Mathematics

(Chapter – 10) (Circles)

(Class X)

Exercise 10.1

Question 1:

How many tangents can a circle have?

Answer 1:

A circle can have infinite number of tangents because a circle have infinite number of points on it and at every point a tangent can be drawn.

Question 2:

Fill in the blanks:

(i) A tangent to a circle intersects it in _________ point (s).

(ii) A line intersecting a circle in two points is called a _________.

(iii) A circle can have _________ parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called _________.

Answer 2:

(i) A tangent to a circle intersects it in ***one*** point (s).

(ii) A line intersecting a circle in two points is called a ***Secant***.

(iii) A circle can have ***two*** parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called ***point of contact***.

Question 3:

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

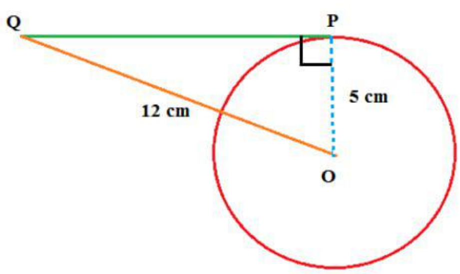
(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm.

Answer 3:

(D) $\sqrt{119}$ cm.

Solution:

In ΔOPQ, angle P is right angle. [Since radius is perpendicular to tangent]



Using Pythagoras theorem, $OQ^2 = PQ^2 + OP^2$

$\Rightarrow 12^2 = PQ^2 + 5^2$

$\Rightarrow 144 = PQ^2 + 25$

$\Rightarrow PQ^2 = 144 - 25 = 119$

$\Rightarrow PQ = \sqrt{119}$

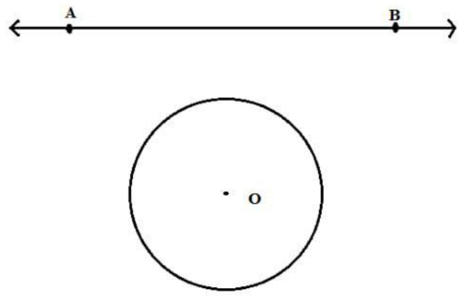
Hence, the option (D) is correct.

Question 4:

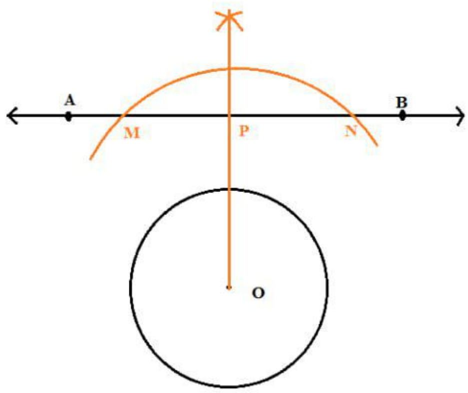
Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Answer 4:

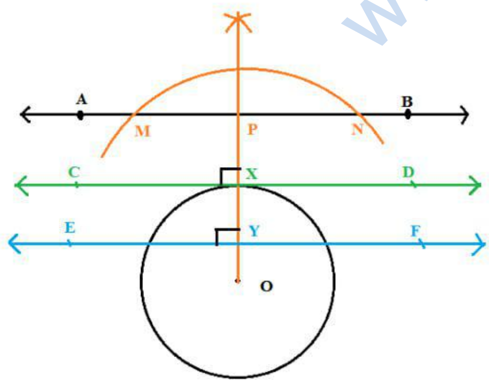
Consider a circle with centre O. Let AB is the given line.



Now draw a perpendicular from O to line AB, which intersect AB at P.



Now take two points on the line PO, one at circle X and another Y inside the circle. Draw lines parallel to AB and passing through X and Y. CD and EF are the required lines.



Mathematics

(Chapter – 10) (Circles)

(Class – X)

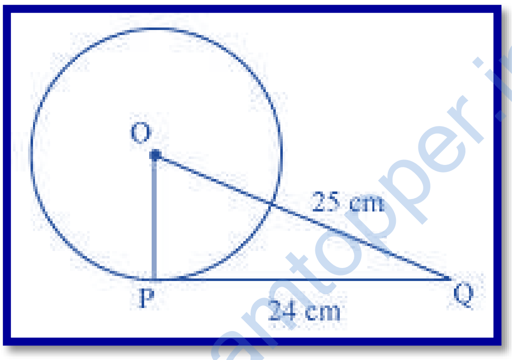
Exercise 10.2

Question 1:

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

Answer 1:



Let O be the centre of the circle.

Given that,

OQ = 25cm and PQ = 24 cm

As the radius is perpendicular to the tangent at the point of contact,

Therefore, OP $\perp$ PQ

Applying Pythagoras theorem in ΔOPQ, we obtain

$OP^2 + PQ^2 = OQ^2$

$OP^2 + 24^2 = 25^2$

$OP^2 = 625 - 576$

$OP^2 = 49$

$OP = 7$

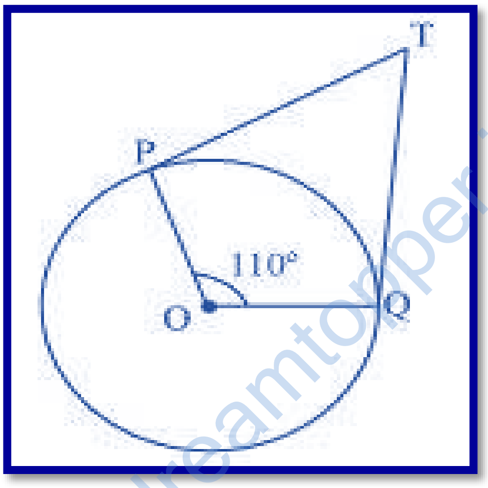
Therefore, the radius of the circle is 7 cm.

Hence, alternative (A) is correct

Question 2:

In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to

(A) 60° (B) 70° (C) 80° (D) 90°



Answer 2:

It is given that TP and TQ are tangents.

Therefore, radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OP \perp TP$ and $OQ \perp TQ$

$\angle OPT = 90^\circ$

$\angle OQT = 90^\circ$

In quadrilateral POQT,

Sum of all interior angles = 360°

$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$

$\Rightarrow 90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$

$\Rightarrow \angle PTQ = 70^\circ$

Hence, alternative (B) is correct

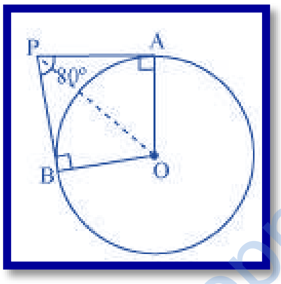
Question 3:

If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of 80°, then ∠POA is equal to

(A) 50° (B) 60° (C) 70° (D) 80°

Answer 3:

It is given that PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.
Thus, OA ⊥ PA and OB ⊥ PB

∠OBP = 90º and ∠OAP = 90º

In AOBP,
Sum of all interior angles = 360°

∠OAP + ∠APB +∠PBO + ∠BOA = 360°

90º + 80º +90º + ∠BOA = 360°

∠BOA = 100°

In ΔOPB and ΔOPA,
AP = BP (Tangents from a point)
OA = OB (Radii of the circle)
OP = OP (Common side)

Therefore, ΔOPB ≅ ΔOPA (SSS congruence criterion)

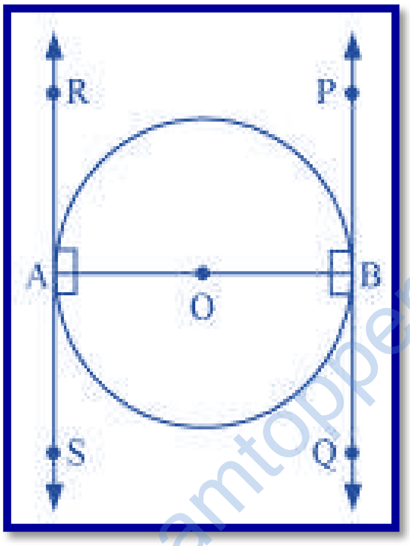
And thus, ∠POB = ∠POA

$$\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$$

Hence, alternative (A) is correct.

Question 4:
Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer 4:



Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA ⊥ RS and OB ⊥ PQ

$\angle OAR = 90^\circ$ $\angle OAS$

$= 90^\circ$

$\angle OBP = 90^\circ$

$\angle OBQ = 90^\circ$

It can be observed that

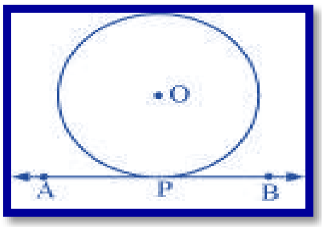
$\angle OAR = \angle OBQ$ (Alternate interior angles)

$\angle OAS = \angle OBP$ (Alternate interior angles)

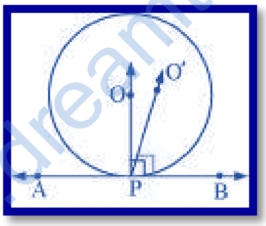
Since alternate interior angles are equal, lines PQ and RS will be parallel.

Question 5:
Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer 5:
Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



We have to prove that the line perpendicular to AB at P passes through centre O. We shall prove this by contradiction method.
Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'. Join OP and O'P.



As perpendicular to AB at P passes through O', therefore,
$\angle O'PB = 90°$ .... (1)

O is the centre of the circle and P is the point of contact. We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.
$\therefore \angle OPB = 90°$ .... (2)

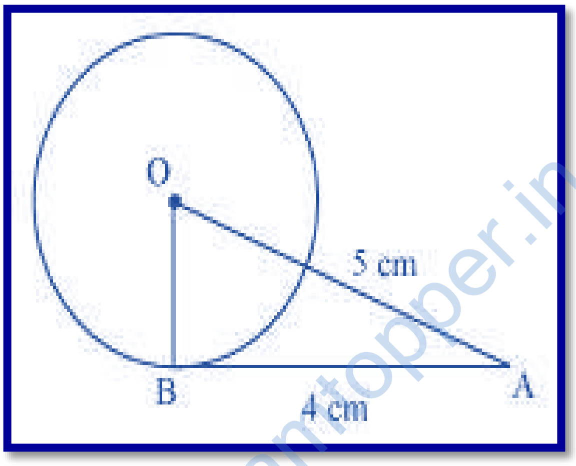
Comparing equations (1) and (2), we obtain
$\angle O'PB = \angle OPB$ .... (3)

From the figure, it can be observed that,
$\angle O'PB < \angle OPB$ .... (4)

Therefore, $\angle O'PB = \angle OPB$ is not possible. It is only possible, when the line O'P coincides with OP.
Therefore, the perpendicular to AB through P passes through centre O.

Question 6:
The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer 6:



Let us consider a circle centered at point O.

AB is a tangent drawn on this circle from point A.

Given that,

OA = 5cm and AB = 4 cm

In ΔABO,

$OB \perp AB$ (Radius $\perp$ tangent at the point of contact)

Applying Pythagoras theorem in ΔABO, we obtain

$AB^2 + BO^2 = OA^2$

$4^2 + BO^2 = 5^2$

$16 + BO^2 = 25$

$BO^2 = 9$

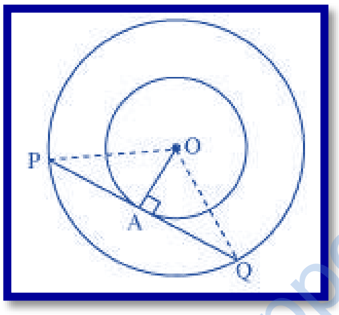
$BO = 3$

Hence, the radius of the circle is 3 cm.

Question 7:

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer 7:



Let the two concentric circles be centered at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A. Therefore, PQ is tangent to the smaller circle.

OA $\perp$ PQ (As OA is the radius of the circle)

Applying Pythagoras theorem in ΔOAP, we obtain

$OA^2 + AP^2 = OP^2$

$3_2 + AP_2 = 5_2$

$9 + AP^2 = 25$

$AP^2 = 16$

$AP = 4$

In ΔOPQ,

Since OA $\perp$ PQ,

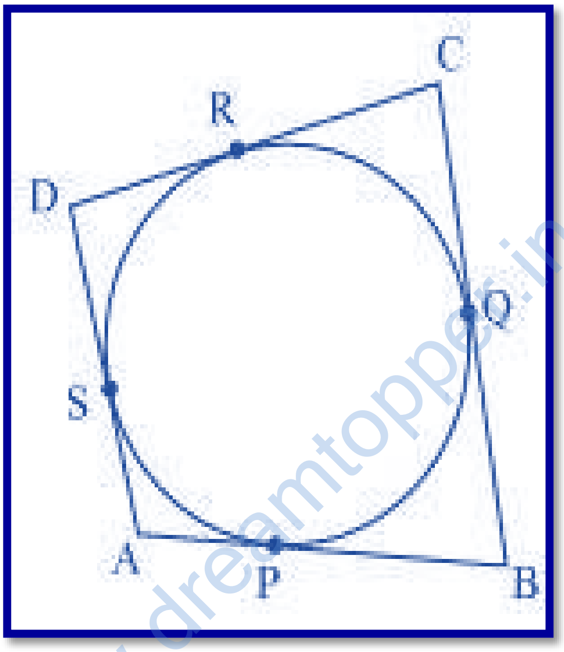
AP = AQ (Perpendicular from the center of the circle bisects the chord)

$\therefore PQ = 2AP = 2 \times 4 = 8$

Therefore, the length of the chord of the larger circle is 8 cm.

Question 8:

A quadrilateral ABCD is drawn to circumscribe a circle (see given figure) Prove that AB + CD = AD + BC



Answer 8:

It can be observed that

DR = DS (Tangents on the circle from point D) ………….. (1)

CR = CQ (Tangents on the circle from point C) …………… (2)

BP = BQ (Tangents on the circle from point B) …………… (3)

AP = AS (Tangents on the circle from point A) …………… (4)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

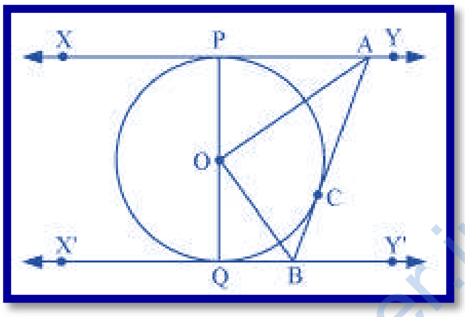
(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

Question 9:

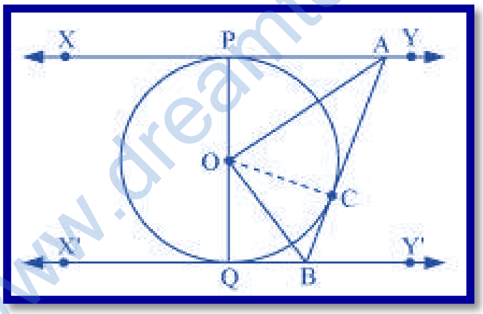
In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B.

Prove that $\angle AOB = 90^\circ$.



Answer 9:

Let us join point O to C.



In ΔOPA and ΔOCA,
OP = OC (Radii of the same circle)
AP = AC (Tangents from point A)
AO = AO (Common side)
ΔOPA ≅ ΔOCA (SSS congruence criterion)
∠POA = ∠COA .... (*i*)

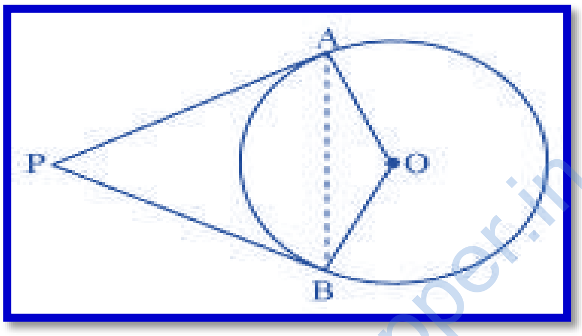
Similarly, ΔOQB ≅ ΔOCB
∠QOB = ∠COB .... (*ii*)

Since POQ is a diameter of the circle, it is a straight line.
Therefore, ∠POA + ∠COA + ∠COB + ∠QOB = 180º
From equations (*i*) and (*ii*), it can be observed that
2∠COA + 2 ∠COB = 180º
∠COA + ∠COB = 90º
∠AOB = 90°

Question 10:

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer 10:



Let us consider a circle centered at point O. Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends ∠AOB at center O of the circle.

It can be observed that

OA (radius) ⊥ PA (tangent)

Therefore, ∠OAP = 90°

Similarly, OB (radius) ⊥ PB (tangent)

∠OBP = 90°

In quadrilateral OAPB,

Sum of all interior angles = 360º

∠OAP +∠APB+∠PBO +∠BOA = 360º

90º + ∠APB + 90º + ∠BOA = 360º

∠APB + ∠BOA = 180º

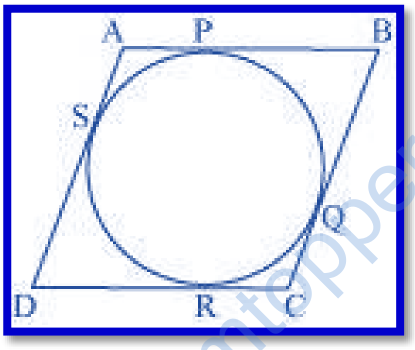
Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Question 11:
Prove that the parallelogram circumscribing a circle is a rhombus.

Answer 11:
Since ABCD is a parallelogram,

AB = CD ……………………………………(1)

BC = AD …………………………………..(2)



It can be observed that

DR = DS (Tangents on the circle from point D)

CR = CQ (Tangents on the circle from point C)

BP = BQ (Tangents on the circle from point B)

AP = AS (Tangents on the circle from point A)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

On putting the values of equations (1) and (2) in this equation, we obtain

2AB = 2BC

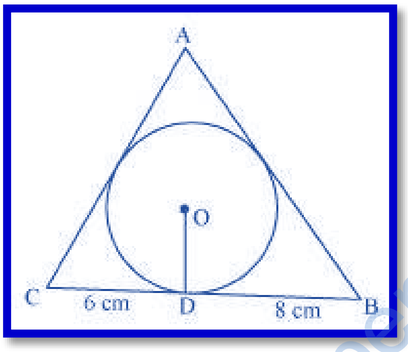
AB = BC ……………………………………(3)

Comparing equations (1), (2), and (3), we obtain

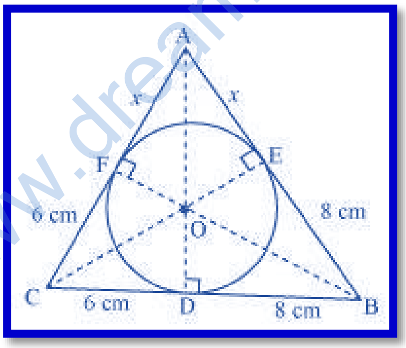
AB = BC = CD = DA

Hence, ABCD is a rhombus.

Question 12:
A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see given figure). Find the sides AB and AC.



Answer 12:



Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of the line segment AF be *x.*

In ΔABC,

CF = CD = 6cm (Tangents on the circle from point C)
BE = BD = 8cm (Tangents on the circle from point B)
AE = AF = x (Tangents on the circle from point A)
AB = AE + EB = x + 8
BC = BD + DC = 8 + 6 = 14
CA = CF + FA = 6 + x
2s = AB + BC + CA

= x + 8 + 14 + 6 + x

= 28 + 2x s = 14 + x

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{\{14+x\}\{(14+x)-14\}\{(14+x)-(6+x)\}\{(14+x)-(8+x)\}}$$
$$= \sqrt{(14+x)(x)(8)(6)}$$
$$= 4\sqrt{3(14x+x^2)}$$

Area of ΔOBC = $\frac{1}{2}\times OD\times BC = \frac{1}{2}\times 4\times 14 = 28$

Area of ΔOCA = $\frac{1}{2}\times OF\times AC = \frac{1}{2}\times 4\times(6+x) = 12+2x$

Area of ΔOAB = $\frac{1}{2}\times OE\times AB = \frac{1}{2}\times 4\times(8+x) = 16+2x$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$$4\sqrt{3(14x+x^2)} = 28 + 12 + 2x + 16 + 2x$$
$$\Rightarrow 4\sqrt{3(14x+x^2)} = 56+4x$$
$$\Rightarrow \sqrt{3(14x+x^2)} = 14+x$$
$$\Rightarrow 3(14x+x^2) = (14+x)^2$$
$$\Rightarrow 42x+3x^2 = 196+x^2+28x$$
$$\Rightarrow 2x^2+14x-196 = 0$$
$$\Rightarrow x^2+7x-98=0$$
$$\Rightarrow x^2+14x-7x-98=0$$
$$\Rightarrow x(x+14)-7(x+14)=0$$
$$\Rightarrow (x+14)(x-7)=0$$

Either x + 14 = 0 or x − 7 =0

Therefore, x = −14 and 7

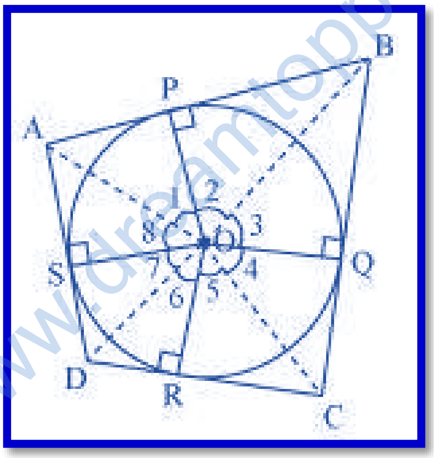
However, $x = -14$ is not possible as the length of the sides will be negative.
Therefore, $x = 7$

Hence, $AB = x + 8 = 7 + 8 = 15$ cm

$CA = 6 + x = 6 + 7 = 13$ cm

Question 13:
Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer 13:



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider ΔOAP and ΔOAS,

AP = AS	(Tangents from the same point)
OP = OS	(Radii of the same circle)
OA = OA	(Common side)
ΔOAP ≅ ΔOAS	(SSS congruence criterion)

thus, $\angle POA = \angle AOS$

$\angle 1 = \angle 8$

Similarly,

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^\circ$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^\circ$

$\angle AOB + \angle COD = 180^\circ$

Similarly, we can prove that BOC + DOA = 180º

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.