Chapter - 11
Area Related to Circles
Exercise No. 11.1

Multiple Choice Questions:

Choose the correct answer from the given four options:

1. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius *R*, then
(A) $R_1 + R_2 = R$
(B) $R_1^2 + R_2^2 = R^2$
(C) $R_1 + R_2 < R$
(D) $R_1^2 + R_2^2 < R^2$

Solution:
(B)

$R_1^2 + R_2^2 = R^2$

Justification:
We have,
Area of circle = Area of first circle + Area of second circle
$$\pi R^2 = \pi R_1^2 + \pi R_2^2$$
$$R^2 = R_1^2 + R_2^2$$
Option B is correct.

2. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius *R*, then
(A) $R_1 + R_2 = R$
(B) $R_1 + R_2 > R$
(C) $R_1 + R_2 < R$
(D) Nothing definite can be said about the relation among R_1, R_2 & R.

Solution:
(A) $R_1 + R_2 = R$

Justification:

We have,
Circumference of circle with radius R = Circumference of first circle with radius R_1 + Circumference of second circle with radius R_2

$$2\pi R = 2\pi R_1 + 2\pi R_2$$
$$R = R_1 + R_2$$

Option A is correct.

3. If the circumference of a circle and the perimeter of a square are equal, then
(A) Area of the circle = Area of the square
(B) Area of the circle > Area of the square
(C) Area of the circle < Area of the square
(D) Nothing definite can be said about the relation between the areas of the circle & square.

Solution:

(B) Area of the circle > Area of the square

Justification:
We have,
Circumference of a circle = Perimeter of square
Let r be the radius of the circle and a be the side of square.
From the given condition,

$2\pi r = 4a$

$11r = 7a$

$a = \frac{11}{7} r$

$r = \frac{7}{11} a \qquad \ldots\ldots\ldots\ldots\ldots \text{(i)}$

Now,
Area of circle = $A_1 = \pi r^2$ and
Area of square = $A_2 = a^2$

From equation (i), we have

$A_1 = \pi \times (\frac{7}{11})^2$

$A_1 = (\frac{22}{7}) \times (\frac{49}{121})a^2$

$= (\frac{14}{11})a^2$

And

$A_2 = a^2$

So,

$A_1 = (\frac{14}{11})\ A_2$

We get,

A1 > A2
Therefore, Area of the circle > Area of the square.
Option B is correct.

4. Area of the largest triangle that can be inscribed in a semi-circle of radius *r* units is

(A) r^2 sq. units

(B) $\frac{1}{2}r^2$ sq. units

(C) $2r^2$ sq. units

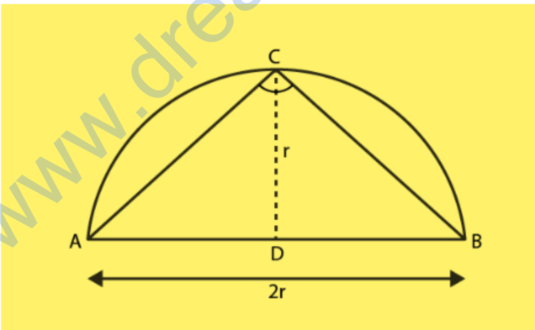
(D) $\sqrt{2}r^2$ sq. units

Solution:

(A) r^2 sq. units

Justification:

The largest triangle that can be inscribed in a semi-circle of radius r units is the triangle having its base as the diameter of the semi-circle and the two other sides are taken by considering a point C on the circumference of the semi-circle and joining it by the end points of diameter A and B.



$\angle$ C = 90° (by the properties of circle)

ΔABC is right angled triangle with base as diameter AB of the circle and height be CD.

Height of the triangle = r

Area of largest ΔABC = $(\frac{1}{2})\times$ Base $\times$ Height

$= (\frac{1}{2})\times AB \times CD$

$= (\frac{1}{2})\times 2r \times r$

$= r^2$ sq. units

Option A is correct.

5. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is
(A) 22 : 7
(B) 14 : 11
(C) 7 : 22
(D) 11: 14

Solution:
(B) 14 : 11

Justification:
Let r be the radius of the circle and a be the side of the square.

We have,
Perimeter of a circle = Perimeter of a square
$2\pi r = 4a$
$a = \frac{\pi r}{2}$
Area of the circle = πr^2 and
Area of the square = a^2

Now,
Ratio of their areas = $\frac{\text{Area of circle}}{\text{Area of square}}$

$$= \frac{\pi r^2}{a^2}$$

$$= \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2}$$

$$= \frac{\pi r^2}{\frac{\pi^2 r^2}{4}}$$

$$= \frac{4}{\pi}$$

$$= \frac{4}{\frac{22}{7}}$$

$$= \frac{14}{11}$$

Option B is correct.

6. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be
(A) 10 m
(B) 15 m
(C) 20 m
(D) 24 m

Solution:
(A)10m

We have,
Area of first Circular Park, whose diameter is 16 m,

$$= \frac{\pi r^2}{2}$$
$$= \pi\left(\frac{16}{2}\right)^2$$
$$= 64\pi \; sq \, metre$$

Area of second Circular Park, whose diameter is 12 m,

$$= \frac{\pi r^2}{2}$$
$$= \pi\left(\frac{12}{2}\right)^2$$
$$= 36\pi \; sq \, metre$$

According to the question,
Area of single circular park = area of 1st circular park + area of 2nd circular park
$\pi R^2 = 36\pi + 64\pi$
$R^2 = 100$
$R = 10m$

7. The area of the circle that can be inscribed in a square of side 6 cm is
(A) 36π cm^2
(B) 18π cm^2
(C) 12π cm^2
(D) 9π cm^2

Solution:
(D) 9π cm^2

We have,
Side of square = 6cm
Side of square = diameter of circle = 6cm
So,
Radius of the circle =3cm
Area of circle = πr^2
$= \pi 3^2$
$= 9\pi \text{ cm}^2$

8. The area of the square that can be inscribed in a circle of radius 8 cm is
(A) 256 cm^2
(B) 128 cm^2
(C) $64\sqrt{2} \text{ cm}^2$
(D) 64 cm^2

Solution:
(B) 128 cm^2

We have,
Radius of circle, r = OC = 8cm.
Diameter of the circle = AC
= 2 x OC
= 2 x 8
= 16 cm
Which is equal to the diagonal of a square.

As square is inscribed in circle,
Diameter of the circle = Diagonal of a square
So,
Area of square,

$$= \frac{Diagonal^2}{2}$$

$$= \frac{16^2}{2}$$

$$= \frac{256}{2}$$

$$= 128 \text{ cm}^2$$

9. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36cm and 20 cm is
(A) 56 cm
(B) 42 cm
(C) 28 cm
(D) 16 cm

Solution:
(C)28cm

Circumference of first circle = $2\pi r$
$= \pi d_1$
$= 36\pi$ cm [given, $d_1 = 36$ cm]
Circumference of second circle = πd_2
$= 20\pi$ cm [given, $d_2 = 20$ cm]
We have,
Circumference of circle = Circumference of first circle + Circumference of second circle
$\pi D = 36\pi + 20\pi$
$D = 56$cm

Required radius of circle=28cm

10. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is
(A) 31 cm
(B) 25 cm
(C) 62 cm
(D) 50 cm

Solution:
(D) 50 cm

Let,
$r_1 = 24$ cm
$r_2 = 7$ cm

Area of first circle = πr_1^2
$= \pi(24)^2$
$= 576\pi$ cm^2
Area of second circle = πr_2^2
$= \pi(7)^2$
$= 49\pi$ cm^2

We have,
Area of circle = Area of first circle + Area of second circle
$\pi R^2 = 576\pi + 49\pi$
$R = 25$cm

Required diameter of circle = 2x 25
= 50 cm

Exercise No. 11.2

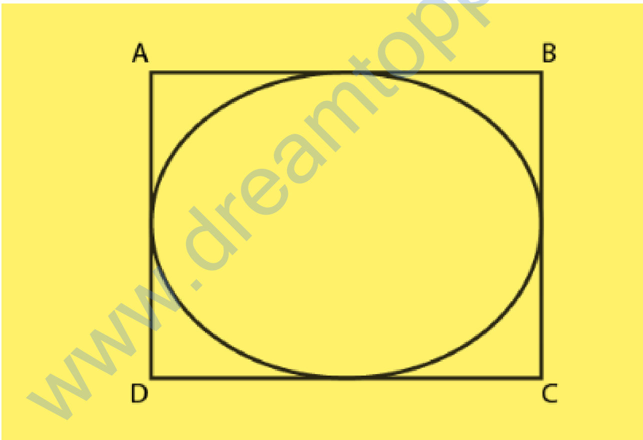
Short Answer Questions with Reasoning:

Questions:

1. Is the area of the circle inscribed in a square of side *a* cm, πa^2 cm^2? Give reasons for your answer.

Solution:
False

Justification:
Let a be the side of square.
We have,
The circle is inscribed in the square.



Diameter of circle = Side of square = a

Radius of the circle = $\frac{a}{2}$

Area of the circle = πr^2

$= \pi(\frac{a}{2})^2$

$= \frac{\pi a^2}{4}$ cm^2

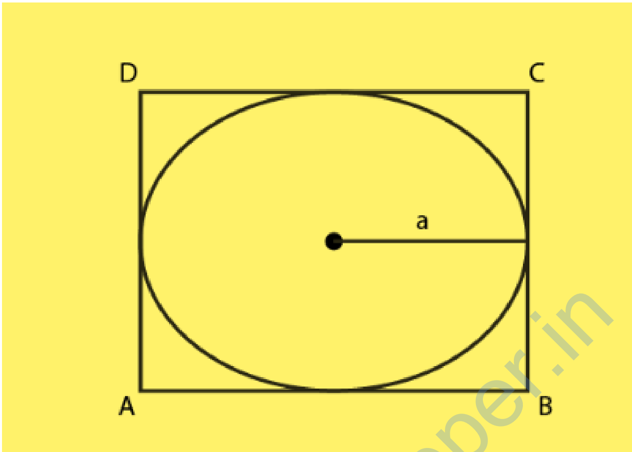
Therefore, area of the circle is $\frac{\pi a^2}{4}$ cm^2

So, the area of the circle inscribed in a square of side a cm is not a^2 cm^2

2. Will it be true to say that the perimeter of a square circumscribing a circle of radius *a* cm is 8*a* cm? Give reasons for your answer.

Solution:
True

Justification:
Let r be the radius of circle = a cm



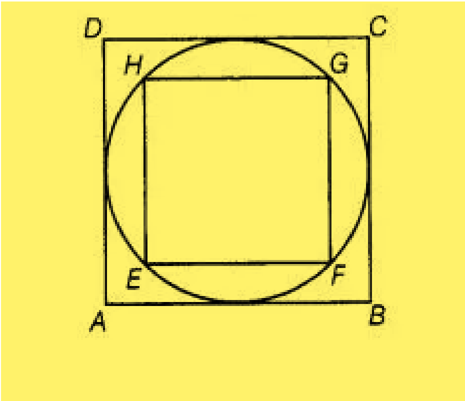
Diameter of the circle = d = 2 × Radius
= 2a cm

As the circle is inscribed in the square,
Side of a square = Diameter of circle
= 2a cm

Therefore, Perimeter of a square = 4 × (side)
= 4 × 2a
= 8a cm

Hence, it will be true to say that the perimeter of a square circumscribing a circle of radius a cm is 8a cm.

3. In Fig., a square is inscribed in a circle of diameter *d* and another square is circumscribing the circle. Is the area of the outer square four times the area of the inner square? Give reasons for your answer.



Solution:
False

Justification:
Diameter of the circle = d
So,
Diagonal of inner square EFGH = Side of the outer square ABCD
= Diameter of circle
= d

Let us take the side of inner square EFGH be ‘a’.

Now in right angled triangle EFG,
$(EG)^2 = (EF)^2 + (FG)^2$

[By Pythagoras theorem]
$d^2 = a^2 + a^2$
$d^2 = 2a^2$
$a^2 = d^2/2$

Area of inner circle = a^2
$= d^2/2$

Also,
Area of outer square = d^2

Therefore, the area of the outer circle is only two times the area of the inner circle.
Thus, area of outer square is not equal to four times the area of the inner square.

4. Is it true to say that area of a segment of a circle is less than the area of its corresponding sector? Why?

Solution:
False

Justification:

It is not true because in case of major segment, area is always greater than the area of its corresponding sector. It is true only in the case of minor segment.

Therefore, we can conclude that it is not true to say that area of a segment of a circle is less than the area of its corresponding sector.

5. Is it true that the distance travelled by a circular wheel of diameter *d* cm in one revolution is $2\pi d$ cm? Why?

Solution:
False

Justification:

Distance travelled by a circular wheel of radius r in one revolution equals the circumference of the circle.
We have,
Circumference of the circle = 2πd [where d is the diameter of the circle]

Hence, it is not true that the distance travelled by a circular wheel of diameter d cm in one revolution is 2 d cm.

6. In covering a distance *s* metres, a circular wheel of radius *r* metres makes $\frac{s}{2\pi r}$ revolutions. Is this statement true? Why?

Solution:
True

Justification:

The distance travelled by a circular wheel of radius r m in one revolution is equal to the circumference of the circle = 2πr

No. of revolutions completed in 2πr m distance = 1

No. of revolutions completed in 1 m distance = $\frac{1}{2\pi r}$

No. of revolutions completed in s m distance = $\left(\frac{1}{2\pi r}\right) \times s$

$$= \frac{s}{2\pi r}$$

Hence, the statement "in covering a distance s metres, a circular wheel of radius r metres makes $\frac{s}{2\pi r}$ revolutions".

7. The numerical value of the area of a circle is greater than the numerical value of its circumference. Is this statement true? Why?

Solution:
False

Justification,

If 0< r< 2, then numerical value of circumference is greater than numerical value of area of circle and if r > 2, area is greater than circumference.

8. If the length of an arc of a circle of radius *r* is equal to that of an arc of a circle of radius 2 *r*, then the angle of the corresponding sector of the first circle is double the angle of the corresponding sector of the other circle. Is this statement false? Why?

Solution:
False

Justification,

Let two circles C_1 and C_2 of radius r and 2r with centres O and O', respectively.
Given,
Length of an arc of a circle of radius r = Length of an arc of a circle of radius 2 r
Or,
Length AB = Length CD
Let,
Angle subtended by arc AB = θ_1
Therefore,

$$AB = \frac{\theta_1}{360} \times 2\pi r$$

Also,

Angle subtended by arc CD $=\theta_2$

Therefore,

$$CD=\frac{\theta_2}{360}\times 2\pi(2r)$$

$$CD=\frac{\theta_2}{360}\times 4\pi r$$

We above equations,

$$\frac{\theta_1}{360}\times 2\pi r=\frac{\theta_2}{360}\times 4\pi r$$

$$\theta_1=2\theta_2$$

Therefore, angle of the corresponding sector of C_1 is double the angle of the corresponding sector of C_2.
It is true statement

9. The areas of two sectors of two different circles with equal corresponding arc lengths are equal. Is this statement true? Why?

Solution:

False

Justification,
It is true for arcs of the same circle. But in different circle, it is not possible.

10. The areas of two sectors of two different circles are equal. Is it necessary that their corresponding arc lengths are equal? Why?

Solution:

False

Justification,
It is true for arcs of the same circle. But in different circle, it is not possible.

11. Is the area of the largest circle that can be drawn inside a rectangle of length *a* cm and breadth *b* cm (*a* >*b*) is πb^2 cm^2? Why?

Solution:

False

Justification,

The area of the largest circle that can be drawn inside a rectangle is $\pi\left(\frac{b}{2}\right)^2$ cm, where $\pi \frac{b}{2}$ is the radius of the circle and it is possible when rectangle becomes a square.

12. Circumferences of two circles are equal. Is it necessary that their areas be equal? Why?

Solution:

True

Justification,
If circumference of two circles are equal, then their corresponding radii are equal. So, their areas will be equal.

13. Areas of two circles are equal. Is it necessary that their circumferences are equal? Why?

Solution:

True

Justification:
If areas of two circles are equal, then their corresponding radii are equal. So, their circumference will be equal.

14. Is it true to say that area of a square inscribed in a circle of diameter *p* cm is p^2 cm^2? Why?

Solution:

False

Justification,
When the square is inscribed in the circle, the diameter of a circle is equal to the diagonal of a square but not the side of the square.

So, we cannot say that area of a square inscribed in a circle of diameter p cm is p^2 cm^2 .

Exercise No. 11.3

Short Answer Questions:

Question:

1. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15 cm and 18 cm.

Solution:

Radius of first circle $= r_1$
$= 15$ cm

Radius of second circle $= r_2$
$= 18$ cm

Circumference of 1st circle $= 2\pi r_1$
$= 30\pi$ cm

Circumference of 2nd circle $= 2\pi r^2$
$= 36\pi$ cm

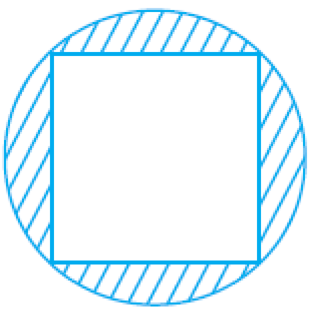
Let the radius of the circle = R

We have,
Circumference of circle = Circumference of first circle + Circumference of second circle
$2\pi R = 2\pi r_1 + 2\pi r_2$
$2\pi R = 30\pi + 36\pi$
$= 66\pi$
$R = 33$

Radius = 33 cm

Therefore, required radius of a circle is 33 cm.

2. In Fig., a square of diagonal 8 cm is inscribed in a circle. Find the area of the shaded region.



Solution:

Let us take a be the side of square.

Diameter of a circle = Diagonal of the square

= 8 cm

In right angled triangle ABC,

Using Pythagoras theorem,

$(AC)^2 = (AB)^2 + (BC)^2$

$(8)^2 = a^2 + a^2$

$64 = 2a^2$

$a^2 = 32$

Area of square = a^2

= 32 cm^2

Radius of the circle = Diameter/2

= 4 cm

Area of the circle = πr^2

$= \pi(4)^2$

= 16 cm^2

So, the area of the shaded region = Area of circle – Area of square

The area of the shaded region = $16\pi - 32$

$= 16 \times (\frac{22}{7}) - 32$

$= \frac{128}{7}$

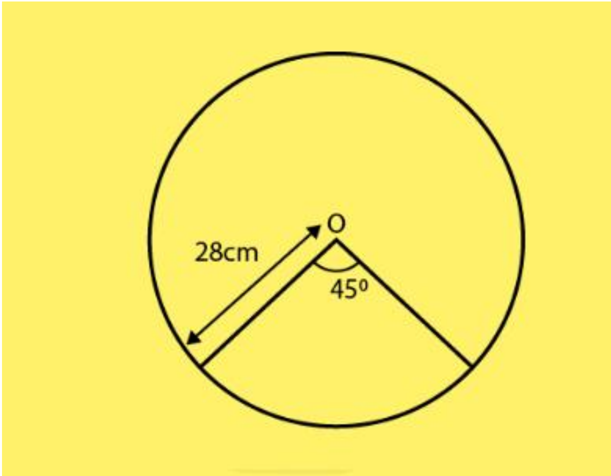
= 18.286 cm^2

3. Find the area of a sector of a circle of radius 28 cm and central angle 45°.

Solution:

Area of a sector of a circle = $(\frac{1}{2})\ r^2\theta$

(In which r = radius and θ = angle in radians subtended by the arc at the center of the circle)



We have,
Radius of circle = 28 cm
Angle subtended at the center = 45°
Angle subtended at the center (in radians) = θ

$$= \frac{45\pi}{180}$$

$$= \frac{\pi}{4}$$

Area of a sector of a circle = $\frac{1}{2} r^2\theta$

$$= \frac{1}{2} \times (28)^2 \times \left(\frac{\pi}{4}\right)$$

$$= 308 \text{ cm}^2$$

Therefore, the required area of a sector of a circle is 308 cm^2.

4. The wheel of a motor cycle is of radius 35 cm. How many revolutions per minute must the wheel make so as to keep a speed of 66 km/h?

Solution:

Radius of wheel = r
= 35 cm

1 revolution of the wheel = Circumference of the wheel

$$= 2\pi r$$

$$= 2 \times \left(\frac{22}{7}\right) \times 35$$

$$= 220 \text{ cm}$$

Speed of the wheel = 66 km/hr

$$= (66\times1000\times100) \times \frac{1}{60} \text{ cm/min}$$

$$= 110000 \text{ cm/min}$$

$\therefore$ Number of revolutions in 1 min = $\dfrac{110000}{220}$

= 500

Thus, required number of revolutions per minute is 500.

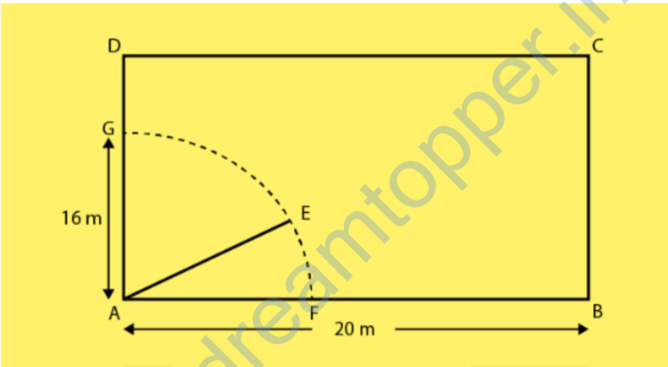
5. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20m × 16m. Find the area of the field in which the cow can graze.

Solution:

ABCD is a rectangular field.

Length of field = 20 m

Breadth of the field = 16 m



A cow is tied at a point A.

Let length of rope be AE = 14 m

Angle subtended at the center of the sector = 90°

Angle subtended at the center (in radians) $\theta = \dfrac{90\pi}{180}$

$= \dfrac{\pi}{2}$

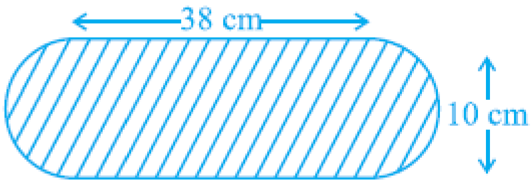
Area of a sector of a circle = $\frac{1}{2} r^2\theta$

$= \frac{1}{2} \times (14)^2 \times (\frac{\pi}{2})$

$= 154 \text{ m}^2$

So, the required area of a sector of a circle is 154 m^2.

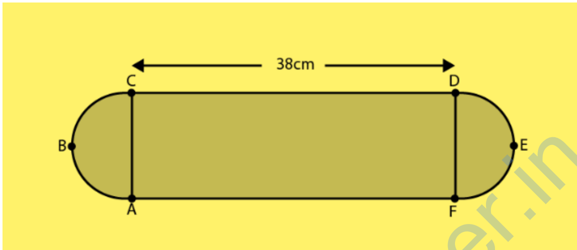
6. Find the area of the flower bed (with semi-circular ends) shown in Fig.



Solution:

Length and breadth of the rectangular portion AFDC of the flower bed are 38 cm and 10 cm

Area of the flower bed = Area of the rectangular portion + Area of the two semi-circles.



Area of rectangle AFDC = Length × Breadth

$= 38 \times 10$

$= 380\ \text{cm}^2$

Both ends of flower bed are semi-circle in shape.

Diameter of the semi-circle = Breadth of the rectangle AFDC

$= 10$ cm

Radius of the semi-circle $= \dfrac{10}{2}$

$= 5$ cm

Area of the semi-circle $= \dfrac{\pi r^2}{2}$

$= 25\left(\dfrac{\pi}{2}\right)\text{cm}^2$

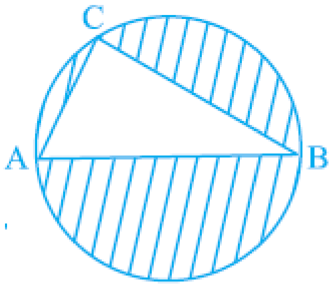
As, there are two semi-circles in the flower bed,

Area of two semi-circles $= 2 \times \left(\dfrac{\pi r^2}{2}\right)$

$= 25\pi\ \text{cm}^2$

Therefore, total area of flower bed $= (380 + 25\pi)\ \text{cm}^2$

7. In Fig., AB is a diameter of the circle, AC = 6 cm and BC = 8 cm. Find the area of the shaded region (Use π= 3.14).



Solution:

AC = 6cm and BC = 8 cm

A triangle in a semi-circle with hypotenuse as diameter is right angled triangle.

By Pythagoras theorem in right angled triangle ACB,

$(AB)^2 = (AC)^2 + (CB)^2$

$(AB)^2 = (6)^2 + (8)^2$

$(AB)^2 = 36 + 64$

$(AB)^2 = 100$

$(AB) = 10$

Diameter of the circle = 10 cm

Radius of the circle = 5 cm

Area of circle $= \pi r^2$

$= \pi(5)^2$

$= 25\pi \text{ cm}^2$

$= 25 \times 3.14 \text{ cm}^2$

$= 78.5 \text{ cm}^2$

Area of the right angled triangle = (½) × Base × Height

= (½) × AC × CB

= (½) × 6 × 8

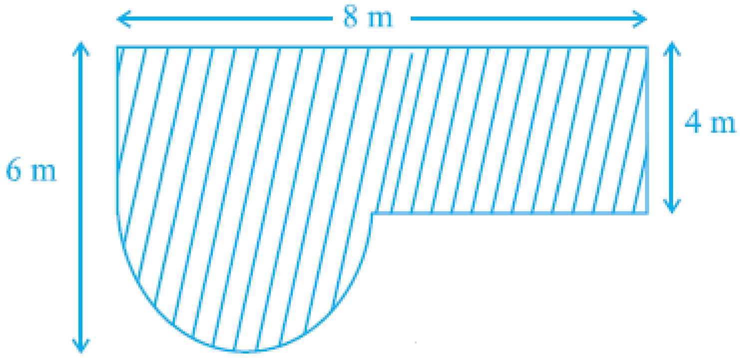
$= 24 \text{ cm}^2$

Area of the shaded region = Area of the circle – Area of the triangle

$= (78.5\text{-}24) \text{ cm}^2$

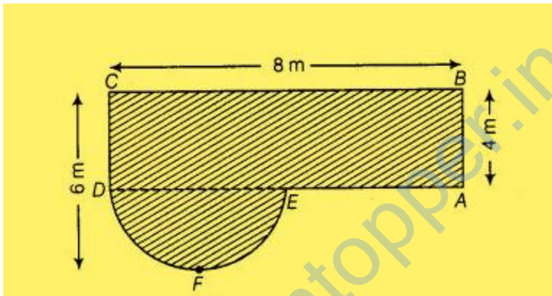
$= 54.5\text{cm}^2$

8. Find the area of the shaded field shown in Fig.



Solution:

Construction : Join ED



Radius of semi-circle DFE = 6-2

= 4 m

Area of rectangle ABCD =BC x AB

= 8 x 4

= 32 m2

Area of semicircle DFE = $\frac{\pi r^2}{2}$

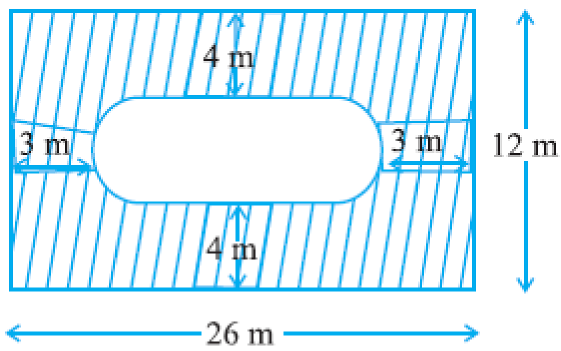
$= \frac{\pi(2)^2}{2}$

$=2\pi$ m^2

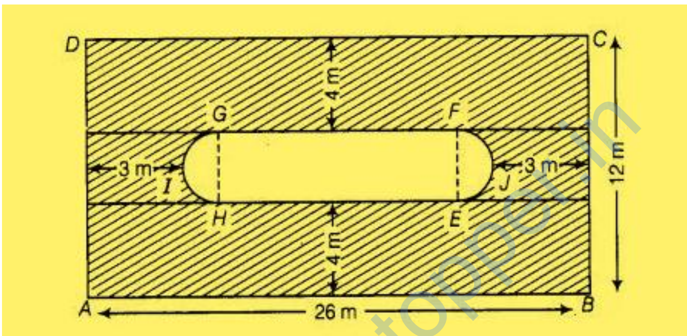
Area of shaded region = Area of rectangle ABCD + Area of semicircle DFE

= (32 +2 π) m2

9. Find the area of the shaded region in Fig.



Solution:



Construction: Join GH and EF
Breadth BC = 12 m
Breadth of inner rectangle EFGH =12-(4+4)
= 4 m
Diameter of semicircle EFJ = 4m
Radius = 2m
Length of inner rectangle EFGH= 26-(5+5)
= 16m

Therefore, area of two semicircles = 2 x $\frac{\pi r^2}{2}$

$= \frac{\pi(2)^2}{2}$

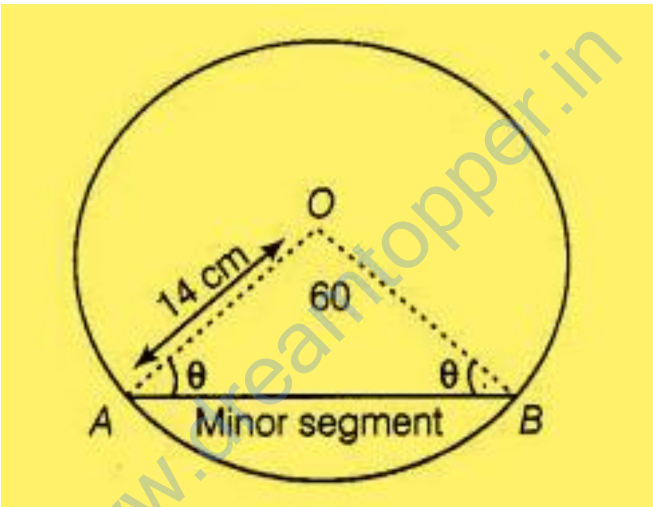
= 4 π m

Now,
Area of inner rectangle (EFGH) = EH x FG
= 16 x 4
= 64 m^2
Area of outer rectangle (ABCD) =26 x 12
= 312 m^2
Area of shaded region =Area of outer rectangle – (area of two semicircles+ area of inner rectangles
= 312 – (64 +4 π)
= (248-4 π) m^2

10. Find the area of the minor segment of a circle of radius 14 cm, when the angle of the corresponding sector is 60°.

Solution:

Given,
Radius of circle (r) = 14 cm
Angle of the corresponding sector ,central angle (θ) = 60°
As,
In ΔAOB,
OA = OB = Radius of circle
So,
ΔAOB is isosceles.



Therefore,
Angle (OAB) = Angle (OBA)
= θ
In ΔOAB,

$$AOB + OAB + OBA = 180^{\circ}$$

$$60^{\circ} + \theta + \theta = 180^{\circ}$$

$$\theta = 60^{\circ}$$

Therefore, ΔOAB is an equilateral triangle.

Also,

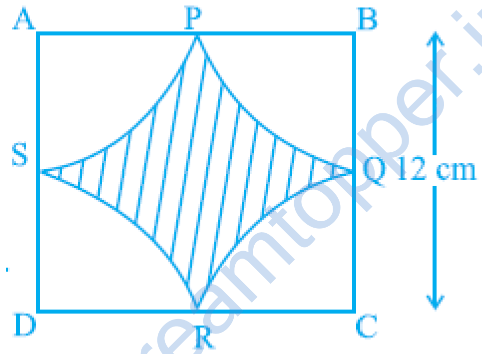
OA=OB=AB=14cm

$$\text{Area of } \Delta OAB = \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4}(14)^2$$

$$= 49\sqrt{3}\text{cm}^2$$

11. Find the area of the shaded region in Fig., where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC,CD and DA, respectively of a square ABCD (Use $\pi = 3.14$).



Solution:

Side of a square BC = 12 cm

Q is a mid-point of BC.

$$\text{Radius } BQ = \frac{12}{2}$$

$$= 6\text{cm}$$

$$\textit{Area of quadrant BPQ} = \frac{\pi r^2}{4}$$

$$= \frac{3.14 \times 6^2}{4}$$

$$= \frac{113.04}{4}$$

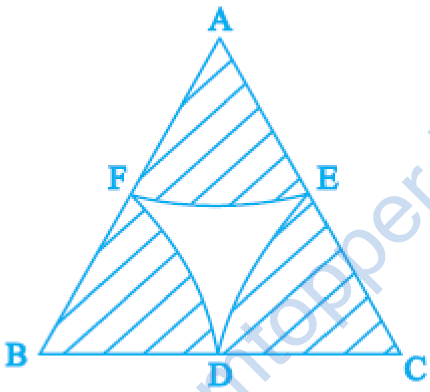
$$\textit{Area of 4 quadrants} = 4 \times \frac{\pi r^2}{4}$$

$$= 4 \times \frac{113.04}{4}$$

$$= 113.04\,cm^2$$

$$Now, Area\ of\ square(ABCD) = (12)^2$$
$$= 144 cm^2$$
$$Area\ of\ shaded\ region = Area\ of\ square - Area\ of\ four\ quadrants$$
$$= 144 - 113.04$$
$$= 30.96 cm^2$$

12. In Fig., arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm. to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region(Use $\pi = 3.14$).



Solution:

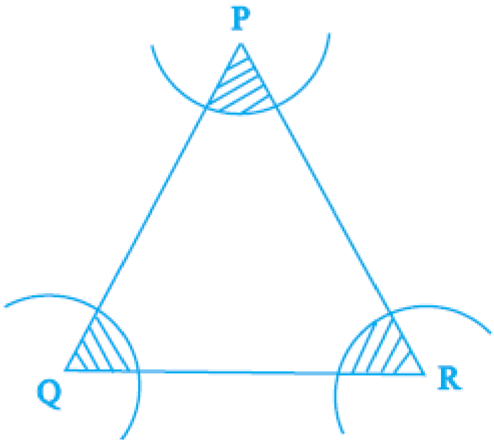
ABC is an equilateral triangle.
And,
AB=BC=CA=10 cm

As, D, E, F are mid points of the sides,
AE=EC=CD=BD=BF=FA=5cm

$\angle A = \angle B = \angle C = 60^o$

$$\text{Area of sector CDE} = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{60}{360} \times \pi (5)^2$$
$$= 13.0833 cm^2$$
$$\text{Area of shaded region} = 3 \times \text{Area of sector CDE}$$
$$= 3 \times 13.0833$$
$$= 39.25 cm^2$$

13. In Fig. 11.12, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



Solution:

Radii of each arc (r) = 14 cm

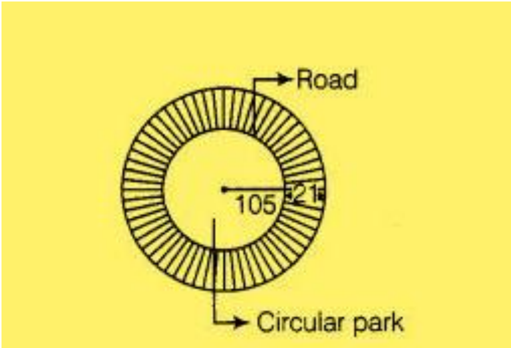
$$\text{Area of sector with } \angle P = \frac{\angle P}{360} \times \pi r^2$$
$$= \frac{\angle P}{360} \times \pi (14)^2$$
$$\text{Area of sector with } \angle Q = \frac{\angle Q}{360} \times \pi r^2$$
$$= \frac{\angle Q}{360} \times \pi (14)^2$$
$$\text{Area of sector with } \angle R = \frac{\angle R}{360} \times \pi r^2$$
$$= \frac{\angle R}{360} \times \pi (14)^2$$

Therefore,

$$\text{Sum of areas of three sectors} = \frac{\angle P}{360} \times \pi (14)^2 + \frac{\angle Q}{360} \times \pi (14)^2 + \frac{\angle R}{360} \times \pi (14)^2$$
$$= \frac{(\angle P + \angle Q + \angle R)}{360^o} \times \pi (14)^2$$
$$= \frac{(180^0) \times 196\pi\, cm^2}{360^o}$$
$$= 308\, cm^2$$

14. A circular park is surrounded by a road 21 m wide. If the radius of the park is105 m, find the area of the road.

Solution:



A circular park is surrounded by a road,
Width of road = 21m
Radius of park = 105 m
Therefore,
Radius of whole circular portion = Park + road
=105+21
=126m

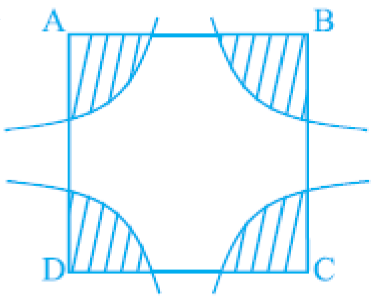
Now,
Area of road = Area of whole circular portion – Area of park

$$= \pi(126^2 - 105^2)$$

$$= \frac{22}{7}(126-105)(126+105)$$

$$= 15246\, m^2$$

15. In Fig., arcs have been drawn of radius 21 cm each with vertices A, B, Cand D of quadrilateral ABCD as centres. Find the area of the shaded region.



Solution:

Radius of each arc (r) = 21 cm

$$\text{Area of sector with } \angle A = \frac{\angle A}{360} \times \pi r^2$$

$$= \frac{\angle A}{360} \times \pi (21)^2$$

$$\text{Area of sector with } \angle B = \frac{\angle B}{360} \times \pi r^2$$

$$= \frac{\angle B}{360} \times \pi (21)^2$$

$$\text{Area of sector with } \angle C = \frac{\angle C}{360} \times \pi r^2$$
$$= \frac{\angle C}{360} \times \pi (21)^2$$
$$\text{Area of sector with } \angle D = \frac{\angle D}{360} \times \pi r^2$$
$$= \frac{\angle D}{360} \times \pi (21)^2$$

Therefore,

$$\text{Sum of areas of four sectors} = \frac{\angle A}{360} \times \pi (21)^2 + \frac{\angle B}{360} \times \pi (21)^2 + \frac{\angle C}{360} \times \pi (21)^2 + \frac{\angle D}{360} \times \pi (21)^2$$
$$= \frac{(\angle A + \angle B + \angle C + \angle D)}{360^o} \times \pi (21)^2$$
$$= \frac{(360^0) \times 1386 cm^2}{360^o}$$
$$= 1386 cm^2$$

16. A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.

Solution:

Length of arc of circle = 20 cm

$\text{Central angle}(\theta) = 60^o$

Now,

$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$
$$20 = \frac{60}{360} \times 2\pi r$$
$$r = \frac{60}{\pi} cm$$

Exercise No. 11.4

Long Answer Questions:

1. The area of a circular playground is $22176\ \text{m}^2$. Find the cost of fencing this ground at the rate of Rs 50 per metre.

Solution:

Area of the circular playground = $22176\ \text{m}^2$

Let r be the radius of the circle.

$$\pi r^2 = 22176$$

$$\frac{22}{7} r^2 = 22176$$

$$r^2 = 22176 \times \left(\frac{22}{7}\right)$$

$$r^2 = 7056$$

$$r = 84$$

Radius of the circular playground = 84 m

Circumference of the circle = $2\pi r$

$$= 2 \times \left(\frac{22}{7}\right) \times 84$$

$$= 528\ \text{m}$$

Cost of fencing 1 meter of ground = Rs 50

Cost of fencing the total ground = Rs 528 × 50

= Rs 26,400

2. The diameters of front and rear wheels of a tractor are 80 cm and 2 m respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions.

Solution:

Diameter of front wheels = d_1

= 80 cm

Diameter of rear wheels = d_2

= 2 m

= 200 cm

Let r_1 be the radius of the front wheels = $\frac{80}{2}$

= 40 cm

Let r_2 be the radius of the rear wheels = $\frac{200}{2}$

= 100 cm

Circumference of the front wheels = $2\pi r$

$= 2 \times (\frac{22}{7}) \times 40$

$= \frac{1760}{7}$ cm

Circumference of the rear wheels = $2\pi r$

$= 2 \times (\frac{22}{7}) \times 100$

$= \frac{4400}{7}$ cm

No. of revolutions made by the front wheel = 1400

∴ Distance covered by the front wheel = $1400 \times (\frac{1760}{7})$

= 352000 cm

Number of revolutions made by rear wheel in covering a distance in which the front wheel makes 1400 revolutions,

$$= \frac{\text{Distance covered by front wheel}}{\text{Circumference of rear wheel}}$$

$$= \frac{352000}{\frac{4400}{7}}$$

$$= \frac{352000 \times 7}{4400}$$

= 560 revolutions.

3. Sides of a triangular field are 15 m, 16 m and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7 m each to graze in the field. Find the area of the field which cannot be grazed by the three animals.

Solution:

Sides of the triangle are 15 m, 16 m, and 17 m.

Now,

Perimeter of the triangle = (15+16+17) m

= 48 m

Semi-perimeter of the triangle = s
= 48/2
= 24 m

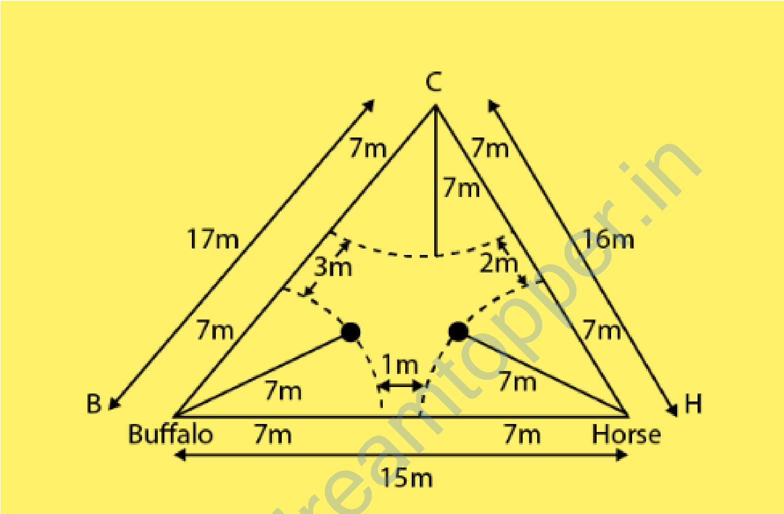
By Heron's formula,

Area of the triangle = $\sqrt{(s(s-a)(s-b)(s-c))}$

(a, b and c are the sides of triangle)

$= \sqrt{(24(24-15)(24-16)(24-17))}$

$= 109.982 \text{ m}^2$



Let B, C and H be the corners of the triangle on which buffalo, cow and horse are tied respectively with ropes of 7 m each.

So, the area grazed by each animal will be in the form of a sector.

Radius of each sector = r
= 7 m

Let x, y, z be the angles at corners B, C, H respectively.

Area of sector with central angle x = ½ $(\frac{x}{180}) \times \pi r^2$

$= (\frac{x}{360}) \times \pi(7)^2$

Area of sector with central angle y = ½ $(\frac{y}{180}) \times \pi r^2$

$= (\frac{y}{360}) \times \pi(7)^2$

Area of sector with central angle z = ½ $\left(\frac{z}{180}\right) \times \pi r^2$

$$= \left(\frac{z}{360}\right) \times \pi(7)^2$$

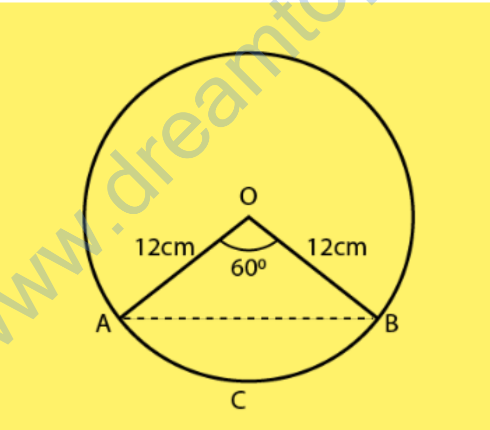
Area of field not grazed by the animals = Area of triangle – (area of the three sectors)
= 109.892 – 77
= 32.982 cm^2

4. Find the area of the segment of a circle of radius 12 cm whose Corresponding sector has a central angle of 60° (Use $\pi = 3.14$).

Solution:
Radius of the circle = r
= 12 cm

OA = OB = 12 cm

∠ AOB = 60° (given)



As triangle OAB is an isosceles triangle,
∠ OAB = ∠ OBA = θ (let)

And, Sum of interior angles of a triangle is 180°,
θ + θ + 60° = 180°
2θ = 120°
θ = 60°

Therefore, the triangle AOB is an equilateral triangle.

AB = OA = OB = 12 cm

Area of the triangle AOB =$\left(\frac{\sqrt{3}}{4}\right) \times a^2$, where a is the side of the triangle.

$= (\frac{\sqrt{3}}{4}) \times (12)^2$

$= (\frac{\sqrt{3}}{4}) \times 144$

$= 36\sqrt{3}$ cm2

$= 62.354 \text{ cm}^2$

Also, Central angle of the sector AOBCA = Ø

$= 60°$

$= (\frac{60\pi}{180})$

$= (\frac{\pi}{3})$ radians

So, area of the sector AOBCA = ½ r^2 Ø

$= ½ \times 12^2 \times \pi/3$

$= 75.36 \text{ cm}^2$

Area of the segment ABCA = Area of the sector AOBCA – Area of the triangle AOB

$= (75.36 – 62.354) \text{ cm}^2$

$= 13.006 \text{ cm}^2$

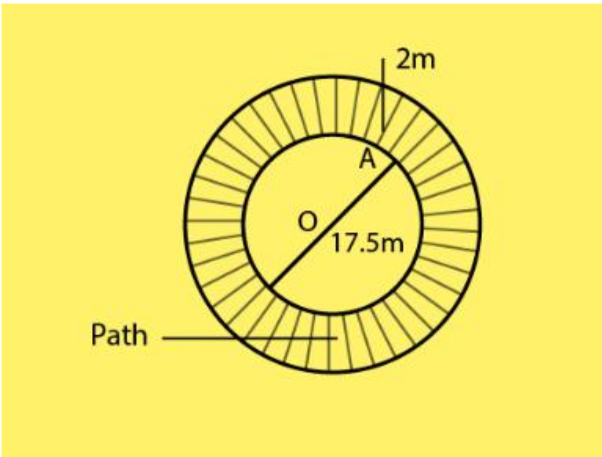
5. A circular pond is 17.5 m is of diameter. It is surrounded by a 2 m wide path. Find the cost of constructing the path at the rate of Rs. 25 per m^2**.**

Solution:

Diameter of the circular pond = 17.5 m

Let r be the radius of the park = r

r = 8.75 m



The circular pond is surrounded by a path of width 2 m.
Radius of the outer circle = R
$= (8.75+2)$ m
$= 10.75$ m

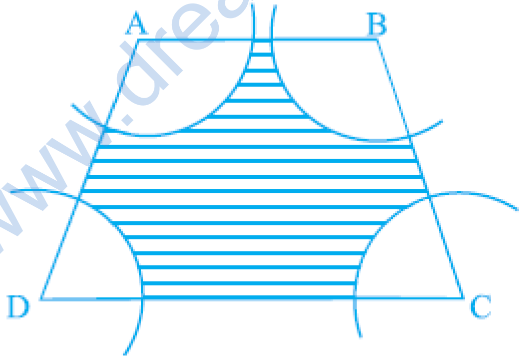
Area of the road = Area of the outer circular path – Area of the circular pond

$= \pi r^2 - \pi R^2$
$= 3.14 \times (10.75)^2 - 3.14 \times (8.75)^2$
$= 3.14 \times ((10.75)^2 - (8.75)^2)$
$= 3.14 \times ((10.75 + 8.75) \times (10.75 - 8.75))$
$= 3.14 \times 19.5 \times 2$
$= 122.46 \text{ m}^2$

So, the area of the path is 122.46 m^2.

Now, Cost of constructing the path per m^2 = Rs. 25
Cost of constructing 122.46m^2 of the path = Rs. 25×122.46
= Rs. 3061.50

6. In Fig., ABCD is a trapezium with AB || DC, AB = 18 cm, DC = 32 cm and distance between AB and DC = 14 cm. If arcs of equal radii 7 cm with centers A, B, C and D have been drawn, then find the area of the shaded region of the figure.



Solution:
AB = 18 cm,
DC = 32 cm
Distance between AB and DC = Height
= 14 cm
Now,
Area of the trapezium = (1/2) × (Sum of parallel sides) × Height
= (½) × (18+32) × 14
$= 350\text{cm}^2$
We have,
AB || DC,
$\angle A + \angle D = 180°$
$\angle B + \angle C = 180°$

Radius of each arc = 7 cm

$$\text{Area of sector with } \angle A = \frac{\angle A}{180} \times \pi r^2$$

$$\text{Area of sector with } \angle B = \frac{\angle B}{360} \times \pi r^2$$

$$\text{Area of sector with } \angle C = \frac{\angle C}{360} \times \pi r^2$$

$$\text{Area of sector with } \angle D = \frac{\angle D}{360} \times \pi r^2$$

Therefore,

$$\text{Sum of total areas of sectors} = \frac{\angle A}{360} \times \pi r^2 + \frac{\angle B}{360} \times \pi r^2 + \frac{\angle C}{360} \times \pi r^2 + \frac{\angle D}{360} \times \pi r^2$$

$$= \frac{(\angle A + \angle B + \angle C + \angle D)}{360^o} \times \pi r^2$$

$$= \frac{(360)}{360^o} \times \pi 7^2$$

$$= 154$$

Area of shaded region = Area of trapezium – (Total area of sectors)

= 350 – 154

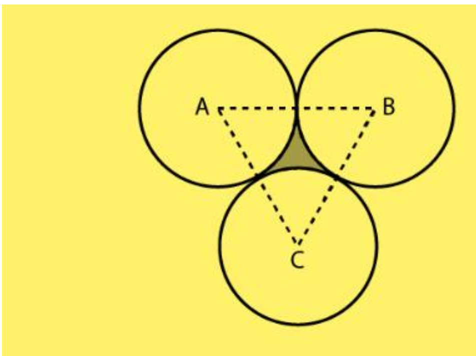
= 196 cm^2

Therefore, the required area of shaded region is 196 cm^2.

7. Three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these circles.

Solution:

The three circles are drawn such that each of them touches the other two.



By joining the centers of the three circles,

AB = BC = CA = 2(radius)

= 7 cm

So, triangle ABC is an equilateral triangle with each side 7 cm.

Area of the triangle = $\left(\frac{\sqrt{3}}{4}\right) \times a^2$,   Where a is the side of the triangle.

$= \left(\frac{\sqrt{3}}{4}\right) \times 7^2$

$= \left(\frac{49\sqrt{3}}{4}\right)$ cm^2

= 21.2176 cm^2

Central angle of each sector = 60°

$= \left(\frac{60\pi}{180}\right)$

$= \frac{\pi}{3}$ radians

Area of each sector = (1/2) $r^2\theta$

$= (1/2) \times (3.5)^2 \times \left(\frac{\pi}{3}\right)$

= 12.25 × (22/ (7×6))
= 6.4167 cm^2
Total area of three sectors = 3 × 6.4167
= 19.25 cm^2

Area enclosed between three circles = Area of triangle ABC – Area of the three sectors
= 21.2176 – 19.25
= 1.9676 cm^2

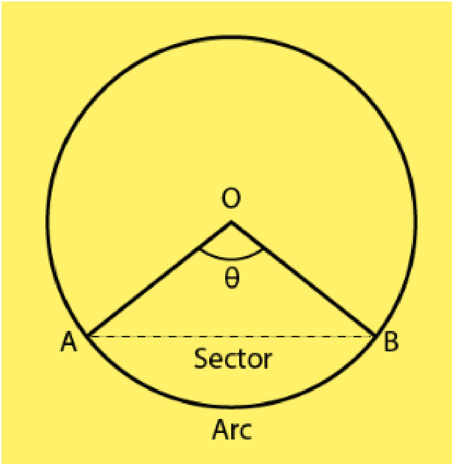
Therefore, the required area enclosed between these circles is 1.967 cm^2 (approx.).

8. Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

Solution:
Radius of the circle = r
= 5 cm

Arc length of the sector = l
= 3.5 cm

Let the central angle (in radians) be θ.

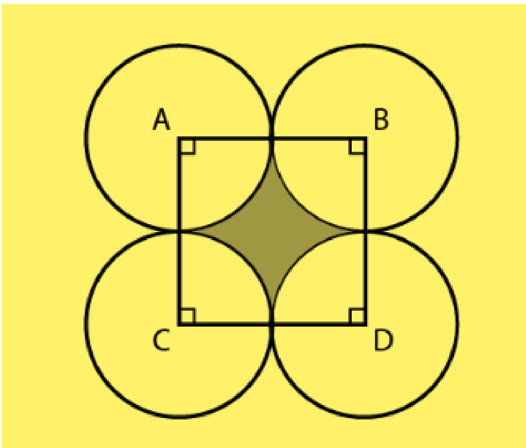


We have,
Arc length = Radius × Central angle (in radians)
Central angle (θ) = Arc length / Radius
= l / r
= 3.5/5
= 0.7 radians

Area of the sector = (½) × r2θ
= (½) × 25 × 0.7
= 8.75 cm^2
Therefore, required area of the sector of a circle is 8.75 cm^2.

9. Four circular cardboard pieces of radii 7 cm are placed on a paper in such a way that each piece touches other two pieces. Find the area of the portion enclosed between these pieces.

Solution:
The four circles are placed such that each piece touches the other two pieces.



By joining the centers of the circles by a line segment, we get a square ABDC with sides,

AB = BD = DC = CA = 2(Radius)
= 2(7) cm
= 14 cm

Area of the square = $(\text{Side})^2$
$= (14)^2$
$= 196\ cm^2$

ABDC is a square,
So, each angle has a measure of 90°.
$\angle A = \angle B = \angle D = \angle C = 90° = \pi/2$ radians
$= \theta$ (let)

Radius of each sector = 7 cm
Area of the sector with central angle A = $(½)r^2\theta$
$= ½\, r^2\, \theta$
$= ½ \times 49 \times \pi/2$
$= ½ \times 49 \times (22/(2\times7))$
$= (77/2)\ cm^2$

As the central angles and the radius of each sector are same, area of each sector is $77/2\ cm^2$.
Area of the shaded portion = Area of square – Area of the four sectors
= 196 – (4 × (77/2))
= 196 – 154
$= 42\ cm^2$

Therefore, required area of the portion enclosed between these pieces is $42\ cm^2$.

10. On a square cardboard sheet of area $784\ cm^2$, four congruent circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of the square sheet is tangent to two circular plates. Find the area of the square sheet not covered by the circular plates.

Solution:

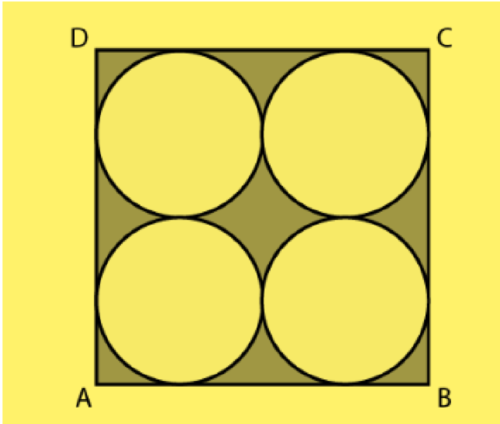
Area of the square = 784 cm2
Side of the square = √Area
= √784
= 28 cm

The four circular plates are congruent,
Then diameter of each circular plate = 28/2
= 14 cm

Radius of each circular plate = 7 cm

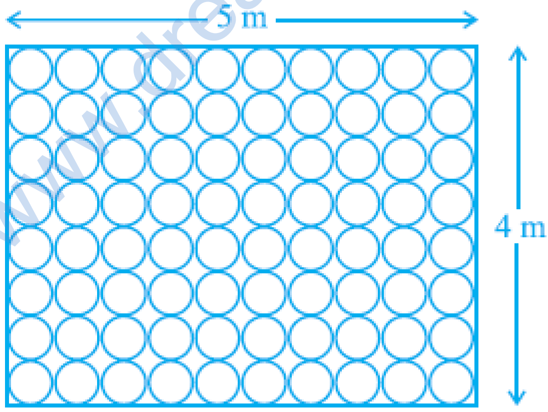
We know that,
Area of the sheet not covered by plates = Area of the square – Area of the four circular plates
As all four circular plates are congruent,
We have,
Area of all four plates equal.
Area of one circular plate = πr^2

$= (22/7) \times 7^2$
$= 154\ cm^2$



Area of four plates = 4×154
$= 616\ cm^2$
Area of the sheet not covered by plates = $784 - 616$
$= 168\ cm^2$

11. Floor of a room is of dimensions 5 m × 4 m and it is covered with circular tiles of diameters 50 cm each as shown in Fig. Find the area of floor that remains uncovered with tiles. (Use $\pi = 3.14$).



Solution:
Length = 5m
Breadth=4m
Area = l x b
$= 20\ m^2$
Diameter of each circular tile = 50cm
Radius=25 cm
=1/4 m
Now,
Area of circular tile;

$$= \pi r^2$$

$$= 3.14 \times \left(\frac{1}{4}\right)^2$$

$$= \frac{3.14}{16}$$

$$\text{Area of 80 circular tiles} = 80 \times \frac{3.14}{16}$$

$$= 1.57 m^2$$

Area which remain uncovered =Area of floor – area of 80 circular tiles

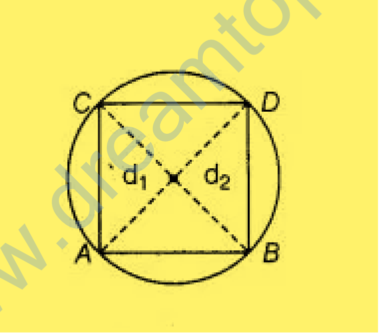
=20- 15.7

=4.3 m^2

12. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is 1256 cm^2. (Use $\pi = 3.14$).

Solution:

Let the radius of the circle be r.



Area of circle = 1256 cm^2

$$\pi r^2 = 1256$$

$$r = 20 cm$$

$$So, Diameter = 40 cm$$

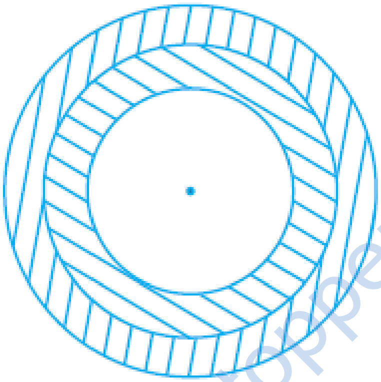
As, all the vertices of a rhombus lie on a circle that means each diagonal of a rhombus must pass through the center of a circle that is why both diagonals are equal and same as the diameter of the given circle.

Let d_1 and d_2 b the diagonals of rhombus,

And, Diameter of circle =40 cm

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$
$$= \frac{1}{2} \times 40 \times 40$$
$$= 800\,cm^2$$

13. An archery target has three regions formed by three concentric circles as shown in Fig.. If the diameters of the concentric circles are in the ratio 1: 2:3, then find the ratio of the areas of three regions.



Solution:

Let the diameters of concentric circles be k, 2k and 3k.

$$Radius\ of\ concentric\ circle\ are = \frac{k}{2}, k, \frac{3k}{2}$$

$$Area\ of\ inner\ circle\left(A_1\right) = \pi\left(\frac{k}{2}\right)^2$$
$$= \frac{k^2\pi}{4}$$

$$Area\ of\ middle\ circle\left(A_2\right) = \pi\left(k\right)^2 - \frac{k^2\pi}{4}$$
$$= \frac{3k^2\pi}{4}$$

$$Area\ of\ outer\ circle\left(A_3\right) = \pi\left(\frac{3k}{2}\right)^2 - \pi\left(k\right)^2$$
$$= \frac{5k^2\pi}{4}$$

$$\text{Re}\,quired\ ratio = A_1 : A_2 : A_3$$
$$= 1:3:5$$

14. The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6 : 05 a m and 6 : 40 a m.

Solution:

We know that in 60 minute hand revolve = 360°

$$\text{In 1 min} = \frac{360}{60} \quad \text{In (6 : 05 a m to 6 : 40 a m)=35 minutes,}$$

$$= \frac{360}{60} \times 35$$

$$= 210$$

$$\text{Length of minute hand}(r) = 5\,cm$$

$$\text{Area of sector AOBA with angle } O = \frac{\pi r^2}{360} \times \angle O$$

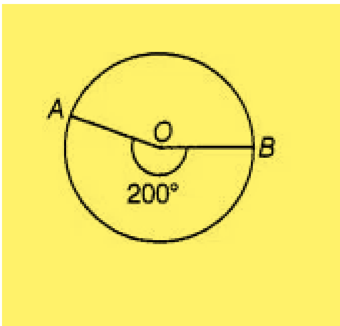
$$= \frac{\pi 5^2}{360} \times 210$$

$$= \frac{275}{6}$$

$$= 45\frac{5}{6}\,cm^2$$

15. Area of a sector of central angle 200° of a circle is $770\ \text{cm}^2$. Find the length of the corresponding arc of this sector.

Solution:



Let the radius of the sector AOBA be r.

Central Angle =200°

Area of sector =770 cm^2

$$\text{Area of sector} = \frac{\pi r^2}{360} \times \theta$$

$$770 = \frac{\pi r^2}{360} \times 200$$

$$r = 21 cm$$

$$\text{Length of corresponding arc of this sector} = \text{Central angle} \times \text{Radius}$$

$$= 200 \times 21 \times \frac{\pi}{180}$$

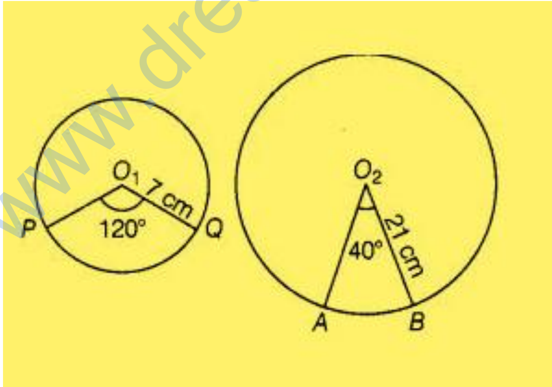
$$= \frac{220}{3}$$

$$= 77\frac{1}{3} cm$$

16. The central angles of two sectors of circles of radii 7 cm and 21 cm are respectively 120° and 40°. Find the areas of the two sectors as well as the lengths of the corresponding arcs. What do you observe?

Solution:

Let the lengths of the corresponding arc be l_1 and l_2.



Radius of the sector PO_1QP=7cm

Radius of the sector AO_2BA=21cm

And,

Central angle of sector $PO_1QP=120^o$

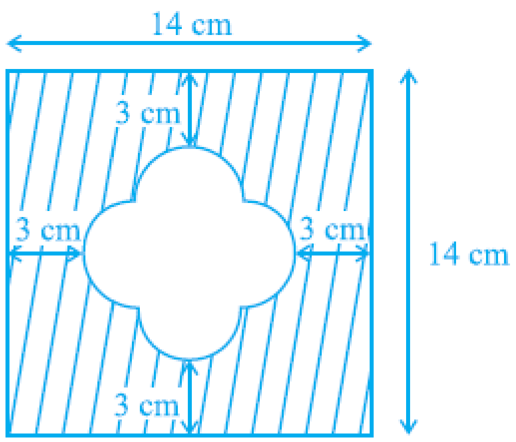
Central angle of sector $AO_2BA=40^o$

$$\text{Area of sector with angle } O_1 = \frac{\pi r^2}{360}\times 120$$
$$= \frac{\pi 7^2}{360}\times 120$$
$$= \frac{154}{3}\, cm^2$$
$$\text{Area of sector with angle } O_2 = \frac{\pi r^2}{360}\times 40$$
$$= \frac{\pi (21)^2}{360}\times 40$$
$$= 154\, cm^2$$

$$\text{Arc length of sector } PO_1QP = \text{Central angle}\times\text{radius}$$
$$= 120\times 7\times \frac{\pi}{180}$$
$$= \frac{44}{3}\, cm$$
$$\text{Arc length of sector } AO_2BA = \text{Central angle}\times\text{radius}$$
$$= 40\times 21\times \frac{\pi}{180}$$
$$= \frac{44}{3}\, cm$$

Hence, we observe that arc lengths of two sectors of two different circles may be equal but their area need not be equal.

17. Find the area of the shaded region given in Fig.

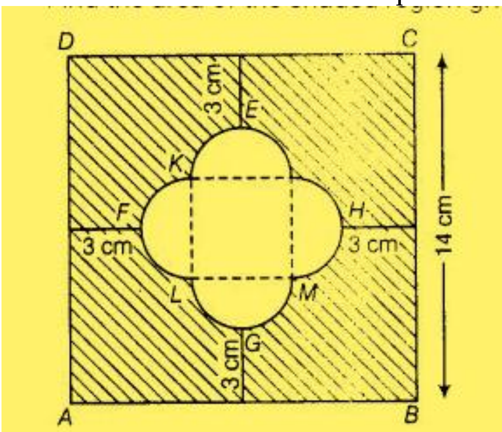


Solution:

Construction:

Join JK, KL, LM and MJ,

There are four equally semi-circles and LMJK formed a square.



FH= 14-(3+3)
=8cm

Side of square =4 cm
Radius of semicircle = 2cm

Now,
Area of square JKLM = 4^2
$= 16\ cm^2$

$$\text{Area of semicircle } HJM = \frac{\pi r^2}{2}$$
$$= \frac{\pi 2^2}{2}$$
$$= 2\pi$$
$$= 6.28\, cm^2$$
$$\text{Area of four semicircle} = 4 \times 6.28$$
$$= 25.12\, cm^2$$

Now,

$$\text{Area of square } ABCD = 14^2$$
$$= 196\, cm^2$$

Area of shaded region = area of square ABCD – (Area of 4 semicircle + area of square JKLM)
$= 196 - (8\pi + 16)$
$= 180 - 8\pi\ cm^2$

18. Find the number of revolutions made by a circular wheel of area $1.54\ m^2$ **in rolling a distance of 176 m.**

Solution:

Let us take the number of revolutions made by a circular wheel be n and the radius of circular wheel be r.

Area of circular wheel = 1.54 m^2

$$\pi r^2 = 1.54$$

$$\frac{22}{7} r^2 = 1.54$$

$$r = 0.7m$$

Distance travelled by circular wheel in one revolution = Circumference of circular wheel

$$= 2\pi r$$

$$= 2 \times \frac{22}{7} \times 0.7$$

$$= 4.4m$$

Distance travelled by circular wheel =176m

$$\text{Number of revolutions} = \frac{\text{Total distance}}{\text{Distance of one revolution}}$$

$$= \frac{176}{4.4}$$

$$= 40$$

19. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending an angle of 90° at the centre.

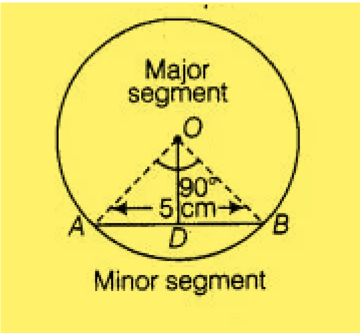
Solution:

Let the radius of the circle be r.

OA=OB= r cm

We have,

AB= 5 cm



Central angle of AOBA= 90^o

$$In\,\Delta AOB, AB^2 = OA^2 + OB^2$$
$$5^2 = r^2 + r^2$$
$$r = \frac{5}{\sqrt{2}}cm$$
$$Also, AD = DB = \frac{5}{2}cm$$

Now, In ΔADO,

$$OA^2 = OD^2 + AD^2$$
$$= \left(\frac{5}{\sqrt{2}}\right)^2 - \left(\frac{5}{2}\right)^2$$
$$= \frac{5}{2}cm$$

$$\text{Area of isosceles } \Delta AOB = \frac{1}{2}\times\text{base}\times\text{height}$$
$$= \frac{1}{2}\times 5\times\frac{5}{2}$$
$$= \frac{25}{4}cm^2$$

$$Area\,of\,\sec tor\,AOBA = \frac{\pi r^2}{360}\times\theta$$
$$= \frac{\pi\left(\frac{5}{\sqrt{2}}\right)^2}{360}\times 90$$
$$= \frac{25\pi}{8}$$

$$\text{Area of minor segment} = \text{Area of sector AOBA-Area of isosceles } \Delta AOB$$
$$= \left(\frac{25\pi}{8} - \frac{25}{4}\right)$$

$$Now, Area\,of\,circle = \pi r^2$$
$$= \pi\left(\frac{5}{\sqrt{2}}\right)^2$$
$$= \frac{25\pi}{2}$$

Area of major segment = Area of circle - Area of minor segment

$$= \frac{25\pi}{2} - \left(\frac{25\pi}{8} - \frac{25}{4}\right)$$

$$= \left(\frac{75\pi}{8} + \frac{25}{4}\right)$$

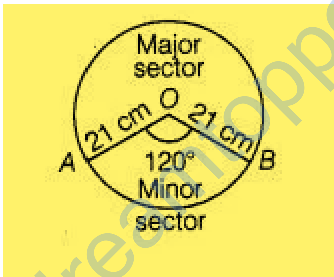
Now, Difference = Area of major segment - Area of minor segment

$$= \left(\frac{75\pi}{8} + \frac{25}{4}\right) - \left(\frac{25\pi}{8} - \frac{25}{4}\right)$$

$$= \frac{25\pi}{4} + \frac{25}{2} cm^2$$

20. Find the difference of the areas of a sector of angle 120° and its corresponding major sector of a circle of radius 21 cm.

Solution:



Radius of the circle (r) = 21 cm and
Central angle of the sector AOBA (θ) = 120°

Area of circle $= \pi r^2$

$$= \frac{22}{7} \times (21)^2$$

$$= 1386\, cm^2$$

Area of minor sector (OABA) $= \dfrac{\pi r^2}{360} \times \theta$

$$= \frac{22}{7} \times \frac{(21)^2}{360} \times 120^o$$

$$= 462\, cm^2$$

Area of major sector = Area of circle − Area of minor sector

$$= 1386 - 432$$

$$= 924\, cm^2$$

Difference of area of major and minor sector,

$$= 924 - 462$$

$$= 462\, cm^2$$