

# PROJECTS

Project work in mathematics may be performed individually by a student or jointly by a group of students. These projects may be in the form of construction such as curve sketching or drawing of graphs, etc. It may offer a discussion of a topic from history of mathematics involving the historical development of particular subject in mathematics/ topics on concepts. Students may be allowed to select the topics of their own choice for projects in mathematics. The teacher may act as a fascilitator by creating interest in various topics. Once the topic has been selected, the student should read as much about the topic as is available and finally prepare the project.

Prior to the first century A.D., there was a lot of development of mathematics in India but the nomenclature of their contributors is not known presently. One of the Indian mathematicians of ancient times about which some definite information is available is **Aryabhat**, and the name of his creation is **Aryabhattiya**.

### TIME AND PLACE OF BIRTH

Aryabhat has said in his creation Aryabhattiya that he was 23 year old, when he wrote Aryabhattiya and upto that time 3600 years of Kaliyug (कलियुग) had elapsed. This works out that he wrote the manuscript in 499 A.D. and his year of birth was 476 A.D.

Aryabhat has also said in the manuscript that he has given the knowledge attained at Kusumpur (Pataliputra) while studying. This gives the impression that he was born at Pattliputra but according to the views of majority he was born in South India (in Ashmak district, which is on the bank of river Godavari). The world famous historian mathematician Dr. Bhou-Daji (भाउ-दाजी) of Maharashtra traced the manuscript of Aryabhattiya in 1864 from South India and published its contents.

Aryabhattiya is in Sanskrit and is divided in four major parts called 'Pads'. The manuscript contains a total of 153 Shlokas and their distribution is given below:

1. Dashgeetika Pad (दशगीतिका पाद) containing 33 shlokas
2. Ganitpad (गणित पाद) containing 25 shlokas
3. Kalkriya Pad (कालक्रिया पाद) containing 25 shlokas
4. Goladhyay (गोलाध्याय) containing 50 shlokas

### CONTRIBUTION

1. Aryabhat created a new method of enumerating numbers using Sanskrit alphabets.

According to this, he gave the following numerical values to 25 consonants (वर्ण अक्षर)

क : 1, ख : 2, ग : 3, घ : 4, ङ : 5, च : 6, छ : 7, ज : 8, झ : 9, ञ : 10, ट : 11, ठ : 12, ड : 13, ढ : 14, ण : 15, त : 16, थ : 17, द : 18, ध : 19, न : 20, प : 21, फ : 22, ब : 23, भ : 24, म : 25, य : 30, र : 40, ल : 50, व : 60, श : 70, ष : 80, स : 90, ह : 100

He gave the following values to vowels (स्वर)

अ:1, ई:100, ऊ:10000, ऋ:1000000, लृ:100000000, ए:10000000000, ऐ:1000000000000, ओ:100000000000000, औ:10000000000000000

As an example, Aryabhat says that in a Mahayug (महायुग), the earth revolves around the Sun 4320000 times. According to the above numerical system, Aryabhat has stated it as खुयुघृ

$$\text{खु: } 2 \times 10000 = 20000$$

$$\text{यु: } 30 \times 10000 = 300000$$

$$\text{घृ: } 4 \times 1000000 = 4000000$$

$$\text{खुयुघृ} = 4320000$$

2. Aryabhat has summarised important principles of arithmetic, geometry and algebra in 33 shlokas of Ganitpad only. In these shlokas, he has given formulae for finding:

- squares and square-roots
- cubes and cube-roots
- area of squares, triangles and circles
- volume of a sphere

His most important contribution was the value of  $\pi$ , the ratio between the circumference and the diameter of a circle upto four places of decimals as 3.1416. He stated that it is the approximate value of  $\pi$ . He was the first Indian mathematician who has stated that it is the approximate value of  $\pi$ .

3. Aryabhat has given methods of drawing a circle, a triangle and quadrilateral and solving of quadratic equations.
4. He has stated and verified Pythagoras theorem through examples.
5. Another important contribution of Aryabhat has been formation of tables of sine and cosine functions at intervals of  $3^{\circ}45'$  each.
6. Aryabhat has also written about astronomy and astrology in his Goladhyay. He was the first mathematician who declared that the earth revolves about its axis and the Nakshtras are still, which was against the mythological statements. He also described about solar and lunar eclipses and reasons for their occurring.

## Project 2 SURFACE AREAS AND VOLUMES OF CUBOIDS

### BACKGROUND

Cuboidal objects are quite useful in our daily life and often we need to know their surface areas and volumes for different purposes. Sometimes, it appears that if there is an increase in the surface area of a cuboid, then its volume will also increase and vice-versa. The present project is a step towards knowing the truth about this statement.

### OBJECTIVE

To explore the changes in behaviours of surface areas and volumes of cuboids with respect to each other.

### DESCRIPTION

#### (A) Cuboids with equal volumes

Let us consider some cuboids with equal volumes, having the following dimensions:

- (i)  $l = 12$  cm,  $b = 6$  cm and  $h = 3$  cm
- (ii)  $l = 6$  cm,  $b = 6$  cm and  $h = 6$  cm
- (iii)  $l = 9$  cm,  $b = 6$  cm and  $h = 4$  cm
- (iv)  $l = 8$  cm,  $b = 6$  cm and  $h = 4.5$  cm.

Now, we calculate the surface area of each of the above cuboids, using the formula

$$\text{surface area} = 2 (lb + bh + hl).$$

$$\text{For (i), surface area} = 2 (12 \times 6 + 6 \times 3 + 3 \times 12) \text{ cm}^2 = 252 \text{ cm}^2$$

$$\text{For (ii), surface area} = 2 (6 \times 6 + 6 \times 6 + 6 \times 6) \text{ cm}^2 = 216 \text{ cm}^2 \rightarrow \text{Minimum}$$

$$\text{For (iii), surface area} = 2 (9 \times 6 + 6 \times 4 + 4 \times 9) \text{ cm}^2 = 228 \text{ cm}^2$$

$$\text{For (iv), surface area} = 2 (8 \times 6 + 6 \times 4.5 + 4.5 \times 8) \text{ cm}^2 = 222 \text{ cm}^2$$

We note that volume of each of the above cuboids

$$= 12 \times 6 \times 3 \text{ cm}^3 = 6 \times 6 \times 6 \text{ cm}^3$$

$$= 9 \times 6 \times 4 \text{ cm}^3 = 8 \times 6 \times 4.5 \text{ cm}^3 = 216 \text{ cm}^3.$$

We also note that **surface area of the cuboid is minimum, in case (ii) above, when the cuboid is a cube.**

### **(B) Cuboids with equal surface areas**

Let us now consider some cuboids with equal surface areas, having the following dimensions:

(v)  $l = 14 \text{ cm}, b = 6 \text{ cm}$  and  $h = 5.4 \text{ cm}$

(vi)  $l = 10 \text{ cm}, b = 10 \text{ cm}$  and  $h = 4.6 \text{ cm}$

(vii)  $l = 8 \text{ cm}, b = 8 \text{ cm}$  and  $h = 8 \text{ cm}$

(viii)  $l = 16 \text{ cm}, b = 6.4 \text{ cm}$  and  $h = 4 \text{ cm}$

Now, we calculate the volume of each of the above cuboids, using the formula

$$\text{volume} = l \times b \times h$$

For (v), volume =  $14 \times 6 \times 5.4 \text{ cm}^3 = 453.6 \text{ cm}^3$

For (vi), volume =  $10 \times 10 \times 4.6 \text{ cm}^3 = 460 \text{ cm}^3$

For (vii), volume =  $8 \times 8 \times 8 \text{ cm}^3 = 512 \text{ cm}^3 \rightarrow$  **Maximum**

For (viii), volume =  $16 \times 6.4 \times 4 \text{ cm}^3 = 409.6 \text{ cm}^3$

We note that surface area of each of the above cuboids

$$= 2 (14 \times 6 + 6 \times 5.4 + 5.4 \times 14) \text{ cm}^2$$

$$= 2 (10 \times 10 + 10 \times 4.6 + 4.6 \times 10) \text{ cm}^2$$

$$= 2 (8 \times 8 + 8 \times 8 + 8 \times 8) \text{ cm}^2$$

$$= 2 (16 \times 6.4 + 6.4 \times 4 + 4 \times 16) \text{ cm}^2 = 384 \text{ cm}^2.$$

We also note that **volume of the cuboid is maximum, in case of (vii), when the cuboid is a cube.**

## CONCLUSION

The statement that if there is an increase in the surface area of cuboid, then its volume also increases and vice versa is not true. In fact, we have:

- (i) **Of all the cuboids with equal volumes, the cube has the minimum surface area.**
- (ii) **Of all the cuboids with equal surface areas, the cube has the maximum volume.**

## APPLICATION

Project is useful in preparing packages with maximum capacity at minimum cost.

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## Project 3 GOLDEN RECTANGLE AND GOLDEN RATIO

### BACKGROUND

'Rectangles' and 'ratios' are the two concepts which have great importance in our day-to-day life. Due to this, they are studied in one form or the other at every stage of school mathematics. It is also a fact that whenever there is some discussion on rectangles and ratios, people start recalling something about 'Golden rectangle' and 'Golden ratio'. Keeping in view the above, it was felt to know something about these two phrases 'Golden rectangle' and 'Golden ratio'.

### OBJECTIVE

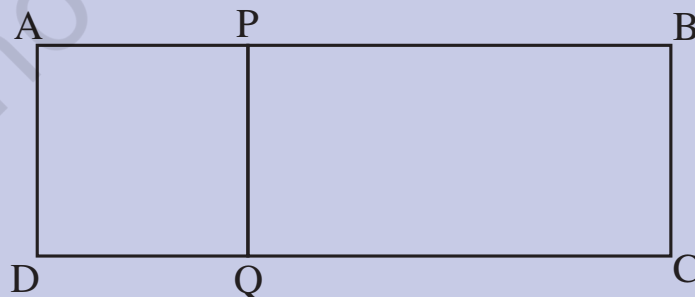
To explore the meanings of 'Golden rectangle' and 'Golden ratio' and their relationship with some other mathematical concepts.

### DESCRIPTION

'Golden rectangle' and 'Golden ratio' are very closely related concepts. To understand this, let us first understand the meaning of a golden rectangle.

#### (A) Golden Rectangle

A rectangle is said to be a golden rectangle, if it can be divided into two parts such that one part is a square and other part is a rectangle similar to the original rectangle. In the following figure, rectangle ABCD has been divided into a square APQD and a rectangle QPBC.





If the rectangle QPBC is similar to rectangle ABCD, then we can say that ABCD is a golden rectangle. Let  $AB = l$  and  $BC = b$ . Therefore,  $QP = b$ .

Now, as  $ABCD \sim QPBC$ , we have

$$\frac{AB}{BC} = \frac{QP}{PB}$$

$$\text{or } \frac{l}{b} = \frac{b}{l-b}$$

$$\text{or } l^2 - lb = b^2$$

$$\text{i.e., } l^2 - lb - b^2 = 0$$

$$\text{or } \frac{l}{b}^2 - \frac{l}{b} - 1 = 0 \quad (1)$$

$$\text{Let } \frac{l}{b} = x$$

So, from (1), we have

$$x^2 - x - 1 = 0$$

$$\text{or } x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times (1) \times (-1)}}{2 \times 1} = \frac{1 \pm \sqrt{5}}{2} \quad (\text{Solving the quadratic equation}).$$

Now, as  $x$  cannot be negative, therefore  $x = \frac{\sqrt{5} + 1}{2}$ .

$$\text{Thus, } \frac{l}{b} = \frac{\sqrt{5} + 1}{2}$$

i.e., for a rectangle to become a golden rectangle, the ratio of its length and

breadth  $\frac{l}{b}$  must be  $\frac{\sqrt{5} + 1}{2}$ .

This ratio  $\frac{\sqrt{5}+1}{2}$  is called the **golden ratio**. Its value is about 1.618.

Thus, it can be seen that **the golden ratio is the ratio of the sides of a golden rectangle**.

### (B) Golden ratio and a continued fraction

Let us consider a continued fraction

$$1 + \frac{1}{1 + \frac{1}{1 + \dots}}$$

We may note that it is an infinite continued fraction. We may write it as

$$x = 1 + \frac{1}{x}$$

So,  $x^2 = x + 1$

or,  $x^2 - x - 1 = 0$

It is the same quadratic equation as we obtained earlier.

So, again we have  $x = \frac{\sqrt{5}+1}{2}$  (Ignoring the negative root).

Thus, it can be said that **the golden ratio is equal to the infinite continued**

**fraction**  $1 + \frac{1}{1 + \frac{1}{1 + \dots}}$  **in the limiting form.**

### (C) Golden Ratio, Continued Fraction and a Sequence

Having seen the relationship between golden ratio and the continued fraction

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

let us examine the value of this fraction at different stages as shown below:  
Considering 1, we get the value as 1;

considering  $1 + \frac{1}{1}$ , we get the value as  $\frac{2}{1}$ ;

considering  $1 + \frac{1}{1 + \frac{1}{1}}$ , we get the value as  $\frac{3}{2}$ ;

considering  $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}$ , we get the value as  $\frac{5}{3}$ ;

considering  $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}$ , we get the value as  $\frac{8}{5}$ ; and so on

Thus, the values obtained at different stages are :

$$1, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \dots$$

The numerators of these values are 1, 2, 3, 5, 8, 13, ...

These values depict the following pattern:

$$3 = 1 + 2, 5 = 2 + 3, 13 = 5 + 8 \text{ and so on}$$

Note that by including 1 in the beginning, it will take the following form:

$$1, 1, 2, 3, 5, 8, 13, \dots$$

This is a famous sequence called the **Fibonacci sequence**.

It can be found that the  $n^{\text{th}}$  term of the Fibonacci sequence is

$$\frac{1}{\sqrt{5}} \left( \frac{\sqrt{5}+1}{2} \right)^n - \frac{-\sqrt{5}+1}{2} \left( \frac{\sqrt{5}+1}{2} \right)^n$$

It can also be seen that in the above expression,  $\frac{\sqrt{5}+1}{2}$  is the **golden ratio**.

### (D) Golden Ratio and Trigonometric Ratios

It can be found that

$$\cos 36^\circ = \sin 54^\circ = \frac{\sqrt{5}+1}{4}$$

That is,  $2 \cos 36^\circ = 2 \sin 54^\circ = 2 \cdot \frac{\sqrt{5}+1}{4} = \frac{\sqrt{5}+1}{2} = \text{Golden Ratio}$

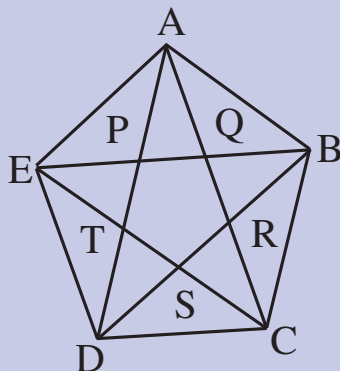
Thus, it can be said that **twice the value of  $\cos 36^\circ$  (or twice the value of  $\sin 54^\circ$ ) is equal to the golden ratio.**

### (E) Golden Ratio and Regular Pentagons

We know that a pentagon having all its sides and all its angles equal is called a regular pentagon. Clearly, each interior angle of a regular pentagon will be

$\frac{540^\circ}{5} = 108^\circ$ . Let us now draw any regular pentagon ABCDE and draw all its

diagonals AC, AD, BD, BE and CE as shown in the following figure:



It can be observed that inside this regular pentagon, another pentagon PQRST is formed.

Further, this pentagon is also a regular pentagon.

It can also be seen that  $\frac{AP}{PQ}, \frac{AQ}{PT}, \frac{AS}{PQ}, \frac{DS}{SR}, \dots$  are all equal to  $\frac{\sqrt{5}+1}{2}$ .

That is, **the ratio of the length of the part of any diagonal not forming the side of the new pentagon on one side and the length of a side of the new pentagon is equal to the golden ratio.** (1)

Further, it can also be seen that  $\frac{AE}{AP}, \frac{AB}{BQ}, \frac{CD}{DS}, \frac{BC}{CS}, \dots$  are all equal to  $\frac{\sqrt{5}+1}{2}$ .

That is, **the ratio of the length of any side of the given regular pentagon and that of the part of the diagonal not forming the side of the new pentagon on one side is equal to the golden ratio.** (2)

Combining the above two results (1) and (2), it can be seen that in the above two regular pentagons ABCDE and PQRST,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \dots = \frac{\sqrt{5}+1}{2}^2, \text{ i.e., (Golden Ratio)}^2$$

That is, **ratio of the corresponding sides of the two regular pentagons ABCDE and PQRST is equal to (golden ratio)<sup>2</sup>.**

We also know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

In fact, it is true for all the similar polygons.

Further, all regular polygons are always similar.

So, it can also be said that

ratio of the areas of above pentagons ABCDE and PQRST =

$$\frac{AB^2}{PQ} = \frac{BC^2}{QR} = \dots$$

Therefore, ratio of the areas of the above two pentagons =

$$\begin{aligned} \frac{AB^2}{PQ} &= \frac{\sqrt{5}+1}{2} \\ &= \frac{\sqrt{5}+1}{2}^4 = (\text{Golden ratio})^4 \end{aligned}$$

Thus, **areas of the above two regular pentagons is equal to (Golden ratio)<sup>4</sup>.**

[**Note** : The above results relating to trigonometric ratios and regular pentagons can be, in fact, proved using simple trigonometrical knowledge of Class XI].

## CONCLUSION

Golden rectangle and golden ratio are very closely related concepts involving other mathematical concepts such as fractions, similarity, quadratic equations, regular pentagons, trigonometry, sequences, etc. After getting some basic understanding of these at the secondary stage, they may be studied in a little details in higher classes at appropriate places and time.

## APPLICATION

Project is useful in designing buildings, architecture and structural engineering.

## Project 4 $\pi$ - WORLD'S MOST MYSTERIOUS NUMBER

### WHAT IS $\pi$ ?

The symbol  $\pi$  is the 16th letter of Greek alphabets. In the old Greek texts,  $\pi$  was used to represent the number 80.

Later on, the letter  $\pi$  was chosen by mathematicians to represent a very important constant value related to a circle. Specifically  **$\pi$  was chosen to represent the ratio of the circumference of a circle to its diameter.**

**Symbolically**  $\pi = \frac{c}{d}$ , where  $c$  represents the circumference and  $d$  represents the length of the diameter of the circle. Since the diameter of a circle is twice the radius,  $d = 2r$ , where  $r$  is the radius.

$$\text{So, } \pi = \frac{c}{2r}.$$

Where the symbol  $\pi$  in Mathematics came from?

According to the well-known mathematics historian Florian Cajori (1859-1930), the symbol  $\pi$  was first used in mathematics by William Oughtred (1575-1660) in 1652 when he referred to the ratio of the circumference of a

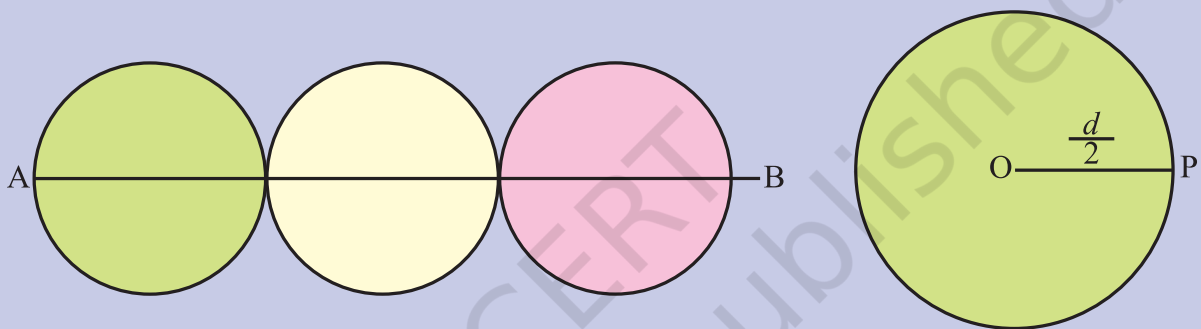
circle to its diameter as  $\frac{\pi}{\delta}$ , where  $\pi$  represented the periphery of a circle and  $\delta$  represented the diameter.

In 1706, William Jones (1675-1749) published his book **Synopsis Palmariorum Matheseos**, in which he used  $\pi$  to represent the ratio of the circumference of a circle to its diameter. This is believed to have been the first time  $\pi$  was used as it is defined/used today. Among others, Swiss mathematician Leonhard Euler also began using  $\pi$  to represent the ratio of circumference of a circle to its diameter.

## VALUE OF $\pi$

It is said that after a wheel was invented, the circumference was probably measured for the sake of comparison. Perhaps in the early days, it was important to measure how far a wheel would travel in one revolution. To measure this distance, it was convenient to measure it by placing the wheel on the distance being measured showing that its length is slightly more than three times the diameter.

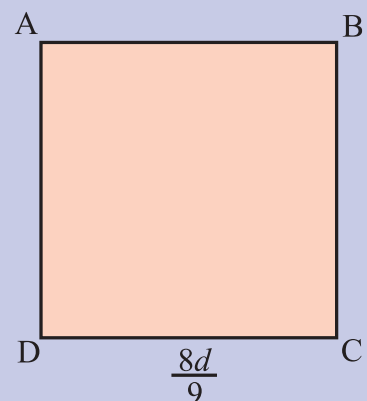
This type of activity repeated with different wheels showed that each time the circumference was just a bit more than three times as long as the diameter.



**Fig. 1 AB: Circumference of the wheel**

This showed that the value of  $\pi$  is slightly more than 3. Frequent measurement also showed that the part exceeding three times the diameter was very close to  $\frac{1}{9}$  of the diameter.

In Rhind Papyrus, written by Ahmes-an Egyptian in about 1650 B.C., it is said to have been mentioned that if a square is drawn with a side whose length is eight-ninths of the diameter of the circle, then the area of the square so formed and the area of circle would be the same.



**Fig. 2**

$$\text{Area of circle} = \pi \frac{d}{2}^2 = \pi \frac{d^2}{4}$$



$$\text{Area of square ABCD} = \frac{8}{9}d^2 = \frac{64}{81}d^2$$

$$\text{So, } \frac{\pi d^2}{4} = \frac{64}{81}d^2 \text{ implies } \pi = \frac{256}{81} = 3.1604938271604938271$$

This gives a reasonably close approximated value of  $\pi$

### ARCHIMEDES CONTRIBUTIONS

Archimedes, born in Syracuse about 287 B.C. gave the following proposition regarding the circle that had a role in the historical development of the value of  $\pi$ .

1. The ratio of the area of a circle to that of a square with side equal to the circle's diameter is close to 11:14.

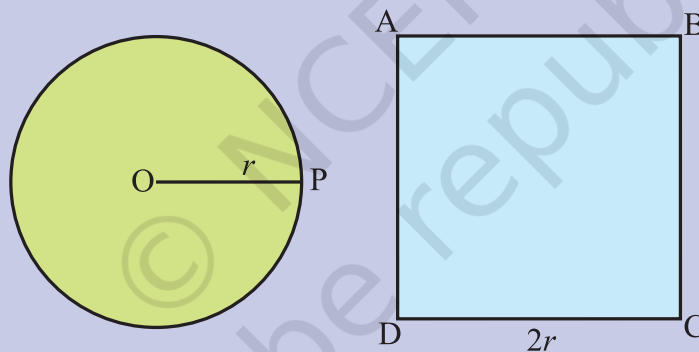


Fig. 3

$$\frac{\pi r^2}{4r^2} = \frac{11}{14}$$

$$\text{i.e., } \pi = \frac{44}{14} = \frac{22}{7}$$

This is again a familiar approximation of  $\pi$  which we often use in the problems related to mensuration.

2. The circumference of a circle is less than  $3\frac{1}{7}$  times of its diameter but more than  $3\frac{10}{71}$  times the diameter, i.e.,  $3\frac{10}{71} < \pi < 3\frac{1}{7}$ .

Let us see how Archimedes actually arrived at this conclusion. What he did was to inscribe a regular polygon (an equilateral triangle, a square, a regular pentagon, a regular hexagon etc.) in a given circle [see Fig. (4)] and also circumscribe the polygon about the same circle.

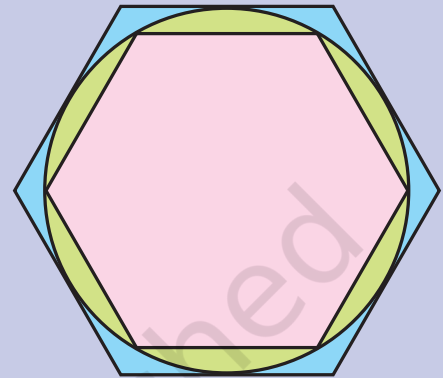


Fig. 4

In both the cases, the perimeter of the polygon gets closer and closer to the circumference of the circle.

He then repeated this process with 12 sided regular polygon, 24 sided regular polygon, 48 sided regular polygon, 96 sided regular polygon, each time getting perimeter closer and closer to circumference of the circle.

Archimedes finally concluded that the value of  $\pi$  is more than  $3\frac{10}{71}$  but less than  $3\frac{1}{7}$ . We know that

$$3\frac{10}{71} = \overline{3.14084507042253521126760563380281690}$$

$$\text{and } 3\frac{1}{7} = \overline{3.142857}$$

Thus, Archimedes gave the value of  $\pi$  which is consistent with what we know as the value of  $\pi$  today.

## CHINESE CONTRIBUTIONS

Liu Hui in 263 also used regular polygons with increasing number of sides to approximate the circle. He used only inscribed circles while Archimedes used both inscribed and circumscribed circles. Liu's approximation of  $\pi$  was

$$\frac{3927}{1250} = 3.1416$$

Zu Chongzhi (429-500), a Chinese astronomer and mathematician found that

$$\pi = \frac{355}{113} = 3. \overline{141592920353982300884} \overline{955752212389380530973} \overline{451327433628318584070796} \overline{460176991150442477876106} \overline{1946902654867256637168}$$

## CONTRIBUTION BY OTHERS

1. John Wallis (1616-1703), a professor of mathematics at Cambridge and Oxford Universities gave the following formula for  $\pi$  :

$$\frac{\pi}{2} = \frac{2 \times 2}{1 \times 3} \times \frac{4 \times 4}{3 \times 5} \times \frac{6 \times 6}{5 \times 7} \times \dots \times \frac{2n \times 2n}{(2n-1)(2n+1)} \times \dots$$

2. Brouncker (1620-1684) obtained the following value of  $\frac{4}{\pi}$  :

$$\frac{4}{\pi} = 1 + \frac{1^2}{2 + \frac{3^2}{2 + \frac{5^2}{2 + \frac{7^2}{2 + \frac{9^2}{2 + \dots}}}}}$$

3. Aryabhata (499) gave the value of  $\pi$  as  $\frac{62832}{20,000} = 3.14156$
4. Brahmagupta (640) gave the value of  $\pi$  as  $\sqrt{10} = 3.162277$
5. Al-Khowarizmi (800) gave the value of  $\pi$  as 3.1416
6. Babylonian used the value of  $\pi$  as  $3 + \frac{1}{8} = 3.125$
7. Yasumasa Kanada and his team at University of Tokyo calculated the value of  $\pi$  to 1.24 trillion decimal places.
8. French mathematician Francois Viete (1540-1603) calculated  $\pi$  correct to nine decimal places. He calculated the value of  $\pi$  to be between the numbers 3.1415926535 and 3.1415926537.
9. S. Ramanujan (1887-1920) calculated the value of  $\pi$  as  $\sqrt[4]{9^2 + \frac{19^2}{22}} = 3.14592652 \dots$  which is correct to eight decimal places.
10. Leonhard Euler came up with an interesting expression for obtaining the value of  $\pi$  as

$$\frac{2}{\pi} = 1 - \frac{1}{4} + 1 - \frac{1}{16} - 1 + \frac{1}{36} + 1 - \frac{1}{64} - 1 + \frac{1}{100} \dots$$

### A $\pi$ Paradox

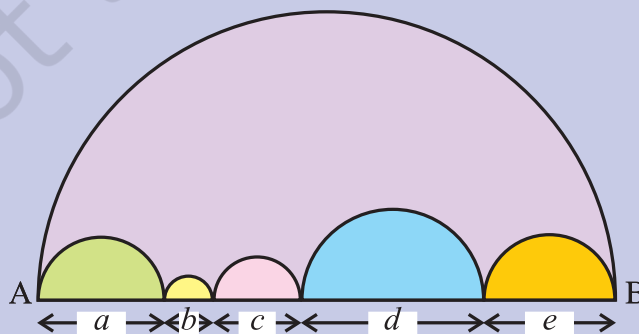


Fig. 5

In the above figure, perimeter of the semi-circle with diameter

$$AB = \frac{\pi}{2}(AB)$$

Sum of the perimeters of smaller semi-circles

$$= \frac{\pi a}{2} + \frac{\pi b}{2} + \frac{\pi c}{2} + \frac{\pi d}{2} + \frac{\pi e}{2} = \frac{\pi}{2}(a+b+c+d+e)$$

This may not 'appear' to be true but it is!

Let us now proceed in the following way. Increase the number of smaller semi-circles along the fixed line segment AB say of 2 units.

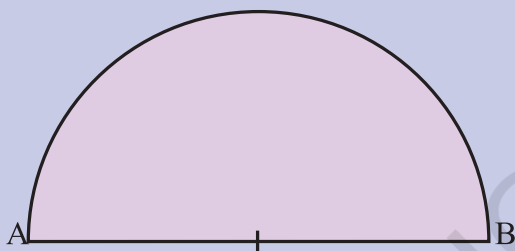


Fig. 6

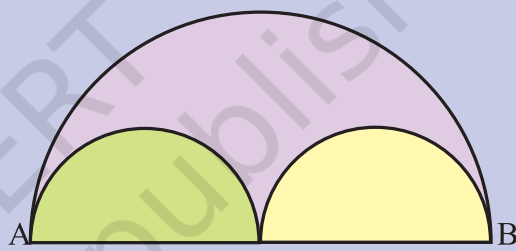


Fig. 7

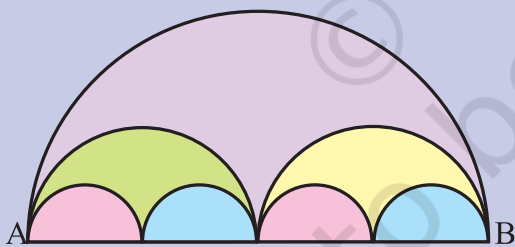


Fig. 8

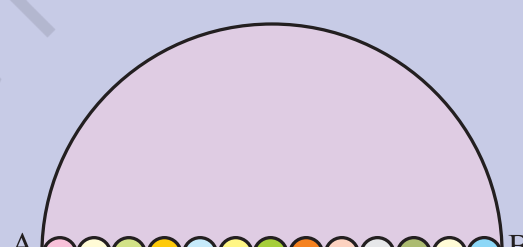


Fig. 9

In the above figures, the sum of the lengths of the perimeters of smaller semicircles "appears" to be approaching the length of the diameter AB but in fact it is not! because the lengths of the perimeters of the smaller

semi-circles is  $\frac{\pi \times 2}{2} = \pi$  while length of AB is 2 units. So, both cannot be the same.

## CONCLUSION

$\pi$  can be seen as a number with unusual properties. It has wide variety of applications in real life.

## APPLICATION

Value of  $\pi$  is used in finding areas and perimeters of designs related to circles and sector of circles. It has applications in the construction of racetracks and engineering equipments.

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## LIST OF PROJECTS

1. To develop Heron's formulae for area of a triangle.
2. Story of  $\pi$ .
3. Development of Number Systems with their needs.
4. Chronology of Indian Mathematicians with their contributions.
5. Chronological development of solution of a quadratic equations.
6. Development of Formula for the area of a cyclic quadrilateral.
7. Pythagoras Theorem-Proofs other than given in the present textbook.
8. Extensions of Pythagoras Theorem.
9. With rectangle of given perimeter finding the one with a maximum area and with rectangle of given area, finding the one with least perimeter.
10. Knowledge and classification of solid figures with respect to surface areas and volumes.
11. Sum of the exterior angles of a polygon taken in an order.
12. Generation of Pythagorean triplets.
13. Magic squares.
14. With cuboids of given surface area, finding the one with maximum volume and with cuboids of given volumes finding one with least surface area.
15. Mathematical designs and patterns.
16. Indian Mathematicians and their contributions.
17. To prepare a list of quotations on mathematics.
18. Ramanujan number (1729)
19. Mathematical Crosswords
20. Application of Geometry in day-to-day life
21. Application of Algebra in day-to-day life.
22. Application of Mensuration in day-to-day life.

## SCHEME OF EVALUATION

The following weightage are assigned for evaluation at Secondary Stage in mathematics:

Theory Examination	:	80 marks
Internal Assessment	:	20 marks

1. Internal assessment of 20 marks, based on school based examination will have following break-up:

Year-end assessment of activities	:	12 marks
Assessment of Project Work	:	5 marks
Viva-voce	:	3 marks

### • Assessment of Activity Work

- (a) Every student will be asked to perform two given activities during the allotted time.
- (b) The assessment may be carried out by a team of two mathematics teachers, including the teacher who is taking practical classes.
- (c) The break-up of 12 marks for assessment for a single activity may be as under:
- |  |          |                 |
|--|----------|-----------------|
| • Statement of objective of the activity | :        | 1 mark          |
| • Material required                      | :        | 1 mark          |
| • Preparation for the activity           | :        | 3 marks         |
| • Conduct of the activity                | :        | 3 marks         |
| • Observation and analysis               | :        | 3 marks         |
| • Results and Conclusion                 | :        | 1 mark          |
| <b>Total</b>                             | <b>:</b> | <b>12 marks</b> |
- (d) The marks for two activities may be added first and then marks calculated out of 12.
- (e) Full record of activities may be kept by each student.

### • Evaluation of Project Work

- (a) Every student will be asked to do at least one project based on the concepts learnt in the classroom.
- (b) The project may be carried out individually (or in a group of two or three students).
- (c) The weightage of 5 marks for the project may be as under :
- |   |   |        |
|---|---|--------|
| • Identification and statement of the project | : | 1 mark |
| • Planning the project                        | : | 1 mark |
| • Procedure adopted                           | : | 1 mark |
| • Observations from data collected            | : | 1 mark |
| • Interpretation and application of result    | : | 1 mark |

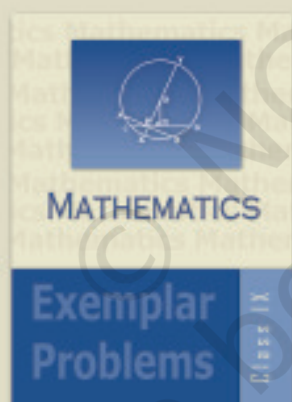
**Total Score out of 20 :** The marks obtained in year-end assessment of activities and project work be added to the marks in viva-voce to get the total score out of 20.

**Note :** Every student should be asked to perform at least twenty activities in one academic year.



## Other Exemplar Problems by NCERT

- Exemplar Problems in Mathematics for Class X
- Exemplar Problems in Physics for Class XI
- Exemplar Problems in Chemistry for Class XI
- Exemplar Problems in Biology for Class XI
- Exemplar Problems in Mathematics for Class XI



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