Chapter 5
Arithmetic Progressions

Exercise No. 5.1

Multiple Choice Questions:

Question: 1

Choose the correct answer from the given four options in the following questions:

1. In an AP, if $d = -4, n = 7, a_n = 4$, then a is

A. 6

B. 7

C. 20

D. 28

Solution:

(D) 28

In an A.P,
$a_n = a + (n - 1)d$
(a = first term, an is nth term and d is the common difference)

4 = a + (7 – 1)(- 4)
4 = a – 24
a = 24 + 4
= 28

www.dreamtopper.in

2. In an AP, if $a = 3.5, d = 0, n = 101$, then a_n will be

A. 0

B. 3.5

C. 103.5

D. 104.5

Solution:

(B) 3.5

In an A.P,
$a_n = a + (n - 1)d$
(a = first term, an is nth term and d is the common difference)

a_n = 3.5 + (101 – 1)0
= 3.5
(Since, d = 0, it's a constant A.P)

3. The list of numbers – 10, – 6, – 2, 2, … is

A. an AP with d = – 16

B. an AP with d = 4

C. an AP with d = – 4

D. not an AP

Solution:

In the given A.P,

$a_1 = -10$

$a_2 = -6$

$a_3 = -2$

$a_4 = 2$

$a_2 - a_1 = 4$

$a_3 - a_2 = 4$

$a_4 - a_3 = 4$

$a_2 - a_1 = a_3 - a_2$

$= a_4 - a_3$

$= 4$

So, it is an A.P with d = 4.

www.dreamtopper.in

4. The 11th term of the AP: $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$ **is**

A. –20

B. 20

C. –30

D. 30

Solution:

According to the given A.P.

$a = -5$

$d = 5 - (-\frac{5}{2})$

$= 5/2$

n = 11

Also,

an = a + (n – 1)d

Here, (a = first term, an is nth term and d is the common difference)

$a_{11} = -5 + (11 - 1)(\frac{5}{2})$

$a_{11} = -5 + 25$

$= 20$

5. The first four terms of an AP, whose first term is –2 and the common difference is –2, are

A. – 2, 0, 2, 4

B. – 2, 4, – 8, 16

C. – 2, – 4, – 6, – 8

D. – 2, – 4, – 8, –16

www.dreamtopper.in

Solution:

First term,

a = – 2

Second Term,

d = – 2

$a_1 = a$

$= -2$

Also,

$a_n = a + (n - 1)d$

Where,

a = first term, a_n is nth term, d is the common difference

Therefore,

$a_2 = a + d$

$= – 2 + (- 2)$

$= – 4$

Similarly,

$a_3 = – 6$

$a_4 = – 8$

So the A.P is– 2, – 4, – 6, – 8.

6. The 21st term of the AP whose first two terms are –3 and 4 is

A. 17

B. 137

C. 143

D. –143

Solution:

First two terms of an AP are $a = – 3$ and $a_2 = 4$.

We know, nth term of an AP is

$a_n = a + (n – 1)d$

Here, a = first term, a_n is nth term, d is the common difference

$a_2 = a + d$

$4 = – 3 + d$

$d = 7$

Common difference,

$d = 7$

$a_{21} = a + 20d$

$= – 3 + (20)(7)$

$= 137$

www.dreamtopper.in

7. If the 2^{nd} term of an AP is 13 and the 5^{th}term is 25, what is its 7^{th} term?

A. 30

B. 33

C. 37

D. 38

Solution:

In an A.P.

$a_n = a + (n – 1)d$

Here, a = first term, a_n is nth term, d is the common difference

$a_2 = a + d$

$= 13$ …..(i)

$a_5 = a + 4d$

$= 25$ … (ii)

From equation (i),

a = 13 – d

Using this in equation (ii),

13 – d + 4d = 25

13 + 3d = 25

3d = 12

d = 4

a = 13 – 4

= 9

$a_7 = a + 6d$

= 9 + 6(4)

= 9 + 24

www.dreamtopper.in

= 33

8. Which term of the AP: 21, 42, 63, 84… is 210?

A. 9th
B. 10th
C. 11th
D. 12th

Solution:

Let nth term of the given AP be 210.

According to question,

First term,

a = 21

Common difference,

d = 42 – 21

= 21

$a_n = 210$

We know that the nth term of an AP is $a_n = a + (n – 1)d$

Where, a = first term, a_n is nth term, d is the common difference

210 = 21 + (n – 1)21

189 = (n – 1)21

n – 1 = 9

n = 10

So, 10th term of an AP is 210.

9. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

A. 5
B. 20
C. 25

www.dreamtopper.in

D. **30**

Solution:

Given, d = 5

Now,

As we know, nth term of an AP is

$a_n = a + (n – 1)d$

Here, a = first term, a_n is nth term, d is the common difference

$a_{18} - a_{13} = a + 17d – (a + 12d)$

$= 5d$

$= 5(5)$

$= 25$

10. **What is the common difference of an AP in which** $a_{18} - a_{14} = 32$?

A. **8**

B. **– 8**

C. **– 4**

D. **4**

www.dreamtopper.in

Solution:

(a)

$a_{18} – a_{14} = 32$

$[a + (18 – 1)d] – [a + (14 – 1)d] = 32$ [$a_n = a + (n – 1)d$]

$a + 17d – a – 13d = 32$

$4d = 32$

$d=8$

So, (a) is the correct answer.

11. Two APs have the same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4^{th} **terms is**

A. -1

B. -8

C. 7

D. -9

Solution:

(c)

According to question,

a_1 (for the first AP)= –1 and a_1 (for the second AP)

= - 8

Let d be the same common difference of two A.Ps.

d_1= d,

$a_n = a + (n – 1)d$

Now,

a_4 of first AP - a_4 of second AP

$(–1 + 3d) – [–8 + 3d]$

$– 1 + 3d + 8 – 3d = 7$

So, the required answer is (c).

12. If 7 times the 7^{th} **term of an AP is equal to 11 times its** 11^{th} **term, then its** 18^{th} **term will be**

A. 7

B. 11

C. 18

D. 0

Solution:

(d)

www.dreamtopper.in

$a_{18} = a + (18 – 1)d$

$= a + 17d$

Also,

$7a_7 = 11a_{11}$ (Given)

$7[a + (7 – 1)d] = 11[a + (11 – 1)d]$

$7[a + 6d] = 11[a + 10d]$

$7a + 42d = 11a + 110d$

$0 = 11a – 7a + 110d – 42d$

$0 = 4a + 68d$

$0 = a + 17d$

$a_{18} = 0$

So, (d) is the correct answer.

13. The 4th term from the end of the AP: –11, –8, –5, ..., 49 is

A. 37

B. 40

C. 43

D. 58

Solution:

(b)
Reversing the A.P., we get
49... –5, –8, and –11

$d = –8 – (–5)$
$= –8 + 5$
$= –3$

$a = 49$ and $n = 4$
$a_n = a + (n – 1)d$
$a_4 = 49 + (4 – 1)(–3)$
$a_4 = 49 + 3(–3)$
$= 49 – 9$

www.dreamtopper.in

$a_4 = 40$
So, the required value of a_4 is 40 and answer is (b).

14. The famous mathematician associated with finding the sum of the first 100 natural numbers is

A. Pythagoras
B. Newton
C. Gauss
D. Euclid

Solution:

(c)

Gauss is the famous mathematician associated with finding the sum of first 100 natural numbers, i.e., 1 + 2 + 3 + 4 + 5 + ... + 100

a = 1, d = 1, n = 100

As, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{100} = \frac{100}{2}[2(1) + (100 - 1)1]$

$= \frac{100}{2}[2 + 99]$

$= 50 \times 101$

$= 5050$

15. If the first term of an AP is –5 and the common difference is 2, then the sum ofthe first 6 terms is

A. 0
B. 5
C. 6
D. 15

Solution:

(a)

a = –5,

www.dreamtopper.in

d = 2,

n = 6

We have,

$S_n = \frac{n}{2}$ [2a + (n – 1)d]

$S_6 = \frac{6}{2}$ [2(–5) + (6 – 1)2]

= 3[–10 + 5 × 2]

= 3[–10 + 10]

= 3[0]

S_6 = 0

So, (a) is the correct answer.

16. The sum of first 16 terms of the AP: 10, 6, 2,... is

A. –320

B. 320

C. –352

D. –400

Solution:

(a)

a = 10,

n = 16,

d = 6 – 10

= –4

S_n = [2a + (n – 1)d]

S_{16} = [2 × 10 + (16 – 1)(–4)]

=8[20 + 15(–4)]

= 8[20 – 60]

= 8 × (– 40)

S_{16} = –320

www.dreamtopper.in

So, the required answer is (a).

17. In an AP if $a = 1, a_n = 20$ and $S_n = 399$, then *n* is

A. 19
B. 21
C. 38
D. 42

Solution:

(c)

S_n = [2a + (n – 1)d]

S_n = [a + a + (n – 1)d]

399 = [a + a_n] (an = last term)

399 = [1 + 20]

n =38

So, (c) is the correct answer.

18. The sum of first five multiples of 3 is

A. 45
B. 55
C. 65
D. 75

Solution:

(a)

1st five multiples of 3 are 3, 6, 9, 12, 15, …..
a = 3,
n = 5,
d = 6 – 3
= 3

$S_5 = \frac{5}{2}$ [2 × 3 + (5 – 1)3]

$S_5 = \frac{5}{2}$ [6 + 12]

www.dreamtopper.in

$= \frac{5}{2} \times 18$

$= 45$

(a) is the correct answer.

www.dreamtopper.in

Exercise No. 5.2

Short Answer Questions with Reasoning:

Question:

1. Which of the following form an AP? Justify your answer.

i. $-1, -1, -1, -1, \ ...$

ii. $0, 2, 0, 2, \ ...$

iii. $1, 1, 2, 2, 3, 3, \ ...$

iv. $11, 22, 33, \ ...$

v. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ ...$

vi. $2, 2^2, 2^3, 2^4, ...$

vii. $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, ...$

Solution:

(i) –1, –1, –1, –1,…

We have

$a_1 = -1$,

$a_2 = -1$,

$a_3 = -1$ and $a_4 = -1$

$a_2 - a_1 = 0$

$a_3 - a_2 = 0$

$a_4 - a_3 = 0$

As the difference of successive terms is same, therefore given list of numbers from an AP.

(ii) 0, 2, 0, 2,…

We have

$a_1 = 0$,

$a_2 = 2$,

www.dreamtopper.in

$a_3 = 0$ and $a_4 = 2$

$a_2 – a_1 = 2$

$a_3 – a_2 = – 2$

$a_4 – a_3 = 2$

The difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iii) 1, 1, 2, 2, 3, 3…

We have

$a_1 = 1$,

$a_2 = 1$,

$a_3 = 2$ and $a_4 = 2$

$a_2 – a_1 = 0$

$a_3 – a_2 = 1$

The difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iv) 11, 22, 33…

We have

$a_1 = 11$,

$a_2 = 22$ and $a_3 = 33$

$a_2 – a_1 = 11$

$a_3 – a_2 = 11$

The difference of successive terms is same, therefore given list of numbers form an AP.

(v) 1/2,1/3,1/4, …

We have

$a_1 = ½$,

www.dreamtopper.in

$a_2 = 1/3$ and $a_3 = ¼$

$a_2 – a_1 = -1/6$

$a_3 – a_2 = -1/12$

The difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vi) $2, 2^2, 2^3, 2^4, \ldots$

We have

$a_1 = 2$,

$a_2 = 2^2$,

$a_3 = 2^3$ and $a_4 = 2^4$

$a_2 – a_1 = 2^2 – 2$

$= 4 – 2$

$= 2$

$a_3 – a_2 = 2^3 – 2^2$

$= 8 – 4$

$= 4$

The difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

We have,

$a_1 = \sqrt{3}$,

$a_2 = \sqrt{12}$,

$a_3 = \sqrt{27}$ and $a_4 = \sqrt{48}$

$a_2 – a_1 = \sqrt{12} – \sqrt{3}$

$= 2\sqrt{3} – \sqrt{3}$

www.dreamtopper.in

$= \sqrt{3}$

$a_3 - a_2 = \sqrt{27} - \sqrt{12}$

$= 3\sqrt{3} - 2\sqrt{3}$

$= \sqrt{3}$

$a_4 - a_3 = \sqrt{48} - \sqrt{27}$

$= 4\sqrt{3} - 3\sqrt{3}$

$= \sqrt{3}$

The difference of successive terms is same, therefore given list of numbers from an AP.

2. Justify whether it is true to say that $-1, -\frac{3}{2}, -2, \frac{5}{2}, \ldots$ forms an AP as $a_2 - a_1 = a_3 - a_2$.

Solution:

It is not true.

$a_1 = -1$,

$a_2 = \frac{-3}{2}$,

$a_3 = -2$

$a_4 = \frac{5}{2}$

$a_2 - a_1 = \frac{-3}{2} - (-1)$

$= \frac{-1}{2}$

$a_3 - a_2 = -2 - (\frac{-3}{2})$

$= \frac{-1}{2}$

www.dreamtopper.in

$a_4 - a_3 = \frac{5}{2} - (-2)$

$= \frac{9}{2}$

As, the difference of successive terms in not same, all though, $a_2 - a_1 = a_3 - a_2$

but $a_4 - a_3 \neq a_3 - a_2$ so, it does not form an AP.

3. For the AP: $-3, -7, -11, \ldots,$ can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.

Solution:

It is true.

We have, a = – 3

$d = a_2 - a_1$

= – 7 – (- 3)

= – 4

$a_{30} - a_{20} = a + 29d - (a + 19d)$

= 10d

= – 40

Therefore, difference between any two terms of an AP is proportional to common difference of that AP.

4. Two APs have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference between their 10th terms is the same as the difference between their 21st terms, which is the same as the difference between any two corresponding terms. Why?

Solution:

Let us assume, there are two AP's with first terms a and their common differences are d and D respectively.

Taking n be any term,

www.dreamtopper.in

$a_n = a + (n – 1)d$

$A_n = A + (n – 1)D$

As common difference is equal for both AP's

We have D = d

So, we have

$A_n – a_n = a + (n – – 1)d – [A + (n – 1)D]$

$= a + (n – 1)d – A – (n – 1)d$

$= a – A$

Since, a – A is a constant value.

So, difference between any corresponding terms will be equal to a – A.

5. Is 0 a term of the AP: 31, 28, 25, ...? Justify your answer.

Solution:

We know, an = a + (n – 1)d

If we put the values of a_n, a, and d in the above equation and if n comes out to be a natural number then, the given a_n will be the term of the given series.

$a_n = 0$, a = 31

$d_1 = 28 – 31$

$= –3,$

$d_2 = 25 – 28$

$= –3$

Hence,

$d_1 = d_2 = –3$

$a_n = a + (n – 1)d$

$0 = 31 + (n – 1) (–3)$

www.dreamtopper.in

$-31 = -(n-1) \times 3$

$(n-1) = \frac{31}{3}$

As n is in fraction and is not a natural number so 0 (an) is not any term of the given A.P.

6. The taxi fare after each km, when the fare is Rs 15 for the first km and Rs 8 for each additional km, does not form an AP as the total fare (in Rs) after each km is $15, 8, 8, 8, \ldots$.
Is the statement true? Give reasons.

Solution:

No, the given statement is false.

15, 8, 8, 8 ... are not the total fare for 1, 2, 3, 4, km respectively.

Total fare for Ist km = Rs 15.

Total fare for 2 km = Rs 15 + Rs 8

= Rs 23

Total fare for 3 km = Rs 23 + Rs 8

= Rs 31

Total fare for 4 km = Rs 31 + Rs 8

= Rs 39

Total fare for 1 km, 2 km, 3km, 4km, ... are Rs15, Rs 23, Rs 31, Rs 39, ... respectively.

Now,

$d_1 = 23 - 15$

$= 8$

$d_2 = 31 - 23$

$= 8$

www.dreamtopper.in

$d_3 = 39 - 31$

$= 8$

Therefore, the total fare for 1 km, 2 km, 3km, 4km, ...from an A.P. as 15, 23, 31, 39, ...

And, fare for each km does not form A.P. as 15, 8, 8, 8,...

7. In which of the following situations, do the lists of numbers involved form an AP?Give reasons for your answers.

i. The fee charged from a student every month by a school for the whole session,when the monthly fee is Rs 400.

ii. The fee charged every month by a school from Classes I to XII, when the monthly fee for Class I is Rs 250, and it increases by Rs 50 for the next higher class.

iii. The amount of money in the account of Varun at the end of every year when Rs 1000 is deposited at simple interest of 10% per annum.

iv. The number of bacteria in a certain food item after each second, when they double in every second.

www.dreamtopper.in

Solution:

(i)

The school charges from a student every month fees =₹400.

So, the fee charged from a student in the whole session is 400, 400, 400, 400, ...

As

$d_1 = d_2 = d_3 = d_{12} = 0$ so, the series of numbers is an A.P.

(ii)

Fee for 1st class = ₹250

Fee for 2nd class = ₹ (250 + 50)

= ₹ 300

Fee for 3rd class = ₹ (300 + 50)

= ₹ 350

Fee for 4rth class = ₹ (350 + 50)

= ₹ 400

So, 250, 300, 350, 400,... is a series consisting of 12 terms.

d_1= 300 – 250 = ₹ 50,

d_2 = 350 – 300 = ₹ 50,

d_3 = 400 – 350 = ₹ 50

d_1 = d_2 = d_3 = ₹ 50

So, the list of numbers 250, 300, 350, 400 ... is in A.P.

(iii)

₹100

So, ₹100 is credited at the end of each year in the account of Varun.

Money in the beginning of 1st year (deposited) = ₹ 1000

Money at the end of 1st year when interest credited = 1000 + 100

= ₹ 1100

Money at the end of 2nd year = 1100 + 100

= ₹ 1200

Money at the end of 3rd year = 1200 + 100

= ₹1300

Money at the end 4th year = 1300 + 100

= ₹1400

So, Amount of money at the end of each year starting initially from Ist year is given by

1000, 1100, 1200, 1300, 1400...

Also,

d_1= d_2 = d_3= d_4= 100

So, the list of numbers is an A.P.

(iv)

Taking the number of bacteria present initially = x

www.dreamtopper.in

So, the number of bacteria preset after 1 sec = 2(x) = 2x

Number of bacteria present after 2 seconds = 2(2x) = 4x

Number of bacteria present after 3 seconds = 2(4x) = 8x

Number of bacteria present after 4 seconds = 2(8x) = 16x

Hence, the number of bacteria from starting to end of each second are given by x, 2x, 4x, 8x, 16x,….

Now,

d_1= 2x – x

= x,

d^2= 4x – 2x

= 2x

Also, $d_1 \neq d_2$

Hence, the list of numbers does not from an A.P.

8. Justify whether it is true to say that the following are the *n*th terms of an AP.

i. $2n-3$

ii. $3n^2+5$

iii. $1+n+n^2$

Solution:

(i) a_n = 2n – 3
a_1= 2(1) – 3
= 2 – 3
= –1,

a_2= 2(2) – 3
= 4 – 3
= 1

a_3= 2(3) – 3
= 6 – 3
= 3,

a_4= 2(4) – 3

www.dreamtopper.in

$= 8 - 3$
$= 5$

Also,
$d_1 = 1 - (-1)$
$= 1 + 1$
$= 2,$

$d_2 = 3 - 1$
$= 2,$

$d_3 = 5 - 3$
$= 2$

$d_1 = d_2 = d_3 = 2,$
Hence, an = 2n – 3 is the nth term of an A.P.

(ii)
$a_n = 3n^2 + 5$

$a_1 = 3(1)^2 + 5$
$= 3 \times 1 + 5$
$= 3 + 5 = 8$

$a_2 = 3(2)^2 + 5$
$= 3 \times 4 + 5$
$= 12 + 5 = 17$

$a_3 = 3(3)^2 + 5$
$= 3 \times 9 + 5$
$= 27 + 5 = 32$

$a_4 = 3(4)^2 + 5$
$= 3 \times 16 + 5$
$= 48 + 5 = 53$

$a_5 = 3(5)^2 + 5$
$= 3 \times 25 + 5$
$= 75 + 5 = 80$

$\therefore d_1 = a_2 - a_1$
$= 17 - 8 = 9,$

$d_2 = a_3 - a_2$
$= 32 - 17 = 15$

$d_3 = a_4 - a_3$
$= 53 - 32 = 21,$

www.dreamtopper.in

$d_4 = a_5 - a_4$
$= 80 - 53 = 27$

As, $d_1 \neq d_2$
Hence, an = $3n^2 + 5$ is not the nth term of an A.P.

(iii) $a_n = 1 + n + n^2$

$a_1 = 1 + (1) + (1)^2$

$= 1 + 1 + 1 = 3$

$a_2 = 1 + (2) + (2)^2$

$= 1 + 2 + 4 = 7$

$a_3 = 1 + (3) + (3)^2$

$= 1 + 3 + 9 = 13$

$a_4 = 1 + (4) + (4)^2$

$= 1 + 4 + 16 = 21$

$a_5 = 1 + (5) + (5)^2$

$= 1 + 5 + 25 = 31$

So,
$d_1 = a_2 - a_1$
$= 7 - 3 = 4$
$d_2 = a_3 - a_2$
$= 13 - 7 = 6$
$d_3 = a_4 - a_3$
$= 21 - 13 = 8$

$d_4 = a_5$ -,a_4
$= 31 - 21 = 10$

As d1$\neq$ d2
Hence, $a_n = 1 + n + n^2$ is not the nth term of an A.P.

www.dreamtopper.in

Exercise 5.3

Short Answer Questions:

Question:

1. Match the APs given in column A with suitable common differences given in column B.

Column A	**Column B**
(A_1) $2, -2, -6, -10, \ldots$	(B_1) $\frac{2}{3}$
(A_2) $a = -18, n = 10, a_n = 0$	(B_2) -5
(A_3) $a = 0, a_{10} = 6$	(B_3) 4
(A_4) $a_2 = 13, a_4 = 3$	(B_4) -4
	(B_5) 2
	(B_6) $\frac{1}{2}$
	(B_7) 5

www.dreamtopper.in

Solution:

(A_1)

AP is 2, – 2, – 6, – 10…

So,

$d = a_2 - a_1$

$= -2 - 2$

$= -4$

$= (B_3)$

(A_2)

First term, a = – 18

No of terms, n = 10

Last term, an = 0

We have,

$a_n = a + (n - 1)d$

$0 = -18 + (10 - 1)d$
$18 = 9d$
$d = 2 = (B_5)$

(A_3)
First term, a = 0
Tenth term, $a_{10} = 6$

We have,
$a_n = a + (n - 1)d$
$a_{10} = a + 9d$
$6 = 0 + 9d$
$d = \frac{2}{3} = (B_6)$

(A_4)
Taking the first term be a and common difference be d

We have,
$a_2 = 13$
$a_4 = 3$
$a_2 - a_4 = 10$
$a + d - (a + 3d) = 10$

$d - 3d = 10$
$-2d = 10$
$d = -5$
$= (B_1)$

2. Verify that each of the following is an AP, and then write its next three terms.

i. $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

ii. $5, \frac{14}{3}, \frac{13}{3}, 4, \ldots$

iii. $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$

iv. $a + b, (a + 1) + b, (a + 1) + (b + 1), \ldots$

v. $a, 2a + 1, 3a + 2, 4a + 3, \ldots$

Solution:
(i)
$0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

$a_1 = 0$

www.dreamtopper.in

$a_2 = \frac{1}{4}$

$a_3 = \frac{1}{2}$

$a_4 = \frac{3}{4}$

$a_2 - a_1 = \frac{1}{4} - 0 = \frac{1}{4}$

$a_3 - a_2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$a_4 - a_3 = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

As, difference of successive terms are equal,

So, 0, $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ … is an AP with common difference ¼.

Therefore, the next three term will be,

$\frac{3}{4} + \frac{1}{4}, \frac{3}{4} + 2(\frac{1}{4}), \frac{3}{4} + 3(\frac{1}{4})$

1, $\frac{5}{4}, \frac{3}{2}$

(ii) 5, $\frac{14}{3}, \frac{13}{3}$, 4…

$a_1 = 5$

$a_2 = \frac{14}{3}$

$a_3 = \frac{13}{3}$

$a_4 = 4$

$a_2 - a_1 = \frac{14}{3} - 5$

www.dreamtopper.in

$= \frac{-1}{3}$

$a_3 - a_2 = \frac{13}{3} - \frac{14}{3}$

$= \frac{-1}{3}$

$a_4 - a_3 = 4 - \frac{13}{3}$

$= \frac{-1}{3}$

As, difference of successive terms are equal,

So, 5, $\frac{14}{3}$, $\frac{13}{3}$, 4… is an AP with common difference -1/3.

Hence, the next three term will be,

$4 + (\frac{-1}{3}), 4 + 2(\frac{-1}{3}), 4 + 3(\frac{-1}{3})$

$\frac{11}{3}, \frac{10}{3}, 3$

(iii)

$\sqrt{3}$, $2\sqrt{3}$, $3\sqrt{3}$,…

$a_1 = \sqrt{3}$

$a_2 = 2\sqrt{3}$

$a_3 = 3\sqrt{3}$

$a_4 = 4\sqrt{3}$

$a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$a_3 - a_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$

$a_4 - a_3 = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$

As, difference of successive terms are equal,

So, $\sqrt{3}$, $2\sqrt{3}$, $3\sqrt{3}$,… is an AP with common difference $\sqrt{3}$.

Hence, the next three term will be,

www.dreamtopper.in

$4\sqrt{3} + \sqrt{3}$, $4\sqrt{3} + 2\sqrt{3}$, $4\sqrt{3} + 3\sqrt{3}$

$5\sqrt{3}$, $6\sqrt{3}$, $7\sqrt{3}$

(iv)
a + b, (a + 1) + b, (a + 1) + (b + 1), …

$a_1 = a + b$

$a_2 = (a + 1) + b$

$a_3 = (a + 1) + (b + 1)$

$a_2 – a_1 = (a + 1) + b – (a + b) = 1$

$a_3 – a_2 = (a + 1) + (b + 1) – (a + 1) – b = 1$

As, difference of successive terms are equal,

So, a + b, (a + 1) + b, (a + 1) + (b + 1), … is an AP with common difference 1.

Hence, the next three term will be,

(a + 1) + (b + 1) + 1, (a + 1) + (b + 1) + 1(2), (a + 1) + (b + 1) + 1(3)

(a + 2) + (b + 1), (a + 2) + (b + 2), (a + 3) + (b + 2)

(v) a, 2a + 1, 3a + 2, 4a + 3,…

$a_1 = a$

$a_2 = 2a + 1$

$a_3 = 3a + 2$

$a_4 = 4a + 3$

$a_2 – a_1 = (2a + 1) – (a)$
$= a + 1$

$a_3 – a_2 = (3a + 2) – (2a + 1)$
$= a + 1$

$a_4 – a_3 = (4a + 3) – (3a+2)$
$= a + 1$

As, difference of successive terms are equal,

www.dreamtopper.in

So, a, 2a + 1, 3a + 2, 4a + 3,… is an AP with common difference a+1.

Hence, the next three term will be,

4a + 3 +(a + 1), 4a + 3 + 2(a + 1), 4a + 3 + 3(a + 1)

5a + 4, 6a + 5, 7a + 6

3. Write the first three terms of the APs when *a* and *d* are as given below:

i. $a = \frac{1}{2}, d = -\frac{1}{6}$

ii. $a = -5, d = -3$

iii. $a = \sqrt{2}, d = \frac{1}{\sqrt{2}}$

Solution:

(i)

$a = \frac{1}{2}$, $d = -\frac{1}{6}$

First three terms of AP are :

a,

a + d,

a + 2d

$\frac{1}{2}, \frac{1}{2} + (-\frac{1}{6}), \frac{1}{2} + 2(-\frac{1}{6})$

$\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

(ii)

a = –5, d = –3

First three terms of AP are:

a, a + d, a + 2d

-5, – 5 + 1 (- 3), – 5 + 2 (- 3)

– 5, – 8, – 11

(iii)

$a = \sqrt{2}$, $d = \frac{1}{\sqrt{2}}$

First three terms of AP are :

a, a + d, a + 2d

www.dreamtopper.in

$\sqrt{2}$, $\sqrt{2}+\frac{1}{\sqrt{2}}$, $\sqrt{2}+\frac{2}{\sqrt{2}}$

$\sqrt{2}$, $\frac{3}{\sqrt{2}}$, $\frac{4}{\sqrt{2}}$

4. Find *a*, *b* and *c* such that the following numbers are in AP: *a*, 7, *b*, 23, *c*.

Solution:

To be a, 7, b, 23, c… in AP.

It should satisfy the condition,

$a_5 - a_4 = a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = d$ (as common difference is same)

$7 - a = b - 7 = 23 - b = c - 23$

So,

$b - 7 = 23 - b$

$2b = 30$

$b = 15$

Also,

$7 - a = b - 7$
$7 - a = 15 - 7$ (putting value of b)
$a = -1$

And,

$c - 23 = 23 - b$

$c - 23 = 23 - 15$

$c - 23 = 8$

$c = 31$

So,
$a = -1$

$b = 15$

www.dreamtopper.in

c = 31

So, we can say that, the sequence – 1, 7, 15, 23, 31 is an AP

5. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

Solution:

As given in the question,
5th term,
$a_5 = 19$

Using the formula,
$a_n = a + (n - 1)d$
We have,
$a + 4d = 19$
$a = 19 - 4d$ …(1)

www.dreamtopper.in

And,
20th term – 8th term = 20
$a + 19d - (a + 7d) = 20$
$12d = 20$
$d = \frac{4}{3}$

Putting $d = \frac{4}{3}$ in equation 1,
We get,
$a = 19 - 4(\frac{4}{3})$
$a = \frac{41}{3}$

The required AP is,
$\frac{41}{3}, \frac{41}{3} + \frac{4}{3}, \frac{41}{3} + 2(\frac{4}{3})$

$\frac{41}{3}, 15, \frac{49}{3}$

6. The 26th, 11th and the last term of an AP are 0, 3 and $-\frac{1}{5}$, respectively. Find the common difference and the number of terms.

Solution:

Given:
$a_{26} = 0$,
$a_{11} = 3$ and
$a_n = -\frac{1}{5}$

$a_{26} = 0$ [Given]
$a + (26 - 1)d = 0$
$a + 25d = 0$ ...(i)
$a_{11} = 3$ [Given]

$a + (11 - 1)d = 3$
$a + 10d = 3$ ...(ii)

$a_n = a + (n - 1)d = -\frac{1}{5}$ ...(iii)

On subtracting eqn. (ii) from eqn. (i), we get
$15d = -3$
$d = -\frac{1}{5}$
From (ii),
$a + 10d = 3$
$a - 2 = 3$
$a = 3 + 2$
$a = 5$

From (iii),
$a + (n - 1)d = -\frac{1}{5}$
$5 + (n - 1) \times -\frac{1}{5} = -\frac{1}{5}$

Multiplying both sides by 5, we get
$25 - (n - 1) = -1$
$25 + 1 = (n - 1)$

$n - 1 = 26$
$n = 27$

So, the common difference and number of terms in the A.P. are $-\frac{1}{5}$ and 27 respectively.

7. The sum of the 5^{th} and the 7^{th} terms of an AP is 52 and the 10^{th} term is 46. Find the AP.

www.dreamtopper.in

Solution:
Let 1st term and common difference of an A.P be a and d
As given in the question,

$a_5 + a_7 = 52$
$a + (5 – 1)d + a + (7 – 1)d = 52$ (an = a + (n – 1)d)
$2a + 4d + 6d = 52$
$2a + 10d = 52$
$a + 5d = 26$ ...(i)

Also,
$a_{10} = 46$ (Given)
$a + (10 – 1)d = 46$
$a + 9d = 46$ ...(ii)

Subtracting (i) from (ii), we get,
$d = 5$

Now,
$a + 5d = 26$ (From (i))
$a + 5 \times 5 = 26$
$a = 26 – 25$
$a = 1$

A.P. is given by a, a + d, a + 2d, ...
So, the required A.P. is given by 1, 6, 11, 16, ...

www.dreamtopper.in

8. Find the 20^{th}term of the AP whose 7^{th}term is 24 less than the 11^{th}term, first termbeing 12.

Solution:
Let us consider an A.P. with first term and common difference are ‘a’ and ‘d’.
We have,
$a_7 = a_{11} – 24$
$a + (7 – 1)d + 24 = a + (11 – 1)d$ [$a_n = a + (n – 1)d$]
$a + 6d + 24 – a = 10d$
$6d – 10d = –24$
$–4d = –24$
$d=6$

Now,
$a_n = a + (n – 1)d$
$a_{20} = 12 + (20 – 1)6$ [As ,n = 20, a = 12, d = 6]
$= 12 + 19 \times 6$
$= 12 + 114$
$a_{20} = 126$

So, the 20th term of the A.P. is 126.

9. If the 9^{th} term of an AP is zero, prove that its 29^{th} term is twice its 19^{th} term.

Solution:

Consider an A.P. whose first term and common difference are ‘a’ and ‘d’ respectively.

$a_9 = 0$ [Given]

$a + (9 – 1)d = 0$ [an = a + (n – 1)d]

$a + 8d = 0$

$a = –8d$ ...(i)

We have to prove that $a_{29} = 2a_{19}$

So, $a_{29} = a + (29 – 1)d$

$= –8d + 28d$ [Using equation (i)]

$a_{29} = 20d$ ...(ii)

Now,

$a_{19} = a + (19 – 1)d$

$a_{19} = –8d + 18d$ [Using (i)]

$a_{19} = 10d$

But, $a_{29} = 20d$ [From (ii)]

$= 2 \times 10d$

$= 2 \times a_{19}$ [$a_{19} = 10d$]

$= 2a_{19}$

$a_{29} = 2a_{19}$

Hence, the 29th term is twice the 19th term in the given A.P.

www.dreamtopper.in

10. Find whether 55 is a term of the AP: 7, 10, 13,--- or not. If yes, find which term it is.

Solution:

55 will be nth term of the given A.P. if value of n is a natural number.

$a = 7$,

$d = 10 – 7$

$= 3$

Let 55 be the nth term of the given A.P.

$a_n = 55$ [assumed]

$7 + (n – 1)3 = 55$ [$\because$ an = a +(n – 1)d]

$(n – 1)3 = 55 – 7$

$n – 1 = 16$

$n = 17$, which is a natural number

So, 55 is the 17th term of the given A.P.

11. Determine *k* so that $k^2+4k+8, 2k^2+3k+6, 3k^2+4k+4$ are three consecutive terms of an AP.

Solution:

Since, $k^2+4k+8, 2k^2+3k+6, 3k^2+4k+4$ and $3k^2 + 4k + 4$ are consecutive terms of an AP.
$2k^2 + 3k + 6 - (k^2 + 4k + 8) = d$

$3k^2 + 4k + 4 - (2k^2 + 3k + 6) = d$

$2k^2 + 3k + 6 - k^2 - 4k - 8 = 3k^2 + 4k + 4 - 2k^2 - 3k - 6$

$k^2 - k - 2 = k^2 + k - 2$
$-k = k$
$2k = 0$
$k = 0$

12. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

Solution:

We know that,
If the sum of three consecutive terms of an AP is given so terms can be considered as (a – d), a, (a + d).
Considering an A.P. whose three consecutive terms are (a – d), a, (a + d).
So,
$(a - d) + a + (a + d) = 207$
$3a = 207$
$a = 69$

Also, $(a - d)(a) = 4623$
$(69 - d)69 = 4623$ (a = 69)

$69 - d = 67$
$d = 69 - 67$
$d = 2$

So,
A.P. = (a – d), a, (a + d)
= (69 – 2), 69, (69 + 2)
= 67, 69, 71
Therefore, 207 can be divided into 67, 69, 71 which form three terms of an A.P.

13. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.

www.dreamtopper.in

Solution:
We know that,
Sum of interior angles of a triangle is 180°.
So, 180° is divided into three parts which are in A.P.
So, the terms of A.P. are (a – d), a, (a + d).

a – d + a + a + d = 180° [Angle sum property of a triangle]
3a = 180°
a = 60°

Also, the greatest angle is twice of the smallest. [Given]

a + d = 2(a – d)
a + d = 2a – 2d
a + d – 2a + 2d = 0
–a + 3d = 0
3d = a

Also, a = 60°
d = 20°

Required parts are a – d, a, a + d
= 60° – 20°, 60°, 60° + 20°
= 40°, 60°, 80°
Hence, the angles of the triangle are 40°, 60° and 80°.

14. If the *n*th terms of the two APs: $9, 7, 5, \ldots$ **and** $24, 21, 18, \ldots$ **are the same, find the value of *n*. Also find that term.**

Solution:
First A.P. is 9, 7, 5, ...
$a_1 = 9$,
d = 7 – 9
= –2
Now,
$a_n = a + (n – 1)d$
= 9 + (n – 1)(–2)
= 9 – 2(n – 1)
= 9 – 2n + 2
$a_n = 11 – 2n$

Second A.P. is 24, 21, 18, ...

$a_n = 24 + (n – 1)(–3)$
= 24 – 3n + 3
= 27 – 3n

www.dreamtopper.in

We have,
11 – 2n = 27 – 3n
3n – 2n = 27 – 11
n = 16

So, 16th term of 1st A.P
$a_{16} = a_1 + (n – 1)d$

$a_{16} = 9 + (16 – 1)(–2)$
$= 9 – 2 \times 15 = 9 – 30$
$a_{16} = –21$

16th term of 2nd A.P.,
= 24 + (16 – 1) (–3)
= 24 – 15 × 3
= 24 – 45
= –21
So, the 16th terms of both A.P.s are equal to – 21.

15. If sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and the 14th terms is –3, find the 10th term.

Solution:

Taking 1st term and common difference 0f an A.P a and d, respectively.
According to the question,
$a_3 + a_8 = 7$ [Given]
a + (3 – 1)d + a + (8 – 1)d = 7 [∵ an = a + (n – 1)d]
a + 2d + a + 7d = 7
2a + 9d = 7 ...(i)

Also, $a_7 + a_{14} = –3$ [Given]
a + (7 – 1)d + a + (14 – 1)d = –3
a + 6d + a + 13d = –3
2a + 19d = –3 ...(ii)

Now, subtracting (i) from (ii), we get
d = –1
Now, 2a + 9d = 7 [Using (i)]
2a + 9(–1) = 7
2a = 7 + 9
a = 8

$a_{10} = a + (10 – 1)d$
$= 8 + 9(–1)$
$a_{10} = –1$
So, the 10th term of A.P. is –1.

www.dreamtopper.in

16. Find the 12^{th} term from the end of the AP: $-2, -4, -6, \ldots, -100$.

Solution:

Considering the given A.P. in reverse order
and finding the term.
i.e.,
–100 ... –6, –4, –2.

Now,
a = –100
$d = a_n + 1 - a_n$
= –4 – (–6)
= –4 + 6
= 2
n = 12

$a_n = a + (n - 1)d$
$a_{12} = -100 + (12 - 1)(2)$
$= -100 + 11 \times 2 = -100 + 22$
$a_{12} = -78$
Therefore, the 12th term from the last of A.P. –2, –4, –6, ... –100 is –78.

17. Which term of the AP: $53, 48, 43, \ldots$ is the first negative term?

Solution:

We have A.P. is 53, 48, 43,...
a = 53,
d = 48 – 53
= –5
Taking the nth term of A.P. is the first negative term.

Then, $a_n < 0$
$a + (n - 1)d < 0$
$53 + (n - 1)(-5) < 0$
$-5(n - 1) < -53$
$5(n - 1) > 53$
$5n - 5 > 53$
$5n > 53 + 5$
$n > 11.6$
$n = 12$
So, the first negative term of A.P. is 12th term,

$a_{12} = a + (n - 1)d$
$= 53 + (12 - 1)(-5)$
$= 53 - 5 \times 11$
$= 53 - 55$

www.dreamtopper.in

$= -2$

18. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

Solution:

The least number between 10 and 300 which leaves remainder 3 after dividing by 4 is 11.
The largest number between 10 and 300 which leaves remainder 3 on dividing by 4 is
296 + 3 =299.

So, 1st term or number = 11,
2nd term or number = 15
3rd term or number = 19,
last term or number = 299

A.P. becomes 11, 15, 19, ..., 299
$a_n = 299$,
$a = 11$,
$d = 15 - 11$
$= 4$,
$n = ?$

Now, $a + (n - 1)d = 299$
$11 + (n - 1)4 = 299$
$(n - 1)4 = 299 - 11$

$n - 1 = 72$
$n = 72 + 1$
$n = 73$
Hence, the required numbers between 10 and 300, which when divided by 4 leave a remainder 3 are 73.

19. Find the sum of the two middle most terms of the AP: $-\frac{4}{3}, -1, -\frac{2}{3}, \ldots, 4\frac{1}{3}$.

Solution:

www.dreamtopper.in

$a = \frac{-4}{3}$

$d = -1 + \frac{4}{3}$

$d = \frac{1}{3}$

Also,

$l = \frac{13}{3}$

$a + (n-1)d = \frac{13}{3}$

So,

$n = 18$

Middle most terms are:

$\frac{n}{2}$ th and $(\frac{n}{2}+1)$th

Which are,

$\frac{18}{2}$ term and $(\frac{18}{2}+1)$ term

that are,

9 th and 10 th terms,

So,

$a_9 = \frac{4}{3}$

$a_{10} = \frac{5}{3}$

$\text{Sum} = a_9 + a_{10}$

$\text{Sum} = 3$

20. The first term of an AP is –5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.

Solution:

Let the first term, common difference and the number of terms of an AP be a, d and n respectively.
Given that,
a = -5
l = 45
Sum of the terms of the AP = 120
$S_n = 120$

www.dreamtopper.in

We know that, if last term of an AP is known, then sum of n terms of an AP is,

$S_n = \frac{n}{2}$ (a + l)

$120 = \frac{n}{2}$ (-5 + 45)

120 × 2 = 40 × n

n = 3 × 2

n = 6

Number of terms of an AP is known, then the nth term of an AP is,

l = a + (n – 1)d

45 = -5 + (6 – 1)d

50 = 5d

d = 10

Hence, the common difference is 10.

So, number of terms and the common difference of an AP are 6 and 10 respectively.

21. Find the sum:

i. $1+(-2)+(-5)+(-8)+...+(-236)$

ii. $4-\frac{1}{n}+4-\frac{2}{n}+4-\frac{3}{n}+...$ **upto *n* terms.**

iii. $\frac{a-b}{a+b}+\frac{3a-2b}{a+b}+\frac{5a-3b}{a+b}+...$ **to 11 terms.**

Solution:

(i)

a = 1 and

d = (-2) – 1

= -3

Sum of n terms of an AP,

$$S_n=\frac{n}{2}(2a+(n-1)d)$$

$$S_n=\frac{n}{2}(2\times1+(n-1)\times-3)$$

$$S_n=\frac{n}{2}(5-3n)$$

We know that, if the last term (l) of an AP is known, then

l = a + (n – 1)d

-236 = 1 + (n – 1) (-3) [∵ l = -236, given]

-237 = -(n – 1) × 3

n – 1 = 79

n = 80

Now, put the value of n in we get ,

www.dreamtopper.in

$S_n = 40[5 – 3 \times 80]$
$= 40[5- 240]$
$= 40 \times (-235)$
$= -9400$
The required sum is -9400.

(ii)

$$a = 4 - \frac{1}{n}$$

$$d = (4 - \frac{2}{n}) - (4 - \frac{1}{n})$$

$$d = -\frac{1}{n}$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{7n-1}{2}$$

(iii)

$$a = \frac{a-b}{a+b}$$

$$d = \frac{3a-2b}{a+b} - \frac{a-b}{a+b}$$

$$d = \frac{2a-b}{a+b}$$

Also,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}(\frac{2an-bn-b}{a+b})$$

So,

$$S_{11} = \frac{11(11a-6b)}{a+b}$$

22. Which term of the AP: –2, –7, –12,... will be –77? Find the sum of this AP up to the term –77.

Solution:

Given, AP : -2, -7, -12, ….
Taking the nth term of an AP is -77
a = -2 and
d = -7 – (-2)

www.dreamtopper.in

= -7 + 2
= -5

nth term of an AP,
$T_n = a + (n – 1)d$
-77 = -2 + (n – 1)(-5)
-75 = -(n – 1) × 5
(n – 1) = 15
n = 16
So, the 16th term of the given AP will be -77.

Now, the sum of n terms of an AP is

$S_n = \frac{n}{2}$ [2a + (n – 1)d]

So, sum of 16 terms i.e., upto the term -77.

$S_{16} = \frac{n}{2}$ [2 × (-2) + (n – 1)(-5)l
= 8[-4 + (16 – 1)(-5)]
= 8(-4 – 75)
= 8 × (-79)
= -632
Therefore, the sum of this AP upto the term -77 is -632.

23. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, ...$ form an AP. Also find S_{20}.

Solution:
Given that, nth term of the series is
a_n = 3 – 4n …(i)

Putting n = 1,
a_1 = 3 – 4(1)
= 3 – 4
= -1

Putting n = 2,
a_2 = 3 – 4(2)
= 3 – 8
= -5

Putting n = 3,
a_3 = 3 – 4(3)
= 3 – 12
= -9

Putting n = 4,
a_4 = 3 – 4(4)
= 3 – 16

www.dreamtopper.in

= -13
So, the series becomes -1, -5, -9, -13,….

We see that,
$a_2 - a_1 = -5 - (-1)$
$= -5 + 1$
$= -4,$

$a_3 - a_2 = -9 - (-5)$
$= -9 + 5$
$= -4,$

$a_4 - a_3 = -13 - (-9)$
$= -13 + 9$
$= -4$

i.e., $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = -4$
Since, the each successive term of the series has the same difference. So, it forms an AP. We know that, sum of n terms of an AP,

$S_n = \frac{n}{2} [2a + (n - 1)d]$

Sum of 20 terms of the AP,
$S_{20} = 10[2(-1) + (20 - 1)(-4)]$
$= 10 [-2 + (19)(-4)]$
$= 10(-2 - 76)$
$= 10 \times (-78) = -780$
So, the required sum of 20 terms i.e., S_{20} is -780

24. In an AP, if $S_n = n(4n+1)$, find the AP.

Solution:

The nth term of an AP is
$a_n = S_n - S_n - 1$
$a_n = n(4n + 1) - (n - 1)[4(n - 1) + 1]$

[as, $S_n = n(4n + 1)$]
$an = 4n^2 + n - (n - 1)(4n - 3)$
$= 4n^2 + n - 4n2 + 3n + 4n - 3$
$= 8n - 3$
Put n = 1,
$a_1 = 8(1) - 3$
$= 5$
Put n = 2,
$a_2 = 8(2) - 3$
$= 16 - 3$
$= 13$

www.dreamtopper.in

Put n = 3,
$a_3 = 8(3) - 3$
$= 24 - 3$
$= 21$
So, the required AP is 5, 13, 21,….

25. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of *k*.

Solution:

We have, nth term of an AP,
$a_n = S_n - S_n - 1$
$= 3n^2 + 5n - 3(n-1)^2 - 5(n-1)$ [As $Sn = 3n^2 + 5n$ (given)]
$= 3n^2 + 5n - 3n^2 - 3 + 6n - 5n + 5$
$a_n = 6n + 2$ …………… (i)
$a_k = 6k + 2$
$= 164$ ($a_k = 164$ (given))
$6k = 164 - 2$
$= 162$

So,
k = 27

26. If S_n denotes the sum of first *n* terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$

Solution:

Sum of n terms of an AP = $\frac{n}{2}(2a + (n-1)d)$

Now,
$S_4 = 4a + 6d$
$S_8 = 8a + 28d$

So,
$S_8 - S_4 = 4a + 22d$
Now,
$S_{12} = \frac{12}{2}(2a + (n-1)d)$
$S_{12} = 3(4a + 22d)$
$S_{12} = 3(S_8 - S_4)$
Proved!!!

27. Find the sum of first 17 terms of an AP whose 4th and 9th terms are –15 and –30 respectively.

Solution:

www.dreamtopper.in

Let us take the first term, common difference and the number of terms in an AP be a, d and n, respectively.
We know that, the nth term of an AP,

$T_n = a + (n – 1)d$ ….. (i)

4th term of an AP,

$T_4 = a + (4 – 1)d$
$= -15$ [given]
$a + 3d = -1$ ….(ii)

and 9th term of an AP,

$T_9 = a + (9 – 1)d = -30$ [given]
$a + 8d = -30$ …………. (iii)

Now, subtract Eq. (ii) from Eq. (iii), we get
$5d = -15$
$d = -3$
Put the value of d in Eq.(ii), we get
$a + 3(-3) = -15$
$a – 9 = -15$
$a = -15 + 9$
$= – 6$

Now putting values of a and d, we get,
$S_{17}= -510$

Hence, the required sum of first 17 terms of an AP is -510.

www.dreamtopper.in

28. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

Solution:

Let a and d be the first term and common difference, of an AP.
Sum of n terms of an AP**,**

Now,
$S_6=36$
So,
$12=2a+5d$ ….1
Also,
$S_{16}=256$
So,
$32=2a+15d$ …..2

Subtracting eqn 1 and 2 we get,
d=2
a=1
Therefore putting value of and d in S_{10}, we get,
S_{10}=100

29. Find the sum of all the 11 terms of an AP whose middle most term is 30.

Solution:
As, the total number of terms (n) = 11 [odd]

Middle most term:

$$\frac{n+1}{2} term$$
$$= \frac{11+1}{2} term$$
$$= 6th\, term$$
$$Also,$$
$$a_6 = 30$$
$$a + 5d = 30$$
$$So,$$
$$S_{11} = \frac{n}{2}[2a + (11-1)d]$$
$$S_{11} = 11(a + 5d)$$
$$S_{11} = 11 \times 30$$
$$S_{11} = 330$$

www.dreamtopper.in

30. Find the sum of last ten terms of the AP: $8, 10, 12, ..., 126$.

Solution:

To find the sum of last ten terms, we write the given AP in reverse order.
i.e., 126, 124, 122,…., 12, 10, 8

a = 126,
d = 124 – 126
= – 2

$$S_{10} = \frac{n}{2} [2a + (10 - 1)d]$$

As,

$$S_n = \frac{n}{2} [2a + (n - 1)d]]$$

$= 5\{2(126) + 9(-2)\}$
$= 5(252 – 18)$
$= 5 \times 234$
$= 1170$

31. Find the sum of first seven numbers which are multiples of 2 as well as of 9.
[Hint: Take the LCM of 2 and 9]

Solution:

To find the sum of first seven numbers which are multiples of 2 as well as of 9.
We take LCM of 2 and 9 which is 18.
Hence, the series becomes 18, 36, 54 ….

a = 18,
d = 36 – 18
= 18
Using the formula of S_n,

$$S_7 = \frac{7}{2}[2\times18+(7-1)18]$$

$$S_7 = \frac{7}{2}[36+6\times18]$$

$$S_7 = 504$$

32. How many terms of the AP: $-15, -13, -11, \ldots$ are needed to make the sum –55? Explain the reason for double answer.

Solution:

Let we assume n number of terms are needed to make the sum -55
a= -15,
d = -13 + 15 = 2
Sum of n terms of an AP,

$$Sn = \frac{n}{2}[2a + (n – 1)d]$$

$$-55 = \frac{n}{2}[2(-15) + (n – 1)2]$$

Also,
Sn = -55 (given)

-55 = -15M + n(n – 1)
$n^2 – 16n + 55 = 0$
$n^2 – 11n – 5n + 55 = 0$
$n(n – 11) – 5(n – 11) = 0$
$(n – 11)(n – 5) = 0$

www.dreamtopper.in

n = 5, 11

Either 5 or 11 terms are needed to make the sum -55 when n = 5,
AP will be -15, -13, -11, -9, -7,

So, resulting sum will be -55 because all terms are negative.
When n = 11,
AP will be -15, -13, -11, -9, -7, -5, -3, -1, 1, 3, 5

Hence, resulting sum will be -55 because the sum of terms 6th to 11th is zero.

33. The sum of the first *n* terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first 2*n* terms of another AP whose first term is – 30 and the common difference is 8. Find *n*.

Solution:
Given,
a = 8
d = 20

Let the number of terms in first AP be n.
Sum of first n terms of an AP,

$$S_n = \frac{n}{2}[2\times 8+(n-1)20]$$

$$S_{31} = \frac{n}{2}(20n-4)$$

$$S_{31} = n(10n-2)$$

www.dreamtopper.in

Now,
first term of the second AP(a’) = – 30
Common difference of the second AP(d’) = 8

Sum of first 2n terms of second AP,

$$S_{2n} = \frac{2n}{2}[2a' + (2n - 1)d']$$

S_{2n} = n[2(-30)+(2n – 1)(8)]
S_{2n} = n[-60 + 16n – 8)]
S_{2n} = n[16n – 68]

Now, by given condition,
Sum of first n terms of the first AP = Sum of first 2n terms of the second AP

S_n — S_{2n}

n(10n – 2) = n(16n – 68)
n[(16n – 68) – (10n – 2)] = 0

n(16n – 68 – 10n + 2) = 0
n(6n – 66) = 0
n = 11

So, the required value of n is 11.

34. Kanika was given her pocket money on Jan 1st, 2008. She puts Re 1 on Day 1,Rs 2 on Day 2, Rs 3 on Day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent Rs 204 of her pocket money, and found that at the end of the month she still had Rs 100 with her. How much was her pocket money for the month?

Solution:

Let her pocket money be ₹ x
If, she puts 11 on day 1, ₹ 2 on day 2, ₹ 3 on day 3 and so on till the end of the month, from this money into her piggy bank.
So,
1 + 2 + 3 + 4 + … + 31
which form an AP in which terms are 31
a = 1,
d = 2 – 1
= 1
Sum of first 31 terms is S_{31}

$$S_{31} = \frac{31}{2}[2\times1+(31-1)1]$$
$$S_{31} = \frac{31\times32}{2}$$
$$S_{31} = 496$$

Hence, Kanika takes ₹ 496 till the end of the month from this money.

Also, she spent ₹ 204 of her pocket money and found that at the end of the month she still has ₹ 100 with her.
So,
(x – 496) – 204 = 100
x – 700 = 100
x = ₹ 800
Therefore, ₹ 800 was her pocket money for the month.

35. Yasmeen saves Rs 32 during the first month, Rs 36 in the second month and Rs 40 in the third month. If she continues to save in this manner, in how many months will she save Rs 2000?

www.dreamtopper.in

Solution:

Yasmeen, during the first month, saves = ₹ 32

During the second month, saves = ₹ 36

During the third month, saves = ₹ 40

Let we take Yasmeen saves Rs 2000 during the n months.

So, we have arithmetic progression 32, 36, 40…

a = 32,

d = 36 – 32

= 4

and she saves total money,

Sn = ₹ 2000

We know that, sum of first n terms of an AP is,

$Sn = \frac{n}{2}][2a + (n-1)d]$

$2000 = \frac{n}{2}[2 \times 32 + (n-1) \times 4]$

2000 = n(32 + 2n – 2)

2000 = n(30 + 2n)

1000 = n (15 + n)

$1000 = 15n + n^2$

$n^2 + 15n - 1000 = 0$

$n^2 + 40n - 25n - 1000 = 0$

n(n + 40) – 25(n + 40) = 0

(n + 40)(n – 25)= 0

n = 25

$n \neq -40$

[As, months cannot be negative]

So, in 25 months she will save ₹ 2000.

www.dreamtopper.in

Exercise 5.4

Long Answer Questions:

Question:

1. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

Solution:

In an A.P. ,
First term = a
Common difference = d
Number of terms of an AP = n

Now, we have,
$S_5 + S_7 = 167$

Using the formula,

$S_n = \frac{n}{2}$ [2a + (n-1)d]

Putting value,

$\frac{5}{2}$ [2a + (5-1)d] + $\frac{7}{2}$ [2a + (7-1)d] = 167

5(2a + 4d) + 7(2a + 6d) = 334

10a + 20d + 14a + 42d = 334

24a + 62d = 334

12a + 31d = 167

12a = 167 – 31d …(i)

Also,
$S_{10} = 235$

$\frac{10}{2}$ [2a + (10-1)d] = 235

5[2a + 9d] = 235

2a + 9d = 47

Multiplying 6 in both the sides,
12a + 54d = 282

From equation (i)
167 – 31d + 54d = 282

23d = 282 – 167

23d = 115

www.dreamtopper.in

$d = 5$

Putting the value of $d = 5$ in equation (i)

$12a = 167 – 31(5)$

$12a = 167 – 155$

$12a = 12$

$a = 1$

Also,

$S_{20} = \frac{n}{2} [2a + (20 – 1)d]$

$= \frac{20}{2} [2(1) + 19 (5)]$

$= 10[2 + 95]$

$= 970$

Hence, the sum of first 20 terms is 970.

2. Find the

i. Sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

ii. Sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

iii. Sum of those integers from 1 to 500 which are multiples of 2 or 5.
[Hint (iii): These numbers will be: multiples of 2 + multiples of 5 – multiples of 2 as well as of 5]

www.dreamtopper.in

Solution:

We know that,

Multiples of 2 as well as of 5 = LCM of (2, 5)
= 10

Also, Multiples of 2 as well as of 5 between 1 and 500 = 10, 20, 30…, 490.

We can conclude that 10, 20, 30…, 490 is an AP with common difference,
$d = 10$

First term,
$a = 10$

Taking the number of terms in this AP = n

Using nth term formula,

$a_n = a + (n – 1)d$

$490 = 10 + (n – 1)10$

$480 = (n – 1)10$

So,
n – 1 = 48
n = 49

Sum of an AP,

$S_n = \left(\frac{n}{2}\right)[a + a_n]$ [a_n is the last term]

$= \left(\frac{49}{2}\right) \times [10 + 490]$

$= \left(\frac{49}{2}\right) \times [500]$

$= 49 \times 250$

$= 12250$

Sum of those integers between 1 and 500 which are multiples of 2 as well as of 5 = 12250

(ii) Sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

We have,
Multiples of 2 as well as of 5 = LCM of (2, 5)
= 10

Multiples of 2 as well as of 5 from 1 and 500 = 10, 20, 30…, 500.

Therefore,

We can conclude that 10, 20, 30…, 500 is an AP with common difference,
d = 10

First term, a = 10
Let the number of terms in this AP = n

Using the formula,
$a_n = a + (n – 1)d$
500 = 10 + (n – 1)10
490 = (n – 1)10
n – 1 = 49
n = 50

Sum of an AP,

$S_n = \left(\frac{n}{2}\right)[a + a_n]$, ($a_n$ is the last term]

www.dreamtopper.in

$= (\frac{50}{2}) \times [10+500]$
$= 25\times [10 + 500]$
$= 25(510)$
$= 12750$

Therefore, sum of those integers from 1 to 500 which are multiples of 2 as well as of 5= 12750

(iii) Sum of those integers from 1 to 500 which are multiples of 2 or 5.

We know that,

Multiples of 2 or 5 = Multiple of 2 + Multiple of 5 – Multiple of LCM (2, 5)

Multiples of 2 or 5 = Multiple of 2 + Multiple of 5 – Multiple of LCM (10)

Multiples of 2 or 5 from 1 to 500 =

List of multiple of 2 from 1 to 500 + List of multiple of 5 from 1 to 500 – List of multiple of 10 from 1 to 500

= (2, 4, 6… 500) + (5, 10, 15… 500) – (10, 20, 30… 500)

Required sum =
sum(2, 4, 6,…, 500) + sum(5, 10, 15,…, 500) – sum(10, 20, 30,., 500)

Consider the first series,
2, 4, 6, …., 500

First term, a = 2
Common difference, d = 2

Let n be no of terms
$a_n = a + (n – 1)d$
$500 = 2 + (n – 1)2$
$498 = (n – 1)2$
$n – 1 = 249$
$n = 250$

Sum of an AP, $S_n = \frac{n}{2} [a + a_n]$

Let the sum of this AP be S_1,
$S_1 = S_{250}$
$= (\frac{250}{2}) \times [2+500]$

www.dreamtopper.in

$S_1 = 125(502)$

$S_1 = 62750$ … (i)

Considering the second series,

5, 10, 15, …., 500

First term,
a = 5
Common difference,
d = 5
Taking number of terms = n

By the formula
$a_n = a + (n – 1)d$
$500 = 5 + (n – 1)$
$495 = (n – 1)5$
$n – 1 = 99$
$n = 100$

Sum of an AP,
$S_n = (\frac{n}{2})$ [a + a_n]

Taking the sum of this AP be S_2,
$S_2 = S_{100}$
$= (\frac{100}{2}) \times [5+500]$

$S_2 = 50(505)$

$S_2 = 25250$ … (ii)

Considering the third series,

10, 20, 30, …., 500

First term,
a = 10
Common difference,
d = 10
Taking number of terms=n
$a_n = a + (n – 1)d$
$500 = 10 + (n – 1)10$
$490 = (n – 1)10$

www.dreamtopper.in

$n - 1 = 49$
$n = 50$

Sum of an AP,

$S_n = \left(\frac{n}{2}\right)[a + a_n]$

Taking the sum of this AP be S_3,

$S_3 = S_{50} = \left(\frac{50}{2}\right) \times [2+510]$

$S_3 = 25(510)$

$S_3 = 12750$ … (iii)

Hence, the required Sum,

$S = S_1 + S_2 - S_3$

$S = 62750 + 25250 - 12750$

$S = 75250$

3. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15thterm.

Solution:

As we know,

First term of an AP = a

Common difference of AP = d

nth term of an AP,

$a_n = a + (n - 1)d$

So,

$a_8 = \frac{1}{2}a_2$

$2a_8 = a_2$

$2(a + 7d) = a + d$

$2a + 14d = a + d$

$a = -13d$ …(i)

Also,

$a_{11} = \frac{1}{3}a_4 + 1$

$3(a + 10d) = a + 3d + 3$

$3a + 30d = a + 3d + 3$

$2a + 27d = 3$

www.dreamtopper.in

Putting a = -13d in the equation,
$2(-13d) + 27d = 3$
$d = 3$

So,
$a = -13(3)$
$= -39$

Now,
$a_{15} = a + 14d$
$= -39 + 14(3)$
$= -39 + 42$
$= 3$
So, 15th term is 3.

4. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

Solution:
First term of an AP = a
Common difference of AP = d

nth term of an AP,
$a_n = a + (n - 1)d$

As,
n = 37 (odd),
Middle term will be $\frac{n+1}{2}$ = 19th term
Hence, the three middle most terms will be, 18th, 19th and 20th terms

Therefore,
$a_{18} + a_{19} + a_{20} = 225$

Using,
$a_n = a + (n - 1)d$
$a + 17d + a + 18d + a + 19d = 225$
$3a + 54d = 225$
$3a = 225 - 54d$
$a = 75 - 18d$ … (i)

Also, we know that last three terms will be 35th, 36th and 37th terms.

$a_{35} + a_{36} + a_{37} = 429$
$a + 34d + a + 35d + a + 36d = 429$
$3a + 105d = 429$

www.dreamtopper.in

$a + 35d = 143$

Putting $a = 75 – 18d$ from equation (i),

$75 – 18d + 35d = 143$ (using (i))

$17d = 68$

$d = 4$

So,

$a = 75 – 18(4)$

$a = 3$

So, the AP is a, a + d, a + 2d….

Which is 3, 7, 11….

5. Find the sum of the integers between 100 and 200 that are

i. divisible by 9

ii. not divisible by 9

[Hint (ii): These numbers will be: Total numbers – Total numbers divisible by 9]

www.dreamtopper.in

Solution:

(i)

The number between 100 and 200 which is divisible by 9 = 108, 117, 126, …198

Taking the number of terms between 100 and 200 which is divisible by 9 = n

$a_n = a + (n – 1)d$

$198 = 108 + (n – 1)9$

$90 = (n – 1)9$

$n – 1 = 10$

$n = 11$

Sum of an AP = S_n

$= (\frac{n}{2})$ [$a + a_n$]

$S_n = (\frac{11}{2}) \times [108 + 198]$

$= (\frac{11}{2}) \times 306$

$= 11(153)$

$= 1683$

(ii)

We know that,

Sum of the integers between 100 and 200 which is not divisible by 9

= (sum of total numbers between 100 and 200) –
(sum of total numbers between 100 and 200 which is divisible by 9)

Sum,
$S = S_1 - S_2$

In this question,
S_1 = sum of AP 101, 102, 103, – – – , 199

S_2 = sum of AP 108, 117, 126, – – – , 198

For AP 101, 102, 103, – – – , 199

First term,
a = 101
Common difference, d = 199
Number of terms = n

Then,
$a_n = a + (n - 1)d$
$199 = 101 + (n - 1)1$
$98 = (n - 1)$
$n = 99$

Sum of an AP = Sn
$= (\frac{n}{2})\ [\ a + a_n]$

www.dreamtopper.in

Sum of AP,
$S_1 = (\frac{99}{2}) \times [199 + 101]$
$= (\frac{99}{2}) \times 300$
$= 99(150)$
$= 14850$

For AP 108, 117, 126, , 198

First term,
a = 108
Common difference, d = 9
Last term, $a_n = 198$

Number of terms = n

Then,
$a_n = a + (n - 1)d$
$198 = 108 + (n - 1)9$
$10 = (n - 1)$
$n = 11$

Sum of an AP = Sn
$= (\frac{n}{2}) [a + a_n]$

Sum of this AP,
$S_2 = (\frac{11}{2}) \times [108 + 198]$
$= (\frac{11}{2}) \times (306)$
$= 11(153)$
$= 1683$

Putting the value of S_1 and S_2 in the equation,
$S = S_1 - S_2$
$= 14850 - 1683$
$= 13167$

6. The ratio of the 11th term to the 18th term of an AP is 2 :3. Find the ratio of the 5th term to the 21st term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

Solution:

Let a and d be the first term and common difference of an AP respectively.
Given,
$a_{11} : a_{18} = 2 : 3$

So, ratio of the sum of the first five terms to the sum of the first 21 terms is
$S_5 : S_{21}$

30d : 29d

www.dreamtopper.in

$$\frac{a+10d}{a+17d}=\frac{2}{3}$$
$$3a+30d=2a+34d$$
$$a=4d$$
$$a_5=a+4d$$
$$=4d+4d$$
$$=8d$$
$$a_{21}=a+20d$$
$$=4d+20d$$
$$=24d$$

So,
ratio = 8d:24d
= 1:3
Now sum,

$$S_5=\frac{5}{2}(2a+(5-1)d)$$
$$=\frac{5}{2}(2(4d)+4d)$$
$$=30d$$
$$S_{21}=\frac{21}{2}(2a+(21-1)d)$$
$$=\frac{21}{2}(2a+20d)$$
$$=294d$$

So, the ratio $S_5:S_{21}$ is 30d:294d or 5:49.

7. Show that the sum of an AP whose first term is *a*, the second term *b* and the last term *c*, is equal to

$$\frac{(a+c)(b+c-2a)}{2(b-a)}$$

Solution:

Given, the AP is a, b, , c

We have,
First term = a,
Common difference = b – a
Last term (l) = a_n = c

a_n = l
= a + (n – 1)d

www.dreamtopper.in

c = a + (n – 1)(b – a)

$$(n-1)=\frac{(c-a)}{(b-a)}$$

$$n=\frac{c+b-2a}{b-a}$$

Now,

$$S_n=\frac{n}{2}(2a+(n-1)d)$$

$$S_n=\frac{c+b-2a}{2(b-a)}[2a+(\frac{c+b-2a}{b-a}-1)(b-a)]$$

$$S_n=\frac{c+b-2a}{2(b-a)}(a+c)$$

Proved!!!

8. Solve the equation

$$-4+(-1)+2+...+x=437$$

Solution:

We have,

-4 + (-1) + 2 + … + x = 437 …(i)

Also,

-4 – 1 + 2 + …+ x forms an AP with,

First term = -4,

Common difference = – 1 – (-4)

= 3

a_n = l = x

nth term of an AP, an = l

= a + (n – 1)d

x = -4 + (n – 1)3 ……………. (ii)

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$S_n=\frac{x+7}{2\times 3}[2(-4)+(\frac{x+4}{3}).3]$$

$$S_n=\frac{(x+7)(x-4)}{6}$$

www.dreamtopper.in

Also,

$S_n = 437$

$$\frac{(x+7)(x-4)}{6} = 437$$

$x^2 + 3x - 2650 = 0$

On, solving,

x=50

x=-53

As x cannot be negative, so x=50

9. Jaspal Singh repays his total loan of Rs 118000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, what amount will be paid by him in the 30th instalment? What amount of loan does he still have to pay after the 30th instalment?

Solution:

Total loan taken by Jaspal Singh= ₹ 118000

He repays his total loan by paying every month

His first installment = ₹ 1000

Second installment = 1000 + 100 = ₹ 1100

Third installment = 1100 + 100 = ₹ 1200 and so on

Let its 30th installment be n,

We have 1000, 1100, 1200… which form an AP,

Here,

a = 1000

d = 1100 – 1000

= 100

nth term of an AP,

Tn = a + (n – 1)d

For 30th instalment,

T_{30} = 1000 + (30 – 1)100

= 100 + 29 × 100

= 1000 + 2900

= 3900

Therefore, ₹ 3900 will be paid by him in the 30th instalment.

He paid total amount upto 30 instalments in ,

1000 + 1100 + 1200 + …………. + 3900

First term (a) = 1000

www.dreamtopper.in

Last term (1) = 3900
S_{30} = 15(1000 + 3900)
= 15 × 4900
= ₹ 73500
Total amount he still have to pay after the 30 installment
= (Amount of loan) – (Sum of 30 installments)
= 118000 – 73500 = ₹ 44500
So, ₹ 44500 still have to pay after the 30th installment.

10. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flagsto be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchikept her books where the flags were stored. She could carry only one flag at atime. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying flag?

Solution:

The number of flags = 27
Distance between each flag = 2 m.

The flags are stored at the position of the middle most flag which is 14th flag and Ruchi was given the responsibility of placing the flags.
Ruchi kept her books, where the flags were stored which is 14th flag and she coluld carry only one flag at a time.

www.dreamtopper.in

Let us assume she placed 13 flags into her left position from middle most flag i.e., 14th flag.

For placing second flag and return her initial position distance travelled = 2 + 2 = 4 m.

Similarly, for placing third flag and return her initial position, distance travelled = 4 + 4 = 8m

For placing fourth flag and return her initial position, distance travelled = 6 + 6 = 12 m

For placing fourteenth flag and return her initial position, distance travelled = 26 + 26 = 52 m

Proceed same manner into her right position from middle most flag i.e., 14th flag.

Total distance travelled in that case = 52 m Also, when Ruchi placed the last flag she return his middle most position and collect her books. This distance also included in placed the flag.

So,

These distance form a series 4 + 8 + 12 + 16 + …. + 52 [for left] and
4 + 8 + 12 + 16 + … + 52 [for right]

Total distance covered by Ruchi for placing these flags,

$= 2 \times (4 + 8 + 12 + \ldots + 52)$

Using,

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= 2 \times [13(4 + 12 \times 2)]$

$= 2 \times 13(4 + 24)$

$= 2 \times 13 \times 28$

$= 728$ m

So, the required distance is 728 m in which she did cover in completing this job and returning back to collect her books.

Now,

Maximum distance she travelled carrying a flag =

Distance travelled by Ruchi during placing the 14th flag in her left position or 27th flag in her right position

$= (2 + 2 + 2 + \ldots + 13 \text{ times})$

$= 2 \times 13$

$= 26$ m

So, the required maximum distance she travelled carrying a flag is 26 m.

www.dreamtopper.in