=0$$
$$(y+5)(y+3)=0$$
$$y=-5$$
$$y=-3$$

Now,
$$PQ=\sqrt{(-10-3)^2+(y-8)^2}$$
$$For\ y=-3$$
$$PQ=\sqrt{290}$$
$$For\ y=-3$$
$$PQ=\sqrt{338}$$

9. Find the area of the triangle whose vertices are (–8, 4), (–6, 6) and (–3, 9).

Solution:

As given in the question the vertices are:
$(x_1, y_1) = (-8, 4)$
$(x_2, y_2) = (-6, 6)$
$(x_3, y_3) = (-3, 4)$

We have,
Area of triangle = $\frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

$= \frac{1}{2}$ (-8(6 – 4) + -6(4 – 4) + -3(4 – 6))

$= \frac{1}{2}$ (-8(2) + -6(0) + -3(-2))

$= \frac{1}{2}$ (-16 + 6)

$= \frac{1}{2}$ (-10)

= 5 units.

10. In what ratio does the x–axis divide the line segment joining the points (– 4, – 6) and $(-1, 7)$? Find the coordinates of the point of division.

Solution:

Let us take the ratio in which x-axis divides the line segment joining (–4, –6) and (–1, 7)
= 1: k.

x-coordinate = (-1 – 4k) / (k + 1)
y-coordinate = (7 – 6k) / (k + 1)

Also,
P lies on x-axis, so,
y coordinate = 0
(7 – 6k) / (k + 1) = 0
7 – 6k = 0
k = 7/6

Hence, the ratio is
1:7/6 = 6:7

So, the coordinates of P are (-34/13, 0).

11. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and $B(2, -5)$.

Solution:

Let P $P\left(\frac{3}{4},\frac{5}{12}\right)$ divide AB internally in the ratio m:n

Using the section formula,

$$\left(\frac{3}{4},\frac{5}{12}\right)=\left(\frac{2m-\frac{n}{2}}{m+n},\frac{-5m+\frac{3}{2}n}{m+n}\right)$$

$$\frac{2m-\frac{n}{2}}{m+n}=\frac{3}{4}$$
$$\frac{4m-n}{m+n}=\frac{3}{2}$$
$$5n-5m=0$$
$$So, m=n$$
$$\frac{-5m+\frac{3}{2}n}{m+n}=\frac{5}{12}$$
$$\frac{-10m+3n}{m+n}=\frac{5}{6}$$
$$65m-13n=0$$
$$5m-n=0$$
$$\frac{m}{n}=\frac{1}{5}$$

So, the required ratio is 1 : 5.

12. If P (9*a* – 2, –*b*) divides line segment joining A (3*a* + 1, –3) and B (8*a*, 5) in the ratio 3 : 1, find the values of *a* and *b*.

Solution:

Let us take P(9a – 2, – b) divides AB internally in the ratio 3:1.

By section formula,
m =3
n =1

So, according to question,

$$9a-2=\frac{3(8a)+1(3a+1)}{3+1} \qquad ....(i)$$
$$9a-2=\frac{24a+3a+1}{4}$$
$$36a-8=27a+1$$
$$9a-9=0$$
$$a=1$$
Also,
$$-b=\frac{3(5)+1(-3)}{3+1} \qquad ....(ii)$$
$$-b=\frac{15-3}{4}$$
$$b=-3$$
The required values of a and b are 1 and – 3.

13. If (*a*, *b*) is the mid-point of the line segment joining the points A (10, –6) andB (*k*, 4) and *a* – 2*b* = 18, find the value of *k* and the distance AB.

Solution:

Since, (a, b) is the mid-point of line segment AB.
$$(a,b)=\left(\frac{10+k}{2},\frac{-6+4}{2}\right)$$
$$(a,b)=\left(\frac{10+k}{2},-1\right)$$

Equating coordinates,
$$a=\frac{10+k}{2} \qquad ..(i)$$
$$b=-1 \qquad ...(ii)$$

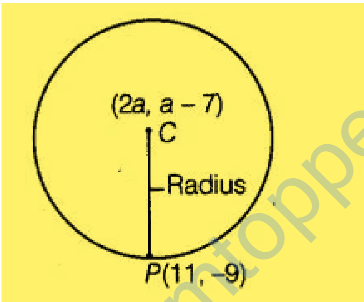
Given :
$$\mathrm{a}-2\mathrm{b}=18$$
$$\mathrm{a}-2(-1)=18$$
$$\mathrm{a}=16$$

From equation 1.
$$16=\frac{10+k}{2}$$
$$k=22$$

Therefore,
$A(10,-6)$
$B(22,4)$
By distance formula,
$AB = 2\sqrt{61}$
The required distance of AB is $2\sqrt{61}$.

14. The centre of a circle is (2*a*, *a* – 7). Find the values of *a* if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units.

Solution:
According to question,



Distance between the centre C(2a, a – 7) and the point P(11, – 9), which lie on the circle = Radius of circle

$$\text{Radius of circle} = \sqrt{(11-2a)^2 + (-9-a+7)^2}$$
$$= \sqrt{(11-2a)^2 + (-2-a)^2} \qquad ...(i)$$
$\text{Diameter} = 10\sqrt{2}$
$\text{So, Radius} = 5\sqrt{2}$
Now from (i),
$$\sqrt{(11-2a)^2 + (-2-a)^2} = 5\sqrt{2}$$

Squaring both sides,
$$(11-2a)^2 + (-2-a)^2 = 50$$
$$5a^2 - 40a + 75 = 0$$
$$a^2 - 8a + 15 = 0$$
$$a = 3, 5$$
The required values of a are 5 and 3.

15. The line segment joining the points A (3, 2) and B (5,1) is divided at the point P in the ratio 1:2 and it lies on the line 3*x* – 18*y* + *k* = 0. Find the value of *k*.

Solution:

We have, the line segment joining the points 4(3,2) and 6(5,1) is divided at the point P in the ratio 1 : 2.

$$\text{Coordinate of } P = \left(\frac{5(1)+3(2)}{1+2}, \frac{1(1)+2(2)}{1+2}\right)$$

$$= \left(\frac{11}{3}, \frac{5}{3}\right)$$

Also,

Point P lies on the line 3x– 18y+k= 0,

so,

$$3\left(\frac{11}{3}\right) - 18\left(\frac{5}{3}\right) + k = 0$$

On solving,

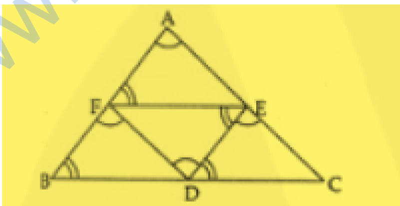
k-19 = 0

k=19

The required value of k is 19.

16. If $D\left(\frac{-1}{2}, \frac{5}{2}\right)$**,** $E(7,3)$ **and** $F\left(\frac{7}{2}, \frac{7}{2}\right)$ **are the midpoints of sides of** ΔABC**, find the area of the** ΔABC**.**

Solution:



In ABC triangle,

E is mid point of AC
F is mid point of AB
D is mid point of BC

As all the triangles are equal so,
ar(ABC)= 4 x ar(DEF)

Also,

$D\left(\frac{-1}{2}, \frac{5}{2}\right), E(7,3)$ and $F\left(\frac{7}{2}, \frac{7}{2}\right)$ are the midpoints of sides of triangle ABC.

Area of triangle DEF =

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$$

$$= \frac{1}{2}[-\frac{1}{2}\left(3 - \frac{7}{2}\right) + 7\left(\frac{7}{2} - \frac{5}{2}\right) + \frac{7}{2}(\frac{5}{3} - 3)$$

On solving,

$$\frac{1}{2}(29 - \frac{7}{4})$$

$$= \frac{22}{8}$$

$$= \frac{11}{4}$$

$$So, area\ of\ ABC\ triangle = 4 \times \frac{11}{4}$$

$$= 11\ square\ units$$

Therefore, Area of triangle ABC= 11 square units.

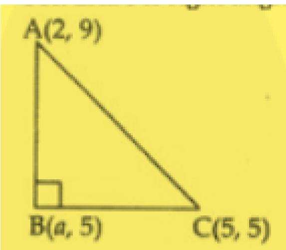
17. The points A (2, 9), B (*a*, 5) and C (5, 5) are the vertices of a triangle ABC right angled at B. Find the values of *a* and hence the area of ΔABC.

Solution:

We have, the points A (2, 9), B(a, 5) and C(5, 5) are the vertices of a ΔABC right angled at B.

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$



$$AB^2 = (a-2)^2 + (5-9)^2$$

$$AB^2 = a^2 - 4a + 20$$

$$BC^2 = (a-5)^2 + (5-5)^2$$

$$BC^2 = a^2 - 10a + 25$$

$$AC^2 = (5-2)^2 + (5-9)^2$$

$$AC^2 = 25$$

$$AC = 5$$

$Now, from\,(i),$

$5 = a^2 - 4a + 20 + a^2 - 10a + 25$

$2a^2 - 14a + 20 = 0$

$On\,solving,$

$a = 2, 5$

Here, a cannot be equal to 5 as at a=5, BC=0, which is not possible.
So, a=2
Therefore the points becomes, A (2, 9), B(2, 5) and C(5, 5)

Area of ABC,

$$= \frac{1}{2}[\mathbf{x_1}(\mathbf{y_2} - \mathbf{y_3}) + \mathbf{x_2}(\mathbf{y_3} - \mathbf{y_1}) + \mathbf{x_3}(\mathbf{y_1} - \mathbf{y_2})$$

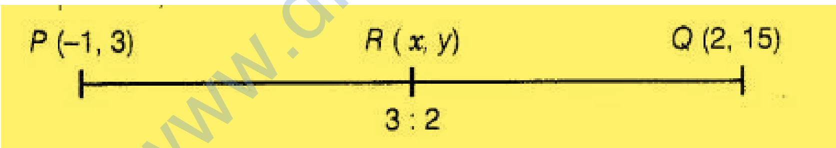
$$= \frac{1}{2}[2(5 - 5) + 2(5 - 9) + 5(9 - 5)$$

$$= 6$$

The required area of ΔABC is 6 sq units.

18. Find the coordinates of the point R on the line segment joining the points $\mathrm{P}(-1,3)$ and $\mathrm{Q}(2,5)$ such that $\mathrm{PR} = \frac{3}{5}\mathrm{PQ}$.

Solution:



Given:

$$\mathrm{PR} = \frac{3}{5}\mathrm{PQ}$$

$$\frac{PQ}{PR} = \frac{5}{3}$$

$$\frac{PR + RQ}{PR} = \frac{5}{3}$$

$$1 + \frac{RQ}{PR} = \frac{5}{3}$$

$$\frac{RQ}{PR} = \frac{2}{3}$$

$$RQ : PR = 2 : 3$$

$$or,$$

$$PR : RQ = 3 : 2$$

Taking R(x,y) be the point which divides line PQ in 3:2,

(x,y)=$\left(\frac{3(2)+2(-1)}{3+2}, \frac{3(5)+2(3)}{3+2}\right)$

(x,y)=$\left(\frac{4}{5}, \frac{21}{5}\right)$

19. Find the values of *k* if the points A (*k* + 1, 2*k*), B (3*k*, 2*k* + 3) and C (5*k* – 1, 5*k*) are collinear.

Solution:

If three points are collinear, then the area of triangle formed by these points is zero.
As, the points A(k + 1,2k), B(3k, 2k + 3) and C(5k -1, 5k) are collinear.

Then, area of ΔABC = 0,

$$\frac{1}{2}[\mathbf{x_1}(\mathbf{y_2} - \mathbf{y_3}) + \mathbf{x_2}(\mathbf{y_3} - \mathbf{y_1}) + \mathbf{x_3}(\mathbf{y_1} - \mathbf{y_2}) = 0$$

We have,

$A(k + 1, 2k)$

$B(3k, 2k + 3)$

$C(5k - 1, 5k)$

So,

$$\frac{1}{2}[k + 1(2k + 3 - 5k) + 3k(5k - 2k) + 5k - 1(2k - 2k + 3)] = 0$$

On solving we get,

$$\frac{1}{2}(6k^2 - 15k + 6) = 0$$

$$2k^2 - 5k + 2 = 0$$

$$(k-2)(2k-1) = 0$$

So,

$$k = 2$$

$$or, k = \frac{1}{2}$$

The required values of k are 2 and 1/2.

20. Find the ratio in which the line 2*x* + 3*y* – 5 = 0 divides the line segment joining the points (8, –9) and (2, 1). Also find the coordinates of the point of division.

Solution:

Taking that the line 2x + 3y – 5 = 0 divides the line segment joining the points A (8, – 9) and B (2,1) in the ratio λ: 1 at point P.

$$P\left(\frac{2\lambda+8}{\lambda+1},\frac{\lambda-9}{\lambda+1}\right)$$

Also,

P lies on 2x+ 3y– 5 = 0, so,

$$2\left(\frac{2\lambda+8}{\lambda+1}\right)+3\left(\frac{\lambda-9}{\lambda+1}\right)-5=0$$

$$2(2\lambda+8)+3(\lambda-9)-5(\lambda+1)=0$$

$$2\lambda-16=0$$

$$\lambda=8$$

So,

$\lambda:1=8:1$

So, point P divides in the ratio, 8:1,

$$P\left(\frac{2(8)+8}{(8)+1},\frac{(8)-9}{(8)+1}\right)$$

$$P\left(\frac{8}{3},\frac{-1}{9}\right)$$

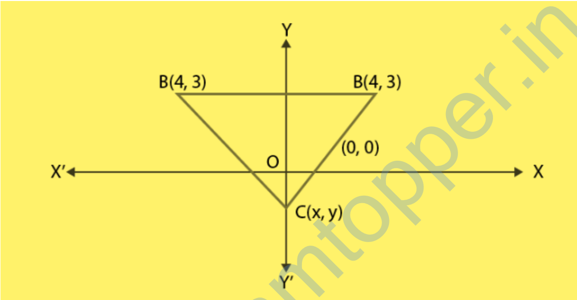
Exercise No. 7.4

Long Answer Questions:

Question:

1. If (– 4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.

Solution:
Let us take the vertices be (x,y)



Distance between (x,y) & (4,3) is = $\sqrt{((x-4)^2 + (y-3)^2)}$ ……(1)
Distance between (x,y) & (-4,3) is = $\sqrt{((x+4)^2 + (y-3)^2)}$ ……(2)
Distance between (4,3) &(-4,3) is =$\sqrt{((4+4)^2 + (3-3)^2)}$
$= \sqrt{(8)^2}$
$= 8$

As given in the question,
Equation (1) = (2)
$(x-4)^2 = (x+4)^2$
$x^2-8x+16 = x^2+8x+16$
$16x = 0$
$x = 0$

Also,
equation (1) = 8
$(x-4)^2+(y-3)^2 = 64$ ……… (3)

Putting the value of x in (3)

We get,
$(0-4)^2+(y-3)^2 = 64$
$(y-3)^2 = 64-16$
$(y-3)^2 = 48$
$y-3 = \sqrt{48}$

$y = 3+4\sqrt{3}, 3-4\sqrt{3}$

We do not take $y = 3+4\sqrt{3}$ as if $y = 3+4\sqrt{3}$ then origin cannot be at interior of triangle
Therefore, the third vertex = $(0, 3-4\sqrt{3})$

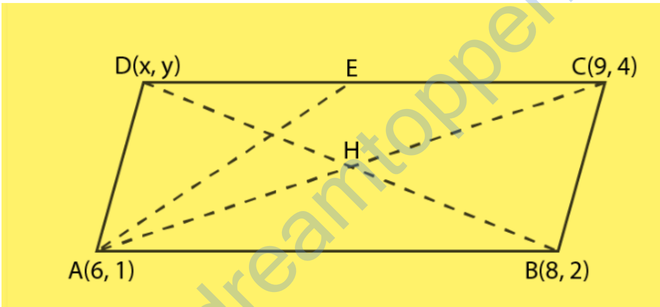
2. A (6, 1), B (8, 2) and C (9, 4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of △ ADE.

Solution:
As given in the question,

The three vertices of a parallelogram ABCD are A (6, 1), B (8, 2) and C (9, 4)

Let the fourth vertex of parallelogram = (x, y),

We know that, diagonals of a parallelogram bisect each other



Mid – point of a line segment joining the points (x_1, y_1) and (x_2, y_2) is given by,

$$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

Midpoint of BD=Midpoint of AC

$$\frac{8+x}{2}, \frac{2+y}{2} = \frac{6+9}{2}, \frac{1+4}{2}$$

$$\frac{8+x}{2}, \frac{2+y}{2} = \frac{15}{2}, \frac{5}{2}$$

So,

$$\frac{8+x}{2} = \frac{15}{2}$$

$$x = 7$$

And

$$\frac{2+y}{2} = \frac{5}{2}$$

$$y = 3$$

So, fourth vertex is D(7,3).

Now, Midpoint of DC,

$$\frac{7+9}{2}, \frac{3+4}{2} = E$$

$$8, \frac{7}{2} = E$$

Now,

Area of ΔABC with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3); $= \frac{1}{2}$ $[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of ΔADE with vertices A (6, 1), D (7, 3) and E (8, $\frac{7}{2}$)

Putting value in the formula we get,

The required area of ΔADE is $\frac{3}{4}$ sq. units.

3. The points A (x_1, y_1), B (x_2, y_2) and C $(x_3\ y_3)$ are the vertices of ABC.
(i) The median from A meets BC at D. Find the coordinates of the point D.
(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1
(iii) Find the coordinates of points Q and R on medians BE and CF, respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1
(iv) What are the coordinates of the centroid of the triangle ABC?

Solution:

Given,

The vertices of ΔABC = A, B and C

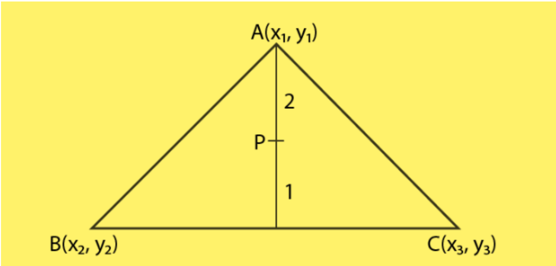
Coordinates of A, B and C = A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)

(i) As given in the question D is the mid – point of BC and it bisect the line into two equal parts.

Coordinates of the mid – point of BC;

$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

(ii) Let the coordinates of a point P be (x, y)



Given,

The ratio in which the point P(x, y), divide the line joining,

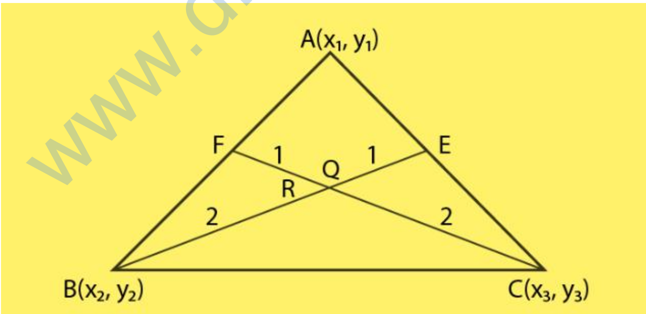
$A(x_1, y_1)$ and $D((x_2+x_3)/2\ ,\ (y_2+y_3)/2)) = 2:1$

Then,
Coordinates of P =

$$P = \left(\frac{2 \times \frac{x_2 + x_3}{2} + 1 \times x_1}{2+1}, \frac{2 \times \frac{y_2 + y_3}{2} + 1 \times y_1}{2+1} \right)$$

$$P = \left(\frac{x_2 + x_3 + x_1}{3}, \frac{y_2 + y_3 + y_1}{3} \right)$$

(iii) Let the coordinates of a point Q be (p, q)



Given,

The point Q (p, q), divides the line B and E in 2:1,

$$Q = \left(\frac{2 \times \frac{x_1 + x_3}{2} + 1 \times x_2}{2+1}, \frac{2 \times \frac{y_1 + y_3}{2} + 1 \times y_2}{2+1} \right)$$

$$Q = \left(\frac{x_2 + x_3 + x_1}{3}, \frac{y_2 + y_3 + y_1}{3} \right)$$

Also,

Midpoint of AC = Coordinates of E (As BE is median of CE),

$$E = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

Also point R divides the line CF in the ratio 2:1,

So coordinates of R =

$$R = \left(\frac{2 \times \frac{x_1 + x_2}{2} + 1 \times x_3}{2+1}, \frac{2 \times \frac{y_1 + y_2}{2} + 1 \times y_3}{2+1}\right)$$

$$R = \left(\frac{x_2 + x_3 + x_1}{3}, \frac{y_2 + y_3 + y_1}{3}\right)$$

Also,

Midpoint of AB = Coordinate of F

$$F = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

(iv) Coordinates of the centroid of the ΔABC;

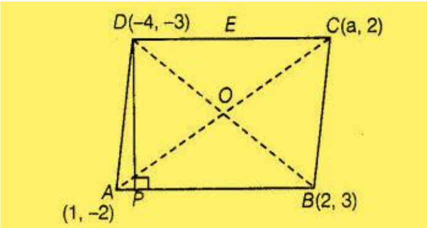
$$R = \left(\frac{Sum\ of\ all\ coordinates\ of\ all\ vertices}{3}, \frac{Sum\ of\ all\ coordinates\ of\ all\ vertices}{3}\right)$$

$$R = \left(\frac{x_2 + x_3 + x_1}{3}, \frac{y_2 + y_3 + y_1}{3}\right)$$

4. If the points A (1, –2), B (2, 3) C (*a*, 2) and D (– 4, –3) form a parallelogram, find the value of *a* and height of the parallelogram taking AB as base.

Solution:

In parallelogram, we know that, diagonals are bisects each other so,
Mid-point of AC = mid-point of BD



$$\left(\frac{1+a}{2},\frac{-2+2}{2}\right)=\left(\frac{2-4}{2},\frac{3-3}{2}\right)$$

$$\frac{1+a}{2}=-\frac{2}{2}$$

$$a=-1$$

So, the required value of a is – 3.

Given that, AS as base of a parallelogram and drawn a perpendicular from D to AS which meet AS at P. So, DP is a height of a parallelogram.

Equation of the points passing through the points (1,-2) and (2,3) is :

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y+2=\frac{3+2}{2-1}(x-1)$$

$$5x-y=7$$

Now, slope of AB,

$$m_1=\frac{y_2-y_1}{x_2-x_1}$$

$$m_1=\frac{3+2}{2-1}$$

$$m_1=5$$

Taking slope of DP m_2, as DP is perpendicular to AB, we can apply the condition,

$$m_1.m_2=-1$$

$$5.m_2=-1$$

$$m_2=\frac{-1}{5}$$

Now, equation of DP having slope m_2, passing through point (-4,-3) is,

$$\text{y-y}_1=m_2(x-x_1)$$

$$y+3=\frac{-1}{5}(x+4)$$

$x + 5y = -19$

Now, on solving both the equations we get,

$$x = \frac{8}{13}$$

$$y = \frac{-51}{13}$$

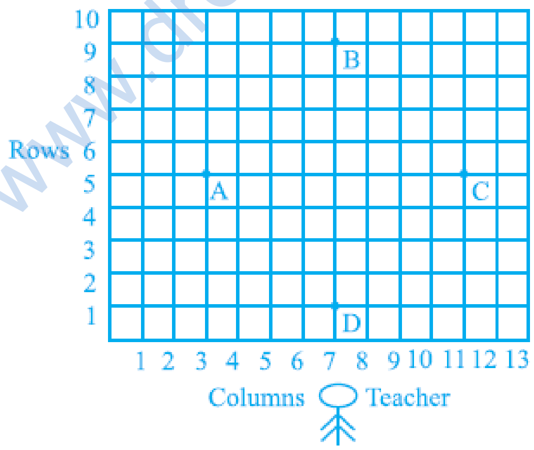
$$Coordinates\ of\ P = \left(\frac{8}{13}, \frac{-51}{13}\right)$$

So, height of parallelogram,

$$\text{DP} = \sqrt{\left(\frac{8}{13} + 4\right)^2 - \left(\frac{-51}{13} + 4\right)^2}$$

$$DP = \frac{12\sqrt{26}}{13}$$

5. Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C and D are the positions of four students as shown in figure. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D? If so, what should be his position?

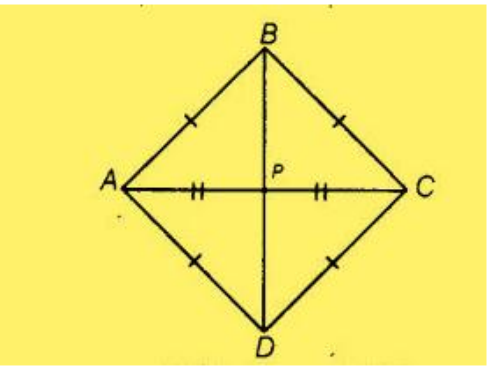


Solution:

Yes, from the figure we observe that the positions of four students A, B, C and D are (3, 5), (7, 9), (11, 5) and (7,1) respectively and these are four vertices of a quadrilateral.

Now, we will find the type of this quadrilateral.

For this, we will find all its sides.



By distance formula,

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(7-3)^2 + (9-5)^2}$

$AB = 4\sqrt{2}$

$BC = \sqrt{(11-7)^2 + (5-9)^2}$

$BC = 4\sqrt{2}$

$CD = \sqrt{(7-11)^2 + (1-5)^2}$

$CD = 4\sqrt{2}$

Also,

$DA = \sqrt{(3-7)^2 + (5-1)^2}$

$DA = 4\sqrt{2}$

Now, length of both diagonals,

$AC = \sqrt{(11-3)^2 + (5-5)^2}$

$AC = 8$

$BD = \sqrt{(7-7)^2 + (1-9)^2}$

$BD = 8$

As,

$AB = BC = CD = DA$

Also, $AC = BD$

We see that, $AB = BC = CD = DA$, *all sides are equal*.

So, it represents a square.

Coordinates of point P = Mid point of AC

$$\left(\frac{3+11}{2}, \frac{5+5}{2}\right)$$

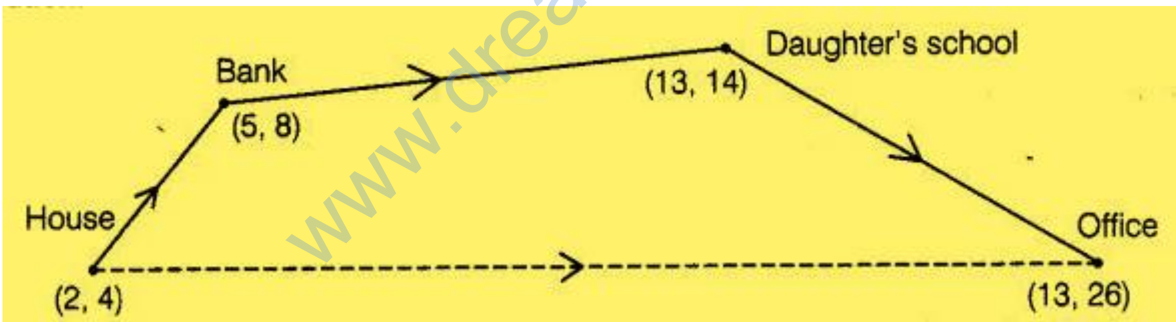
$$(7,5)$$

The required position of jaspal is (7,5).

6.Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching his office? (Assume that all distances covered are in straight lines).

If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km.

Solution:



In the figure we had located every place with its coordinates and direction,

By distance formula,

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance between house and the bank $= \sqrt{(5-2)^2 + (8-4)^2}$

$= 5$

Distance between bank and the daughter school $= \sqrt{(13-5)^2 + (14-8)^2}$

$= 10$

Distance between the daughter school and office $= \sqrt{(13-13)^2 - (26-14)^2}$

$= 12$

Now,
Total distance = 5+10+12
=27 units

Now,

$$\text{Distance between house to offices} = \sqrt{(13-2)^2 + (26-4)^2}$$
$$= 24.6\,km$$

Therefore, extra distance travelled by Ayush in reaching his office;
= 27 – 24.6
= 2.4 km

Hence, the required extra distance travelled by Ayush is 2.4 km.