Chapter - 9
Circles

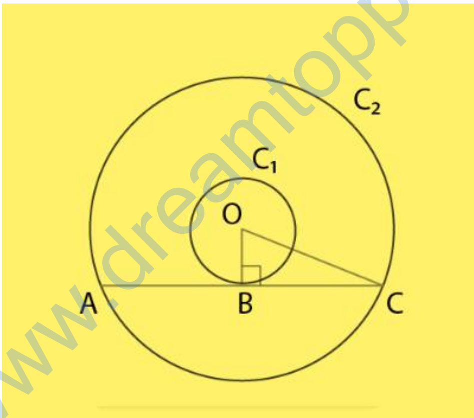
Exercise No. 9.1

Multiple Choice Questions:

Choose the correct answer from the given four options:

1. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is
(A) 3 cm
(B) 6 cm
(C) 9 cm
(D) 1 cm

Solution:

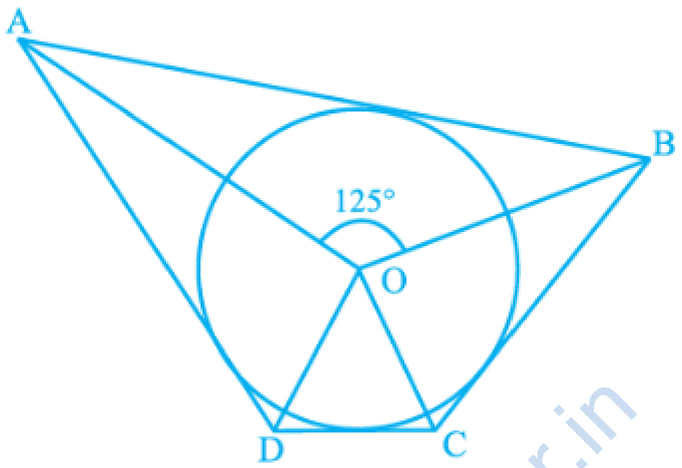


As given in the question,
OA = 4cm,
OB = 5cm
And,
OA ⊥ BC

Therefore,
$OB^2 = OA^2 + AB^2$
$5^2 = 4^2 + AB^2$
AB = 3cm
And,
BC = 2AB
= 2 × 3cm
= 6cm

2. In Fig., if AOB = 125°, then COD is equal to

(A) 62.5°
(B) 45°
(C) 35°
(D) 55°



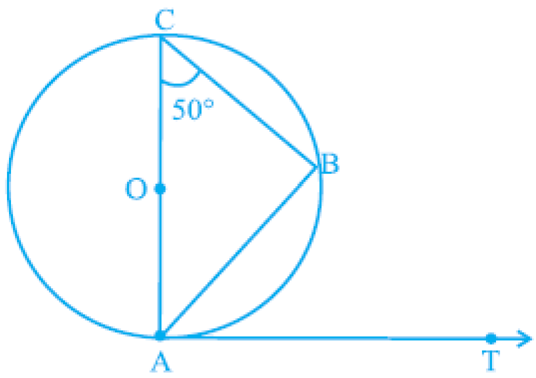
Solution:

ABCD is a quadrilateral circumscribing the circle

And we know that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the center of the circle.

So,
$\angle AOB + \angle COD = 180°$
$125° + \angle COD = 180°$
$\angle COD = 55°$

3. In Fig., AB is a chord of the circle and AOC is its diameter such that ∠ACB = 50°. If AT is the tangent to the circle at the point A, then ∠∠BAT **is equal to**
(A) 65°
(B) 60°
(C) 50°
(D) 40°



Solution:

As given in the question,

A circle with centre O, diameter AC and
$\angle ACB = 50°$

AT is a tangent to the circle at point A

As, angle in a semicircle is a right angle
$\angle CBA = 90°$

So using angle sum property of a triangle,

$\angle ACB + \angle CAB + \angle CBA = 180°$
$50° + \angle CAB + 90° = 180°$
$\angle CAB = 40°$ … (1)

As, tangent to at any point on the circle is perpendicular to the radius through point of contact,

We get,
$OA \perp AT$
$\angle OAT = 90°$
$\angle OAT + \angle BAT = 90°$
$\angle CAT + \angle BAT = 90°$
$40° + \angle BAT = 90°$ [from equation (1)]

$\angle BAT = 50°$

4. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is
(A) 60 cm^2
(B) 65 cm^2

(C) 30 cm^2
(D) 32.5 cm^2

Solution:

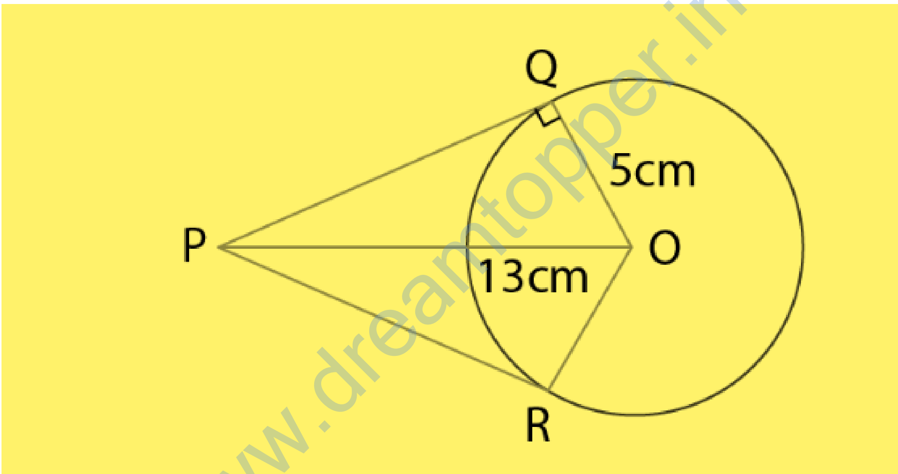
Construction: Draw a circle of radius 5 cm with center O.

Let P be a point at a distance of 13 cm from O.

Draw a pair of tangents, PQ and PR.

OQ = OR = radius = 5cm …equation (1)

And OP = 13 cm



Also, tangent to at any point on the circle is perpendicular to the radius through point of contact,

We get,

$OQ \perp PQ$ and $OR \perp PR$

$\triangle POQ$ and $\triangle POR$ are right-angled triangles.

By using Pythagoras Theorem in $\triangle PQO$,

$(Base)^2 + (Perpendicular)^2 = (Hypotenuse)^2$

$(PQ)^2 + (OQ)^2 = (OP)^2$
$(PQ)^2 + (5)^2 = (13)^2$
$(PQ)^2 + 25 = 169$
$(PQ)^2 = 144$

PQ = 12 cm

Tangents through an external point to a circle are equal.

So,

PQ = PR = 12 cm … (2)

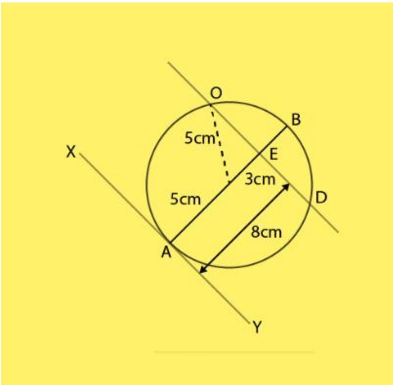
Therefore,
Area of quadrilateral PQRS, A = area of △POQ + area of △POR

Area of right angled triangle = ½ x base x height

A = (½ x OQ x PQ) + (½ x OR x PR)
A = (½ x 5 x 12) + (½ x 5 x 12)
A = 30 + 30
= 60 cm^2

5. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY isdrawn to the circle. The length of the chord CD parallel to XY and at adistance 8 cm from A is
(A) 4 cm
(B) 5 cm
(C) 6 cm
(D) 8 cm

Solution:



As given the question,

Radius of circle,
AO=OC = 5cm

AM=8CM
AM=OM+AO

OM =AM-AO

Putting these values in the equation,

OM= (8-5)
=3CM

OM is perpendicular to the chord CD.

In ΔOCM $<OMC=90^\circ$

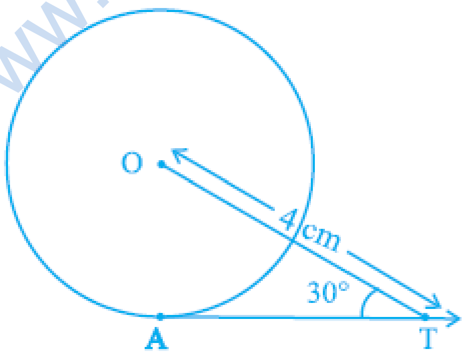
By Pythagoras theorem,

$OC^2=OM^2+MC^2$

Therefore,

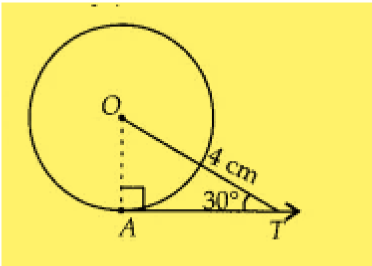
CD= 2 ×CM
= 8 cm

6. In Fig., AT is a tangent to the circle with centre O such that OT = 4 cm and ∠ $\angle OTA = 30^\circ$. Then AT is equal to
(A) 4 cm
(B) 2 cm
(C) $2\sqrt{3}$ cm
(D) $4\sqrt{3}$ cm



Solution:

(C)
Join OA
We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.



$\angle OAT = 90^o$

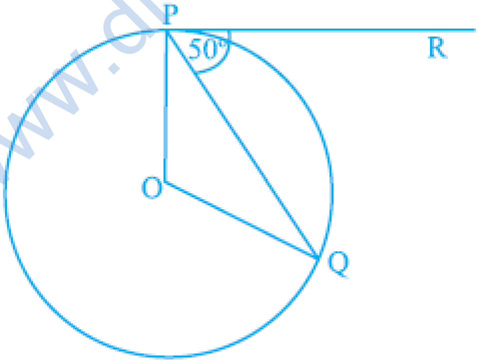
$In\, OAT,$

$\cos 30^o = \frac{OT}{AT}$

$\frac{\sqrt{3}}{2} = \frac{AT}{4}$

$AT = 2\sqrt{3}$ cm

7. In Fig., if O is the centre of a circle,PQ is a chord and the tangent PR at Pmakes an angle of 50° with PQ, then∠POQ is equal to
(A) 100°
(B) 80°
(C) 90°
(D) 75°



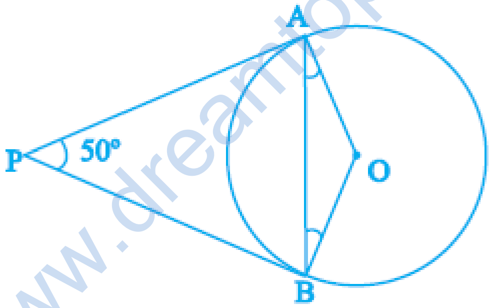
Solution:

(A)
Given,
∠QPR = 50°
We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.
∠OPR = 90°
∠OPQ + ∠QPR = 90°
∠OPQ = 90° – 50°
= 40° [as, ∠QPR = 50°]

Now,
OP = OQ = radius of circle
$\angle OQP = \angle OPQ$
$= 40°$
[Angles opposite to equal sides are equal]

In ΔOPQ,
$\angle O + \angle OPQ + \angle Q = 180°$ [Angle sum property]
$\angle POQ = 180° – (40 + 40°)$
$= 180° – 80°$
[$\angle OPQ = 40° = \angle Q$]
$\angle POQ = 100°$

8. In Fig., if PA and PB are tangents to the circle with centre O such that $\angle$ $\angle APB = 50°$, then $\angle\angle OAB$ is equal to
(A) 25°
(B) 30°
(C) 40°
(D) 50°



Solution:

(A)
Given,
PA and PB are tangent lines.
PA = PB [as, Length of tangents drawn from an external point to a circle is equal]
Let,
$\angle PBA = \angle PAB = \theta$

In ΔPAB,
$\angle P + \angle A + \angle B = 180°$ [Angle sum property]
$50° + \theta + \theta = 180°$
$2\theta = 180° – 50° = 130°$
$\theta = 65°$
Also,
$OA \perp PA$
[as,tangent at any point of a circle is perpendicular to the radius through the point of contact]

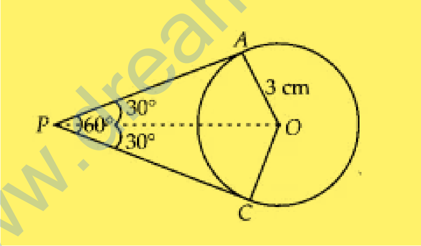
So,
$\angle PAO = 90°$
$\angle PAB + \angle BAO = 90°$
$65° + \angle BAO = 90°$
$\angle BAO = 90° - 65°$
$= 25°$
$\angle OAB = 25°$

9. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to

(A) $\frac{3}{2}\sqrt{3}$ cm
(B) 6 cm
(C) 3 cm
(D) $3\sqrt{3}$ cm

Solution:

(D)
Let P be an external point and a pair of tangents is drawn from point P such that the angle between two tangents is 60°.



Join OA and OP.
Also,
OP is a bisector line of $\angle APC$.
$\angle APO = \angle CPO = 30°$
And,
$OA \perp AP$
[Tangent at any point of a circle is perpendicular to the radius through the point of contact.]
$\angle OAP = 90°$

In right angled ΔOAP,

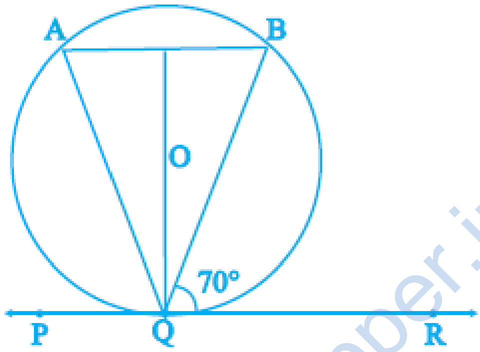
$$\tan 30^o = \frac{OA}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{3}{AP}$$

$$AP = 3\sqrt{3}\,cm$$

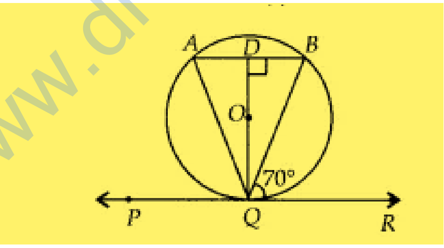
So, the length of each tangent is 3√3 cm.

10. In Fig., if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70°$, then∠ $\angle AQB$ is equal to
(A) 20°
(B) 40°
(C) 35°
(D) 45°



Solution:

(B) Given,
AB ll PR



Therefore,
∠ABQ = ∠BQR = 70° [Alternate angles]
Also,
QD is perpendicular to AB and QD bisects AB.

In ΔQDA and ΔQDB,
∠QDA = ∠QDB [90° each]
AD = BD
QD = QD [Common side]
So,
ΔADQ ≅ ΔBDQ [by SAS congruency]

Therefore,
∠QAD = ∠QBD [CPCT] …(i)
But,

$\angle QBD = \angle ABQ = 70°$
$\angle QAD = 70°$ [From (i)]

Now, in ΔABQ,
$\angle A + \angle B + \angle AQB = 180°$. [Angle sum property]
$\angle AQB = 180° – (70° + 70°)$
$= 40°$

Exercise No. 9.2

Short Answer Questions with Reasoning:

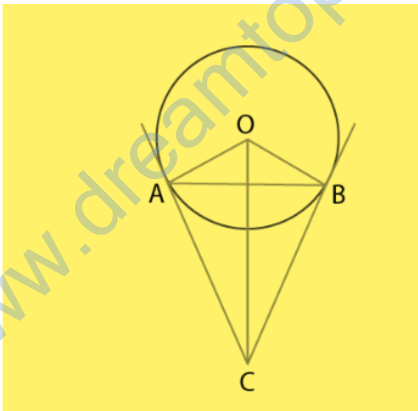
Write 'True' or 'False' and justify your answer in each of the following:

1. If a chord AB subtends an angle of 60° at the centre of a circle, then angle between the tangents at A and B is also 60°.

Solution:

False

Explanation:
Let us consider the given figure. In which we have a circle with centre O and AB a chord with $\angle AOB = 60°$



As, tangent to any point on the circle is perpendicular to the radius through point of contact,

We get,

$OA \perp AC$ and $OB \perp CB$
$\angle OBC = \angle OAC = 90°$ … eq(i)

Using angle sum property of quadrilateral in Quadrilateral AOBC,
We get,

$\angle OBC + \angle OAC + \angle AOB + \angle ACB = 360°$
$90° + 90° + 60° + \angle ACB = 360°$
$\angle ACB = 120°$

Therefore, the angle between two tangents is 120°.

And, we can conclude that,the given statement is false.

2. The length of tangent from an external point on a circle is always greater than the radius of the circle.

Solution:

False

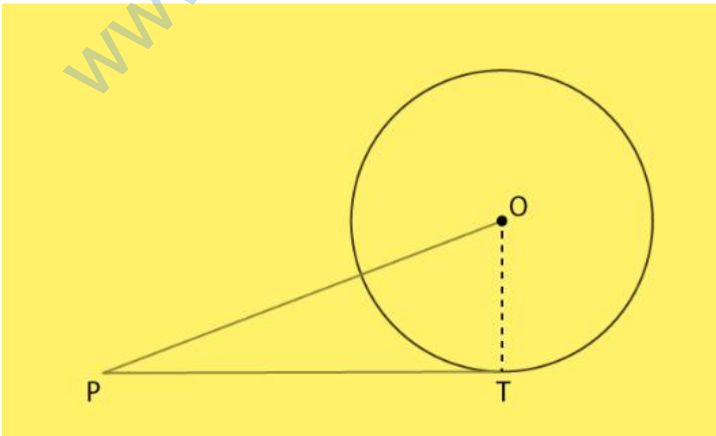
Explanation:
Length of tangent from an external point P on a circle may or may not be greater than the radius of the circle.

3. The length of tangent from an external point P on a circle with centre O is always less than OP.

Solution:

True

Explanation:
Consider the figure of a circle with centre O.
Let PT be a tangent drawn from external point P.
Now, Joint OT.
OT $\perp$ PT



We know that,
Tangent at any point on the circle is perpendicular to the radius through point of contact
Therefore, OPT is a right-angled triangle formed.

We also know that,
In a right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.

So,
OP > PT or PT < OP
Hence, length of tangent from an external point P on a circle with center O is always less than OP.

4. The angle between two tangents to a circle may be 0°.

Solution:

True

Explanation:
The angle between two tangents to a circle may be 0°only when both tangent lines coincide or are parallel to each other.

5. If angle between two tangents drawn from a point P to a circle of radius a and centre O is 90°, then $OP = a\sqrt{2}$.

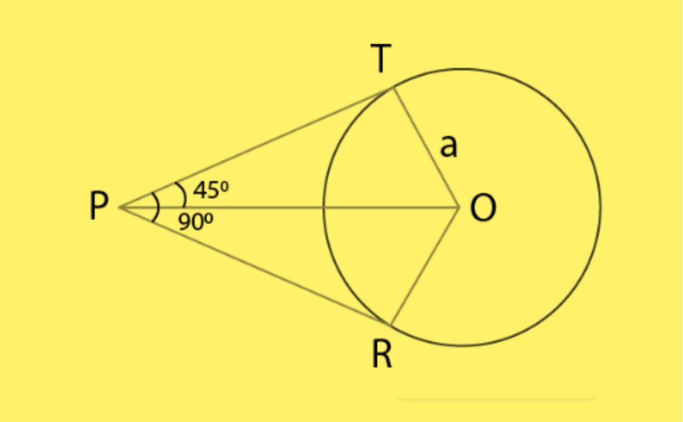
Solution:

True.

Tangent is always perpendicular to the radius at the point of contact.
So, ∠OTP = 90

If 2 tangents are drawn from an external point, then they are equally inclined to the line segment joining the centre to that point.

Let us consider the following figure,



From point P, two tangents are drawn.
Given,
OT = a

Also, line OP bisects the ∠RPT
∠TPO = ∠RPO = 45°
Also,
OT ⊥ TP
∠OTP = 90°
In right angled ΔOTP,

$$\sin 45^o = \frac{OT}{OP}$$

$$\frac{1}{\sqrt{2}} = \frac{a}{OP}$$

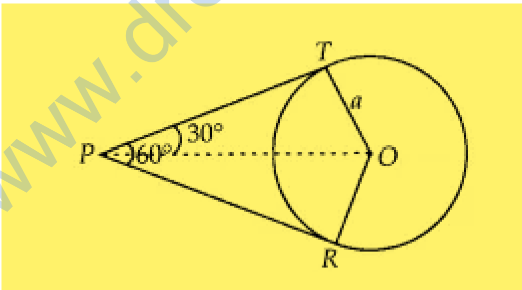
$$OP = a\sqrt{2}$$

6. If angle between two tangents drawn from a point P to a circle of radius *a* and centre O is 60°, then $OP = a\sqrt{3}$.

Solution:

False

Explanation:
From point P, two tangents are drawn.
Given,
OT= a



Also, line OP bisects the ∠RPT.
∠TPO = ∠RPO = 30°
Also,
OT ⊥ PT
∠OTP = 90°
In right angled ΔOTP,

$$\sin 30^o = \frac{OT}{OP}$$

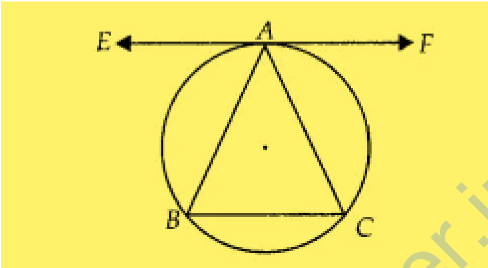
$$\frac{1}{2} = \frac{a}{OP}$$

OP = 2a
OP = 2a

7. The tangent to the circumcircle of an isosceles triangle ABC at A, in which AB = AC, is parallel to BC.

Solution:

True
Explanation:
Let EAF be tangent to the circumcircle of ΔABC.



To prove: EAF ∥ BC
We have, ∠EAB = ∠ACB …(i)
[Angle between tangent and chord is equal to angle made by chord in the alternate segment]

Here, AB = AC
∠ABC = ∠ACB …(ii)

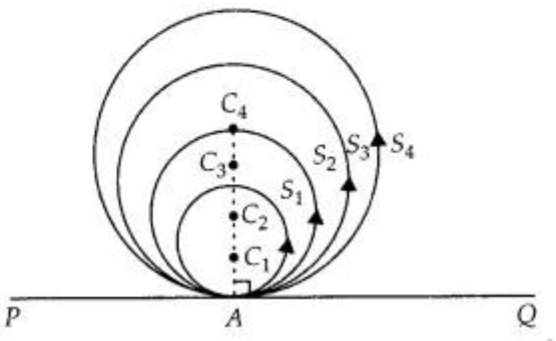
From equation (i) and (ii), we get
∠EAB = ∠ABC
Alternate angles are equal.
EAF ∥ BC

8. If a number of circles touch a given line segment PQ at a point A, then their centres lie on the perpendicular bisector of PQ.

Solution:

False

Explanation:
Given that PQ is any line segment and S_1, S_2, S_3, S_4, …….. circles touch the line segment PQ at a point A. Let the centres of the circles S_1, S_2, S_3, S_4, ……….. be C_1, C_2, C_3, C_4, ……… respectively.



To prove: Centres of the circles lie on the perpendicular bisector of PQ.
Joining each centre of the circles to the point A on the line segment PQ by line segment i.e., C_1A, C_2A, C_3A, C_4A,… and so on.

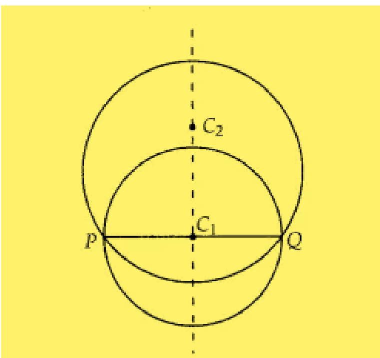
We know that, if we draw a line from the centre of a circle to its tangent line, then the line is always perpendicular to the tangent line. But it does not bisect the line segment PQ.

9. If a number of circles pass through the end points P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.

Solution:

True

Explanation:
We draw two circles with centre C_1 and C_2 passing through the end points P and Q of a line segment PQ. We know that the perpendicular bisector of a chord of circle always passes through the centre of the circle.



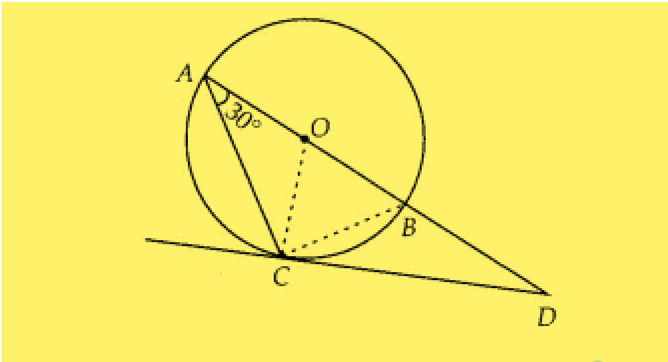
Thus, perpendicular bisector of PQ passes through C_1 and C_2 . Similarly, all the circle passing through the end points of line segment PQ, will have their centres on the perpendicular bisector of PQ.

10. AB is a diameter of a circle and AC is its chord such that ∠BAC = 30°. If the tangent at C intersects AB extended at D, then BC = BD.

Solution:

True

To prove: BC = BD



Join BC and OC.

Given,

∠BAC = 30°

∠BCD = 30°

[Angle between tangent and chord is equal to angle made by chord in the alternate segment]

OC ⊥ CD and

OA = OC = radius

∠OAC = ∠OCA = 30°

So,

∠ACD = ∠ACO + ∠OCD

= 30° + 90°

= 120°

Now,

In ΔACD,

∠DAC + ∠ACD + ∠CDA = 180° [Angle sum property]

30° + 120° + ∠CDA = 180°

∠CDA = 180° – (30° + 120°)

= 30°

∠CDA = ∠BCD

BC = BD

[as, Sides opposite to equal angles are equal]

Exercise No. 9.3

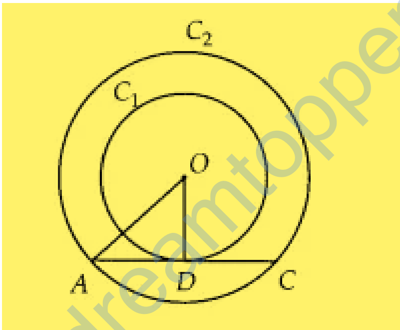
Short Answer Questions:

Question:

1. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

Solution:

Let C_1 and C_2 be the two circles having same centre O. AC is a chord which touches C_1 at point D.



Join OD.
Also, OD ⊥ AC
AD = DC = 4 cm
[Perpendicular line OD bisects the chord]

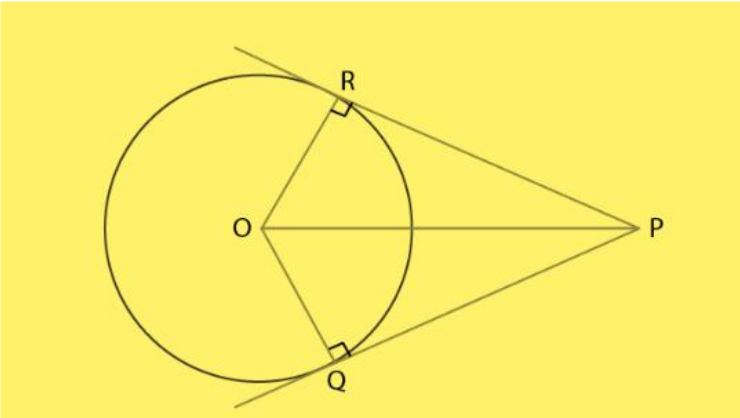
In right angled ΔAOD,
$OA^2 = AD^2 + OD^2$ [By Pythagoras theorem]
$OD^2 = 5^2 – 4^2$
$OD^2 = 25 – 16$
$= 9$
OD = 3 cm
Radius of the inner circle is OD = 3 cm

2. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

Solution:

We know that,

Radius ⊥ Tangent = OR ⊥ PR



$\angle ORP = 90°$

Similarily,

Radius $\perp$ Tangent = $OQ \perp PQ$
$\angle OQP = 90°$

In quadrilateral ORPQ,

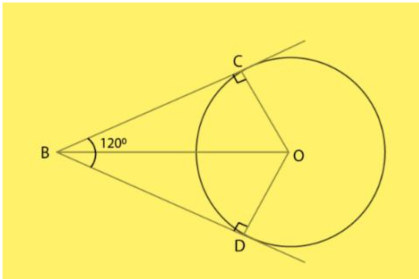
Sum of all interior angles = 360^o
$\angle ORP + \angle RPQ + \angle PQO + \angle QOR = 360^o$
$90^o + \angle RPQ + 90^o + \angle QOR = 360^o$

So,
$\angle O + \angle P = 180^o$
PROQ is a cyclic quadrilateral.

3. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle \angle DBC = 120°$, prove that BC + BD = BO, i.e., BO = 2BC.

Solution:

As given in the question,
By RHS rule,
ΔOBC and ΔOBD are congruent {By CPCT}



∠OBC and ∠ OBD are equal
Therefore,
∠OBC = ∠OBD =60°

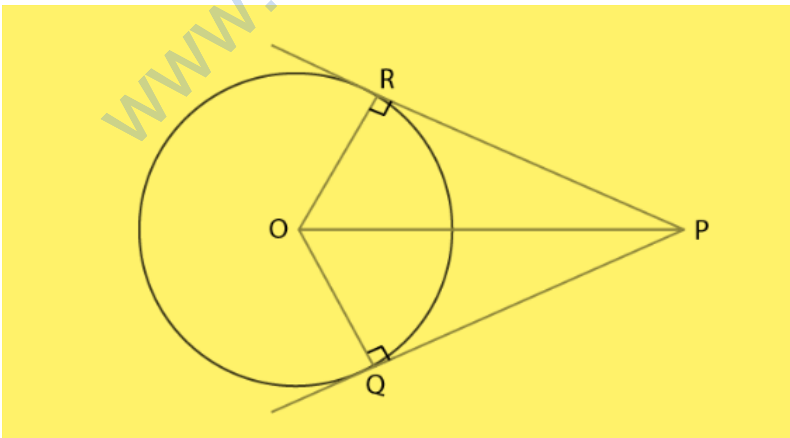
In triangle OBC,
cos 60°= BC/OB
½ = BC/OB
OB = 2BC

Hence proved.

4. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

Solution:

Let us take the lines be l_1 and l_2.



Assume that O touches l_1 and l_2 at M and N,
So,
OM = ON (Radius of the circle)

Therefore,
From the centre "O" of the circle, it has equal distance from l_1 & l_2.

In Δ OPM & OPN,

OM = ON (Radius of the circle)

∠OMP = ∠ONP (As, Radius is perpendicular to its tangent)

OP = OP (Common sides)

Therefore,

Δ OPM = ΔOPN (SSS congruence rule)

By C.P.C.T,

∠MPO = ∠NPO

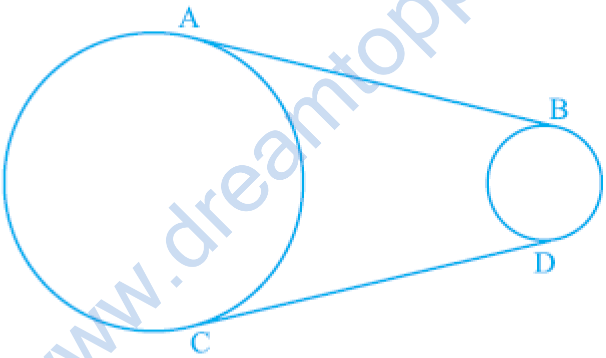
So, l bisects ∠MPN.

Hence, O lies on the bisector of the angle between l_1 & l_2 .

Also,

Centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

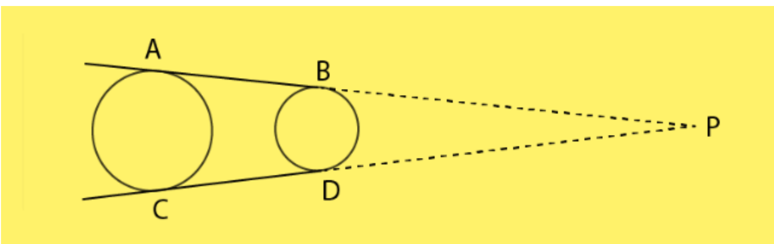
5. In Fig. 9.13, AB and CD are common tangents to two circles of unequal radii.
Prove that AB = CD.



Solution:

As given in the question,

AB = CD



Construction: Produce AB and CD, to intersect at P.

Proof:

Consider the circle with greater radius.

Tangents drawn from an external point to a circle are equal

AP = CP ...(i)

Also,
Consider the circle with smaller radius.
Tangents drawn from an external point to a circle are equal
BP = BD ...(ii)

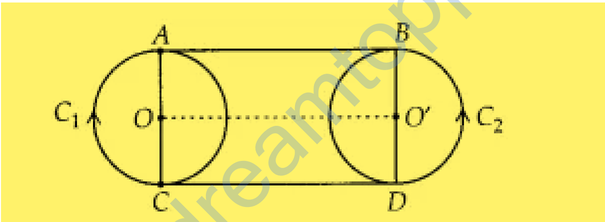
Subtract Equation (ii) from (i),

AP – BP = CP – BD
AB = CD

Hence Proved.

6. In Question 5 above, if radii of the two circles are equal, prove that AB = CD.

Solution:



Join OO'
Since, OA = O'B [Given]
And,
∠OAB = ∠O'BA = 90°
[Tangent at any point of a circle is perpendicular to the radius at the point of contact]

Since, perpendicular distance between two straight lines at two different points is same.
AB is parallel to OO'

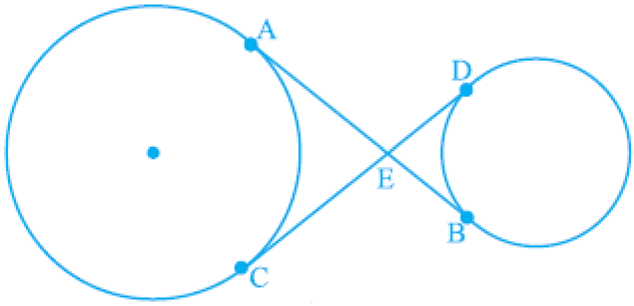
Also,
CD is parallel to OO'
AB ll CD

Now,
∠OAB = ∠OCD = ∠O'BA = ∠O'DC = 90°
ABCD is a rectangle.

Hence,
AB = CD.

7. In Fig., common tangents AB and CD to two circles intersect at E.

Prove that AB = CD.



Solution:

Given, common tangents AB and CD to two circles intersecting at E.

To prove:
AB = CD

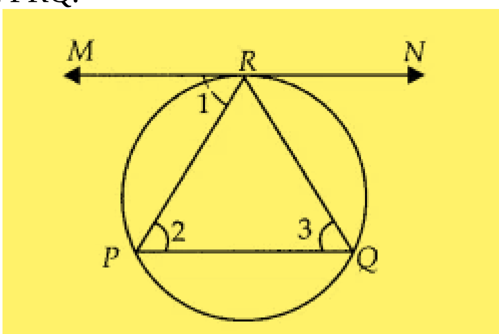
We have,
The length of tangents drawn from an external point to a circle is equal
EB = ED and
EA = EC

On adding, we get
EA + EB = EC + ED
AB = CD

8. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

Solution:

Given, chord PQ is parallel to tangent at R.
To prove : R bisects the arc PRQ.



$\angle 1 = \angle 2$ [Alternate interior angles]
$\angle 1 = \angle 3$
[Angle between tangent and chord is equal to angle made by chord in alternate segment]
$\angle 2 = \angle 3$

PR = QR [Sides opposite to equal angles are equal]

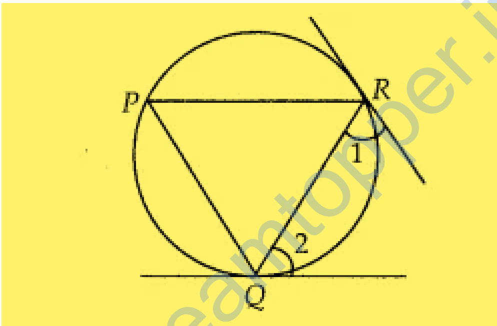
As,
arc PR = arc QR

Therefore, R bisects PQ.

9. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

Solution:

To prove : ∠1 = ∠2
Let RQ be a chord of the circle. Tangents are drawn at the points R and Q.

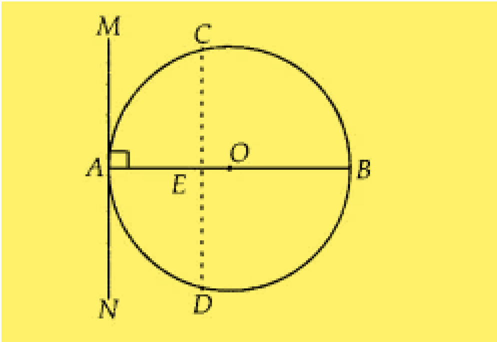


Let P be another point on the circle, then join PQ and PR.
As, at point Q, there is a tangent.
∠2 = ∠P [Angles in alternate segments are equal]
Also, at point R, there is a tangent.
∠1 = ∠P [Angles in alternate segments are equal]
∠1 = ∠2 = ∠P
∠1 = ∠2

10. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

Solution:

Given, AB is a diameter of the circle.
A tangent is drawn at point A.
Draw a chord CD parallel to tangent MAN.



Therefore, CD is a chord of the circle and OA is radius of the circle.
$\angle MAO = 90°$
[tangent at any point of a circle is perpendicular to the radius through the point of contact]
$\angle CEO = \angle MAO$ [Corresponding angles]
$\angle CEO = 90°$

So,
OE bisects CD,
[Perpendicular from centre of circle to chord bisects the chord]
Similarily,
Diameter AB bisects all chords which are parallel to the tangent at the point A.

Exercise No. 9.4

Long Answer Questions:

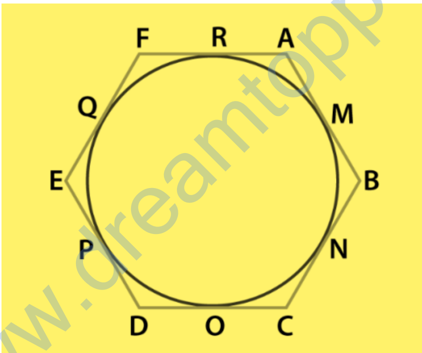
Question:

1. If a hexagon ABCDEF circumscribe a circle, prove that AB + CD + EF = BC + DE + FA.

Solution:

As given in the question,

A Hexagon ABCDEF circumscribe a circle.



To prove:

AB + CD + EF = BC + DE + FA

We know that,
Tangents drawn from an external point to a circle are equal.
So,

AM = RA … i [tangents from point A]
BM = BN … ii [tangents from point B]
CO = NC … iii [tangents from point C]
OD = DP … iv [tangents from point D]
EQ = PE … v [tangents from point E]
QF = FR … vi [tangents from point F]

Now,adding,
[eq i]+[eq ii]+[eq iii]+[eq iv]+[eq v]+[eq vi]

AM + BM + CO + OD + EQ + QF = RA + BN + NC + DP + PE + FR

On solving, we get,

(AM + BM) + (CO + OD) + (EQ + QF) = (BN + NC) + (DP + PE) + (FR + RA)

AB + CD + EF = BC + DE + FA

Hence Proved!

2. Let *s* denote the semi-perimeter of a triangle ABC in which BC = *a*, CA = *b*, AB = *c*. If a circle touches the sides BC, CA, AB at D, E, F, respectively, prove that BD = *s* – *b*.

Solution:

As given in the question,

A triangle ABC with BC = a , CA = b and AB = c . Also, a circle is inscribed which touches the sides BC, CA and AB at D, E and F respectively and s is semi- perimeter of the triangle

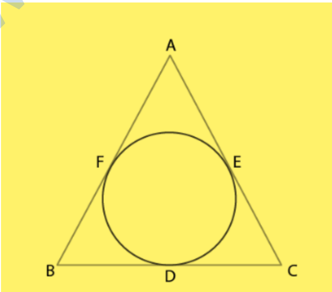
To Prove: BD = s – b

We have,

Semi Perimeter = s

Perimeter = 2s

2s = AB + BC + AC [i]



We know that,

Tangents drawn from an external point to a circle are equal

So,

AF = AE [ii] [Tangents from point A]

BF = BD [iii] [Tangents From point B]

CD = CE [iv] [Tangents From point C]

Adding [ii] [iii] and [iv],

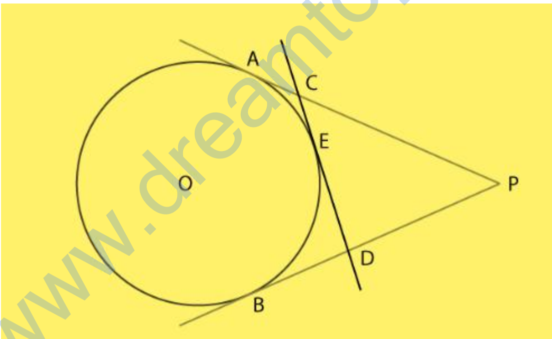
AF + BF + CD = AE + BD + CE
AB + CD = AC + BD

Adding BD both side,
AB + CD + BD = AC + BD + BD
AB + BC – AC = 2BD
AB + BC + AC – AC – AC = 2BD
2s – 2AC = 2BD [From i]
2BD = 2s – 2b [as AC = b]
BD = s – b

Hence Proved.

3. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of the triangle PCD.

Solution:



As given in the question,

From an external point P, two tangents, PA and PB are drawn to a circle with center O. At a point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. And PA = 10 cm

To Find : Perimeter of △PCD

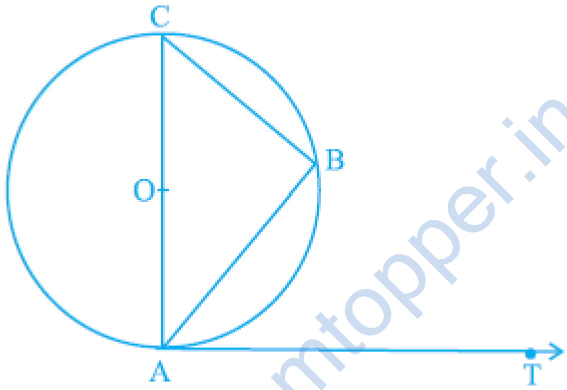
As we know that, Tangents drawn from an external point to a circle are equal.
So, we have,

AC = CE [i] [Tangents from point C]
ED = DB [ii] [Tangents from point D]
Now,
Perimeter of Triangle PCD= PC + CD + DP
= PC + CE + ED + DP

= PC + AC + DB + DP   [From i and ii]
= PA + PB

Now,
PA = PB = 10 cm as tangents drawn from an external point to a circle are equal
So ,
Perimeter = PA + PB
= 10 + 10
= 20 cm

4. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in Fig.. Prove that ∠BAT = ∠ACB.



Solution:

As given in the question,

A circle with center O and AC as a diameter and AB and BC as two chords also AT is a tangent at point A
To Prove : ∠BAT = ∠ACB

Proof :

∠ABC = 90°   [Angle in a semicircle is a right angle]

In △ABC By angle sum property of triangle

∠ABC + ∠ BAC + ∠ACB = 180 °

∠ACB + 90° = 180° – ∠BAC
∠ACB = 90 – ∠BAC   [i]

Now,
OA ⊥ AT
[Tangent at a point on the circle is perpendicular to the radius through point of contact]

$\angle OAT = \angle CAT = 90°$
$\angle BAC + \angle BAT = 90°$
$\angle BAT = 90° - \angle BAC$ [ii]

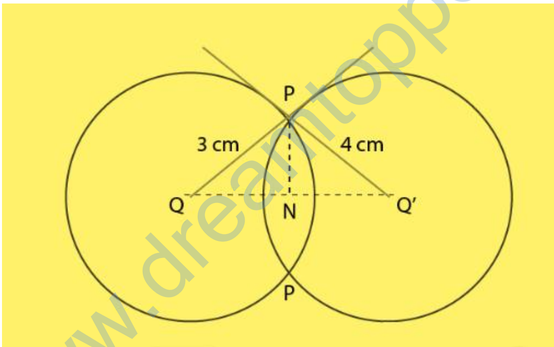
From [i] and [ii],
$\angle BAT = \angle ACB$ [Proved]

5. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

Solution:

We have,

Two circles with centers O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O'P are tangents to the two circles and PQ is a common chord.



To Find: Length of common chord PQ

$\angle OPO' = 90°$
[Tangent at a point on the circle is perpendicular to the radius through point of contact]

Therefore,
OPO is a right-angled triangle at P

By Pythagoras in $\triangle$ OPO', we have

$(OO')^2 = (O'P)^2 + (OP)^2$
$(OO')^2 = (4)^2 + (3)^2$
$(OO')^2 = 25$
$OO' = 5$ cm

Let ON = x cm and
NO' = 5 – x cm

In right angled triangle ONP

$(ON)^2 + (PN)^2 = (OP)^2$
$x^2 + (PN)^2 = (3)^2$
$(PN)^2 = 9 - x^2$ [i]

In right angled triangle O'NP

$(O'N)^2 + (PN)^2 = (O'P)^2$
$(5 - x)^2 + (PN)^2 = (4)^2$
$25 - 10x + x^2 + (PN)^2 = 16$
$(PN)^2 = -x^2 + 10x - 9$ [ii]

From [i] and [ii]

$9 - x^2 = -x^2 + 10x - 9$
$10x = 18$
$x = 1.8$

From (1) we have

$(PN)^2 = 9 - (1.8)^2$
$= 9 - 3.24$
$= 5.76$
$PN = 2.4$ cm

$PQ = 2PN$
$= 2(2.4)$
$= 4.8$ cm

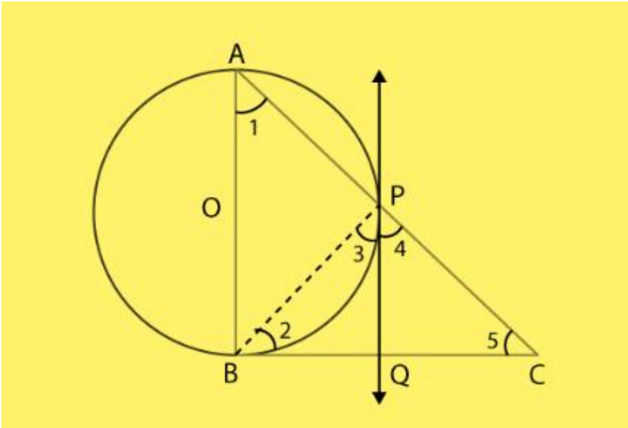
6. In a right triangle ABC in which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.

Solution:

As given in the question,

In a right angle ΔABC is which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Also PQ is a tangent at P

To Prove: PQ bisects BC or, BQ = QC



We have,

$\angle APB = 90°$ [Angle in a semicircle is a right-angle]
$\angle BPC = 90°$ [Linear Pair]
$\angle 3 + \angle 4 = 90$ [i]

Now,
$\angle ABC = 90°$

In $\triangle ABC$,
$\angle ABC + \angle BAC + \angle ACB = 180°$
$90 + \angle 1 + \angle 5 = 180$
$\angle 1 + \angle 5 = 90$ [ii]

Now,

$\angle 1 = \angle 3$
[angle between tangent and the chord equals angle made by the chord in alternate segment]

Using this in [ii] we have,

$\angle 3 + \angle 5 = 90$ [iii]

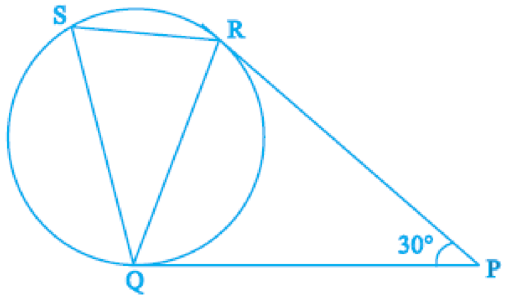
From [i] and [iii] we have

$\angle 3 + \angle 4 = \angle 3 + \angle 5$
$\angle 4 = \angle 5$

QC = PQ [Sides opposite to equal angles are equal]

Also
PQ = BQ [Tangents drawn from an external point to a circle are equal]
So,
BQ = QC
Therefore,PQ bisects BC .

7. In Fig., tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find the ∠RQS.
[Hint: Draw a line through Q and perpendicular to QP.]



Solution:

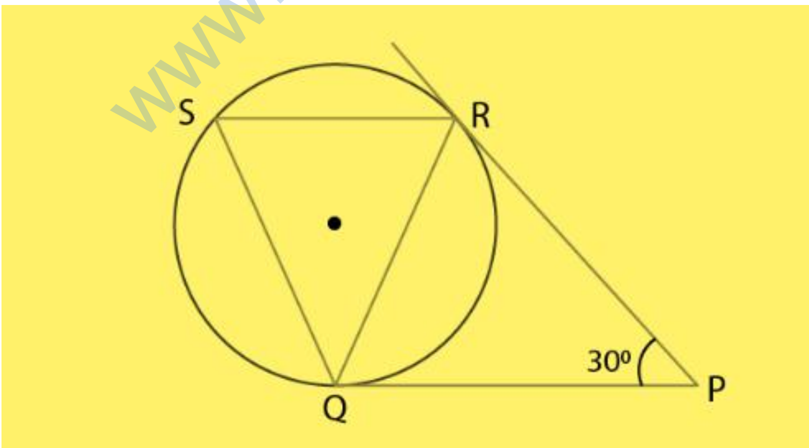
As given in the question,

Tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ.

To Find : ∠RQS

PQ = PR [Tangents drawn from an external point to a circle are equal]

∠PRQ = ∠PQR [Angles opposite to equal sides are equal] ..[i]



In △PQR

∠PRQ + ∠PQR + ∠QPR = 180°

∠PQR + ∠PQR + ∠QPR = 180° [Using 1]

2∠PQR + ∠RPQ = 180°

$$2\angle PQR + 30 = 180$$

$$2\angle PQR = 150$$

$\angle PQR = 75°$

$\angle QRS = \angle PQR = 75°$ [Alternate interior angles]

$\angle QSR = \angle PQR = 75°$

[angle between tangent and the chord equals angle made by the chord in alternate segment]

Now

In ΔRQS

$\angle RQS + \angle QRS + \angle QSR = 180$

$\angle RQS + 75 + 75 = 180$

$\angle RQS = 30°$

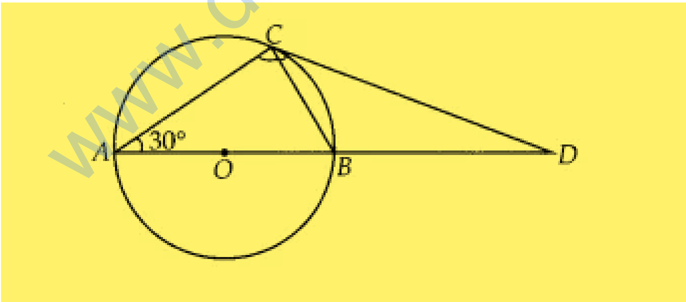
8. AB is a diameter and AC is a chord of a circle with centre O such that $\angle BAC = 30°$. The tangent at C intersects extended AB at a point D. Provethat BC = BD.

Solution:

Given,

AB is a diameter and AC is a chord of circle with centre O, $\angle BAC = 30°$

To prove : BC = BD



Construction: Join BC

$\angle BCD = \angle CAB$ [Angles in alternate segment]

$\angle CAB = 30°$ [Given]

$\angle BCD = 30°$ …(i)

$\angle ACB = 90°$ [Angle in semi-circle]

In ΔABC,

$\angle A + \angle B + \angle C = 180°$ [Angle sum property]

$30° + \angle CBA + 90° = 180°$

$\angle CBA = 60°$

Also,

∠CBA + ∠CBD = 180° [Linear pair]
∠CBD = 180° – 60°
= 120°
[as, ∠ CBA = 60°]

Now,
In ACBD,
∠CBD + ∠BDC + ∠DCB = 180°
120° + ∠BDC + 30° = 180°
∠BDC = 30° …(ii)
From (i) and (ii),
∠BCD = ∠BDC
BC = BD
[Sides opposite to equal angles are equal]

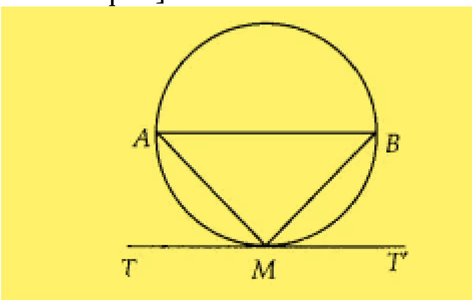
9. Prove that the tangent drawn at the mid-point of an arc of a circle is parallelto the chord joining the end points of the arc.

Solution:

Let us take the mid-point of an arc AMB be M and TMT' be the tangent to the circle.
Join AB, AM and MB.

Since,
arc AM = arc MB =3
Chord AM = Chord MB

In ΔAMB,
AM = MB
∠MAB = ∠MBA …(i)
[Sides opposite to equal angles are equal]

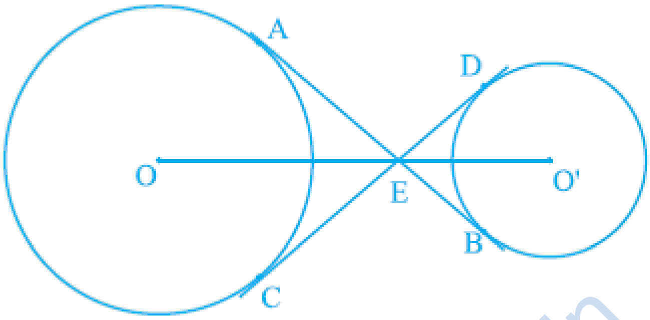


Since, TMT' is a tangent line.
Therefore,
∠AMT = ∠MBA [Angles in alternate segments are equal]
= ∠MAB [from equation (i)]

But, ∠AMT and ∠MAB are alternate angles, which is possible only when AB is parallel to TMT

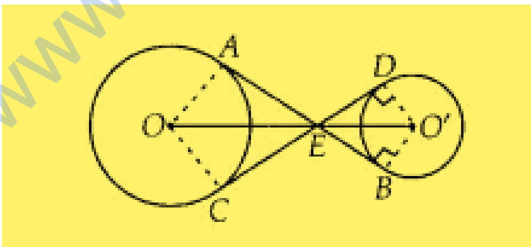
Hence, the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

10. In Fig., the common tangent, AB and CD to two circles with centres Oand O′ intersect at E. Prove that the points O, E, O′ are collinear.



Solution:

Join AO, OC and O’D, O’B.
Now,
In ΔEO’D and ΔEO’B,
O’D = O’B
O’E = O’E
ED = EB
[Tangents drawn from an external point to the circle are equal in length]



EO’D ≅ Δ EO’B [By SSS congruence criterion]
∠O’ED = ∠O’EB …(i)
Therefore,
O’E is the angle bisector of ∠DEB.

Similarly,
OE is the angle bisector of ∠AEC.

Now, in quadrilateral DEBO’.

∠O’DE = ∠O’BE = 90°
[CED is a tangent to the circle and O’D is the radius, i.e., O’D ⊥ CED]

$\angle O'DE + \angle O'BE = 180°$
$\angle DEB + \angle DO'B = 180°$

[as, DEBO' is cyclic quadrilateral] …(ii)

Since,
AB is a straight line.

$\angle AED + \angle DEB = 180°$
$\angle AED + 180° - \angle DO'B = 180°$ [from (ii)]
$\angle AED = \angle DO'B$ …(iii)

Similarly,
$\angle AED = \angle AOC$ …(iv)

Again from eq. (ii),
$\angle DEB = 180° - \angle DO'B$
Dividing by 2 on both sides, we get,

$$\frac{1}{2}\angle DEB = 90^o - \frac{1}{2}\angle DO'B$$

$$\angle DEO' = 90^o - \frac{1}{2}\angle DO'B \quad ...(v)$$

Similarily,

$$\angle AEC = 180^o - \angle AOC$$

Dividing 2 on both sides,

$$\frac{1}{2}\angle AEC = 90^o - \frac{1}{2}\angle AOC$$

$$\angle AEO = 90^o - \frac{1}{2}\angle AOC \quad ....(vi)$$

Now,

$$\angle AED + \angle AEO + \angle DEO' = \angle AED + 90^o - \frac{1}{2}\angle DO'B + 90^o - \frac{1}{2}\angle AOC$$

$$= \angle AED + 180^o - \frac{1}{2}(\angle DO'B + \angle AOC)$$

$$= \angle AED + 180^o - \frac{1}{2}(\angle AED + \angle AED) \quad \text{(from iii and iv)}$$

$$= \angle AED + 180^o - \angle AED$$

$$= 180^o$$

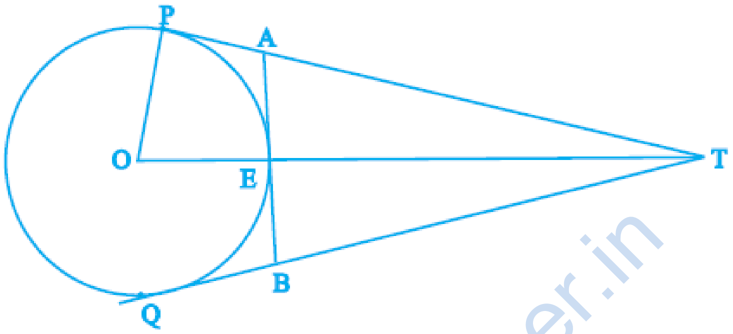
So,

$$\angle AED + \angle AEO + \angle DEO' = 180^o$$

So,
OEO’ is straight line.

Hence, O, E and O’ are collinear.

11. In Fig. 9.20. O is the centre of a circle of radius 5 cm, T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find the length of AB.



Solution:

OP is perpendicular to PT.
In ΔOPT,
$OT^2 = OP^2 + PT^2$
$PT^2 = OT^2 - OP^2$
$PT^2 = (13)^2 - (5)^2$
$= 169 - 25$
$= 144$
PT= 12 cm

Since, the length of pair of tangents from an external point T is equal.

So,
QT= 12 cm

Now,
TA = PT – PA
TA = 12 -PA …(i)
and
TB = QT – QB
TB = 12 – QB …(ii)

Also,
PA = AE and QB = EB …(iii) [Pair of tangents]
ET = OT – OE [as, OE = 5 cm = radius]
ET = 13 – 5
ET = 8 cm

Since, AB is a tangent and OE is the radius.

$OE \perp AB$,
$\angle OEA = 90°$
$\angle AET = 180° - \angle OEA$ [Linear pair]
$\angle AET = 90°$

Now, in right angled ΔAET,
$(AT)^2 = (AE)^2 + (ET)^2$ [by Pythagoras theorem]
$(12 - PA)^2 = (PA)^2 + (8)^2$

On solving,
$144 + (PA)^2 - 24\ PA = (PA)^2 + 64$
$24\ PA = 80$

$PA = \frac{10}{3}$

So,

$AE = \frac{10}{3}$

We join OQ,

Similarily,

$BE = \frac{10}{3}$

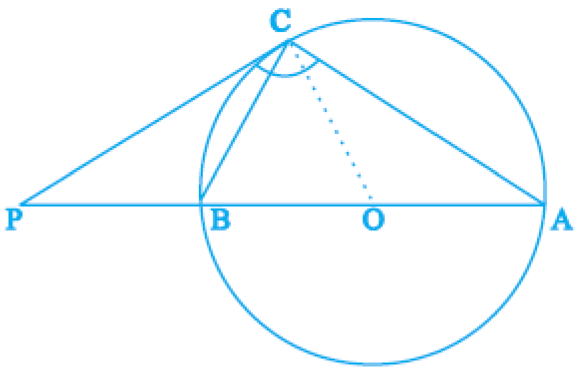
Also,

$AB = AE + BE$

$= \frac{10}{3} + \frac{10}{3}$

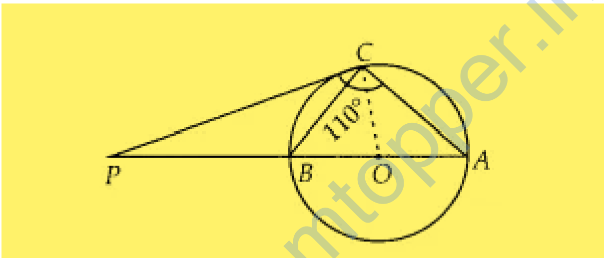
$= \frac{20}{3}$

12. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110^o$, find CBA.
[Hint: Join C with centre O.]



Solution:

Join OC. In this, OC is the radius.



We know that, tangent at any point of a circle is – perpendicular to the radius through the point of contact.

Therefore,
OC ⊥ PC

Now,
∠PCA = 110° [Given]
∠PCO + ∠OCA = 110°
90° + ∠OCA = 110°
∠OCA = 20°

Also,
OC = OA = radius of circle
∠OCA = ∠OAC = 20° [Sides opposite to equal angles are equal]

Since, PC is a tangent,
∠BCP = ∠CAB = 20° [Angles in alternate segment]

In ΔPAC,
∠P + ∠C + ∠A = 180°

So,
∠P = 180° – (∠C + ∠A)

$= 180° - (110° + 20°)$
$= 180° – 130° = 50°$

In ΔPBC,
$\angle BPC + \angle PCB + \angle CBP = 180°$
$50° + 20° + \angle PBC = 180°$
$\angle PBC = 180° – 70°$
$\angle PBC = 110°$
Since, ABP is a straight line.

Therefore,
$\angle PBC + \angle CBA = 180°$
$\angle CBA = 180° – 110°$
$= 70°$

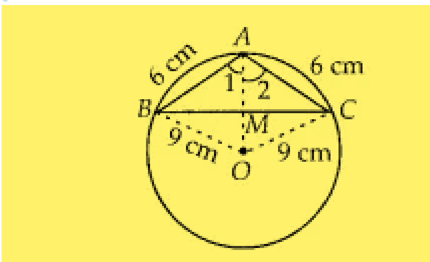
13. If an isosceles triangle ABC, in which AB = AC = 6 cm, is inscribed in a circleof radius 9 cm, find the area of the triangle.

Solution:

Join OB, OC and OA.

In ΔABO and ΔACO,
AB = AC [Given]
BO = CO [Radii of same circle]
AO = AO [Common side]

ΔABO ≅ ΔACO [By SSS congruence criterion]



$\angle 1 = \angle 2$ [CPCT]

Now,

In ΔABM and ΔACM,
AB = AC [Given]
$\angle 1 = \angle 2$ [proved above]
AM = AM [Common side]

So,

$\Delta AMB \cong \Delta AMC$ [By SAS congruence criterion]
$\angle AMB = \angle AMC$ [CPCT]

Also,
$\angle AMB + \angle AMC = 180°$ [Linear pair]
$\angle AMB = 90°$

We know that a perpendicular from the centre of circle bisects the chord.

So, OA is a perpendicular bisector of BC.

Let AM = x, then OM = 9 – x [as, OA = radius = 9 cm]

In right angledd ΔAMC,
$AC^2 = AM^2 + MC^2$ [By Pythagoras theorem]
$MC^2 = 6^2 - x^2$ …(i)

In right angle ΔOMC,
$OC^2 = OM^2 + MC^2$ [By Pythagoras theorem]
$MC^2 = 9^2 - (9 - x)^2$

From equation (i) and (ii),
$6^2 - x^2 = 9^2 - (9 - x)^2$
$36 - x^2 = 81 - (81 + x^2 - 18x)$
$36 = 18x$
$x = 2$
So,
AM = 2 cm
From equation (ii),
$MC^2 = 9^2 - (9 - 2)^2$
$MC^2 = 81 - 49$
$= 32$
$MC = 4\sqrt{2}$ cm

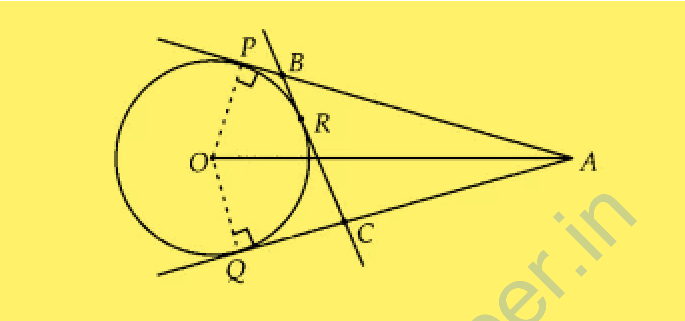
So,
BC = 2 MC
$= 8\sqrt{2}$ cm

$$\text{Area of ABC} = \frac{1}{2}\times\text{base}\times\text{height}$$
$$= \frac{1}{2}\times BC\times AM$$
$$= \frac{1}{2}\times 8\sqrt{2}\times 2$$
$$= 8\sqrt{2}\,\text{cm}^2$$

The required area of ΔABC is 8√2 cm2.

14. A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm.AP and AQ are the tangents to the circle at P and Q. If a tangent BC isdrawn at a point R lying on the minor arc PQ to intersect AP at B and AQ atC, find the perimeter of the ΔABC.

Solution:



We have,
∠OPA = 90°
[Tangent at any point of a circle is perpendicular to the radius through the point of contact]

In ΔOAP,
$OA^2 = OP^2 + PA^2$
$13^2 = 5^2 + PA^2$
PA = 12 cm

Now,
Perimeter of ΔABC = AB + BC + CA
= AB + BR + RC + CA
= (AB + BR) + (RC + CA)
= (AB + BP) + (CQ + CA)
[As,BR = BP, RC = CQ i.e., tangents from external point to a circle are equal]

Perimeter of ΔABC = AP + AQ
= 2AP [as, AP = AQ]
= 2 x 12
= 24 cm

Hence, the perimeter of ΔABC = 24 cm.