Chapter – 5

Arithmetic Progressions

Arithmetic Progressions

Many things in nature follow a certain pattern, like

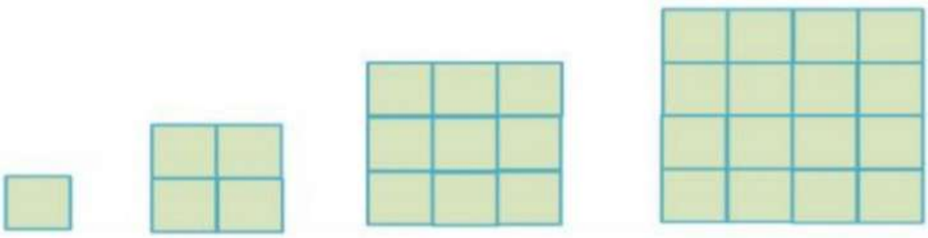
- pattern on a cactus

- holes of a honey comb

- petals of a sunflower

- spirals on a pineapple

There are some patterns which occur in our daily life also If we have a savings account at the bank and we start with Rs.2000 and we deposit Rs.500 every month then the account balance in a year will be Rs.2000, Rs.2500, Rs. 3000...............

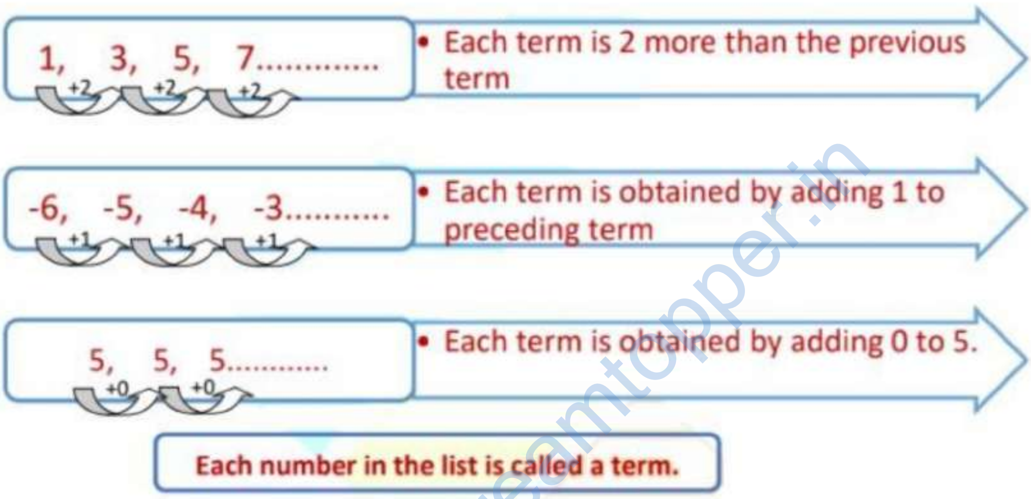


The numbers of unit squares in squares with side 1, 2, 3, 4 Units are respectively 1^2, 2^2, 3^2, 4^2.

Now, we will study the patterns in which the succeeding terms are obtained by adding the fixed term.

Arithmetic Progressions

Here is a list of numbers, observe the pattern of the numbers.



Here, we see that each successive term is obtained by adding a fixed number to the preceding term except the first term. Such a list of numbers is said to be in arithmetic progression.

An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

This fixed number is called the common difference of the AP. This number can be positive, negative or zero.

Let us denote the first term of an AP by a_1, second term by a_2,.........nth term by a_n and the common difference by d. Then the AP becomes a_1 , a_2, a_3, ... a_n.

So, $a_1 - a_2 = a_3 - a_2 =$.........$= a_n - a_{n-1} = d$

If there are a finite number of terms in an AP then it is called finite AP otherwise it is called an infinite AP.

General form of an AP ⇒ $a, a+d, a+2d, a+3d$ …. …

a →first term and d →common difference

Common difference (d) is the difference between any two consecutive terms, $d = a_2 - a_1$ or $a_3 - a_2$ or $a_4 - a_3$

Example: In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(REFERENCE: NCERT)

a) The cost of digging a well after every meter of digging when it costs Rs. 150 for the first meter and rises by Rs. 50 for each subsequent meter.

Cost of digging the first meter = Rs.150

Cost of digging second metre = Rs.(150 + 50) = Rs.200

Cost of digging third metre = Rs.(150 + 2× 50) = Rs. 250

In this case, each term is obtained by adding Rs. 50 to the preceding term. Hence, they make an AP.

b) The amount of water present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

Let the amount of air present in the cylinder be x units.

According to the question,

Amount of air left in the cylinder after using vacuum pump first time = y $- \frac{y}{4} = \frac{3y}{4}$

Amount of air left in the cylinder after using vacuum pump the second time $= \frac{3y}{4} - \left(\frac{1}{4} \times \frac{3y}{4}\right) = \frac{3y}{4} - \frac{3y}{16} = \frac{12y - 3y}{16} = \frac{9y}{4}$

List of numbers y, $\frac{3y}{4}$, $\frac{9y}{4}$, ...

$a_2 - a_1 = \frac{3y}{4} - y = \frac{-y}{4}$

$a_3 - a_2 = \frac{9y}{4} - \frac{3y}{4} = \frac{3y}{4}$

As the common difference of the terms is not the same, they do not form AP.

Example: Find the common difference of the AP: $\frac{1}{p}$, $\frac{1-2p}{p}$, $\frac{1-4p}{p}$, ...

Common difference (d) = Second Term – First Term

$d = \frac{1-2p}{p} - \frac{1}{p} = \frac{1-2p-1}{p} = \frac{1-1-2p}{p} = -2$

Common Difference = -2

Example: For what value of k will k + 10 , 2k, and 2k + 8 are the consecutive terms of an AP.

If k + 10 , 2k, and 2k + 8 are in AP then,
$a_1 = k + 10$, a2 = 2k and a3 = 2k + 8
Common Difference (d) $= a_2 - a_1 = a_3 - a_2$

$$2k - (k + 10) = 2k + 8 - (2k)$$

$$k - 10 = 2k + 8 - 2k$$

$$k - 10 = 8$$

$$k = 18$$

Example: If the numbers 2n − 2, 3n + 1, and 6n − 2 are in AP, then find n and hence find the numbers.

The numbers 2n − 2, 3n + 1, and 6n − 2 are in AP then,

$a_1 = 2n - 2$, $a_2 = 3n + 1$ and $a_3 = 6n - 2$

Common difference (d) $= a_2 - a_1 = a_3 - a_2$

$$d = 3n + 1 - (2n - 2) = 6n - 2 - (3n + 1)$$

$$3n + 1 - 2n + 2 = 6n - 2 - 3n - 1$$

$$n + 3 = 3n - 3 \Rightarrow 3n - n = 3 + 3$$

$$2n = 6 \Rightarrow n = 3$$

Therefore, the value of n is 3.

The numbers are,

$$2n - 2 = 2 \times 3 - 2 = 6 - 2 = 4$$

$$3n + 1 = 3 \times 3 + 1 = 9 + 1 = 10$$

$$6n - 2 = 6 \times 3 - 2 = 16$$

nth term of an AP

n^{th} term of an AP

Let a_1 , a_2, a_3, … … … … … an be an AP whose first term a_1 is a and the common difference is d. Then,

First term a_1	$a + (1 - 1)d = a + 0d$	a
Second term a_2	$a + (2 - 1)d = a + 1d$	$a + d$
Third term a_3	$a + (3 - 1)d = a + 2d$	$a + 2d$
Last term a_n	$a + (n - 1)d$	$a + (n - 1)d$

The n^{th} term an of the AP with the first term a and common difference d is given by

$a_n = a + (n - 1)d$

Example: Find the 25th term of the AP: $-5, -\frac{5}{2}, 0, \frac{5}{2}$

Here, $a = -5$

Common Difference (d) = $a_2 - a_1$

$$= -\frac{5}{2} - 5 = -\frac{5}{2} + 5 = \frac{-5+10}{2} = \frac{5}{2}$$

We know, $a_n = a + (n - 1)d$

25^{th} term, $a_{25} = -5 + (25 - 1) \times \frac{5}{2} \Rightarrow -5 + \frac{24X5}{2} = -5 + 60 = 55$

25^{th} term of the given AP is 55.

Example: For an AP, if $a_{18} - a_{14} = 36$, then find the common difference d.

We know, $an = a + (n - 1)d$

$$a_{18} = a + (18 - 1)d = a + 17d$$

$$a_{14} = a + (14 - 1)d = a + 13d$$

$$a_{18} - a14 = 36$$

$a + 17d - (a + 13d) \Rightarrow a + 17d - a - 13d$

$$4d = 36$$
$$d = 9$$

Therefore, the common difference (d) = 9

Example: If $a_n = 6 - 11n$, then find the common difference.

Given: $a_n = 6 - 11n$ .... (1)

Replacing n by n+1 in eq (1) we get

$a(n+1) = 6 - 11(n+1)$

$a(n+1) = 6 - 11n-11$

$a(n+1) = - 11n - 5$

Common difference, $d = a(n+1) - an$

$d = -11n-5 - (6-11n)$

$d = -11$

Example: Find the 7th term of the sequence whose nth term is given by $a_n = (-1)^{n-1}.\ n^2$

Given: $an = (-1)^{n-1}.\ n^2$

The 7th term of the sequence, $a_7 = (-1)^{7-1}.\ 7^2$

$= (-1)^6.\ 7^2 = 49$

Example: An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

(REFERENCE: NCERT)

Let a be the first term and d be the common difference of the given AP.

Given: $a_3 = 12$ and $a_{50} = 106$

We know, $an = a + (n - 1)d$

$$a_3 = a + (3 - 1)d \Rightarrow a_3 = a + 2d$$

$a_3 = a + 2d = 12 \rightarrow$Eq 1

$$a_{50} = a + (50 - 1)d \Rightarrow a_{50} = a + 49d$$

$a_{50} = a + 49d = 106 \rightarrow$Eq 2

Subtracting Eq 1 from Eq 2

$$a + 49d - (a + 2d) = 106 - 12$$

$$a + 49d - a - 2d = 94$$

$$47d = 94 \Rightarrow d = 2$$

Putting the value of d in Eq 1 we get,

$$a + 2 \times 2 = 12$$

$$a = 12 - 4 = 8$$

29th term, $a_{29} = a + 28d = 8 + 28 \times 2 = 8 + 56 = 64$

Example: Find how many two-digit numbers are divisible by 7.

Two-digit numbers are 10, 11, 12, 13, ...97, 98, 99, 100.

Here 14, 21, 28...................... 91, 98 are divisible by 7.

This list of numbers forms an AP, where $a = 14$ and

$$d = 21 - 14 = 7$$

Let the number of terms be n, then $an = 98$

$$98 = 14 + (n - 1)7 \Rightarrow 98 = 14 + 7n - 7$$

$$7n + 7 = 98 \Rightarrow 7n = 98 - 7$$

$$7n = 91$$

$$n = 13$$

Hence, 13 two-digit numbers are divisible by 7.

Sum of first n terms of an AP

Sum of First n^{th} terms of an AP

Sum of first n terms of an AP is given by

$$S = \frac{n}{2}[2a + (n - 1)d]$$

Now, this can also be written as

$$S = \frac{n}{2}[a + a + (n - 1)d]$$

We know $a_n = a + (n - 1)d$

$$S = \frac{n}{2}[a + a_n]$$

If the number of terms in the AP is n then, a_n is the last term and $a_n = l$

Therefore, $S = \frac{n}{2}[a + l]$

This form of the result is useful when only the first and the last term are given and the common difference is not given.
Example: If the n^{th} term of an AP is $(2n + 2)$, find the sum of first n terms of the AP.

We have,

$$a_n = 2n + 2 \Rightarrow a_1 = 2 \times 1 + 2 = 4$$

Therefore, $a_1 = a$ is the first term and $a_n = l$ is the last term of the AP.

As we know the first and the last term of the AP, the sum of n terms is given by,

$S = \frac{n}{2}[a + l]$

$S = \frac{n}{2}[4 + 2n + 2] = \frac{n}{2}[2n + 6] = \frac{2n}{2}[n + 3] = n[n + 3]$

Sum of first n terms of the given AP is $n[n + 3]$

Example: Find the sum of the first 25 terms of an AP, whose n^{th} term is given by $a_n = 6 - 3n$.

Given: $an = 6 - 3n$

$a_1 = 6 - 3 \times 1 = 3$
$a_{25} = 6 - 3 \times 25 = -69$

Therefore, $a_1 = a = 3$ is the first term and $a_{25} = l = -69$

$S = \frac{n}{2}[a + l]$

$S = \frac{25}{2}[3 + (-69)] = \frac{25}{2}[3 - 69] = \frac{25}{2}[-66]$

$= 25 \times (-33) = -825$

Therefore, the sum of the first 25 terms of the given AP is -825.

Example: Find the sum of all two-digit odd positive numbers.
Two-digit odd positive numbers are 11, 13, 15..................99 which form an AP.
Here, First term a = 11, last term (l) = 99 and common difference (d) = 13 – 11 = 2
Now we have to find the number of terms.
We know, $l = a_n = a + (n - 1)d$

$99 = 11 + (n - 1) \times 2$
$99 = 11 + 2n - 2 \Rightarrow 99 = 9 + 2n$

$99 - 9 = 2n \Rightarrow 90 = 2n$

$n = 45$

$S = \frac{n}{2}[a + l]$

$S = \frac{45}{2}[11 + 99] = \frac{45}{2}[110] = 45 \times 55 = 2475$

Therefore, the sum of all two-digit odd positive numbers is 2475.