04

Electricity and magnetism have been known to us for more than 2000 years and we treated them as two separate subjects. The first evidence for the existence of relationship between electricity and magnetism was observed in 1820 by Hans Oersted, the man who himself used to demonstrate that electricity and magnetism had got no relationship with each other.

MOVING CHARGES AND MAGNETISM

So, in this chapter, we will be going to study magnetism produced by a moving charge and further we will proceed with Ampere's circuital law and its applications and at last, the chapter will be ended with magnetic force and torque between two parallel conductors. All the topics mentioned above are discussed in detail, so it will be more interesting to understand them very carefully after going through each and every sentence very thoroughly.

CHAPTER CHECKLIST

- Magnetic Field and Its Applications
- Ampere's Circuital Law and Moving Charges
- Magnetic Force and Torque Experienced by a Current Loop

|TOPIC 1|
Magnetic Field and Its Applications

MAGNETIC FIELD

In electrostatics, we studied that a static charge produces an electric field. Similarly, a moving charge or current flowing through a conductor produces a magnetic field.

The space in the surroundings of a magnet or a current carrying conductor in which its magnetic influence can be experienced is called magnetic field.

The SI unit of magnetic field is tesla (T) or weber/metre^2 (Wbm^{-2}) or $NA^{-1}m^{-1}$ and its CGS unit is gauss (G).

$$1 \text{ tesla} = 10^4 \text{ gauss}$$

Oersted's Experiment

In the summer of 1820, HC Oersted by his experiment concluded that a current carrying conductor deflects magnetic compass needle placed near it. He found that the alignment of magnetic needle is tangential to an imaginary circle which has the straight wire as its centre and has its plane perpendicular to the wire as shown in Fig. (a).

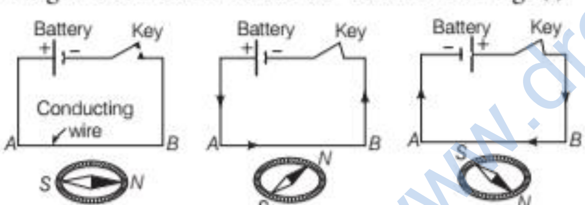

(a) (b) (c)

If the current is reversed, the needle is deflected in opposite direction as shown in Fig. (b). The deflection of the needle indicates that a magnetic field is established around a current carrying wire. On increasing the current in the wire or bringing the needle closer to the wire, the deflection of the needle increases. He also found that the iron filings sprinkled around the wire arrange themselves in concentric circles with the wire as the centre as shown in Fig. (c).

This experiment shows that the magnetic field is produced due to electric current. Electric current means moving charge, so it can be concluded that moving charges produce magnetic field in the surroundings.

Note A current or field (electric or magnetic) emerging out of the plane of the paper is represented by a dot (⊙) and going into the plane of the paper is represented by a cross (⊗).

Consider a conducting wire AB be placed over the magnetic needle parallel to it. It will be found that the North pole of needle gets deflected towards the West as shown in Fig. (e). If the direction of current is reversed, then the North pole of needle gets deflected towards East as shown in Fig. (f).



(d) (e) (f)

Direction of deflection of magnetic needle

The direction of deflection of magnetic needle due to current in the wire is given by Ampere's swimming rule.

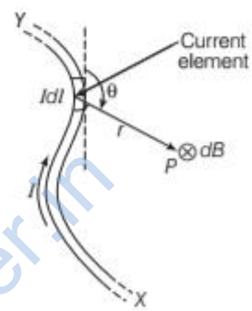
Ampere's Swimming Rule

According to this rule, if we imagine a man is swimming along the wire in the direction of current with his face turned towards the magnetic needle, so that the current enters through his feet and leaves at his head, then the North pole of the needle will be deflected towards his left hand. This rule can be recollected with the help of the word SNOW. It means, current from South to North, in a wire over the magnetic needle, the North pole of the needle is deflected towards West.

MAGNETIC FIELD DUE TO A CURRENT ELEMENT : BIOT-SAVART'S LAW

Biot-Savart's law is an experimental law predicted by Biot and Savart. This law deals with the magnetic field induction at a point due to a small current element (a part of any conductor, carrying current).

Let XY be current carrying conductor, I be current in the conductor, dl be infinitesimal small element of the conductor, dB be magnetic field at point P at a distance r from the element.



According to Biot-Savart's law, the magnitude of magnetic field induction (dB) at a point P due to a current element depends on the following factors

(i) $dB \propto I$ (i.e. magnetic field is directly proportional to the current flowing through the conductor).

(ii) $dB \propto dl$ (i.e. magnetic field is directly proportional to the length of the element).

(iii) $dB \propto \sin\theta$ (i.e. magnetic field is directly proportional to the sine of angle between the length of element and line joining the element to point (P).

(iv) $dB \propto \frac{1}{r^2}$ (i.e. magnetic field is inversely proportional to the square of distance between the element and point P). Combining all the above relations,

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

This relation is called **Biot-Savart's law.**

If conductor is placed in air or vacuum, then magnetic field is given by

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$$

where, $\frac{\mu_0}{4\pi}$ is a proportionality constant, μ_0 is the permeability of free space. $\mu_0 = 4\pi \times 10^{-7}$ Tm/A (or Wb/A-m), its dimensions are $[MLT^{-2}A^{-2}]$.

In vector form, Biot-Savart's law can be written as

$$d\mathbf{B} \propto \frac{Id\mathbf{l} \times \mathbf{r}}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{Id\mathbf{l} \times \mathbf{r}}{r^3} \quad \text{...(i)}$$

From Eq. (i), the direction of $d\mathbf{B}$ would be the direction of the cross-product vector $(d\,\mathbf{l} \times \mathbf{r})$, which is represented by the right handed screw rule or right hand thumb rule.

Here, $d\,\mathbf{B}$ is perpendicular to the plane containing $d\,\mathbf{l}$ and $\mathbf{r}$ is directed inwards (since, point P is to the right of the current element).

Magnetic field induction at point P due to current through entire wire is

$$\mathbf{B} = \int \frac{\mu_0}{4\pi} \cdot \frac{Id\mathbf{l} \times \mathbf{r}}{r^3}$$

or

$$B = \int \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2}$$

Biot-Savart's law in terms of current density J can be written as

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{\mathbf{J} \times \mathbf{r}}{r^3} dV \quad \left[\because J = \frac{I}{A} = \frac{Idl}{Adl} = \frac{Idl}{dV}\right]$$

where, J = current density at any point on the current element and dV = volume of the element.

Biot-Savart's law in terms of charge (q) and its velocity (v) can be written as

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{q(\mathbf{v} \times \mathbf{r})}{r^3} \left[\because Id\mathbf{l} = \frac{q}{dt} d\mathbf{l} = q\frac{d\mathbf{l}}{dt} = q\mathbf{v}\right]$$

Biot-Savart's law in terms of magnetising force or magnetising intensity (H) of the magnetic field is in SI or MKS system,

$$d\mathbf{H} = \frac{d\mathbf{B}}{\mu_0} = \frac{1}{4\pi} \cdot \frac{Id\mathbf{l} \times \mathbf{r}}{r^3} = \frac{1}{4\pi} \cdot \frac{Id\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

$$\therefore \quad d\mathbf{H} = \frac{1}{4\pi} \cdot \frac{Idl \sin\theta}{r^2}$$

In CGS units, $d\mathbf{H} = \dfrac{Id\mathbf{l} \times \mathbf{r}}{r^3}$ and $dH = \dfrac{Idl \sin\theta}{r^2}$

Features of Biot-Savart's Law

Some important features of Biot-Savart's law are as follows

(i) This law is analogous to Coulomb's law in electrostatics.

(ii) The direction of $d\mathbf{B}$ is perpendicular to both $Id\mathbf{l}$ and $\mathbf{r}$. It is given by right hand thumb rule.

(iii) If $\theta = 0°$, i.e. the point P lies on the axis of the linear conductor carrying current (or on the wire carrying current), then

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 0°}{r^2} = 0$$

It means that there is no magnetic field induction at any point on the thin linear current carrying conductor.

(iv) If $\theta = 90°$, i.e. the point P lies at a perpendicular position with respect to current element, then

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2}, \text{ which is maximum.}$$

If $\theta = 180°$, then $dB = 0$, which is minimum.

Similarities and Differences between Biot-Savart's Law and Coulomb's Law

The Biot-Savart's law for the magnetic field has certain similarities as well as differences with the Coulomb's law for the electrostatic field. Some of these are as follows

(i) Both are long range, since both depend inversely on the square of distance from the source to the point of interest. The principle of superposition applies to both fields. (In this connection, note that the magnetic field is linear in the source $Id\mathbf{l}$ just as the electrostatic field is linear in its source, the electric charge.)

(ii) The electrostatic field is produced by a scalar source, namely, the electric charge. The magnetic field is produced by a vector source $Id\mathbf{l}$.

(iii) The electrostatic field is along the displacement vector joining the source and the field point. The magnetic field is perpendicular to the plane containing the displacement vector $\mathbf{r}$ and the current element $Id\mathbf{l}$.

(iv) There is an angle dependence in the Biot-Savart's law, which is not present in the electrostatic case. The magnetic field at any point in the direction of $d\mathbf{l}$ is zero. Along this line, $\theta = 0°$, $\sin 0° = 0$, so $|d\mathbf{B}| = 0$.

Permittivity and Permeability

Electric permittivity ε_0 is the physical quantity that determines the degree of interaction of electric field with medium. However, magnetic permeability μ_0 is the physical quantity that measures the ability of a substance to acquire magnetisation in magnetic field, i.e. the degree of penetration of matter by B.

The relation between ε_0 and μ_0 is always a constant, i.e.

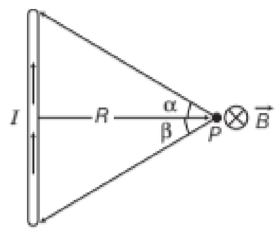
$$\mu_0 \varepsilon_0 = (4\pi\varepsilon_0) \times \left(\frac{\mu_0}{4\pi}\right) = \frac{1 \times 10^{-7}}{9 \times 10^9} = \frac{1}{(3 \times 10^8)^2} = \frac{1}{c^2}$$

MAGNETIC FIELD DUE TO A CURRENT CARRYING CONDUCTOR

Radial magnetic field created by a current element is perpendicular to both current element dl and position vector **r**.

Magnetic field B for a straight wire of finite length is given by

$$B = \frac{\mu_0 I}{4\pi R}(\sin\alpha + \sin\beta)$$



According to right hand thumb rule, the direction of magnetic field in this case is perpendicular to the plane of paper and directed inwards.

Magnetic field B for infinitely long wire,

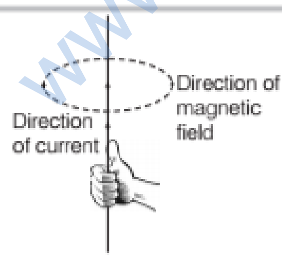
As, $\alpha = \beta = \pi/2$

then $\boxed{B = \dfrac{\mu_0 I}{2\pi R}}$

The direction of the magnetic field associated with a current carrying conductor can be determined by right hand thumb rule or Maxwell's cork screw rule.

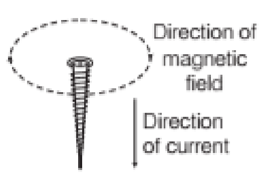
Right Hand Thumb Rule

According to this rule, if we imagine a linear wire conductor to be held in the grip of the right hand such that the thumb points in the direction of current, then the curvature of the fingers around the conductor will give the direction of magnetic field lines.



Maxwell's Cork Screw Rule

According to this rule, if we imagine a right handed cork screw placed along the current carrying wire conductor, rotated such that the screw moves in the direction of current, then the direction of rotation of the screw gives the direction of magnetic field lines.

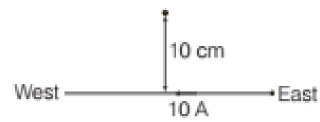


EXAMPLE |1| A current 10 A is flowing East to West in a long wire kept horizontally in the East-West direction. Find the magnitude and direction of magnetic field in a horizontal plane at a distance of 10 cm North.

Sol. Given, current, $I = 10$ A (East to West)

Distance, $r = 10 \text{ cm} = 10 \times 10^{-2}$ m

Magnetic field, $|\mathbf{B}| = ?$



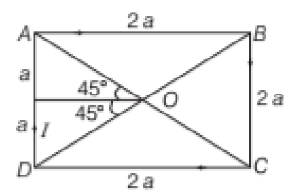
The magnitude of magnetic field $|\mathbf{B}|$ for infinite length of wire $= \dfrac{\mu_0 I}{2\pi r}$

$$\Rightarrow \quad |B| = \frac{4\pi \times 10^{-7} \times 10}{2 \times \pi \times 10 \times 10^{-2}} = 2 \times 10^{-5}\text{ T}$$

The direction of magnetic field is given by right hand thumb rule or Maxwell's cork screw rule. So, the direction of magnetic field at point 10 cm North due to flowing current is perpendicularly inwards to the plane of paper.

EXAMPLE |2| Find an expression for the magnetic field at the centre of a coil bent in the form of square of side $2a$, carrying current I as shown in the figure.

Sol. Given, $\theta_1 = 45°, \theta_2 = 45°$



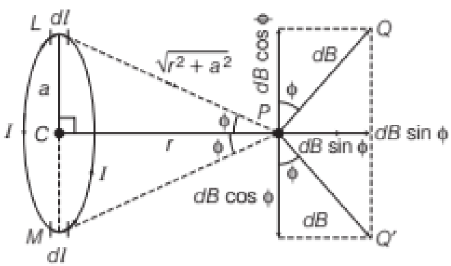
Total magnetic field due to each side at point O is given by

$$B = 4\frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2) = 4\frac{\mu_0 I}{4\pi a}(\sin 45° + \sin 45°)$$

$$= \frac{\mu_0 I}{\pi a}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}\mu_0 I}{\pi a}$$

MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT CARRYING LOOP

Let us consider a circular loop of radius a with centre C. Let the plane of the coil be perpendicular to the plane of the paper and current I be flowing in the direction shown. Suppose P is any point on the axis of a coil at a distance r from the centre C.



Now, consider a current element *Idl* on top (*L*), where current comes out of paper normally, whereas at bottom (*M*), current enters into the plane paper normally.

$\because \qquad LP \perp dl$

Also, $\qquad MP \perp dl$

$\therefore \qquad LP = MP = \sqrt{r^2 + a^2}$

The magnetic field at point *P* due to the current element *Idl*, according to Biot-Savart's law is given by

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{(r^2 + a^2)} = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{(r^2 + a^2)}$$

where, a = radius of circular loop

and $\quad r$ = distance of point *P* from the centre *C* along the axis.

According to right hand screw rule, the direction of *dB* is perpendicular to *LP* and along *PQ*, where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element *Idl* at the bottom and direction is along PQ', where $PQ' \perp MP$.

Now, resolving *dB* due to current element at *L* and *M*. So, $dB \cos \phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin \phi = \oint \frac{\mu_0}{4\pi} \left(\frac{Idl}{r^2 + a^2} \right) \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

$$\left[\because \text{In } \Delta PCL, \sin \phi = \frac{a}{\sqrt{r^2 + a^2}} \right]$$

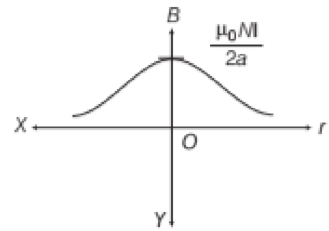
$$= \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a)$$

or $\quad$ $$B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}} \qquad \ldots \text{(ii)}$$

For *N* turns, the net magnetic field is given by

$$B = \frac{\mu_0 N I a^2}{2(r^2 + a^2)^{3/2}}$$

The direction of **B** is along the axis and away from the loop, when current in the coil is in anti-clockwise direction.



Variation of magnetic field induction (*B*) with distance *r*

EXAMPLE |3| A circular coil of 120 turns has a radius of 18 cm and carries a current of 3 A. What is the magnitude of the magnetic field at a point on the axis of the coil at a distance from the centre equal to the radius of the circular coil?

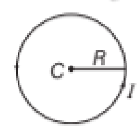
Sol. Given, number of turns $N = 120$, current $I = 3$ A, radius of coil, $r = 18$ cm $= 0.18$ m and distance from the centre to a point on axis, $a = r = 0.18$ m

As, $$B = \frac{\mu_0 N I a^2}{2(a^2 + r^2)^{3/2}} = \frac{4\pi \times 10^{-7} \times 120 \times 3 \times (0.18)^2}{2[(0.18)^2 + (0.18)^2]^{3/2}}$$

$\Rightarrow \quad B = 4.4 \times 10^{-4}$ T

MAGNETIC FIELD AT THE CENTRE OF A CURRENT CARRYING CIRCULAR LOOP

Consider a circular loop of radius *R* carrying current *I*. Magnetic field at its centre *C* is given by



$$B = \frac{\mu_0 I}{2R}$$

is obtained by setting $r = 0$ in previous relation (ii).

If we take a coil having *N* number of turns, then magnetic field at its centre is

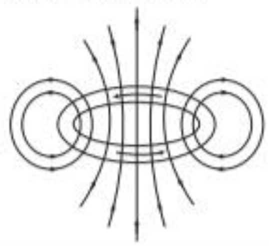
$$B = \frac{\mu_0 N I}{2R}$$

The direction of magnetic field at the centre of circular loop is given by **right hand rule.**

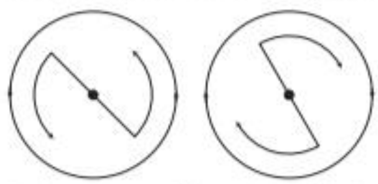
Similarly, magnetic field at the centre of a semi-circular wire of radius *R*, carrying current *I* is given by , $B = \frac{\mu_0 I}{4R}$

Right Hand Rule

According to this rule, if we hold the thumb of right hand mutually perpendicular to the grip of fingers such that the curvature of fingers depicts the direction of current in circular wire loop, then the thumb will point in the direction of magnetic field near the centre of loop.



Note As current carrying loop has the magnetic field lines around it thus, it behaves as a magnet with two mutually opposite poles.



The anti-clockwise flow of current behaves like a North pole, whereas clockwise flow as South pole.

EXAMPLE |4| A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field B at the centre of the coil? **NCERT**

Sol. Here, $n = 100$, $r = 8\,\text{cm} = 8\times10^{-2}\,\text{m}$ and $I = 0.40\,\text{A}$

$\therefore$ Magnetic field B at the centre,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi In}{r} = \frac{10^{-7}\times 2\times 3.14\times 0.40\times 100}{8\times10^{-2}} = 3.1\times10^{-4}\,\text{T}$$

EXAMPLE |5| The magnetic field B due to a current carrying circular loop of radius 12cm at its centre is 0.5×10^{-4} T. Find the magnetic field due to this loop at a point on the axis at a distance of 5.0 cm from the centre.

Sol. Magnetic field at the centre of a circular loop,

$$B_1 = \frac{\mu_0 I}{2R}$$

and that at an axial point, $B_2 = \dfrac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}$

Thus, $\dfrac{B_2}{B_1} = \dfrac{R^3}{(R^2+x^2)^{3/2}}$ or $B_2 = B_1\left[\dfrac{R^3}{(R^2+x^2)^{3/2}}\right]$

Substituting the values, we have

$$B_2 = (0.5\times10^{-4})\left[\frac{(12)^3}{(144+25)^{3/2}}\right] = 3.9\times10^{-5}\text{ T}$$

EXAMPLE |6| An electric current is flowing in a circular coil of radius a. At what distance from the centre on the axis of the coil will the magnetic field be $\frac{1}{8}$th of its value at the centre?

Sol. Magnetic field induction at a point on the axis at distance x from the centre of the circular coil carrying current is

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi nIa^2}{(a^2+x^2)^{3/2}}$$

Magnetic field induction at the centre of the circular coil carrying current is

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi nI}{a}$$

But as per question, $B_1 = \dfrac{B_2}{8}$

$$\Rightarrow \frac{\mu_0}{4\pi}\cdot\frac{2\pi nIa^2}{(a^2+x^2)^{3/2}} = \frac{\mu_0}{4\pi}\cdot\frac{2\pi nI}{a}\times\frac{1}{8}$$

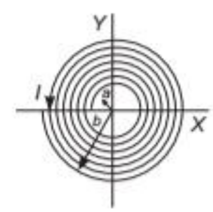
$$\Rightarrow \frac{a^2}{(a^2+x^2)^{3/2}} = \frac{1}{8a} \quad\Rightarrow\quad 8a^3 = (a^2+x^2)^{3/2}$$

$$\Rightarrow 2a = (a^2+x^2)^{1/2}$$

$$\Rightarrow 4a^2 = a^2+x^2$$

$$\Rightarrow x = \sqrt{3}a$$

EXAMPLE |7| A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the XY-plane and a steady current I flows through the wire. Find the Z-component of the magnetic field at the centre of the spiral.



Sol. If we take a small strip of dr at distance r from centre, then number of turns in this strip would be

$$dN = \left(\frac{N}{b-a}\right)dr$$

Magnetic field due to this element at the centre of the coil will be

$$dB = \frac{\mu_0(dN)I}{2r} = \frac{\mu_0 NI}{2(b-a)}\,\frac{dr}{r}$$

$$\therefore \quad B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

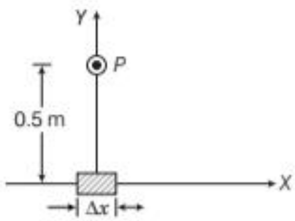
TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. A magnetic field can be produced
(a) only by moving charge
(b) only by changing electric field
(c) Both (a) and (b)
(d) None of the above

2. Biot-Savart law indicates that the moving electrons (velocity v) produce a magnetic field **B** such that **NCERT Exemplar**
(a) **B** is perpendicular to v
(b) **B** is parallel to v
(c) it obeys inverse cube law
(d) it is along to the line joining the electron and point of observation

3. An element $\Delta l = \Delta x\,\hat{\mathbf{i}}$ is placed at the origin and carries a current $I = 10$ A.



If $\Delta x = 1$ cm, magnetic field at point P is
(a) $4\times10^{-8}\,\hat{\mathbf{k}}$ T (b) $4\times10^{-8}\,\hat{\mathbf{i}}$ T
(c) $4\times10^{-8}\,\hat{\mathbf{j}}$ T (d) $-4\times10^{-8}\,\hat{\mathbf{j}}$ T

4. There is a thin conducting wire carrying current. What is the value of magnetic field induction at any point on the conductor itself?
(a) 1 (b) Zero
(c) −1 (d) Either (a) or (b)

5. A helium nucleus moves in a circle of 0.8 m radius in one second. The magnetic field produced at the centre of circle will be
(a) $\mu_0\times10^{-19}$ (b) $\mu_0\times10^{+19}$
(c) $2\mu_0\times10^{-19}$ (d) $\dfrac{2\times10^{-19}}{\mu_0}$

VERY SHORT ANSWER Type Questions

6. In what respect does a wire carrying a current differ from a wire, which carries no current?

7. How can you justify that a current carrying wire produces magnetic field?

8. Give the dependence of magnetic field produced by a current conductor.

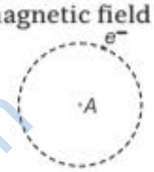
9. State Biot-Savart's law and express this law in the vector form. **All India 2017**

10. Among Biot-Savart's law and Coulomb's law, which one is angle dependent?

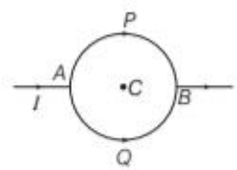
11. Name the kind of magnetic field produced by an infinitely long current carrying conductor.

12. Draw the magnetic field lines due to a current carrying loop. **Delhi 2013**

13. An electron is revolving around a circular loop as shown in the figure. What will be the direction of magnetic field at the point A?

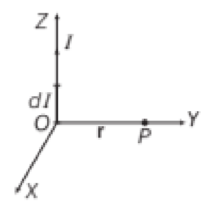


14. There is a circuit given below, where APB and AQB are semi-circles. What will be the magnetic field at the centre C of the circular loop?

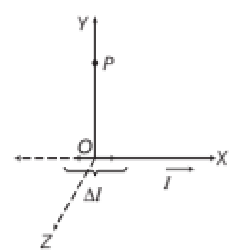


SHORT ANSWER Type Questions

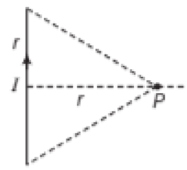
15. State Biot-Savart's law. A current I flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element $d\mathbf{l}$ at a point P situated at a distance $\mathbf{r}$ from the element as shown in the figure. **Delhi 2009**



16. An element $\Delta l = \Delta x\,\hat{\mathbf{i}}$ is placed at the origin (as shown in figure) and carries a current $I = 2$A. Find out the magnetic field at a point P on the Y-axis at a distance of 1.0 m due to the element $\Delta x = w$ cm. Also, give the direction of the field produced. **Delhi 2009**



17. Find the magnetic field at point P due to the current carrying conductor of current I as shown in the figure.



18. If a current loop of radius R carrying an anti-clockwise current I is placed in a plane parallel to YZ-plane.Then, what will be the magnetic field at a point on the axis of the loop?

19. A circular coil of closely wound N turns and radius r carries a current I in the clockwise direction. Find

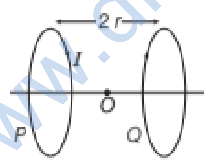
(i) the direction of magnetic field at its centre.

(ii) the magnitude of magnetic field at the centre. **All India 2012**

20. A straight wire of length L is bent into a semi-circular loop. Use Biot-Savart's law to deduce an expression for the magnetic field at its centre due to the current I passing through it. **Delhi 2011C**

21. A wire of length L is bent round in the form of a coil having N turns of same radius. If a steady current I flows through it in clockwise direction, then find the magnitude and direction of the magnetic field produced at its centre. **Foreign 2009**

22. Two identical circular loops P and Q, each of radius r and carrying equal currents are kept in the parallel planes having a common axis passing through O. The direction of current in P is clockwise and in Q is anti-clockwise as seen from O, which is equidistant from the loops P and Q. Find the magnitude of the net magnetic field at O. **Delhi 2012**



LONG ANSWER Type I Questions

23. Use Biot-Savart's law to derive the expression for the magnetic field on the axis of a current carrying circular loop of radius R.

Draw the magnetic field lines due to a circular wire carrying current (I). **Delhi 2016**

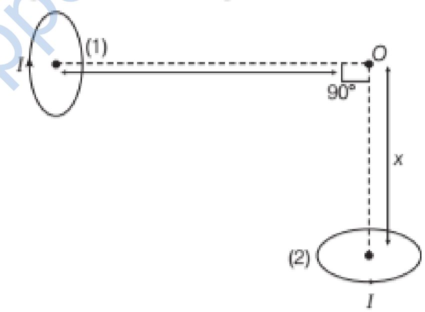
24. Using Biot-Savart's law, write the expression for the magnetic field **B** due to an element $d\mathbf{l}$ carrying current I at a distance **r** from it in a vector form.

Hence, derive the expression for the magnetic field due to a current carrying loop of radius R at a point P and distance x from its centre along the axis of the loop. **Delhi 2015**

LONG ANSWER Type II Questions

25. State Biot-Savart's law expressing it in the vector form. Use it to obtain the expression for the magnetic field at an axial point distance d from the centre of a circular coil of radius a carrying current I. Also, find the ratio of the magnitudes of the magnetic field of this coil at the centre and at an axial point for which $d = a\sqrt{3}$. **Delhi 2013C**

26. Two very small identical circular loops (1) and (2) carrying equal current I are placed vertically (with respect to the plane of the paper) with their geometrical axes perpendicular to each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point O. **Delhi 2014**



NUMERICAL PROBLEMS

27. A current of 5 A is flowing from South to North in a straight wire. Find the magnetic field due to a 1cm piece of wire at a point 1 m North-East from the piece of wire. **All India 2011**

28. An element $d\mathbf{l} = dx\,\hat{\mathbf{i}}$ (where, $dx = 1\,\text{cm}$) is placed at the origin and carries a large current $I = 10$ A. What is the magnetic field on the Y-axis at a distance of $0.5\,\text{m}$? **NCERT**

29. A long straight wire in the horizontal plane carries a current of 50 A in North to South direction.Give the magnitude and direction of B at a point 2.5 m East of the wire. **NCERT**

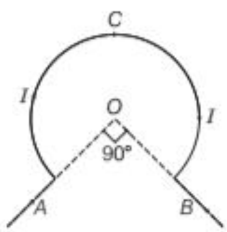
30. Two wires A and B have the same length equal to 44 cm and carry a current of 10 A each. Wire A is bent into a circle and wire B is bent into a square.

(i) Obtain the magnitudes of the fields at the centres of the two wires.

(ii) Which wire produces a greater magnetic field at its centre?

31. A tightly wound 100 turns coil of radius 10 cm is carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil? **NCERT**

32. The wire shown in the figure carries a current of 10 A. Determine the magnitude of magnetic field induction at the centre O. Given the radius of bent coil is 3 cm.

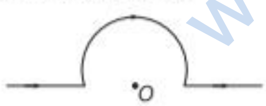


33. Two identical circular coils, P and Q each of radius R, carrying currents 1 A and $\sqrt{3}$ A respectively, are placed concentrically and perpendicular to each other lying in the XY and YZ-planes. Find the magnitude and direction of the net magnetic field at the centre of the coils. **All India 2017**

34. A straight wire carrying a current of 10 A is bent into a semi-circular arc of radius 2.0 cm as shown in the figure. What is the magnetic field at O due to

(i) straight segments

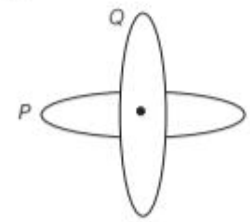
(ii) the semi-circular arc?



35. Two concentric circular coils x and y of radii 16 cm and 10 cm respectively lie in the same vertical plane containing the North to South direction. Coil x has 20 turns and carries a current of 16 A, coil y has 25 turns and carries a current of 18 A. The sense of the current in x is anti-clockwise and clockwise in y, for an observer looking at the coils facing West. Find the magnitude and direction of the net magnetic field due to the coils at their centre.

NCERT

36. Two identical loops P and Q each of radius 5 cm are lying in perpendicular planes such that they have a common centre as shown in the figure. Find the magnitude and direction of the net magnetic field at the common centre of the two coils, if they carry currents equal to 3 A and 4 A, respectively. **Delhi 2017**



HINTS AND SOLUTIONS

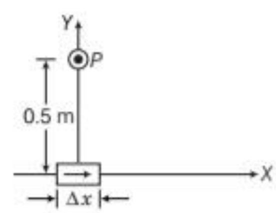
1. (a) Electric current or moving charges produce magnetic field around them.

2. (a) In Biot-Savart's law, magnetic field $\mathbf{B} \parallel id\mathbf{l} \times \mathbf{r}$ and $id\mathbf{l}$ due to flow of electron is in opposite direction of $\mathbf{v}$ and by direction of cross product of two vectors, i.e.

$$\mathbf{B} \perp \mathbf{v}$$

3. (a) The magnitude of magnetic field,

$$|d\mathbf{B}| = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$$



i.e., $$|d\mathbf{B}| = \frac{10^{-7} \times 10 \times 10^{-2}}{25 \times 10^{-2}} = 4 \times 10^{-8}\text{ T}$$

As, $$Id\mathbf{l} \times \mathbf{r} = \Delta x\hat{\mathbf{i}} \times y\hat{\mathbf{j}} = y\,\Delta x(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) = y\,\Delta x\hat{\mathbf{k}}$$

So, the direction of the field is in the + Z-direction.

4. (b) $$|d\mathbf{B}| = \frac{\mu_0}{4\pi}\left|\frac{Id\mathbf{l} \times \mathbf{r}}{r^3}\right| = \frac{\mu_0}{4\pi} \times \frac{Idl\sin\theta}{r^2}$$

If point lies on the conductor, then $\theta = 0°$ or $180°$. So, $\sin\theta = 0$, thus $dB = 0$. Hence, the magnetic field induction at any point on the conductor itself is zero.

5. (c) The magnetic field at the centre of circle, $B = \frac{\mu_0 i}{2r}$

The charge on helium nucleus is $2e$, so

Current, $$i = \frac{q}{t} = \frac{2e}{t}$$

$$\Rightarrow \quad B = \frac{\mu_0 \times 1.6 \times 10^{-19} \times 2}{2 \times 0.8}$$

$$= 2\mu_0 \times 10^{-19}\text{ N/A-m}$$

6. A current carrying wire produces magnetic field but wire which does not carry current has no magnetic field.

7. It can be justified by placing a magnetic needle around current carrying wire, which shows deflection of needle.

8. Magnetic field produced by a current conductor is

(i) Directly proportional to the current flowing through the conductor, length of the element and sine of the angle between the length of the element and line joining the element to the point

(ii) Inversely proportional to the square of the distance between the element and the point.

9. Biot-Savart's law states that, the magnitude of magnetic field intensity (dB) at a point P due to current element is given by

$$dB \propto \frac{I\,dl \sin\theta}{r^2}$$

or $$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

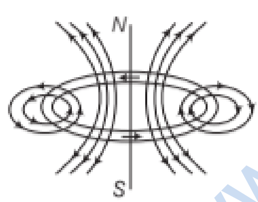
Thus, in vector notation,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l}\times\mathbf{r}}{r^3}$$

10. Biot-Savart's law is an angle dependent law.

11. An infinitely long current carrying conductor produces magnetic field in the form of concentric circular loops in a plane of straight conductor.

12. Magnetic field lines due to a current carrying loop is given by



13. As, electron is revolving clockwise, therefore conventional current due to the motion of electron will be in anti-clockwise direction.

So, according to right hand rule, magnetic field at point A will be in outward direction.

14. Magnetic field due to loop APB at the centre is given by

$$B_1 = \frac{\mu_0 I}{4a}\odot$$

Magnetic field due to loop AQB at the centre is given by

$$B_2 = \frac{\mu_0 I}{4a}\odot.$$

So, net magnetic field at centre = $B_1 + B_2 = 0$ (zero)

15. Refer to text on pages 168 and 169.

16. Biot-Savart's law states that

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$

Here $\Delta x = w$ cm

$\because$ $\Delta l = \Delta x i$

$\Rightarrow$ $I = 2$A, $r = 1$ m

$\therefore$ $$dB = \frac{\mu_0}{4\pi}\cdot\frac{(2w\hat{i}\times\hat{j})}{(1)^2}$$

$Idl = 2\times w\hat{\mathbf{i}}$

$\because$ $\hat{\mathbf{r}} = \hat{\mathbf{j}} \Rightarrow |\mathbf{r}| = 1$ m

$\therefore$ $$dB = \frac{\mu_0 w}{2\pi}\hat{\mathbf{k}}$$

$\Rightarrow$ $$|d\mathbf{B}| = \frac{\mu_0 w}{2\pi}$$

and direction along $+Z$-axis.

17. Refer to text on page 170.

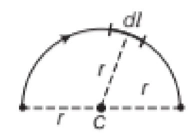
$$B_P = \frac{\mu_0 I}{4\pi r}(\sin 45° + \sin 45°) = \frac{\mu_0 I}{4\pi r}\times\sqrt{2} = \frac{\mu_0 I}{2\sqrt{2}\pi r}$$

18. Refer to text on pages 170 and 171.

19. (i) Inward

(ii) Refer to text on page 171.

20. According to the questions the wire will now look like.



$\because$ Length L is bent into semi-circular loop.

$\therefore$ Length of wire = Circumference of semi-equal circular wire

$\Rightarrow$ $L = \pi r$

$\Rightarrow$ $r = \dfrac{L}{\pi}$ ...(i)

Considering a small element dl on current loop. The magnetic field dB due to small current element Idl at centre C. Using Biot-Savart's law, we have

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin 90°}{r^2} \quad [\because Idl \perp \mathbf{r}, \therefore \theta = 90°]$$

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl}{r^2}$$

$\therefore$ Net magnetic field at C due to semi-circular loop,

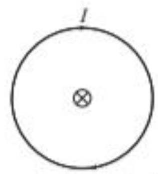
$$B = \int_{\text{semi-circle}}\frac{\mu_0}{4\pi}\frac{Idl}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int_{\text{semi-circle}} dl$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{I}{r^2}L \qquad \left[\text{but } r = \frac{L}{\pi}\right]$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{IL}{(L/\pi)^2} = \frac{\mu_0}{4\pi}\times\frac{IL}{L^2}\times\pi^2 = \frac{\mu_0 I\pi}{4L}$$

which is the required expression.

21. When a straight wire is bent into the form of a circular coil of N turns, then the length of the wire is equal to circumference of the coil multiplied by the number of turns. Let the radius of coil be r.



As, the wire is bent round in the form of a coil having N turns.

$\therefore$ $N \times$ circumference of the coil = Length of the wire

$$\Rightarrow \quad (2\pi r) \times N = L$$

$$\Rightarrow \quad r = \frac{L}{2\pi N} \quad \text{...(i)}$$

Magnetic field at the centre due to N turns of a coil is given by

$$B = \frac{\mu_0 (NI)}{2r} = \frac{\mu_0 (NI)}{2\left(\frac{L}{2\pi N}\right)} \quad \text{[from Eq. (i)]}$$

$$= \frac{\mu_0 \pi N^2 I}{L}$$

The direction of magnetic field is perpendicular to the plane of loop and entering into it.

22. Magnetic field at O due to two loops will be in same direction ($Q \to P$, along the axis) and of equal magnitude.

$$B = B_1 + B_2 \text{ but } B_2 = B_1$$

$$\Rightarrow \quad B = 2B_1 = 2\left[\frac{\mu_0 I r^2}{2(r^2 + r^2)^{3/2}}\right]$$

$$= \frac{\mu_0 I r^2}{(2r^2)^{3/2}} = \frac{\mu_0 I r^2}{2^{3/2} r^3}$$

$$= \frac{\mu_0 I}{2^{3/2} r}$$

23. Refer to text on pages 170 and 171.

For magnetic field lines Refer to Sol. 12.

24. Refer to text on pages 168, 169, 170 and 171.

25. Refer to text on pages 168, 169, 170 and 171.

In this answer, put $r = d$.

Magnetic field induction at the centre of the circular coil carrying current is

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{a}, \quad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi a^2 I}{(a^2 + d^2)^{3/2}}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{a^2 \times a}{(a^2 + d^2)^{3/2}} = \frac{a^3}{(a^2 + d^2)^{3/2}}$$

$$= \frac{a^3}{(a^2 + 3a^2)^{3/2}} \quad [\because d = a\sqrt{3}]$$

$$= \frac{a^3}{(4a^2)^{3/2}} = \frac{a^3}{8a^3}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{1}{8}$$

26. The magnetic field at a point due to a circular loop is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + r^2)^{3/2}}$$

where, I = current through the loop,

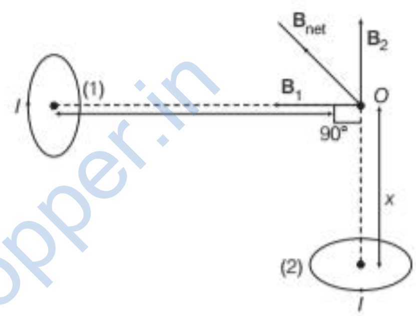
a = radius of the loop

and r = distance of O from the centre of the loop.

Since I, a and $r = x$ are the same for both the loops, the magnitude of B will be the same and is given by

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + x^2)^{3/2}}$$

The direction of magnetic field due to loop (1) will be away from O and that of the magnetic field due to loop (2) will be towards O as shown. The direction of the net magnetic field will be as shown below



The magnitude of the net magnetic field is given by

$$B_{net} = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2\pi\sqrt{2} I a^2}{(a^2 + x^2)^{3/2}}$$

27. Here, $I = 5$ A, $dl = 1$ cm $= 0.01$ m, $r = 1$ m, $\theta = 45°$

[$\because$ direction is North-East]

$$\therefore \quad dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2}$$

$$= 10^{-7} \times \frac{5 \times 0.01 \times \sin 45°}{(1)^2} = 3.54 \times 10^{-9} \text{ T}$$

Its direction is vertically downwards.

28. Here, $dl = dx = 1$ cm $= 10^{-2}$ m

$I = 10$ A, $r = 0.5$ m

Using Biot-Savart's law,

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\mathbf{l} \times \mathbf{r}}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{Idx}{r^2}(\hat{\mathbf{i}} \times \hat{\mathbf{j}})$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{Idx}{r^2}\hat{\mathbf{k}} \quad [\because \hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}]$$

$$= \frac{10^{-7} \times 10 \times 10^{-2}}{(0.5)^2}\hat{\mathbf{k}}$$

$$= 4 \times 10^{-8}\hat{\mathbf{k}} \text{ T}$$

29. Refer to Example 1 on page 179. [**Ans.** 4×10^{-6} T]

30. Given, $I = 10$ A, length of each wire = 44 cm

(i) Let r be the radius of wire A when it is bent into a circle.

$$\Rightarrow \quad 2\pi r = 44 \Rightarrow r = \frac{7}{100} \text{ m}$$

Magnetic field at the centre of the circular coil carrying current is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} = 10^{-7} \times 2 \times \frac{22}{7} \times 10 \times \frac{100}{7} = 9 \times 10^{-5} \text{ T}$$

When another wire is bent into a square of each side L, then

$$4L = 44 \Rightarrow L = 11 \text{cm} = 0.11 \text{m}$$

Since, magnetic field induction at a point, at perpendicular distance a from the linear conductor carrying current is given by

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$

$$B = 4 \times \frac{\mu_0 I}{4\pi a}(\sin 45° + \sin 45°)$$

$$= 4 \times 10^{-7} \times \frac{10}{(11/100)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= 10.3 \times 10^{-5} \text{ T}$$

(ii) The magnetic field due to a square will be more than that due to a circle of same perimeter.

31. Refer to Example 4 on page 172. [**Ans.** 6.28×10^{-4} T]

32. Here, $I = 10$ A, $r = 3$ cm, $r = 3 \times 10^{-2}$ m

Angle subtended by coil at the centre,

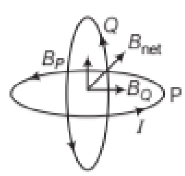
$$\theta = 360° - 90° = 270° = \frac{3\pi}{2} \text{ rad}$$

Magnetic field induction at O due to current through circular path ACB is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}\theta = 10^{-7} \times \frac{10}{(3 \times 10^{-2})} \times \frac{3\pi}{2}$$

$$= 1.57 \times 10^{-4} \text{ T}$$

33. Magnetic field due to circular wire P,



$$B_P = \frac{\mu_0}{4\pi} \times \frac{2\pi I_1}{R}$$

[along vertically upwards]

$$= \frac{\mu_0 I_1}{2R}$$

Magnetic field due to circular wire Q,

$$B_Q = \frac{\mu_0}{4\pi} \times \frac{2\pi I_2}{R} \quad \text{[along horizontal towards left]}$$

$$= \frac{\mu_0 I_2}{2R}$$

Net magnetic field at the common centre of the two coils,

$$B = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{\left(\frac{\mu_0 I_1}{2R}\right)^2 + \left(\frac{\mu_0 I_2}{2R}\right)^2}$$

$$= \sqrt{\left(\frac{\mu_0}{2R}\right)^2 (I_1^2 + I_2^2)}$$

$$= \frac{\mu_0}{2R}\sqrt{I_1^2 + I_2^2}$$

$$= \frac{4\pi \times 10^{-7}}{2 \times R}\sqrt{(1)^2 + (\sqrt{3})^2}$$

$$= \frac{4\pi \times 10^{-7}}{R} \text{ T}$$

Resultant magnetic field makes an angle θ with direction of B_Q, which is given by

$$\tan\theta = \frac{B_P}{B_Q} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \theta = 30°$$

34. (i) Magnetic field due to straight segment is

$$\mathbf{B} = \int \frac{\mu_0}{4\pi} \cdot \frac{I\, d\mathbf{l} \times \mathbf{r}}{r^3}$$

For point O, $d\mathbf{l}$ and $\mathbf{r}$ for each element of straight segments PQ and RS are parallel.

Therefore, $d\mathbf{l} \times \mathbf{r} = 0$.

Thus, magnetic field due to straight segment is zero.

(ii) Magnetic field at centre O due to semi-circular arc

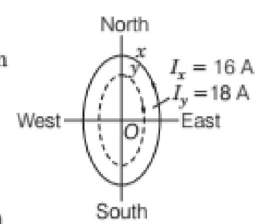
$$= \frac{\text{Magnetic field at centre of circular coil}}{2}$$

$$= \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right) = \frac{\mu_0 I}{4r} = \frac{(4\pi \times 10^{-7}) \times 10}{4 \times 2 \times 10^{-2}}$$

[given, $I = 10$ A and $r = 2.0$ cm $= 2 \times 10^{-2}$ m]

$$= 5\pi \times 10^{-5} \text{ T}$$

35. **For coil x**



Radius of coil, $r_x = 16$ cm $= 0.16$m

Number of turns, $n_x = 20$

Current in the coil, $I_x = 16$ A (anti-clockwise)

For coil y

Radius of coil, $r_y = 10$ cm $= 0.1$ m

Number of turns, $n_y = 25$

Current in the coil, $I_y = 18$ A (clockwise)

The magnitude of the magnetic field at the centre of coil x,

$$B_x = \frac{\mu_0}{4\pi} \cdot \frac{2 I_x \pi n_x}{r_x}$$

$$= \frac{10^{-7} \times 2 \times 16 \times \pi \times 20}{0.16} = 4\pi \times 10^{-4} \text{ T}$$

The direction of magnetic field due to the coil x at centre O is towards right, i.e. East, according to right hand thumb rule. The magnitude of the magnetic field at the centre of coil y,

$$B_y = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I_y n_y}{r_y} = \frac{10^{-7}\times 2\times\pi\times 18\times 25}{0.1}$$

$$= 9\pi\times 10^{-4}\,\text{T}$$

The direction of magnetic field due to coil y at centre O is towards left, i.e. West, according to right hand thumb rule. Here, the magnitude of B_y is greater than B_x, so the resultant magnetic field will be in the direction of B_y, i.e. left (West).

Net magnetic field at the centre, $B = B_y - B_x$

$$= (9\pi - 4\pi)10^{-4}\,\text{T}$$

$$= 5\pi\times 10^{-4}\,\text{T}$$

[$\because B_y$ and B_x are opposite to each other]

$$= 1.6\times 10^{-3}\,\text{T} \quad \text{[towards West]}$$

36. $$B_{\text{net}} = \sqrt{B_P^2 + B_Q^2} = \sqrt{\left(\frac{\mu_0 i_P}{2r}\right)^2 + \left(\frac{\mu_0 I_Q}{2r}\right)^2}$$

$$= \frac{\mu_0}{2r}\sqrt{i_P^2 + i_Q^2} = \frac{4\pi\times 10^{-7}}{2\times 5\times 10^{-2}}\times 5$$

$$= 2\pi\times 10^{-5}\,\text{T}$$

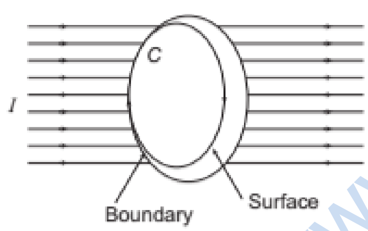
Resultant magnetic field makes an angle θ with B_Q which is given by,

$$\tan\phi = \frac{B_P}{B_Q} = \frac{i_P}{i_Q} = \frac{3}{4}$$

|TOPIC 2|
Ampere's Circuital Law and Moving Charges

AMPERE'S CIRCUITAL LAW

According to this law, the line integral of a magnetic field **B** around any closed path in vacuum is μ_0 times the net current I_{net} enclosed by the curve.



Mathematically,

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{net}}$$

Ampere's law is applicable only for an Amperian loop as the Gauss's law is used for Gaussian surface in electrostatics.

The choice of an Amperian loop has to be such that, at each point of the loop either

(i) **B** is tangential to the loop and is a non-zero constant

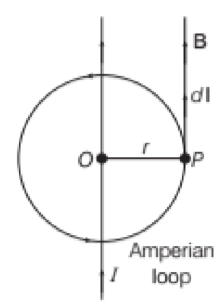
(ii) **B** is normal to the loop

(iii) **B** vanishes

Ampere's circuital law has same content as the Biot-Savart's law. Both of these relate magnetic field and current and express the same physical consequences of a steady electrical current. Ampere's circuital law holds for any loop but does not always facilitate. Ampere's circuital law can be conveniently applied in situations of high symmetry. e.g. To find magnetic field of a straight wire, magnetic field of solenoid and toroid as discussed in coming sections.

Magnitude of Magnetic Field of a Straight Wire using Ampere's Law

Magnetic field due to a straight conductor at a point P at a distance (r) is in the form of a circle of radius (r) which is taken as closed path for Amperian loop.



Angle between **B** and $d\mathbf{l}$ is zero, everywhere in this path. Hence, on applying Ampere's law to this closed path, we get

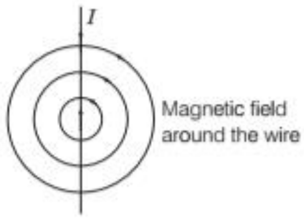
$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I \quad \text{or} \quad \oint B\,dl\cos 0^\circ = \mu_0 I$$

$\Rightarrow \quad B \oint dl = \mu_0 I$

$\Rightarrow \quad B \times 2\pi r = \mu_0 I$ or $\boxed{B = \dfrac{\mu_0 I}{2\pi r}}$

From the result, some important points can be derived.

(i) The magnetic field at every point on a circle of radius r is same in magnitude. The magnetic field around a wire possesses cylindrical symmetry.



(ii) The field direction at any point on this circle is tangential to it. The lines of constant magnitude of magnetic field forms concentric circles. The circular lines are called **magnetic field lines**.

(iii) Even though the wire is of infinite length, the field due to it at a non-zero distance is not infinite.

EXAMPLE |1| A straight wire carries a current of 3 A. Calculate the magnitude of the magnetic field at a point 15 cm away from the wire.

Sol. Here, current, $I = 3\,A$, point where magnetic field is to be determined, $a = 15$ cm $= 0.15\,m$

$\therefore$ Magnitude of magnetic field, $B = \dfrac{\mu_0 2I}{4\pi a}$

$$= \frac{10^{-7} \times 2 \times 3}{0.15}$$

$$= 4 \times 10^{-6}\ \text{T}$$

THE SOLENOID AND THE TOROID

A **solenoid** is an insulated long wire closely wound in the form of a helix. Its length is very long as compared to its diameter. The toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound. The solenoid and the toroid are two equipments which are used to produce magnetic fields.

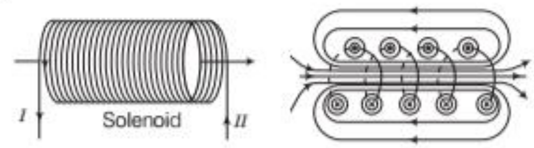
In television, we make use of solenoid to generate magnetic field needed for the deflection of electrons in picture tube.

Toroid is used in devices such as synchroton in which high magnetic field is required, which is generated by either a toroid or combination of both solenoid and toroid.

Both solenoid and toroid have symmetrically geometric shapes, therefore we can apply Ampere's law conveniently to find the magnetic field.

Magnetic Field of a Solenoid

A long coil of wire consisting of closely packed loops is called solenoid, whose magnetic field resembles that of a bar magnet of south(S) and north(N) poles as shown in the figure given below.



Magnetic field of a solenoid

Inside the solenoid, magnetic field is uniform and parallel to the solenoid axis. Outside the solenoid, magnetic field is assumed to be zero.

Consider an air cored solenoid having closely packed coils in which I is current, n is number of turns per unit length and B is magnetic field inside the solenoid.

Applying Ampere's circuital law to determine magnetic field (**B**) inside the solenoid, we choose rectangular closed path $PQRS$, where $PQ = L$ and the line integral of **B** over closed path $PQRS$ is

$$\oint_{PQRS} \mathbf{B} \cdot d\mathbf{l} = \int_P^Q \mathbf{B} \cdot d\mathbf{l} + \int_Q^R \mathbf{B} \cdot d\mathbf{l} + \int_R^S \mathbf{B} \cdot d\mathbf{l} + \int_S^P \mathbf{B} \cdot d\mathbf{l}$$

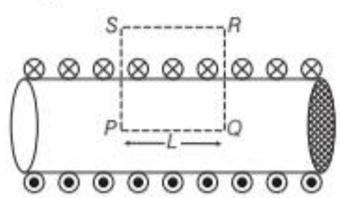
Now, $\int_Q^R \mathbf{B} \cdot d\mathbf{l} = \int_S^P \mathbf{B} \cdot d\mathbf{l} = 0$

[because along QR and PS, the field B is at right angles to dl, so that $\mathbf{B} \cdot d\mathbf{l} = Bdl\ \cos 90° = 0$]

$\int_R^S \mathbf{B} \cdot d\mathbf{l} = 0$ [$\because$ **B** is zero at points outside the solenoid]

$$\therefore \oint_{PQRS} \mathbf{B} \cdot d\mathbf{l} = \int_P^Q \mathbf{B} \cdot d\mathbf{l} = \int_P^Q B\,dl \cos 0°$$

$$= \int_P^Q B\,dl = BL$$



Hence, from Ampere's law,

$$\oint_{PQRS} \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times (\text{current enclosed by } PQRS)$$

Here, number of turns per unit length is n, the number of turns in length L is nL. The current in each turn is I, so net current enclosed by the loop is nLI.

Total current, $I_{\text{net}} = nLI$

$$\therefore \quad \oint_{PQRS} \mathbf{B} \cdot d\,\mathbf{l} = \mu_0 \times nIL$$

$$\Rightarrow \quad BL = \mu_0 nIL$$

$$\Rightarrow \quad \boxed{B = \mu_0 nI}$$

From the expression, it is clear that B is independent of the length and diameter of the solenoid and is uniform over the cross-section of the solenoid.

If a material of permeability μ_r is used as a core, then B inside the solenoid is $\mu_0 \mu_r n I$.

At points near the end of air closed solenoid,

$$\boxed{B = \frac{1}{2}\mu_0 nI}$$

Note The formula for magnetic field $B = \mu_0 nI$ is only valid when length of the solenoid (l) is much larger than its radius(r), i.e. $l >> r$.

EXAMPLE |2| The length of a solenoid is 0.2 m and it has 120 turns. Find the magnetic field in its interior, if a current of 2.5 A is flowing through it.

Sol. Here, $l = 0.2$ m, $N = 120$, $I = 2.5$A

Magnetic field in the interior of the solenoid,

$$B = \mu_0 nI = \mu_0 \frac{N}{l} I$$

$$= 4\pi \times 10^{-7} \times \frac{120}{0.2} \times 2.5$$

$$= 1.85 \times 10^{-3} \text{ T}$$

EXAMPLE |3| A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of magnetic field inside the solenoid? **NCERT**

Sol. Given, total number of turns, $N = 500$

Length of solenoid, $l = 0.5$ m

Current, $I = 5$ A

and radius $r = 1 \text{ cm} = 10^{-2}$ m

Here, $\frac{l}{r} = \frac{0.5}{10^{-2}} = 50$

$\Rightarrow \quad l >> r$

$$\therefore \quad B = \mu_0 nI = \frac{\mu_0 NI}{l}$$

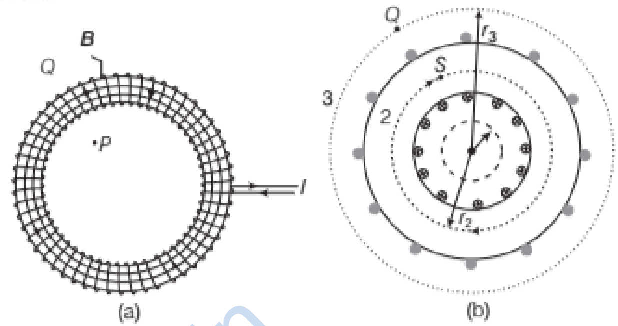
$$= 4\pi \times 10^{-7} \times \frac{500}{0.5} \times 5$$

$$= 6.28 \times 10^{-3} \text{T}$$

Magnetic Field of a Toroid

An endless solenoid in the form of a ring is called a toroid. Magnetic field lines inside the toroid are circular, concentric with the centre of toroid.

Let I be the current, r be the mean radius, n be the number of turns per unit length and B be the magnetic field inside the toroid.



(a) A toroid carrying a current I (b) A sectional view of the toroid. The magnetic field can be obtained at an arbitrary distance r from the centre O of the toroid by Ampere's circuital law. The dashed lines labelled 1, 2 and 3 are three circular Amperian loops

The line integral of magnetic field around closed path of circle of radius r is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \oint B\, dl \cos 0° = B \times 2\pi r$$

Now, from Ampere's law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times \text{current enclosed by closed path}$$

$$\Rightarrow \quad B \times 2\pi r = \mu_0\, n\,(2\pi r)\, I \quad \text{or} \quad \boxed{B = \mu_0\, nI}$$

If the toroid is a material cored of relative permeability μ_r, then magnetic field inside the toroid,

$$\boxed{B = \mu_0 \mu_r n\, I}$$

FORCE ON A MOVING CHARGE IN A UNIFORM MAGNETIC FIELD

When a charged particle (q) moves with velocity ($\mathbf{v}$) inside a uniform magnetic field $\mathbf{B}$, then force acting on it is

$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

Force due to magnetic field depends on $q, \mathbf{v}, \mathbf{B}$. The magnetic force will be zero, if the particle is at rest. The reason is that for the charged particle at rest $|\mathbf{v}| = 0$, which will turn the expression for magnetic force, $q\,(\mathbf{v} \times \mathbf{B})$, into zero. Only moving charges feel the magnetic force. The magnetic force is at its maximum value, when $\mathbf{v}$ and $\mathbf{B}$ are perpendicular to each other because in this case the angle $\theta = 90°$, in the expression $F = qvB \sin\theta$ and the maximum force will be

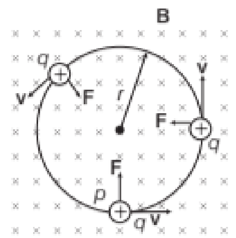
$$F_{\max} = q\,vB \sin 90° = qvB \quad [\because \sin 90° = 1]$$

The magnetic force includes the cross product of velocity (**v**) of the particle and magnetic field (**B**). Thus, the magnetic force will be zero, if the velocity vector and magnetic field vector are either parallel or anti-parallel to each other.

The force ($F_{magnetic}$) acting on a charged particle moving with velocity (**v**) through a magnetic field (**B**) is always perpendicular to **v** and **B**.

From right hand thumb rule, the force F is perpendicular to velocity (**v**) and magnetic field (**B**). Hence, it changes its path continuously.

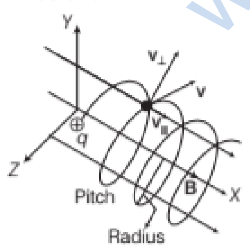
In case of motion of a charge in a magnetic field, the magnetic force is perpendicular to the velocity of the particle. So, no work is done and no change in the magnitude of the velocity is produced.



Circular motion

Magnetic force acts as a centripetal force and produces a circular motion perpendicular to the magnetic field. If **v** and **B** are perpendicular to each other, then particle will describe a circle.

If a charged particle has a velocity not perpendicular to **B**, then component of velocity along **B** remains unchanged as the motion along the magnetic field will not be affected by the magnetic field. Then, the motion of the particle in a plane perpendicular to **B** is as before a circular one, thereby producing a helical motion.



Helical motion

As, centripetal force required for circular motion is provided by magnetic force, so

$$\frac{mv_\perp^2}{r} = qBv_\perp \text{ or } \boxed{r = \frac{mv_\perp}{qB} = \frac{p}{qB}} \quad \text{...(i)}$$

where, m = mass of charged particle

and r = radius of circular path.

where, $\mathbf{v}_\perp$ and $\mathbf{v}_\parallel$ are perpendicular and parallel components of the velocity **v**.

Radius of the path of the charged particle is proportional to the momentum ($p = mv$) of the particle and inversely proportional to the magnitude of charge (q) and magnetic field (B).

In terms of kinetic energy (K), the equation may be expressed as, $r = \frac{\sqrt{2mK}}{qB}$ $\quad [\because p = \sqrt{2mK}]$

Time period (T) of the motion is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \times \frac{mv}{qB}$$

or $$\boxed{T = \frac{2\pi m}{qB}} \quad \text{...(ii)}$$

Angular frequency, $\omega = \frac{2\pi}{T} = \frac{Bq}{m}$

Frequency, $\nu = \frac{Bq}{2\pi m}$ $\quad [\because \omega = 2\pi\nu]$

The angular speed ω of the particle is given by

$$\omega = v/r \text{ or } \omega = (qB)/m$$

For helical path, the distance moved along the magnetic field in one rotation is called **pitch** (P).

$$\boxed{P = v_\parallel T = \frac{2\pi m v_\parallel}{qB}}$$

Note One tesla (1 T) is defined as the field which produces a force of one newton (1 N) when a charge of one coulomb (1 C) moves perpendicularly in the region of the magnetic field at a velocity of 1 ms^{-1}.

Aurora Borealis

During a solar flare, a large number of electrons and protons are ejected from the sun. Some of them get trapped in the earth's magnetic field and move in helical paths along the field lines, which come closer near magnetic poles and collide with atoms and molecules of the atmosphere. Excited oxygen atoms emit green light and excited nitrogen atoms emit pink light and this phenomenon is called Aurora Borealis in Physics.

Note This topic is very important as it has been asked frequently in the previous years 2017, 2016, 2015, 2014, 2012, 2011, 2010.

EXAMPLE |4| A proton and an α-particle, accelerated through same potential difference, enter in a region of uniform magnetic field with their velocities perpendicular to the field. Compare the radii of circular paths followed by them.

Sol. Let mass of proton = m, charge of proton = e

Now, mass of α-particle = 4 m, charge of α-particle = 2e

When a charge q is accelerated by V volts, it acquires a kinetic energy $E_K = qV$

$\therefore$ Momentum is given by $mv = \sqrt{2mE_K} = \sqrt{2mqV}$

$$\text{Radius, } r = \frac{mv}{qB} \text{ or } r = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{2mV}{qB^2}}$$

$$\text{Thus, } \frac{r_p}{r_\alpha} = \sqrt{\frac{2mV}{eB^2}} \times \sqrt{\frac{2eB^2}{2(4m)V}} = \frac{1}{\sqrt{2}}$$

EXAMPLE |5| A beam of protons with a velocity of 4×10^5 ms^{-1} enters in a region of uniform magnetic field of 0.3 T. The velocity makes an angle of 60° with the magnetic field. Find the radius of the helical path taken by the proton beam and the pitch of the helix.

Sol. Velocity component along the field

$$v_{\parallel} = 4 \times 10^5 \times \cos 60° = 2 \times 10^5 \text{ ms}^{-1}$$

and velocity component perpendicular to the field.

$$v_{\perp} = (4 \times 10^5) \sin 60° = 2\sqrt{3} \times 10^5 \text{ ms}^{-1}$$

Proton will describe a circle in plane perpendicular to magnetic field with radius,

$$r = \frac{mv_{\perp}}{qB} = \frac{(1.67 \times 10^{-27} \text{ kg}) \times (2\sqrt{3} \times 10^5 \text{ ms}^{-1})}{(1.6 \times 10^{-19} \text{C}) \times (0.3 \text{ T})} = 1.2 \text{ cm}$$

Time taken to complete one revolution is

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2 \times 3.14 \times 0.012}{2\sqrt{3} \times 10^5} \text{ s}$$

Because of $v_{\parallel}$ protons will also move in the direction of magnetic field.

$\therefore$ Pitch of helix = $v_{\parallel} \times T$

$$= \frac{2 \times 10^5 \times 2 \times 3.14 \times 0.012}{2\sqrt{3} \times 10^5} \text{ m} = 0.044 \text{ m} = 4.4 \text{ cm}$$

FORCE ON A MOVING CHARGE IN A UNIFORM MAGNETIC AND ELECTRIC FIELD (LORENTZ FORCE)

Suppose a point charge (q) is moving in the presence of both electric and magnetic fields. Let q be the magnitude of the charge, **v** be velocity of the point charge, **B** be the magnetic field and **E** be the electric field. We have studied two kinds of forces that can be exerted on an electrically charged particle. The electric force is given by $\mathbf{F} = q\mathbf{E}$ and the magnetic force is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.

The sum of these forces represents the net force that can be exerted on a particle due to its electric charge (q), this sum is called the Lorentz force and is given by

$$\mathbf{F}_{\text{Lorentz}} = \mathbf{F}_{\text{electric}} + \mathbf{F}_{\text{magnetic}}$$
$$= q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$
$$= q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

Force on negative charge is opposite to that of positive charge.

MOTION OF CHARGED PARTICLE IN COMBINED ELECTRIC AND MAGNETIC FIELD

Velocity Selector

Net force in presence of magnetic and electric field is

$$\mathbf{F} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

Consider that electric and magnetic fields are perpendicular to each other and also perpendicular to the velocity of particle. Suppose we have

$$\mathbf{E} = E\,\hat{\mathbf{j}},\ \mathbf{B} = B\,\hat{\mathbf{k}} \text{ and } \mathbf{v} = v\,\hat{\mathbf{i}}$$

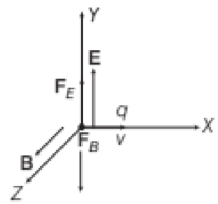
Then, $\mathbf{F}_E = q\mathbf{E} = qE\,\hat{\mathbf{j}}$

and $\mathbf{F}_B = q(\mathbf{v} \times \mathbf{B}) = q(v\hat{\mathbf{i}} \times B\hat{\mathbf{k}}) = -qB\,\hat{\mathbf{j}}$

$\therefore$ $\mathbf{F} = \mathbf{F}_E + \mathbf{F}_B = qE\,\hat{\mathbf{j}} + (-qvB\hat{\mathbf{j}})$

$= q(E - vB)\,\hat{\mathbf{j}}$

Thus, electric and magnetic forces are in opposite directions as shown in the figure given below.



Point charge q is moving in the presence of perpendicular electric and magnetic fields

If we adjust the value of **E** and **B** such that magnitude of the two forces are equal, then total force on the charge is zero and the charge will move in the fields undeflected. This happens, when

$$qE = qvB$$

$$\Rightarrow \quad \boxed{v = \frac{E}{B}}$$

The above condition can be used to select a charged particle of a particular velocity from charges moving with different speeds. Therefore, it is called velocity selector.

EXAMPLE |6| A proton beam passes without deviation through a region of space, where there are uniform transverse mutually perpendicular electric and magnetic fields with $E = 220$ kV/m and $B = 50$ mT. Then, the beam strikes a grounded target. Find the force imparted by the beam on the target, if the beam current is equal to $I = 0.80$ mA.

Sol. Since, proton is moving in a straight line, hence net force is zero.

$$\therefore \quad qE = Bqv \Rightarrow v = \frac{E}{B}$$

Also, current associated with the beam,

$$I = ne$$

$$\Rightarrow \quad n = I/e$$

where, n is number of protons/time.

Momentum of a proton $= mv$

$\therefore$ Force, $F = n\,m\,v$

$$= \frac{I\,m\,E}{eB} = \frac{0.80\times 10^{-3} \times 1.67\times 10^{-27} \times 220\times 10^{3}}{1.6\times 10^{-19} \times 50\times 10^{-3}}$$

$$= 3.6\times 10^{-5}\ \text{N}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. For a toroid, magnetic field strength in the region enclosed by wire turns is given by
   (a) $B = \mu_0 n I$, where n = number of turns
   (b) $B = \mu_0 I/n$, where n = number of turns per metre
   (c) $B = \frac{\mu_0 I}{2r}$, where r = mean radius
   (d) $B = \frac{\mu_0 NI}{2\pi r}$, $\left\{\begin{array}{l}\text{where, } N = \text{ number of turns} \\ \text{and } r = \text{ radius of toroid.}\end{array}\right.$

2. The value of force F acting on charge q moving with velocity perpendicular to the magnetic field B will be
   (a) $F = qvB$ (b) $F = \frac{qv}{B}$
   (c) $F = \frac{qB}{v}$ (d) $F = \frac{Bv}{q}$

3. An electron of charge (e) is moving parallel to uniform magnetic field B with constant velocity v. The force acting on electron is
   (a) Bev (b) Be/v
   (c) B/ev (d) zero

4. In a uniform magnetic field, an electron (or charge particle) enters perpendicular to the field. The path of electron will be
   (a) ellipse (b) circular
   (c) parabolic (d) linear

5. If the velocity of charged particle is doubled and value of magnetic field is reduced to half, then the radius of path of charged particle will be
   (a) 8 times (b) 4 times (c) 3 times (d) 2 times

6. An electron is projected with uniform velocity along the axis of a current carrying long solenoid. Which of the following is true? **NCERT Exemplar**
   (a) The electron will be accelerated along the axis
   (b) The electron path will be circular about the axis
   (c) The electron will experience a force at 45° to the axis and hence execute a helical path
   (d) The electron will continue to move with uniform velocity along the axis of the solenoid

VERY SHORT ANSWER Type Questions

7. Magnetic field lines can be entirely confined within the core of toroid, but not within a straight solenoid. Why?

8. An electron does not suffer any deflection while passing through a region of uniform magnetic field, what is the direction of the magnetic field? **All India 2009**

9. A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal to the initial speed, if it suffered no collisions with the environment? **NCERT**

10. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. What would be the ratio of the radii of the circular path described by them? **Foreign 2011**

11. A loop of irregular shape carrying current is located in an external magnetic field.
    If the wire is flexible, why does it change to a circular shape?

12. A solenoid tends to contract when a current passes through it. Justify the given statement.

13. A proton and an electron travelling along parallel paths enter a region of uniform magnetic field, acting perpendicular to their paths. Which of them will move in a circular path with higher frequency? **CBSE 2018**

SHORT ANSWER Type Questions

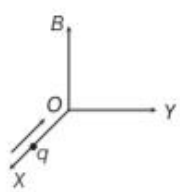
14. A long solenoid of length L having N turns carries a current I. Deduce the expression for the magnetic field in the interior of the solenoid. **All India 2011C**

15. Obtain with the help of a necessary diagram, the expression for the magnetic field in the interior of a toroid carrying current. **All India 2011C**

16. Define one tesla using the expression for the magnetic force acting on a particle of charge q moving with velocity v in a magnetic field **B**. **Foreign 2014**

17. A particle of charge q and mass m is moving with velocity v. It is subjected to a uniform magnetic field B directed perpendicular to its velocity. Show that it describes a circular path. Write the expression for its radius. **Foreign 2012**

18. A charge q moving along the X-axis with a velocity **v** is subjected to a uniform magnetic field **B** acting along the Z-axis as it crosses the origin O.



(i) Trace trajectory.

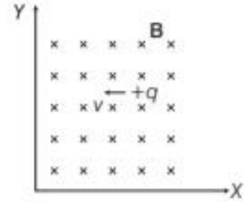
(ii) Does the charged particle gain kinetic energy as it enters the magnetic field? Justify your answer. **Delhi 2009**

19. Write the expression in the vector form for the Lorentz magnetic force **F** due to a charge moving with velocity **v** in a magnetic field **B**. What is the direction of the magnetic force? **Delhi 2014**

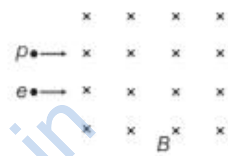
20. Write the expression for Lorentz magnetic force on a particle of charge q moving with velocity v in a magnetic field **B**. Show that no work is done by this force on the charged particle. **All India 2011**

21. Find the condition under which the charged particles moving with different speeds in the presence of electric and magnetic field vectors can be used to select charged particles of a particular speed. **All India 2017**

22. A point charge is moving with a constant velocity perpendicular to a uniform magnetic field as shown in the figure. What should be magnitude and direction of the electric field so that the particle moves undeviated along the same path? **Foreign 2009**



23. An electron and a proton moving with the same speed enter the same magnetic field region at right angles to the direction of the field. Show the trajectory followed by the two particles in the magnetic field. Find the ratio of the radii of the circular paths which the particles may describe. **Foreign 2010**



24. An iron ring of relative permeability μ_r has windings of insulated copper wire of n turns per metre. When the current in the windings is I, find the expression for the magnetic field in the ring. **CBSE 2018**

LONG ANSWER Type I Questions

25. Explain, how Biot-Savart's law enables one to express the Ampere's circuital law in the integral form, *viz.*

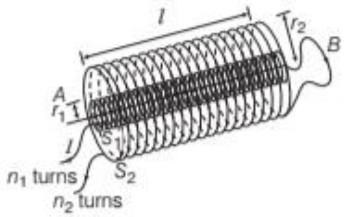
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

where, I is the total current passing through the surface. **Delhi 2015**

26. A long straight solid metal wire of radius R carries a current I uniformly distributed over its circular cross-section. Find the magnetic field at a distance r from the axis of wire (i) inside (ii) outside the wire.

27. (i) State Ampere's circuital law expressing it in the integral form.

(ii) Two long coaxial insulated solenoids, S_1 and S_2 of equal lengths are wound one over the other as shown in the figure. A steady current I flows through the inner solenoid S_1 to the other end B, which is connected to the outer solenoid S_2 through which the same current I flows in the opposite direction, so as to come out at end A. If n_1 and n_2 are the number of turns per unit length, find the magnitude and direction of the net magnetic field at a point (a) inside on the axis and (b) outside the combined system. **Delhi 2014**



28. (i) Write the expression for the force **F** acting on a particle of mass m and charge q moving with velocity v in a magnetic field **B**. Under what conditions will it move in
(a) a circular path and
(b) a helical path?

(ii) Show that the kinetic energy of the particle moving in magnetic field remains constant.
Delhi 2017

29. (i) Write the expression for the magnetic force acting on a charged particle moving with velocity v in the presence of magnetic field **B**.

(ii) A neutron, an electron and an alpha particle moving with equal velocities, enter a uniform magnetic field going into the plane of the paper as shown in the figure. Trace their paths in the field and justify your answer.

Delhi 2016

30. A uniform magnetic field **B** is set up along the positive X-axis. A particle of charge q and mass m moving with a velocity v enters the field at the origin in XY-plane such that it has velocity components both along and perpendicular to the magnetic field **B**. Trace, giving reason, the trajectory followed by the particle. Find out the expression for the distance moved by the particle along the magnetic field in one rotation. **Delhi 2015**

LONG ANSWER Type II Questions

31. (i) Using Ampere's circuital law, derive the expression for the magnetic field in the vector form at a point on the axis of a solenoid.

(ii) What does a toroid consist of ? Find out the expression for the magnetic field inside a toroid for N turns of the coil having the average radius r and carrying a current I. Show that the magnetic field in the open space interior and exterior to the toroid is zero.

32. Answer the following questions.

(i) A magnetic field that varies in magnitude from point to point but has a constant direction (East to West) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

(ii) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal to the initial speed, if it suffered no collisions with the environment ?

(iii) An electron travelling West to East enters a chamber having a uniform electrostatic field in North to South direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting its straight line path. **NCERT**

NUMERICAL PROBLEMS

33. A solenoid of length 50 cm having 100 turns carries a current of 2.5 A. Find the magnetic field (i) in the interior of the solenoid, (ii) at one end of the solenoid. **Delhi 2010**

34. A solenoid of length 1.0 m and 3.0 cm diameter has 5 layers of windings of 850 turns each and carries a current of 5 A. What is the magnetic field at the centre of solenoid? Also, calculate the magnetic flux from a cross-section of the magnetic flux solenoid at the centre of solenoid. **All India 2011**

35. A magnetic field of 100 G $(1\text{ G} = 10^{-4}\text{T})$ is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.
NCERT

36. An electron of energy 2000 eV describes a circular path in magnetic field of flux density 0.2 T. What is the radius of path? Take, $e = 1.6 \times 10^{-19}$ C, $m = 9 \times 10^{-31}$ kg.

HINTS AND SOLUTIONS

1. (d) For toroid, applying Ampere's circuital law,

$$B(2\pi r) = \mu_0 NI \Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

where, B = magnetic field of a toroid,

N = number of turns of toroidal coil

and r = radius of toroid.

2. (a) The force on charge q, $F = qvB$.

3. (d) The force on electron, $F = qvB\sin\theta$.

The electron is moving parallel to the magnetic field,

so $\theta = 0°$

$\therefore$ $F = qvB\sin 0° = 0$

4. (b) When the charged particle enters in the magnetic field perpendicular to it, then the force due to magnetic field,

$$F = qvB\sin 90° = qvB$$

The direction of this force is always perpendicular to movement of charged particle. The charged particle is moving under the influence of constant force but its direction is continuously changing. So, the particle will move in a circular path with constant velocity v.

5. (b) In first case, the radius of path, $r = \frac{mv}{qB}$

In second case, the radius of path, $r' = \frac{m'v'}{q'B'} = \frac{m \times 2v}{q \times B/2}$

10. For given momentum of charged particle, radius of circular path depends on charge and magnetic field as

$$r = \frac{mv}{qB} \Rightarrow r \propto \frac{1}{qB}$$

For given momentum, $r_{proton} : r_{deuteron} = 1:1$

As, they have same momentum, charges are moving in small magnetic field.

11. Forces on the loop due to magnetic field act is in all directions. Thus, the loop attains a circular shape.

12. The turns of the solenoid are parallel to each other and carry current in the same direction. As we know that two parallel current carrying conductors in the same direction attract each other. Thus, the solenoid tends of contract.

13. As we know that in a circular path, frequency of a charged particle is given by

$$\nu = \frac{qB}{2\pi m}$$

or $\nu \propto \frac{1}{m}$

Since, $m_p > m_e$, therefore electron will move in circular path with higher frequency.

14. Refer to text on pages 180 and 181.

15. Refer to text on page 181.

16. Refer to text on page 183.

$\because$ $F = qvB$

$\Rightarrow$ $B = \frac{F}{qv}$

$\Rightarrow$ $1\text{ T} = \frac{1\text{N}}{(1\text{C})(1\text{ms}^{-1})}$

17. Refer to text on page 182.

36. An electron of energy 2000 eV describes a circular path in magnetic field of flux density 0.2 T. What is the radius of path? Take, $e = 1.6 \times 10^{-19}$ C, $m = 9 \times 10^{-31}$ kg.

HINTS AND SOLUTIONS

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$$F = qvB\sin 90° = qvB$$

The direction of this force is always perpendicular to movement of charged particle. The charged particle is moving under the influence of constant force but its direction is continuously changing. So, the particle will move in a circular path with constant velocity v.

5. (b) In first case, the radius of path, $r = \frac{mv}{qB}$

In second case, the radius of path, $r' = \frac{m'v'}{q'B'} = \frac{m \times 2v}{q \times B/2}$

$= 4r$

6. (d) Magnetic Lorentz force on electron projected with uniform velocity along the axis of a current carrying long solenoid $F = -evB\sin 180° = 0\ (\theta = 0°)$ as magnetic field and velocity are parallel. So, the electron will continue to move with uniform velocity along the axis of the solenoid.

7. The magnetic field lines always form closed loops. As, the turns of the wires in a toroidal solenoid are wound over its core in circular form, the field lines are confined within the core of toroid. In a straight solenoid, the magnetic field lines cannot form closed loops within the solenoid.

8. As $|\mathbf{F}| = qvB\sin\theta$

$\therefore$ If $\theta = 0°$ or $180°$, then $\text{F} = 0$

When the particle moves parallel or anti-parallel to the magnetic field, then it does not experience any deflection.

9. Yes, the final speed is equal to its initial speed as the magnetic force acting on the charged particle only changes the direction of velocity of charged particle but cannot change the magnitude of velocity of charged particle.

10. For given momentum of charged particle, radius of circular path depends on charge and magnetic field as

$$r = \frac{mv}{qB} \Rightarrow r \propto \frac{1}{qB}$$

For given momentum, $r_{proton} : r_{deuteron} = 1:1$

As, they have same momentum, charges are moving in small magnetic field.

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or $$\nu \propto \frac{1}{m}$$

Since, $m_p > m_e$, therefore electron will move in circular path with higher frequency.

14. Refer to text on pages 180 and 181.

15. Refer to text on page 181.

16. Refer to text on page 183.

$\because$ $F = qvB$

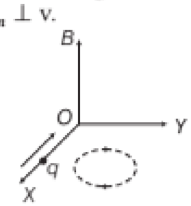
$\Rightarrow$ $B = \frac{F}{qv}$

$\Rightarrow$ $1\ \text{T} = \frac{1\text{N}}{(1\text{C})\ (1\text{ms}^{-1})}$

17. Refer to text on page 182.

18. (i) As, the charged particle is moving perpendicularly to the magnetic field. So, it will perform circular motion in XY-plane.

(ii) No, the charged particle does not gain any KE as Lorentz force acting on it does not perform any work as $\mathbf{F}_m \perp$ v.

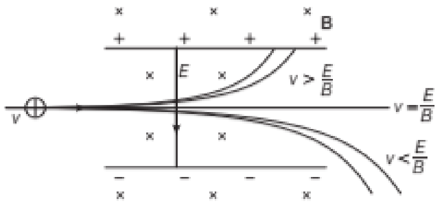


19. The expression in vector form is given by $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. The direction of the magnetic force is in the direction of $(\mathbf{v} \times \mathbf{B})$, i.e. perpendicular to the plane containing $\mathbf{v}$ and $\mathbf{B}$

20. Refer to text on page 183.

Lorentz magnetic force always acts perpendicular to the direction of motion of the particle. Thus, work done by this force is zero.

21. A diagram in which particle moves in magnetic and electric field is shown below



Forces on a charged particle are

F_e = electric force = qE,

F_m = magnetic force = Bqv

For a particle to go straight without any deflection

$$F_e = F_m \Rightarrow qE = Bqv \Rightarrow v = \frac{E}{B}$$

In this way, particles having speed, $v = \frac{E}{B}$ are separated.

22. As, $\mathbf{v} = -v\hat{\mathbf{i}}$

[$\because$ the particle is moving along X-direction]

$\mathbf{B} = -B\hat{\mathbf{k}}$

[$\because$ the magnetic field is perpendicular to the plane of the paper directed inwards, i.e. negative Z-direction]

$\therefore$ Force acting due to magnetic field,

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}) = q[-v\hat{\mathbf{i}} \times (-B\hat{\mathbf{k}})],$$

$$\mathbf{F}_m = -qvB\hat{\mathbf{j}} \qquad [\because \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}]$$

$\Rightarrow$ Magnitude of $\mathbf{F}_m = |\mathbf{F}_m| = qvB$

The direction of Fm is along negative Y-direction.For the undeflected motion of particle,

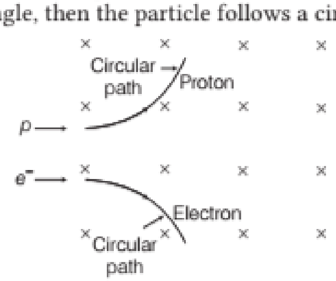
Force due to electric field = Force due to magnetic field,

$$q\mathbf{E} = q(\mathbf{v} \times \mathbf{B})$$

$$\therefore \qquad \mathbf{E} = \mathbf{v} \times \mathbf{B}$$

Magnitude of electric field, $|E| = |\mathbf{v} \times \mathbf{B}|$ and direction of magnetic field will be perpendicular to both $\mathbf{v}$ and $\mathbf{B}$, i.e. along Y-axis.

23. When a charged particle enters in the magnetic field at right angle, then the particle follows a circular path.



Radius of the circular path, $r = \frac{mv}{qB}$

For same speed v, magnitude of charge and magnetic field

$$r \propto m$$

$$\Rightarrow \qquad \frac{r_e}{r_p} = \frac{m_e}{m_p}$$

where, m_e and m_p are masses of electron and proton, respectively.

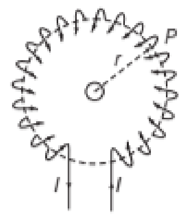
$\because \qquad m_e < m_p$

i.e. Proton is much heavier than electron.

$\Rightarrow \qquad r_e < r_p$

The curvature of path of electron is much more than curvature of path of proton.

24. An iron ring having insulated copper winding is also called a toroid. Magnetic field lines inside the toroid are circular, concentric with the centre of toroid.



Let I be the current, r be the mean radius, n be the number of turns per unit length and B be the magnetic field inside the toroid.

The line integral of magnetic field around closed path of circle of radius r is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \oint B\, dl \cos 0° = B \times 2\pi r$$

Now, from Ampere's law,

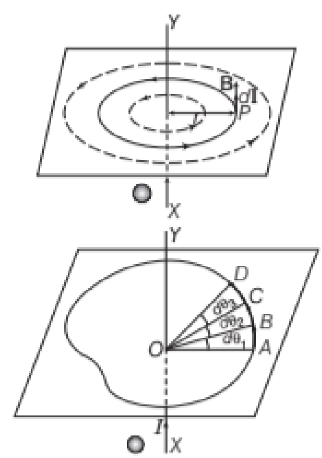
$$\oint B \cdot dl = \mu_0 \times \text{current enclosed by closed path}$$

$$B \times 2\pi r = \mu_0 n (2\pi r) I \quad \text{or} \quad B = \mu_0 nI$$

If the toroid has a material core of relative permeability μ_r, then magnetic field is given by

$$B = \mu_0 \mu_r n I$$

25.



Consider any arbitrary closed path perpendicular to the plane of paper around a long straight conductor XY carrying current from X to Y, lying in the plane of paper.

Let the closed path be made of large number of small elements, where

$$AB = dl_1,\ BC = dl_2,\ CD = dl_3$$

Let $d\theta_1, d\theta_2, d\theta_3$, be the angles subtended by the various elements at point O through which conductor is passing. Then

$$d\theta_1 + d\theta_2 + d\theta_3 + \ldots = 2\pi$$

Suppose these small elements $AB, BC, CD,$...... are small circular arcs of radii r_1, r_2, r_3 ...... respectively.

Then $d\theta_1 = \frac{d\mathbf{l}_1}{r_1}, d\theta_2 = \frac{d\mathbf{l}_2}{r_2}, d\theta_3 = \frac{d\mathbf{l}_3}{r_3}$

If $\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3$ are the magnetic field inductions at a point along the small elements $d\mathbf{l}_1, d\mathbf{l}_2, d\mathbf{l}_3$ ......, then from Biot-Savart's law we know that for the conductor of infinite length, magnetic field is given by

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r_1}, B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r_2}, B_3 = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r_3}$$

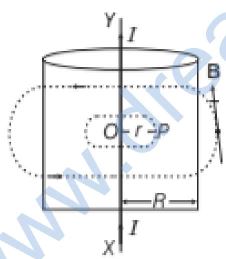
In case of each element, the magnetic field induction **B** and current element vector $d\mathbf{l}$ are in the same direction.

Line integral of **B** around closed path is

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mathbf{B}_1\cdot d\mathbf{l}_1 + \mathbf{B}_2\cdot d\mathbf{l}_2 + \mathbf{B}_3\cdot d\mathbf{l}_3 + \dots$$
$$= B_1(dl_1) + B_2(dl_2) + B_3(dl_3) + \dots$$
$$= \frac{\mu_0}{4\pi}\cdot\frac{2I}{r_1}dl_1 + \frac{\mu_0}{4\pi}\cdot\frac{2I}{r_2}dl_2 + \frac{\mu_0}{4\pi}\cdot\frac{2I}{r_3}dl_3 + \dots$$
$$= \frac{\mu_0 2I}{4\pi}\left[\frac{dl_1}{r_1} + \frac{dl_2}{r_2} + \frac{dl_3}{r_3} + \dots\right]$$
$$= \frac{\mu_0 2I}{4\pi}[d\theta_1 + d\theta_2 + d\theta_3 + \dots]$$
$$= \frac{\mu_0}{4\pi} 2I \times 2\pi = \mu_0 I$$

which is an expression of Ampere's circuital law.

26. (i) Let the point P be lying inside the wire at a perpendicular distance r from the axis of the wire. Consider a circular path of radius r around the axis of the wire. By symmetry, the magnetic field produced due to current flowing in the wire at any point over this path is tangential to it and equal in magnitude at all points on this path.



Current enclosed by the closed path,

$$I' = \frac{I}{\pi R^2}\times \pi r^2 = \frac{Ir^2}{R^2}$$

Applying Ampere's circuital law,

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\mu_r I'$$

$$\Rightarrow B(2\pi r) = \mu_0\mu_r\frac{Ir^2}{R^2}$$

$$\Rightarrow \quad B = \frac{\mu_0\mu_r Ir^2}{2\pi R^2 r}$$

$$\Rightarrow \quad B = \frac{\mu_0\mu_r Ir}{2\pi R^2}$$

(ii) When point P is outside the wire, $r > R$, so that the current enclosed by closed path = I

Using Ampere's circuital law, $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$

$$B\times 2\pi r = \mu_0 I \text{ or } B = \frac{\mu_0 I}{2\pi r}$$

27. (i) Refer to text on page 179.

(ii) According to Ampere's circuital law, the net field is given by $B = \mu_0 nI$

(a) The net magnetic field is given by

$$B_{net} = B_2 - B_1 = \mu_0 n_2 I - \mu_0 n_1 I \qquad [\because I_2 = I_1 = I]$$
$$= \mu_0 I(n_2 - n_1)$$

The direction is from B to A.

(b) As the magnetic fields due to S_1 is confined solely inside S_1 as the solenoids are assumed to be very long. So, there is no magnetic field outside S_1 due to current in S_1, similarly, there is no field outside S_2.

$$\therefore \quad B_{net} = 0$$

28. (i) Refer to text on pages 181 and 182.

(ii) Since, force always adjusts itself in a direction which becomes perpendicular to velocity, so only direction of velocity changes not the magnitude . Hence, the kinetic energy of the particle always remains constant.

29. (i) Refer to text on pages 181 and 182.

(ii) According to question, magnetic force on a charge **F** particle is given by

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$$

The direction of force on the charged particle is given by $(\mathbf{v}\times\mathbf{B})$ with the sign of charged particle, i.e. **for α-particle,** charge is positive and direction of **v** is $+\hat{\mathbf{i}}$ and direction of **B** is $-\hat{\mathbf{k}}$.

So, direction of force is $+(\hat{\mathbf{i}}\times -\hat{\mathbf{k}})$, i.e. $+\hat{\mathbf{j}}$.

It describes a circle with anti-clockwise motion.

For neutron

It is a neutral particle so, it goes undeviated.

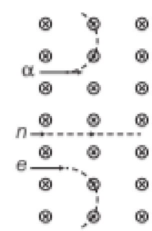
As $\quad \mathbf{F} = q(\mathbf{v}\times\mathbf{B}) = 0$

For electron

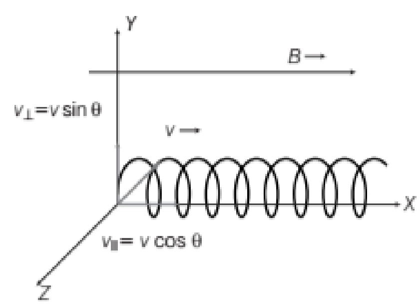
Force is given by $\mathbf{F} = -e(\mathbf{v}\times\mathbf{B})$

So, direction $= -(\hat{\mathbf{i}}\times -\hat{\mathbf{k}}) \Rightarrow -\hat{\mathbf{j}}$

e^- describes a circle with clockwise motion



30.



The path of the charged particle will be helix. As, the charge moves linearly in the direction of the magnetic field with velocity $v\cos\theta$ and also describe the circular path due to velocity $v\sin\theta$.

Time taken by the charge to complete one circular rotation, $T = \dfrac{2\pi r}{v_\perp}$ ...(i)

$\Rightarrow \quad f = qv_\perp B$

and $\quad \dfrac{mv_\perp^2}{r} = qv_\perp B$

$\Rightarrow \quad \dfrac{v_\perp m}{qB} = r$ ...(ii)

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad T = \frac{2\pi v_\perp m}{qB \cdot v_\perp} = \frac{2\pi m}{Bq}$$

Distance moved by the particle along the magnetic field in one rotation (pitch of the helix path)

$$= v_\parallel \times T \qquad [\because v_\parallel = v_{\text{parallel}}]$$

$$= v\cos\theta \times \frac{2\pi m}{Bq}$$

$$P = \frac{2\pi m v\cos\theta}{qB}$$

31. (i) Refer to text on pages 180 and 181.

(ii) Refer to text on page 181.

Magnetic field inside the open space interior of the toroid Let the loop 2 be shown in the figure, experience magnetic field **B**.

No current threads the loop 2 which lie in the open space inside the toroid.

$\therefore$ By Ampere's circuital law,

$$\oint_{\text{loop 2}} \mathbf{B}\cdot d\mathbf{l} = \mu_0 (\mathbf{I}) = 0$$

$$\Rightarrow \quad \mathbf{B} = 0$$

Magnetic field in the open space exterior of the toroid Let us consider a coplanar loop 3 in the open space of exterior of toroid. Here, each turn of toroid threads the loop two times in opposite directions.

Therefore, net current threading the loop

$$= NI - NI = 0$$

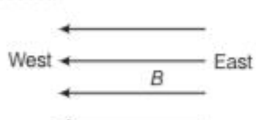
$\therefore$ By Ampere's circuital law,

$$\oint_{\text{loop 3}} \mathbf{B}\cdot d\mathbf{l} = \mu_0 (NI - NI) = 0$$

$$\Rightarrow \quad \mathbf{B} = 0$$

Thus, there is no magnetic field in the open space interior and exterior of the toroid.

32. (i) The magnetic field is in constant direction from East to West. According to the question, a charged particle travels undeflected along a straight path with constant speed. It is only possible, if the magnetic force experienced by the charged particle is zero.



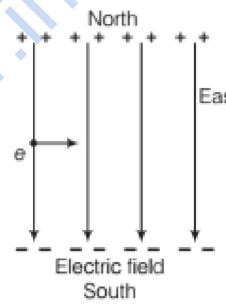
The magnitude of magnetic force on a moving charged particle in a magnetic field is given by $F = qv\,B\sin\theta$ (where θ is the angle between v and B).

Here $F = 0$, if and only if $\sin\theta = 0$ (as $v \neq 0$, $q \neq 0$, $B \neq 0$). This indicates the angle between the velocity and magnetic field is 0° or 180°.

Thus, the charged particle moves parallel or anti-parallel to the magnetic field **B**.

(ii) Yes, the final speed be equal to its initial speed as the magnetic force acting on the charged particle only changes the direction of velocity of charged particle but cannot change the magnitude of velocity of charged particle.

(iii) As, the electric field is from North to South, that means the plate in North is positive and in South is negative. Thus, the electrons (negatively charged) attract towards the positive plate that means move towards North. If we want that there should be no deflection in the path of electron, then the magnetic force should be in South direction.



By $\mathbf{F} = -e(\mathbf{v}\times\mathbf{B})$, the direction of velocity is West to East, the direction of force is towards South, by using the Fleming's left hand rule, the direction of magnetic field (**B**) is perpendicularly inwards to the plane of paper.

33. Here, $I = 2.5$ A, $n = \dfrac{100}{0.50} = 200$

(i) $B = \mu_0 nI = 4\pi \times 10^{-7} \times 200 \times 2.5$

$B = 6.28 \times 10^{-4}$ T

(ii) $B = \dfrac{\mu_0 nI}{2} = \dfrac{4\pi \times 10^{-7} \times 200 \times 2.5}{2}$

$= 3.14 \times 10^{-4}$ T

34. Number of turns, $N = 850 \times 5$, $l = 1$ m, $I = 5$ A

Area of cross-section, $A = \pi r^2 = \dfrac{22}{7}\left(\dfrac{3}{2}\times 10^{-2}\right)^2$ m^2

Magnetic field at the centre of solenoid, $B = \mu_0 NI/l$

$$= 4\pi \times 10^{-7} \times (850 \times 5) \times 5/1$$

$$= 2.671 \times 10^{-2}\ \text{T}$$

$\therefore$ Magnetic flux $= BA$

$$= 2.671 \times 10^{-2} \times \frac{22}{7} \times \left(\frac{3}{2}\times 10^{-2}\right)^2$$

$$= 1.89 \times 10^{-5}\ \text{Wb}$$

35. Magnetic field, $B = 100\text{ G} = 100 \times 10^{-4}\text{ T} = 10^{-2}\text{ T}$

To design the solenoid, let us find the product of current and number of turns in the solenoid.

The magnitude of magnetic field, $B = \mu_0 nI$

or $nI = \dfrac{B}{\mu_0} = \dfrac{10^{-2}}{4} \times 3.14 \times 10^{-7} \Rightarrow nI = 7961 = 8000$

Here, the product of nI is 8000.

Current, $I = 8$ A and number of turns, $n = 1000$

The other design is $I = 10$ A and $n = 800$/m. This is the most appropriate design as per the requirement.

36. Here, energy of electron, $E' = 2000$ eV

$$E' = 2000 \times 1.6 \times 10^{-19}\text{ J} = 3.2 \times 10^{-16}\text{ J}$$

Now, $B = 0.2$ T, $r = ?$, $E' = \frac{1}{2}mv^2$

$$\Rightarrow \quad v = \sqrt{\frac{2E'}{m}}$$

$$Bev = \frac{mv^2}{r}$$

$$\therefore \quad r = \frac{mv}{Be} = \frac{m}{Be}\sqrt{\frac{2E'}{m}} = \frac{\sqrt{2E'm}}{Be}$$

$$= \frac{\sqrt{2 \times 3.2 \times 10^{-16} \times 9 \times 10^{-31}}}{0.2 \times 1.6 \times 10^{-19}}$$

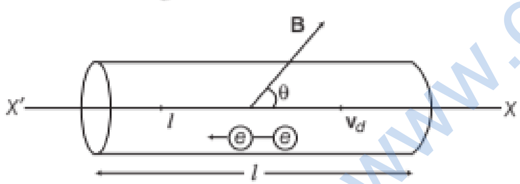
$$= 7.5 \times 10^{-4}\text{ m}$$

|TOPIC 3|

Magnetic Force and Torque Experienced by a Current Loop

FORCE ON A CURRENT CARRYING CONDUCTOR IN A UNIFORM MAGNETIC FIELD

When a current carrying conductor is placed in a uniform magnetic field, then due to motion of free electrons inside the conductor, a magnetic force acts on it.



Current carrying conductor in a uniform magnetic field

Let us consider a portion of length l and cross-sectional area A of a straight conductor carrying a current I.

Let the magnetic field **B** be in the plane of the paper directed upwards and making an angle θ with the direction of velocity of electrons.

Let n be the number of free electrons per unit volume in the conductor and v_d be the drift velocity of the electrons. From the relation $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$, where $(q = e)$ is the charge of an electron. If however, the conductor makes an angle θ with the magnetic field **B** measured from the conductor towards the field **B**, then the magnitude of the force on each electron is

$$F' = ev_d\, B\sin\theta$$

The number of electrons in the length l of the conductor is

$$N = n\,A\,l$$

The total force F on the free electrons and hence on the length l of the conductor is, therefore

$$F = F' \times N = (ev_d\, B\sin\theta)(n\,A\,l)$$
$$= (n\,e\,A\,v_d)Bl\sin\theta$$

But current flowing through a conductor, $I = neA\,v_d$

$$\therefore \quad F = I\,B\,l\sin\theta$$

or

$$\mathbf{F} = I(\mathbf{l} \times \mathbf{B})$$

If $\theta = 0°$ or $180°$, then $F = I\,Bl\sin 0° = 0$

[$\because \sin 0° = 0$ and $\sin 180° = 0$]

It means a conductor placed parallel to direction of magnetic field, experiences no force due to magnetic field.

If $\theta = 90°$, then force is maximum.

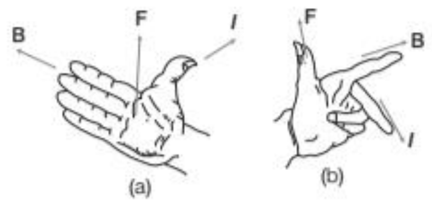
$$F_{max} = IBl\sin 90°$$
$$= IBl \qquad [\because \sin 90° = 1]$$

It means a conductor placed perpendicular to direction of magnetic field, experiences maximum force.

Rules to Find Out the Direction of Force

The direction of the force acting on a current carrying conductor in a magnetic field can be found by any of the following two rules.

(i) **Right Hand Palm Rule** If we stretch our right hand palm such that the thumb points in the direction of the current (I) and the stretched fingers in the direction of the magnetic field **B**, then the force **F** on the conductor will be perpendicular to the palm in the direction of pushing by the palm as shown in the Fig. (a).



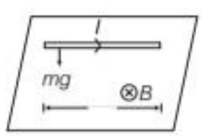
(ii) **Fleming's Left Hand Rule** If the fore-finger, the middle-finger and the thumb of the left hand are stretched mutually at right angles to one another such that the fore-finger points in the direction of the magnetic field **B** and the middle-finger in the direction of the current I, then the thumb will point in the direction of the force **F** on the conductor as shown in the Fig. (b).

EXAMPLE |1| A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

Sol. Magnetic force on the wire,

$$F = BIl\sin\theta = BIl\sin 90^\circ$$
$$= 0.27 \times 10 \times 3 \times 10^{-2}$$
$$= 8.1 \times 10^{-2}\,\text{N} \qquad [\because \theta = 90^\circ]$$

EXAMPLE |2| A straight wire of mass 200 g and length 1.5 m carries a current of 4 A. It is suspended in mid air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field?



Sol. Applying Fleming's rule, we find that upward force F of magnitude IlB acts. For mid air suspension this must be balanced by the force due to gravity.

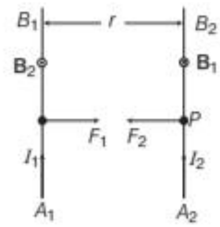
$$\therefore \qquad mg = IlB \Rightarrow B = \frac{mg}{Il}$$

Given, $m = 200\text{ g} = 0.2\text{ kg}$, $g = 9.8\text{ ms}^{-2}$, $I = 4\text{A}$, $l = 1.5\text{ m}$

We have, $B = \dfrac{0.2 \times 9.8}{4 \times 1.5} = 0.325\text{ T}$

FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS

To find the force on a current carrying wire due to a second current carrying wire, first find the magnetic field due to second wire at the site of first wire. Then, find the force on the first wire due to that field.



Two parallel current carrying conductors

Let us consider A_1B_1 and A_2B_2 are two infinite long straight conductors.

I_1 and I_2 are the currents flowing through them and these are at r distance apart.

Magnetic field induction at a point P on conductor A_2B_2 due to current

I_1 passing through A_1B_1 is $B_1 = \dfrac{\mu_0 2I_1}{4\pi r}$

The unit length of A_2B_2 will experience a force as

$$F_2 = B_1 I_2 \times l = B_1 I_2 l$$

or
$$F_2 = \frac{\mu_0}{4\pi}\cdot\frac{2I_1I_2}{r}\cdot l$$

Conductor A_1B_1 also experiences the same amount of force, directed towards the wire A_2B_2.

Therefore, force between two current carrying parallel conductors per unit length is

$$\boxed{F = \frac{\mu_0}{4\pi}\cdot\frac{2I_1I_2}{r}}$$

Two linear parallel conductors carrying currents in the same direction attract each other while carrying currents in opposite direction they repel each other.

Definition of Ampere (In terms of the force)

One ampere is the current which flows through each of the two parallel uniform long linear conductors, which are placed in free space at a distance of 1 m from each other and which attract or repel each other with a force of 2×10^{-7} N/m of their lengths.

EXAMPLE |3| Calculate the force per unit length on a long straight wire carrying current of 4 A due to a parallel wire carrying 6 A current, if the distance between the wires is 3 cm.

Sol. Given, $I_1 = 4$ A, $I_2 = 6$ A, $r = 3$ cm $= 0.03$ m

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r}$$

$$= \frac{10^{-7} \times 2 \times 4 \times 6}{0.03}$$

$$= 1.6 \times 10^{-4} \text{N/m}$$

EXAMPLE |4| A short conductor of length 5 cm is placed parallel to a long conductor of length 1.5 m near its centre. The conductors carry currents 4 A and 3 A respectively in the same direction. What is the total force experienced by the long conductor when they are 3 cm apart?

Sol. Force on long conductor is equal and opposite to the force on small conductor $= \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2 l}{r}$

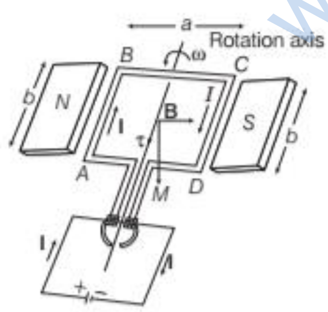
Given, $I_1 = 4$ A, $I_2 = 3$ A, $r = 3 \times 10^{-2}$ m, $l = 5 \times 10^{-2}$ m

$$\Rightarrow \quad F = \frac{4\pi \times 10^{-7} \times 2 \times 4 \times 3 \times 5 \times 10^{-2}}{4\pi \times 3 \times 10^{-2}}$$

$$= 4 \times 10^{-6} \text{ N}$$

TORQUE EXPERIENCED BY A CURRENT LOOP IN UNIFORM MAGNETIC FIELD (MAGNETIC DIPOLE)

Consider a rectangular loop *ABCD* be suspended in a uniform magnetic field **B**. Let $AB = CD = b$ and $AD = BC = a$. Let I be the current flowing through the loop.



A rectangular current carrying coil in uniform magnetic field

***Case* I** The rectangular loop is placed such that the uniform magnetic field B is in the plane of loop.

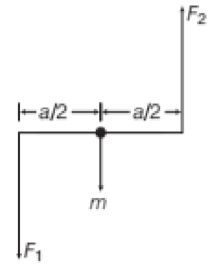
No force is exerted by the magnetic field on the arms AD and BC ($\because$ they are parallel to the magnetic field).

Magnetic field exerts a force F_1 on arm AB,

$$\therefore \qquad F_1 = IbB$$

Magnetic field exerts a force F_2 on arm CD,

$$F_2 = IbB = F_1$$



F_1 and F_2 are equal and opposite, so net force on the loop is zero. But line of action of F_1 and F_2 are opposite and parallel, so they form a couple.

The torque produced due to couple on the loop rotates the loop in anti-clockwise direction.

Torque, $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

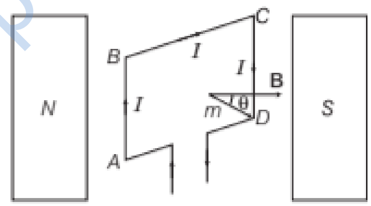
So, $\quad \tau = F_1 \frac{a}{2} + F_2 \frac{a}{2} \qquad [\because \sin 90° = 1]$

($\because$ Torque = Force × Perpendicular distance of line of action)

$$\Rightarrow \quad \tau = IbB \frac{a}{2} + IbB \frac{a}{2} = I(ab)B = IAB$$

where, b be breadth of the rectangular coil, a be length of the rectangular coil and $A = ab$ (area of the coil).

***Case* II** The plane of the loop is not along the magnetic field, but makes an angle with it.



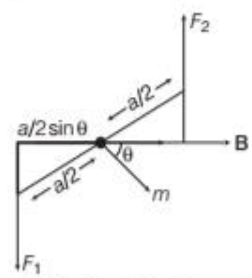
The area vector of the loop *ABCD* makes an arbitrary angle θ with the magnetic field

Angle between the field and the normal to the coil is θ. Forces on BC and DA are equal and opposite and they cancel each other as they are collinear.

Force on AB is F_1 and force on CD is F_2.

and $\qquad F_1 = F_2 = IbB$

Magnitude of torque on the loop is as shown in figure below:



Top view of the loop. The forces F_1 and F_2 acting on the area *AB* and *CD* are indicated

$$\tau = F_1 \frac{a}{2} \sin\theta + F_2 \frac{a}{2} \sin\theta$$

$$\tau = IabB \sin\theta = IAB \sin\theta$$

where, $A = ab$ (area of coil)

The torque on the loop can be expressed as the vector product of the magnetic moment of the coil and the magnetic field

$$\tau = MB\sin\theta\hat{n} = \mathbf{M}\times\mathbf{B}$$

where, $\mathbf{M} = NI\mathbf{A}$ is magnetic moment of the loop, its unit is ampere-metre^2.

This is analogous to the electrostatic case (electric dipole of dipole moment p in an electric field E)

When **M** and **B** are parallel, then current loop is in stable equilibrium. Any small rotation of the loop produces a torque which brings it back to its original position. When **M** and **B** are anti-parallel, then current loop is in unstable equilibrium.

Magnetic moment of the loop of N turns,

$$M = NIA$$

The total torque on the coil is given by

$$\tau = NIAB\sin\theta = (NIA)\,B\sin\theta = BINA\sin\theta$$

The presence of this torque is also the reason why a small magnet or any magnetic dipole aligns itself with the external magnetic field.

EXAMPLE |5| A circular coil of 20 turns and radius 10 cm carries a current of 5 A. It is placed in a uniform magnetic field of 0.10 T. Find the torque acting on the coil, when the magnetic field is applied in the plane of coil.

Sol. Given, total number of turns, $N = 20$

Radius, $r = 10\text{ cm} = 10\times 10^{-2}\text{ m}$

Current, $I = 5\text{ A}$

Angle, $\theta = 90°$

External uniform magnetic field $(B) = 0.10\text{ T}$

Torque, $\tau = ?$

As, torque, $\tau = BINA\sin\theta$

$\Rightarrow \quad = 0.10\times 5\times 20\times 0.0314\times\sin 90°$

$= 0.314\text{ N-m}$

EXAMPLE |6| Calculate the torque of a 100 turns rectangular coil of length 40 cm and breadth 20 cm, carrying a current of 10 A, when placed making an angle of 60° with a magnetic field of 5 T.

Sol. Given, $I = 10\text{A}$, $N = 100$, $l = 40\text{ cm}$, $b = 20\text{ cm}$

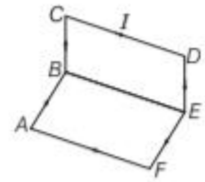
$B = 5\text{ T}$, $\theta = 60°$

$$A = l\times b = 40\times 20 = 800\text{ cm}^2 = 8\times 10^{-2}\text{ m}^2$$

$$\because \quad \tau = NBIA\sin\theta = 100\times 5\times 10\times 8\times 10^{-2}\times\sin 60°$$

$$= 346.41\text{ N-m}$$

EXAMPLE |7| Find the magnitude of magnetic moment of the current carrying loop *ABCDEFA*. Each side of the loop is 10 cm long and current in the loop is $I = 2.0$ A.



Sol. By assuming two equal and opposite currents in *BE*, two current carrying loops (*ABEFA* and *BCDEB*) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other. Hence,

$$M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

$$\because \quad M = IA = 2\times(l\times b)$$

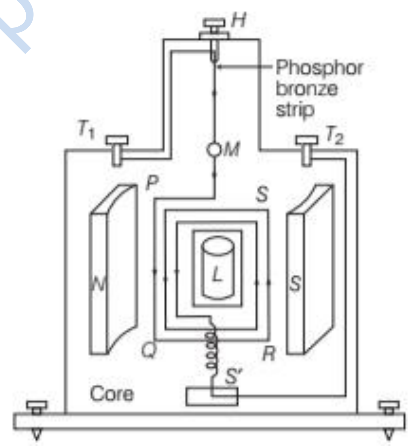
$$= 2\times(0.1)\,(0.1) = 0.02\text{ A-m}^2$$

$$\therefore \quad M_{net} = (\sqrt{2})\,(0.02) = 0.028\text{ A-m}^2$$

MOVING COIL GALVANOMETER

Principle

Its working is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.



Schematic arrangement of moving coil galvanometer

Construction

The moving coil galvanometer consists of a coil with many turns free to rotate about a fixed axis, in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. When a current flows through the coil, a torque acts on it.

Working

Suppose the coil *PQRS* is suspended freely in the magnetic field. Let l be length *PQ* or *RS* of the coil, b be breadth *QR* or *SP* of the coil, N be number of turns in the coil and area of each turn of the coil, $A = l\times b$. Let B be strength of the magnetic field in which coil is suspended and I is current passing through the coil in the direction $PQRS$.

Let at any instant, α be the angle, which normal drawn on the plane of the coil makes with the direction of magnetic field. The rectangular coil carrying current when placed in the magnetic field experiences a torque whose magnitude is given by $\tau = NIBA \sin \alpha$.Due to deflecting torque, the coil rotates and suspension wire gets twisted. A restoring torque is set up in the suspension wire.

Let θ be the twist produced in the phosphor bronze strip due to rotation of the coil and k be the restoring torque per unit twist of the phosphor bronze strip. Then,

Total restoring torque produced $= k\theta$

In equilibrium position of the coil,

Deflecting torque = Restoring torque

$$NIBA = k\theta$$

$$\Rightarrow \quad I = \frac{k}{NBA}\theta = G\theta$$

where, $\frac{k}{NBA} = G$

It is known as galvanometer constant.

i.e. $\theta \propto I$. It means that the deflection produced is proportional to the current flowing through the galvanometer.

Current sensitivity of the galvanometer is the deflection per unit current flowing through it.

It is given by $I_S = \frac{\theta}{I} = \frac{NAB}{k}$

Its unit is rad/A or div/A.

Voltage sensitivity is the deflection per unit voltage.

It is given by

$$V_S = \frac{\theta}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V}$$

or $$V_S = \frac{NAB}{k} \times \frac{I}{IR} = \frac{NAB}{kR}$$

[$\because$ according to Ohm's law, $V = IR$]

Its unit is rad/V or div/V.

Note Dead beat galvanometer is one in which the coil comes to rest at once after the passage of current through it. The deflection can be noted in no time.

EXAMPLE |8| In order to increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased so that the new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change?

Sol. Increased current sensitivity, $I'_S = I_S + \frac{50\, I_S}{100} = \frac{150\, I_S}{100}$

$$= \frac{3\, I_S}{2},\ R' = 2R$$

Initial voltage sensitivity, $V_S = \frac{I_S}{R}$ ...(i)

New voltage sensitivity, $V'_S = \frac{I'_S}{R'} = \frac{\frac{3}{2} I_S}{2R}$

$$\Rightarrow \quad V'_S = \frac{3}{4} V_S \quad \text{[from Eq. (i)]}$$

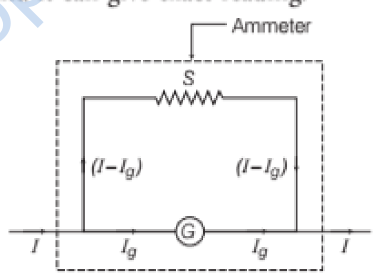
$\therefore$ % decrease in voltage sensitivity $= \frac{V_S - V'_S}{V_S} \times 100$

$$= \left(1 - \frac{3}{4}\right) \times 100 = 25\%$$

Note This topic has been frequently asked in the previous year exams, i.e. in year 2016, 2015, 2014, 2010, 2009.

Conversion of a Galvanometer into Ammeter

To convert a galvanometer into ammeter, its resistance needs to be lowered, so that maximum current can pass through it and it can give exact reading.



A shunt (low resistance) is connected in parallel with the galvanometer.

$$S = \left(\frac{I_g}{I - I_g}\right) G$$

where, I = total current in circuit,

G = resistance of the galvanometer,

S = resistance of the shunt (low resistance)

and I_g = current through the galvanometer.

EXAMPLE |9| A galvanometer of resistance 15 Ω gives full scale deflection for a current of 2 mA. Calculate the shunt resistance needed to convert it to an ammeter of range 0 to 5 A.

Sol. Given,

$$G = 15\ \Omega,\ I_g = 2\text{ mA}$$
$$= 2 \times 10^{-3}\text{ A},\ I = 5\text{ A}$$

$\therefore$ Shunt resistance, $S = \dfrac{I_g G}{I - I_g}$

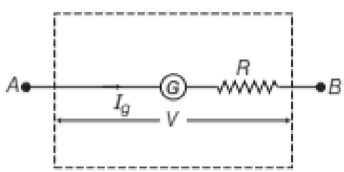
$$= \frac{2\times10^{-3}\times15}{5-2\times10^{-3}}$$

$$= 0.006\ \Omega$$

This resistance $S = 0.006\ \Omega$ is connected in parallel with the galvanometer. The small resistance is connected in parallel, because we have to decrease the resistance of the galvanometer so that most of the current passes through it and it gives the exact value of the current.

Conversion of a Galvanometer into Voltmeter

To convert a galvanometer into voltmeter, its resistance needs to be increased, so that there is no potential drop across it because with high resistance no current passes through it.



Voltmeter

A high resistance is connected in series with the galvanometer, then the value of R is given by

$$R = \frac{V}{I_g} - G$$

where, V = potential difference across the terminals A and B

I_g = current through the galvanometer (full scale deflection current),

R = high resistance

and G = resistance of the galvanometer.

EXAMPLE |10| The full scale deflection current of a galvanometer of resistance $1\ \Omega$ is 5 mA. How will you convert it into a voltmeter of range 5 V?

Sol. From the relation, $V = I_g(G+R)$, we have

$$R = \frac{V}{I_g} - G$$

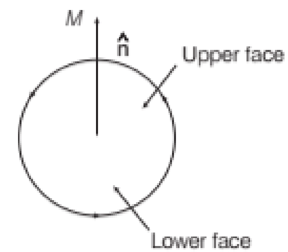
$$= \left(\frac{5}{5\times10^{-3}}\right) - 1$$

$$= 999\ \Omega$$

i.e., a resistance of 999 Ω should be connected in series with the galvanometer to convert it into a voltmeter of desired range.

CIRCULAR CURRENT LOOP AS A MAGNETIC DIPOLE

A current loop behaves as a magnetic dipole. If we look at the upper face, current is anti-clockwise, so it has North polarity. If we look at lower face, current is clockwise, so it has South polarity.



That means current loop behaves as a system of two equal and opposite magnetic poles hence, it acts as a magnetic dipole.

Magnetic dipole moment of loop, $M = NIA$

where, I = current flowing through the loop

A = area enclosed by the loop

and N = number of turns in the coil

The magnitude of magnetic field on the axis of a circular loop of radius R, carrying steady current I is given by

$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

For $x >> R$,

$$B = \frac{\mu_0 I R^2}{2x^3} = \frac{\mu_0 I A}{2\pi x^3} \quad [\because A = \pi R^2]$$

$$\Rightarrow \quad B = \frac{\mu_0 M}{2\pi x^3} \quad [\because M = IA]$$

MAGNETIC DIPOLE MOMENT OF A REVOLVING ELECTRON

An electron being a charged particle, constitutes a current while moving in its circular orbit around the nucleus ($\because$ Moving charge constitutes a current as well as magnetic field). If T is the time period of revolution, then current constituted by electron is $I = \dfrac{e}{T}$ ... (i)

where, e = charge of electron.

If r is the orbital radius of electron and its orbital speed is v, then

$$T = \frac{2\pi r}{v} \Rightarrow I = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r} \quad \text{[from Eq. (i)]}$$

Magnetic moment of revolving electron, $M = IA$

$$M = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}$$

The direction of this magnetic moment is into the plane of the paper.

$$M = \frac{e}{2m_e}(m_e vr) = \frac{e}{2m_e} l \text{ or } M = \frac{-el}{2m_e}$$

where, $l = m_e vr$ is angular momentum of the electron.

$\frac{M}{l} = \frac{e}{2m_e}$ is a constant, called **gyromagnetic ratio**, its value is 8.8×10^{10} C/kg for an electron.

From Bohr's hypothesis, angular momentum can have only some discrete values, $l = \frac{nh}{2\pi}$

where, h = Planck's constant and n is natural number

i.e. $n = 1, 2, 3, ... \Rightarrow M = \frac{e}{2m_e} \cdot \frac{nh}{2\pi} = \frac{e}{4\pi m_e} nh$

For $n = 1$, M will be minimum.

$$\therefore \quad M_{\min} = \frac{eh}{4\pi m_e}$$

It is Bohr's magneton, which is defined as magnetic moment of revolving electron in its first orbit. Its value is 9.27×10^{-24} A-m^2.

EXAMPLE |1| An electron in a hydrogen atom is moving with a speed of 2.3×10^6 ms^{-1} in an orbit of radius 0.53 Å. Calculate the magnetic moment of the revolving electron.

Sol. Given, $v = 2.3 \times 10^6$ ms^{-1},

$r = 0.53$ Å $= 0.53 \times 10^{-10}$ m

Equivalent current, $I = \frac{e}{T} = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r}$

$$= \frac{1.6 \times 10^{-19} \times 2.3 \times 10^6}{2 \times 3.14 \times 0.53 \times 10^{-10}}$$

$$= 1.105 \times 10^{-3} \text{ A}$$

$\therefore$ Magnetic moment,

$M = IA = I(\pi r^2) = 1.105 \times 10^{-3} \times 3.14 \times (0.53 \times 10^{-10})^2$

$= 9.75 \times 10^{-24}$ A-m^2

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. Two parallel wires are placed 1 m apart and 1 A and 3 A currents are flowing in the wires in opposite direction. The force acting per unit length of both the wires will be

(a) 6×10^{-7} N/m attractive
(b) 6×10^{-5} N/m attractive
(c) 6×10^{-7} N/m repulsive
(d) 6×10^{-5} N/m repulsive

2. A circular loop of area A, carrying current I, is placed in a magnetic field B perpendicular to the plane of the loop. The torque on the loop due to magnetic field is

(a) BIA (b) $2\,BIA$ (c) $\frac{1}{2}BIA$ (d) zero

3. The area of a circular ring is 1 cm^2 and current of 10 A is passing through it. If a magnetic field of intensity 0.1 T is applied perpendicular to the plane of the ring. The torque due to magnetic field on the ring will be

(a) zero (b) 10^{-4} N-m (c) 10^{-2} N-m (d) 1 N-m

4. A circular current loop of magnetic moment M is in an arbitrary orientation in an external magnetic field B. The work done to rotate the loop by 30° about an axis perpendicular to its plane is **NCERT Exemplar**

(a) MB (b) $\sqrt{3}\frac{MB}{2}$
(c) $\frac{MB}{2}$ (d) zero

5. The current i is flowing in a coil of area A with the number of turns N, then the magnetic moment of the coil M will be

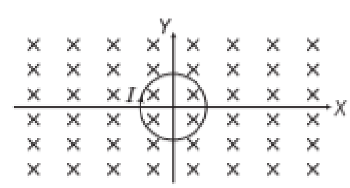
(a) NiA (b) Ni/A
(c) $Ni/\sqrt{A}$ (d) N^2Ai

6. A galvanometer of resistance 25 Ω shows full scale deflection for current of 10 mA. To convert it into 100 V range voltmeter, the required series resistance is

(a) 9975 Ω (b) 10025 Ω
(c) 10000 Ω (d) 975 Ω

VERY SHORT ANSWER Type Questions

7. A conducting loop carrying a current I is placed in a uniform magnetic field, pointing into the plane of the paper as shown in the figure, then the loop will have a tendency to expand. Explain.



8. Give the magnitude of torque which acts on a coil carrying current placed in a uniform radial magnetic field.

9. Write the underlying principle of a moving coil galvanometer. **Delhi 2016**

10. Why should the spring/suspension wire in a moving coil galvanometer have low torsional constant?

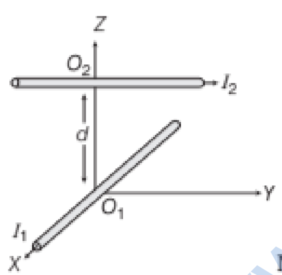
11. Why is a coil wrapped on a conducting frame in a galvanometer?

12. The coils in certain galvanometers, have a fixed core made of a non-magnetic metallic material. Why does the oscillating coil come to rest so, quickly in such a core?

13. A voltmeter, an ammeter and a resistance are connected in series with a lead accumulator. The voltmeter gives some deflection but the deflection of ammeter is zero. Comment.

SHORT ANSWER Type Questions

14. Two long wires carrying currents I_1 and I_2 are arranged as shown in the figure. One carrying current I_1 is along the X-axis. The other carrying current I_2 is along a line parallel to Y-axis, given by $x = 0$ and $z = d$. Find the force exerted at point O_2 because of the wire along the X-axis.



NCERT Exemplar

15. Two long parallel wires carrying a current I, separated by a distance r are exerting a force F on each other. If the distance between them is increased to $2r$ and current in each wire is reduced from I to $I/2$, then what will be the force between them?

16. (i) Two long straight parallel conductors a and b carrying steady currents I_a and I_b respectively are separated by a distance d. Write the magnitude and direction, what is the nature and magnitude of the force between the two conductors?

(ii) Show with the help of a diagram, how the force between the two conductors would change when the currents in them flow in the opposite directions. **Foreign 2014**

17. A rectangular coil of sides l and b carrying a current I is subjected to a uniform magnetic field **B** acting perpendicular to its plane. Obtain the expression for the torque acting on it. **Delhi 2014C**

18. Define current sensitivity and voltage sensitivity of galvanometer. Increasing the current sensitivity may not necessarily increase the voltage sensitivity of a galvanometer, justify your answer. **All India 2009**

19. How is a moving coil galvanometer converted into a voltmeter? Explain giving the necessary circuit diagram and the required mathematical relation used. **All India 2011C**

20. A galvanometer gives full scale deflection with the current I_g.

Can it be converted into an ammeter of range $I < I_g$?

LONG ANSWER Type I Questions

21. Draw a labelled diagram of a moving coil galvanometer and explain its working. What is the function of radial magnetic field inside the coil? **Foreign 2012**

22. Answers the following questions.

(i) Write two reasons why a galvanometer cannot be used as such to measure the current in a given circuit. Name any two factors on which the current sensitivity of a galvanometer depends.

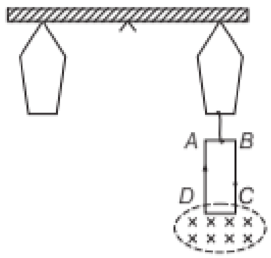
(ii) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer?

23. State the principle of working of a galvanometer.

A galvanometer of resistance G is converted into a voltmeter to measure upto V volts by connecting a resistance R_1 in series with the coil. If a resistance R_2 is connected in series with it, then it can measure upto $V/2$ volts. Find the resistance, in terms of R_1 and R_2, required to be connected to convert it into a voltmeter that can read upto 2V. Also, find the resistance G of the galvanometer in terms of R_1 and R_2. **All India 2015**

24. A 100 turns rectangular coil $ABCD$ (in XY-plane) is hung from one arm of a balance figure. A mass 500 g is added to the other arm to balance

the weight of the coil. A current 4.9 A passes through the coil and a constant magnetic field of 0.2 T acting inward (in XZ-plane) is switched ON such that only arm CD of length 1 cm lies in the field. How much additional mass m must be added to regain the balance? **NCERT Exemplar**



LONG ANSWER TypeII Questions

25. Explain using a labelled diagram, the principle and working of a moving coil galvanometer. What is the function of

(i) uniform radial magnetic field (ii) soft iron core?

Also, define the terms

(iii) current sensitivity and

(iv) voltage sensitivity of a galvanometer?

Why does increasing the current sensitivity not necessarily increase voltage sensitivity? **Delhi 2015**

26. (i) Draw a labelled diagram of a moving coil galvanometer. Describe briefly its principle and working.

(ii) Answer the following are as follows

(a) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer?

(b) Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. Explain giving reason. **All India 2014**

27. (i) Explain giving reasons, the basic difference in converting a galvanometer into

(a) a voltmeter and (b) an ammeter

(ii) Two long straight parallel conductors carrying steady currents I_1 and I_2 are separated by a distance d. Explain briefly, with the help of a suitable diagram, how the magnetic field due to one conductor acts on the other. Hence, deduce the expression for the force acting between the two conductors. Mention the nature of this force. **All India 2012**

NUMERICAL PROBLEMS

28. What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A making an angle of 30° with the direction of a uniform magnetic field of 0.15 T? **NCERT**

29. A long straight wire carrying current of 25 A rests on a table shown in the figure. Another wire PQ of length 1 m, mass 2.5 g carries the same current but in the opposite direction. The wire PQ is free to slide up and down. To what height will PQ rise? **NCERT Exemplar**

30. A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire? **NCERT**

31. A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction parallel to the axis along East to West. A wire carrying current of 7.0 A in the North to South direction passes through this region. What is the magnitude and direction of the force on the wire, if

(i) the wire intersect the axis?

(ii) the wire is turned from North-South to North East-North West direction?

(iii) the wire in the North-South direction is lowered from the axis by a distance of 6.0 cm? **NCERT**

32. A solenoid 60 cm long and of radius 4.0 cm has 3 layers of winding of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis, both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? ($g = 9.8\ m/s^2$) **NCERT**

33. A conductor of length 2 m carrying current of 2 A is held parallel to an infinitely long conductor carrying current of 10 A at a distance of 100 mm. Find the force on a small conductor. **Delhi 2010**

34. Two long and parallel straight wires *A* and *B* carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire *A*. **NCERT**

35. A wire *AB* is carrying a steady current of 12 A and is lying on the table. Another wire *CD* carrying 5 A is held directly above *AB* at a height of 1 mm.
Find the mass per unit length of the wire *CD*, so that it remains suspended at its position, when left free. Give the direction of the current flowing in *CD* with respect to that in *AB*. (Take the value of $g = 10\ \text{ms}^{-2}$) **All India 2013**

36. A circular coil of 100 turns, radius 10 cm carries a current of 5 A. It is suspended vertically in a uniform magnetic field of 0.5 T, the field lines making an angle of 60° with the plane of the coil. Calculate the magnitude of the torque that must be applied to it to prevent it from turning.

37. A square coil of side 10 cm consists of 20 turns and carries current of 12 A. The coil is suspended vertically and normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil? **NCERT**

38. (i) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle 60° with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.
(ii) Would your answer change, if the circular coil were replaced by a planar coil of some irregular shape that encloses the same area? All other particulars are also unaltered. **NCERT**

39. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.1 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the (i) total torque on the coil, (ii) total force on the coil (iii) average force on each electron in the coil due to the magnetic field?
(The coil is made of copper wire of cross-sectional area $10^{-5}\ \text{m}^2$ and the free electron density in copper is given to be about $10^{29}/\text{m}^3$). **NCERT**

40. A rectangular coil of area $2\times10^{-4}\ \text{m}^2$ and 40 turns is pivoted about one of its vertical sides. The coil is in a radial horizontal field of 60 G. What is the torsional constant of the hair springs connected to the coil, if a current of 4.0 mA produces an angular deflection of 16°?

41. Two moving coil meters M_1 and M_2 having the following particulars
$R_1 = 10\ \Omega$, $N_1 = 30$, $A_1 = 3.6\times10^{-3}\ \text{m}^2$,
$B_1 = 0.25$ T
$R_2 = 14\ \Omega$, $N_2 = 42$, $A_2 = 1.8\times10^{-3}\ \text{m}^2$,
$B_2 = 0.50$ T (The spring constants are identical for the two meters). Determine the ratio of (i) current sensitivity and (ii) voltage sensitivity of M_2 and M_1. **NCERT**

42. A galvanometer coil has a resistance of 15 Ω and the meter shows full scale deflection for a current of 4 mA. How will you convert the meter into an ammeter of range 0 to 6 A? **NCERT**

43. When a galvanometer having 30 division scale and 100 Ω resistance is connected in series to the battery of emf 3 V through a resistance of 200 Ω, shows full scale deflection. Find the figure of merit of the galvanometer in microampere.

44. A galvanometer coil has a resistance of 12 Ω and the meter shows full scale deflection for a current of 3 mA. How will you convert the meter into a voltmeter of range 0 to 18 V? **NCERT**

HINTS AND SOLUTIONS

1. (c) The force acting per unit length,

$$\frac{F}{L} = \frac{\mu_0}{2\pi}\cdot\frac{i_1 i_2}{r} = 2\times10^{-7}\times\frac{1\times3}{1} = 6\times10^{-7}$$

If the currents are in opposite direction, then the wires will repel each other.

2. (d) Torque experienced by a current loop in a uniform magnetic field, $\tau = NIBA\sin\theta$

When $\theta = 0°$, $\tau = 0$ $\quad$ ($\because N = 1$ and $\sin 0° = 0$)

3. (a) Given, $A = 1\,cm^2 = 1 \times 10^{-4}\,m^2$,

$I = 10\,A$, $B = 0.1\,T$, $\theta = 0°$

The moment of force or torque acting on the circular ring,

$\tau = IBA\sin\theta$

$= 10 \times 0.1 \times 1 \times 10^{-4} \times \sin 0° = 0$

4. (d) The rotation of the loop by 30° about an axis perpendicular to its plane make no change in the angle made by axis of the loop with the direction of magnetic field, therefore, the work done to rotate the loop is zero.

5. (a) The magnetic moment of a current-carrying coil $M = iA$. If there are N turns, then $M = NiA$.

6. (a) To convert a galvanometer into a voltmeter by connecting a high resistance in series, required series resistance will be, $R = \frac{V}{I_g} - G$

which restricts the current to safe limit I_g.

where, G = resistance of galvanometer = 25 Ω,

I_g = current with which galvanometer gives full scale deflection = 10 mA,

$= 10 \times 10^{-3}\,A$

V = required range of voltmeter = 100 V

$\Rightarrow R = \frac{100}{10 \times 10^{-3}} - 25 = 9975\,\Omega$

7. We can see that magnetic field is perpendicular to paper and current in the loop is in clockwise direction. So,by Fleming's left hand rule, force on each element of the loop is radially outwards, so loop will have a tendency to expand.

8. Torque, $\tau = NBIA\sin\theta$, where the terms have their usual meanings.

9. The principle of moving coil galvanometer is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.

10. Low torsional constant facilitates greater deflection θ in coil for given value of current and hence, sensitivity of galvanometer increases.

11. In order to produce electromagnetic damping i.e., by producing eddy currents in conducting frame which helps in stopping the coil soon.

12. Due to eddy currents produced in core which opposes the cause (deflection of coil), that produces it. This further helps in stopping the coil so on, i.e. in making the galvanometer dead beat.

13. Voltmeter and resistance being very high when connected in series, makes the effective resistance of the circuit very high. Due to this, current in the circuit becomes extremely small.

14. Here, first we have to find the direction of magnetic field at point O_2 due to the wire carrying current I_1. Use Maxwell's right hand grip (cork screw) rule, the direction of magnetic field at point O_2 due to current I_1 is along Y-axis.

Here, the wire at point O_2 is placed along Y-axis. Now, by the formula, $\mathbf{F} = I_2\,(\mathbf{l} \times \mathbf{B})$

Angle between **l** and **B** is 0°, both are at Y-axis, i.e.

$F = IlB\sin 0° = 0$

So, the force exerted at point O_2 because of wire along X-axis is zero.

15. Force per unit length is

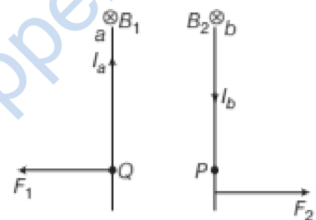
$$F = \frac{\mu_0 2I^2}{4\pi r} \qquad [\because I_1 = I_2 = I]$$

If r is increased to $2r$ and I is reduced to $\frac{I}{2}$, then new force per unit length is $F' = \frac{\mu_0}{4\pi} \times \frac{2(I/2)^2}{2r}$

$$= \frac{1}{8}\left(\frac{\mu_0}{4\pi} \cdot \frac{2I^2}{r}\right) \Rightarrow F' = \frac{F}{8}$$

$\therefore$ Force per unit length between them is $\frac{F}{8}$.

16. Refer to text on page 192.



Now, let the direction of current in conductor b be reversed. The magnetic field B_2 at point P due to current I_a flowing through a will be downwards. Similarly, the magnetic field B_1 at point Q due to current I_b passing through b will also be downward as shown. The froce on a will be therefore towards the left. Also, the force on b will be towards the right. Hence, the two conductors will repel each other as shown.

17. Equivalent magnetic moment of the coil, $\mathbf{M} = IA\hat{\mathbf{n}}$

$\therefore \quad \mathbf{M} = Ilb\,\hat{\mathbf{n}}$

where, $\hat{\mathbf{n}}$ = unit vector ⊥ to the plane of the coil.

$\therefore$ Torque $= \mathbf{M} \times \mathbf{B} = Ilb(\hat{\mathbf{n}} \times \mathbf{B}) = 0$

As $\hat{\mathbf{n}}$ and **B** are parallel or anti-parallel to each other.

18. Refer to text on pages 195.

Increasing the current sensitivity may not necessarily increase the voltage sensitivity, because the current sensitivity increases with the increase of number of turns of the coil but the resistance of coil also increases which affect adversely on voltage sensitivity.

19. Refer to text on page 196.

20. The resistance of an ideal ammeter is zero or very low in practical condition, so to convert a galvanometer into ammeter its resistance needs to be decreased which can be done by connecting a low resistance in its parallel order.

A moving coil galvanometer of range I_s and resistance G can be converted into ammeter by connected very low resistance shunt in parallel with galvanometer.

21. Refer to text on pages 194 and 195.

22. (i) The galvanometer cannot be used to measure the current because

(a) all the currents to be measured passes through coil and it gets damaged easily.

(b) its coil has considerable resistance because of length and it may affect original current.

Current sensitivity of galvanometer depends on

(a) the magnetic field

(b) the value of torsional constant.

(ii) It is necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer because magnetic field is increased, so its sensitivity increases and magnetic field becomes radial. So, angle between the plane of coil and magnetic line of force is zero in all orientations of coil.

23. According to the principle of working of a moving coil galvanometer, when a current carrying coil is placed in a magnetic field, it experiences a torque.

A high resistance that is connected in series with the galvanometer to convert into voltmeter. The value of the resistance is given by, $R = \frac{V}{I_g} - G$

where, V = potential difference across the terminals of the voltmeter, I_g = current through the galvanometer and G = resistance of the galvanometer.

When resistance R_1 is connected in series with the galvanometer, then

$$R_1 = \frac{V}{I_g} - G \quad \text{...(i)}$$

When resistance R_2 is connected in series with the galvanometer, then $R_2 = \frac{V}{2I_g} - G$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{V}{2I_g} = R_1 - R_2$$

and $G = R_1 - 2R_2$

The resistance R_3 required to convert the given galvanometer into voltmeter of range 0 to 2V is given by

$$R_3 = \frac{2V}{I_g} - G$$

$$\Rightarrow \quad R_3 = 4(R_1 - R_2) - (R_1 - 2R_2)$$

$$= 3R_1 - 2R_2$$

G in terms of R_1 and R_2 is given by $G = R_1 - 2R_2$

24. For equilibrium balance, net torque should also be equal to zero. When the field is off, $\Sigma\tau = 0$ considering the separation of each hung from mid-point be l.

$\because$ The magnetic force applied on CD by magnetic field must balance the weight.

$$\therefore \quad Mgl = W_{coil}\ l$$

$$\Rightarrow \quad 500\ gl = W_{coil}\ l \Rightarrow W_{coil} = 500 \times 9.8\ \text{N}$$

Taking moment of force about mid-point, we have the weight of coil. When the magnetic field is switched ON.

$$Mgl + mgl = W_{coil}\ l + IBL \sin 90°\ l \Rightarrow mgl = BIL\ l$$

$\therefore$ Additional mass,

$$m = \frac{BIL}{g} = \frac{0.2 \times 4.9 \times 1 \times 10^{-2}}{9.8}$$

$$= 10^{-3}\ \text{kg} = 1\ \text{g}$$

Thus, 1g of additional mass must be added to regain the balance.

25. For principle and working of galvanometer,

Refer to text on pages 194 and 195.

(i) **Cylindrical soft iron** core which not only makes the field radial but also increases the strength of the magnet.

(ii) **Radial magnetic field** is a field in which coil of the galvanometer always remains parallel to the field even on large deflection.

(iii) and (iv) refer to text on page 195.

Current sensitivity does not depend upon resistance (R), whereas voltage sensitivity does, as evident from their expression. Current sensitivity can be increased by increasing the number of turns of the coil. However, this increases the resistance of the coil, since voltage sensitivity decreases with increase in the resistance of the coil the effect of increase in number of turns is nullified in the case of voltage sensitivity

26. (i) Refer to text on pages 194 and 195.

(ii) (a) Refer to Sol. 22 (ii).

(b) Refer to Sol. 25 (iv).

27. (i) A galvanometer of range I_g and resistance G can be converted into

(a) a voltmeter of range V, by connecting a high resistance R in series with galvanometer whose value is given by

$$R = \frac{V}{I_g} - G$$

(b) an ammeter of range I, by connecting a very low resistance (shunt) in parallel with galvanometer whose value is given by

$$S = \frac{I_g G}{I - I_g}$$

(ii) Refer to text on page 192.

Thus, the nature of force is attractive.

When direction of flow of current is in opposite direction, the nature of force becomes repulsive.

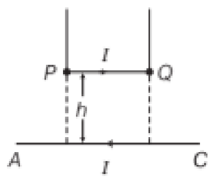
28. Here, $I = 8$ A, $\theta = 30°$, $B = 0.15$ T, $F = ?$, $l = 1$ m

We know that, $F = BIl \sin\theta$

$$\frac{F}{l} = BI \sin\theta = 0.15 \times 8 \times \sin 30°$$

$$= 0.15 \times 8 \times (1/2) = 0.6 \text{ N m}^{-1}$$

29. Mass of wire PQ, $m = 2.5$ g $= 2.5 \times 10^{-3}$ kg



Length of wire PQ, $l = 1$ m

Current in wire PQ and AC, $I = 25$ A

Let the wire PQ rises upto a height h.

The magnetic field on wire PQ due to wire AC is B.

By using the formula of magnetic field due to an infinite length of wire,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r} = \frac{\mu_0}{4\pi} \times \frac{2 \times 25}{h}$$

$$= \frac{10^{-7} \times 50}{h} = \frac{50 \times 10^{-7}}{h} \quad \text{...(i)}$$

The direction of magnetic field **B** on wire PQ is perpendicularly inwards to the plane of paper (by using Maxwell's right hand rule).

Force on wire PQ, $\mathbf{F} = I(\mathbf{l} \times \mathbf{B})$

[$\because$ angle between **l** and **B** is 90°]

$$\Rightarrow F = IlB \sin 90° = 25 \times 1 \times \frac{50 \times 10^{-7}}{h} \times 1 \text{ [From Eq.(i)]}$$

$$\Rightarrow F = \frac{1250 \times 10^{-7}}{h} \quad \text{...(ii)}$$

The wire will lift, if the weight of the wire is balanced by force due to wire AC.

i.e. $F = mg$

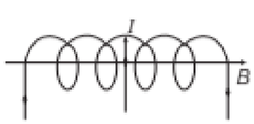
$$\Rightarrow \frac{1250 \times 10^{-7}}{h} = 2.5 \times 10^{-3} \times 9.8 \quad \text{[from Eq. (ii)]}$$

$$\therefore h = \frac{1250 \times 10^{-7}}{2.5 \times 9.8 \times 10^{-3}} = 51.02 \times 10^{-4} \text{ m}$$

$$= 51.02 \times 10^{-2} \text{ cm} = 0.51 \text{ cm}$$

Thus, the wire PQ will rise upto a height of 0.51 cm.

30. Here, the angle between the magnetic field and the direction of flow of current is 90°. Because the magnetic field due to a solenoid is along the axis of the solenoid and the wire is placed perpendicular to the axis.



Given, $l = 3$ cm $= 3 \times 10^{-2}$ m

$I = 10$ A, $B = 0.27$ T

The magnitude of magnetic force on the wire,

$$F = IlB \sin 90°$$

$$= 10 \times 3 \times 10^{-2} \times 0.27 \times \sin 90°$$

$$= 8.1 \times 10^{-2} \text{ N}$$

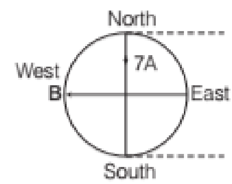
According to right hand palm rule, the direction of magnetic force is perpendicular to plane of paper inwards.

31. (i) Uniform magnetic field, $B = 1.5$ T

Radius, $r = 10.0$ cm $= 0.1$ m

Current in the wire, $I = 7.0$ A

The magnitude of force on the wire,



$$\mathbf{F} = I(\mathbf{l} \times \mathbf{B}) = IlB \sin 90°$$

[$\because$ angle between **l** and **B** is 90° and the length of wire is equal to the diameter of the cylindrical region]

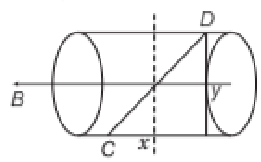
$\therefore$ Force on the wire,

$$F = I \times 2r \times B$$

$$= 7 \times 2 \times 0.1 \times 1.5 = 2.1 \text{ N}$$

According to Fleming's left hand rule, the direction of force is vertically inwards to the plane of paper.

(ii) Now, we take the component of length of wire. The horizontal component experiences no force as B is parallel to length.



The vertical component,

y = Diameter of the cylinder

So, force $F = IlB \sin 90°$

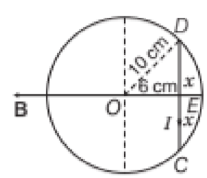
$$= 7 \times 0.1 \times 1.5 \times 2 \times 1$$

$$= 2.1 \text{ N}$$

According to the Fleming's left hand rule, the direction of force is perpendicularly inwards to the plane of paper.

(iii) Let the wire be shifted by 6 cm and the position of wire is CD.

$$OE = 6 \text{ cm}, OD = 10 \text{ cm}, DE = EC = x$$



In ΔODE, $OD^2 = OE^2 + DE^2$

$$\Rightarrow 100 = 36 + DE^2$$

$$\Rightarrow DE^2 = 64 \text{ or } DE = 8 \text{ cm}$$

and $l' = CD = 2DE = 16$ cm $= 0.16$ m

Magnitude of force,

$\mathbf{F}' = I(\mathbf{l} \times \mathbf{B}) = 7\ (0.16 \times 1.5 \times \sin 90°) = 1.68$ N

According to Fleming's left hand rule, the direction of force is vertically downwards to the plane of the paper.

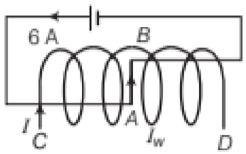
32. **For solenoid**

Given, length $l = 60$ cm,

Radius = 4 cm

Number of layers =3

Number of turns in each layer = 300



For wire

Given, length, $l_w = 2$ cm

Mass, $m = 2.5$ g, current $I_w = 6$ A

Let I be the current passing through the solenoid, so the magnetic field due to the solenoid.

$$B = \mu_0 nI \qquad \left[\because n = \frac{\text{Number of turns}}{\text{length}} = \frac{300 \times 3}{0.6}\right]$$

$$= 4\pi \times 10^{-7} \times \frac{300 \times 3}{0.6} \times I \qquad \text{...(i)}$$

Force on the wire, $\mathbf{F} = I_w(\mathbf{l}_w \times \mathbf{B}) = I_w(l_w B \sin\theta)$

[$\because$ angle between l_w and B is 90°]

This force balances by the weight of wire = mg

$$\therefore \qquad I_w l_w B \sin 90° = mg$$

$$6 \times 0.02 \times \frac{4\pi \times 10^{-7} \times 300 \times 3}{0.6} I = 2.5 \times 10^{-3} \times 9.8$$

[from Eq. (i)]

$$\text{Current, } I = \frac{2.5 \times 10^{-3} \times 9.8 \times 0.6}{108 \times 4\pi \times 10^{-7}} = 108.36 \text{ A}$$

33. 8×10^{-5} N ; refer to Example 4 on page 193.

34. 2×10^{-5} N; refer to Example 3 on page 192.

35. Force per unit length between the current carrying wires is given as

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}$$

where, I_1 = current in wire AB = 12 A

I_2 = current in wire CD = 5 A

and r = distance between wires = 1 mm = 1×10^{-3} m

$$\therefore \qquad \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} = mg$$

where, m = mass per unit length.

$$\Rightarrow \quad 10^{-7} \times \frac{2 \times 12 \times 5}{1 \times 10^{-3}} = m \times 10$$

$$\Rightarrow \qquad m = 10^{-7} \times \frac{2 \times 12 \times 5}{1 \times 10^{-3}} \times \frac{1}{10}$$

$$m = 1.2 \times 10^{-3} \text{ kg/m}$$

Current in CD should be in opposite direction to that in AB.

36. Refer to Example 5 on page 194. [**Ans.** 3.927 N-m]

37. Given, $N = 20$, $I = 12$ A, $B = 0.80$ T,

$$l = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}, \theta = 30°$$

$\because$ Area, $A = l^2 = (10 \times 10^{-2})^2 = 100 \times 10^{-4}$ m^2

As, $\qquad \tau = NBIA \sin\theta$

$$\Rightarrow \qquad \tau = 20 \times 0.80 \times 12 \times 100 \times 10^{-4} \times \sin 30°$$

$$= 9600 \times 10^{-4} = 0.96 \text{ N-m}$$

38. Here, $N = 30$, $R = 8.0$ cm $= 8 \times 10^{-2}$ m,

$$I = 6.0 \text{ A}, \theta = 60° \text{ and } B = 1.0 \text{ T}$$

(i) The magnitude of the counter torque

= magnitude of the deflecting torque

$= NAIB \sin\theta = N \cdot (\pi R^2) IB \sin\theta$

$= 30 \times 3.14 \times (8 \times 10^{-2})^2 \times 6.0 \times 1.0 \times \sin 60°$

$= 3.14$ N-m

(ii) The answer would not change as area enclosed by the coil as well as all other particulars remain unaltered and the formula, $\tau = NAIB \sin\theta$ is true for planar coil for any shape.

39. Given, number of turns, $N = 20$

Radius of circular coil, $r = 10$ cm $= 0.1$ m

Magnitude of magnetic field, $B = 0.1$ T

The angle between the area vector and magnetic field is 0°.

$$\Rightarrow \qquad \theta = 0°$$

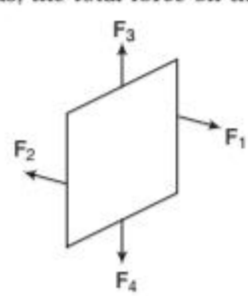
Current in the coil, $I = 5.0$ A

(i) Torque on the coil, $\tau = NIAB \sin\theta$

$$= 20 \times 5 \times \pi (0.1)^2 \times 0.1 \times \sin 0° = 0$$

[$\because \sin 0° = 0$]

(ii) The forces on the planar loop are in pairs, i.e. the forces on two opposite sides are equal and opposite to each other and on the other two opposite sides, they are same. Thus, the total force on the coil is zero.



($\because \mathbf{F}_1 = -\mathbf{F}_2$ and $\mathbf{F}_3 = -\mathbf{F}_4$)

(iii) Number density of electrons, $N = 10^{29}/\text{m}^3$

Area of cross-section of copper wire, $A = 10^{-5}\text{m}^2$

The magnitude of magnetic force, $F = e(\mathbf{v}_d \times \mathbf{B})$

$\because \quad I = neAv_d$

$$\therefore \quad v_d = \frac{I}{neA} \Rightarrow \quad F = e \cdot \frac{I}{NeA} \cdot B \sin 90°$$

$$= \frac{0.1 \times 5}{10^{-5} \times 10^{29}} \text{ N} \qquad [\because \sin 90° = 1]$$

$$= 5 \times 10^{-25}$$

40 Here, $B = 60$ G, $A = 2 \times 10^{-4}$ m^2, $N = 40$

$$I = 4 \text{ mA} = 4 \times 10^{-3} \text{ A}, \theta = 16°$$

$$\because \quad I = \frac{k}{NBA}\theta \Rightarrow k = \frac{NBAI}{\theta}$$

$$= \frac{40 \times 60 \times 2 \times 10^{-4} \times 4 \times 10^{-3}}{16}$$

$$= 1.2 \times 10^{-4} \text{ N-m per degree}$$

41. Given, $R_1 = 10\ \Omega$, $N_1 = 30$, $A_1 = 3.6 \times 10^{-3}\text{m}^2$,

$B_1 = 0.25$ T, $R_2 = 14\ \Omega$, $N_2 = 42$,

$A_2 = 1.8 \times 10^{-3}\text{m}^2$, $B_2 = 0.50$ T

$k_1 = k_2$ (spring constants are same) ...(i)

(i) Using the formula of current sensitivity, $I = \frac{NAB}{k}$

$$\therefore \quad \frac{I_{S_2}}{I_{S_1}} = \frac{N_2 B_2 A_2 k_1}{N_1 B_1 A_1 k_2} = \frac{42 \times 0.50 \times 1.8 \times 10^{-3}}{30 \times 0.25 \times 3.6 \times 10^{-3}}$$

$= 1.4$ [From Eq. (i)]

(ii) Using the formula of voltage sensitivity,

$$V = \frac{NAB}{kR}$$

$$\therefore \quad \frac{V_{S_2}}{V_{S_1}} = \frac{N_2 B_2 A_2 k_1 R_1}{k_2 R_2 N_1 B_1 A_1}$$

$$= \frac{42 \times 0.50 \times 1.8 \times 10^{-3} \times 10}{14 \times 30 \times 0.25 \times 3.6 \times 10^{-3}}$$

$= 1$ [from Eq. (i)]

42. Refer to Example 9 on page 195. [**Ans.** $0.01\ \Omega$]

43. Here, $n = 30$, $G = 100\ \Omega$, $E = 3$V, $R = 200\ \Omega$, $k = ?$

Total resistance $= G + R = 100 + 200 = 300\ \Omega$

$$I_g = \frac{E}{G + R} = \frac{3}{300} = \frac{1}{100} \text{ A}$$

$$k = \frac{I_g}{n} = \frac{1/100}{30} = \frac{1}{3} \times 10^{-3} \text{ A/div}$$

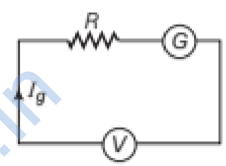
$$k = \frac{1}{3} \times 10^{-3} \times 10^6 \mu\text{A/div}$$

Figure of merit (Restoring torque per unit twist)

$$k = 333.3\ \mu\text{A/div}$$

44. Given, resistance of galvanometer coil, $G = 12\ \Omega$

Current in galvanometer,



$$I_g = 3 \text{ mA} = 3 \times 10^{-3} \text{ A}$$

and potential difference,

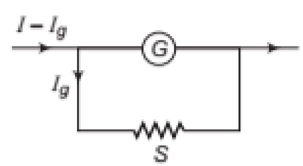
$$V = 18 \text{ V}$$

We can convert the galvanometer into voltmeter by using a large resistance R in series. The resistance can be calculated using the formula,

$$R = \frac{V}{I_g} - G$$

$$R = \frac{18}{3 \times 10^{-3}} - 12 = 5988\ \Omega$$

This resistance ($R = 5988\ \Omega$) is connected in series with the galvanometer. The resistance is connected in series because we have to increase the resistance of the galvanometer, so that almost no current flows through it and it gives an exact value of potential difference.



SUMMARY

- **Magnetic Field** The space in the surroundings of a magnet or a current carrying conductor in which its magnetic influence can be experienced is called magnetic field.
- **Oersted's Experiment** HC Oersted by his experiment observed that a current carrying conductor deflects magnetic compass needle placed near it.
- **Ampere's Swimming Rule** If a man is swimming along the wire in the direction of current with his face always turned towards the needle, so that the current enters through his feet and leaves at his head, then the *N*-pole of the magnetic needle will be deflected towards his left hand.
- **Biot-Savart's Law** This law deals with the magnetic field induction at a point due to a small current element, i.e.
$$dB \propto \frac{Idl\sin\theta}{r^2}$$
- **Permittivity and Permeability** Electric permittivity (ε_0), the degree of interaction of electric field with medium. Magnetic permeability, the ability of a substance to acquire magnetisation in a magnetic field.
- **Right Hand Thumb Rule** When the thumb of right hand is placed along the direction of current, the fingers curl around the conductor in the direction of magnetic field lines.
- Magnetic field at any point along the **axis of circular current carrying conductor** is $B = \dfrac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}$
- Magnetic field at the **centre of a circular current carrying conductor/ coil**
$$B = \frac{\mu_0 I}{2r}$$
- **Ampere's Circuital Law** According to this law, the line integral of the magnetic field **B** around any closed path in vacuum is equal to μ_0 times the net current enclosed by the curve,
i.e. $$\oint \mathbf{B} \times d\mathbf{l} = \mu_0 I$$
- **Magnitude of a Magnetic Field of a Straight Wire** It is given by $B = \dfrac{\mu_0 I}{2\pi r}$
- **Solenoid** It is an insulated long wire closely wound in the form of a helix.
- **Magnetic Field due to Straight Solenoid** At any point inside the solenoid, $B = \mu_0 nI$
At points near the end of air closed solenoid, $B = (\mu_0 nI/2)$.
- **Force on Moving Charge in a Uniform Magnetic Field** When a charged particle (q) moves with a velocity (v) inside a uniform magnetic field, then force acting on it is given by $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.
- **Magnetic Force On a Charged Particle** It is given by
$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B}).$$
- When charged particle enters into a magnetic field perpendicularly, then
(i) $\dfrac{mv^2}{r} = qvB$ (ii) $r = \dfrac{mv}{qB}$ (iii) $T = \dfrac{2\pi m}{qB}$
(iv) $\nu = \dfrac{qB}{2\pi m}$ (v) $KE = \dfrac{q^2B^2r^2}{2m}$
- **Lorentz Force** The sum of the electric force and magnetic force that can be exerted on a charged particle due to its electric charge (q) is called Lorentz force.
It is $$\mathbf{F} = q\,(\mathbf{E} + v \times \mathbf{B})$$
- **Force on a Current Carrying Conductor in a Uniform Magnetic Field** It is given by, $F = IlB\sin\theta$
- **Fleming's Left Hand Rule** If the forefinger, middle finger and the thumb of the left hand are stretched mutually at right angles to one another such that the forefinger points in the direction of magnetic field, middle finger in the direction of current, then thumb will point in the direction of force on the conductor.
- **Force between Two Parallel Current Carrying Conductors**
It is given by $$F = \left[\frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}\right]$$
- **Torque Experienced by a Current Loop in a Uniform Magnetic Field** It is given by $\tau = BINA\sin\theta$
- **Moving Coil Galvanometer** It is an instrument which is based on the fact that when a current carrying coil is placed in a magnetic field, then it experiences a torque.
- **Circular Current Loop as Magnetic Dipole** The magnitude of the magnetic field on the axis at a distance x from the centre of a circular loop of radius R carrying a steady current I is
$$B = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}}$$
- Torque on a bar magnet in a uniform magnetic field is
$$\tau = MB\sin\theta = \mathbf{M} \times \mathbf{B}$$

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. Vector form of Biot-Savart's law is

(a) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{\mathbf{l} \times d\mathbf{l}}{r^2}$

(b) $d\mathbf{B} = \frac{I\, d\mathbf{l} \times \mathbf{r}}{r^3}$

(c) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\, d\mathbf{l} \times \mathbf{r}}{r^3}$

(d) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\, d\mathbf{l} \times \mathbf{r}}{r^2}$

2. A polygon shaped wire is inscribed in a circle of radius R. The magnetic induction at the centre of polygon, when current flows through the wire is

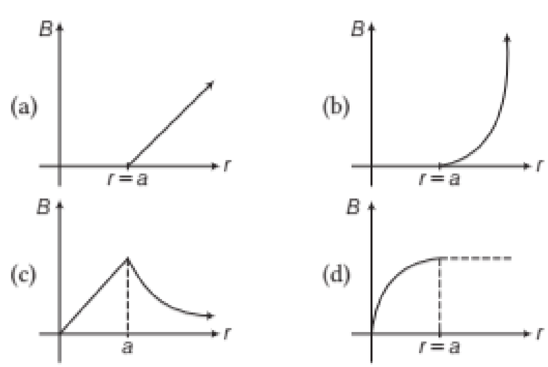
(a) $\frac{\mu_0 n I}{2\pi R}\tan\left(\frac{2\pi}{n}\right)$

(b) $\frac{\mu_0 n I}{2\pi R}\tan\left(\frac{4\pi}{n}\right)$

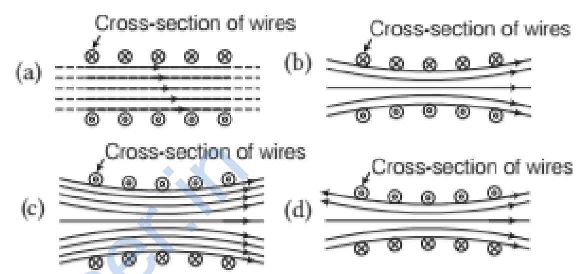
(c) $\frac{\mu_0 n I}{2\pi R}\tan\left(\frac{\pi}{n}\right)$

(d) $\frac{\mu_0 n I}{2\pi R}\tan\left(\frac{\pi}{n^2}\right)$

3. For a cylindrical conductor of radius a, which of the following graphs shows a correct relationship of B *versus* r?



4. Which of the following represent a correct figure to display of magnetic field lines due to a solenoid?



5. A long solenoid has 20 turns cm^{-1}. The current necessary to produce a magnetic field of 20 mT inside the solenoid is approximately

(a) 1A (b) 2 A (c) 4 A (d) 8 A

6. An electron is travelling horizontally towards East. A magnetic field in vertically downward direction exerts a force on the electron along

(a) East (b) West (c) North (d) South

7. Which of the following statements is correct?
CBSE 2021 (Term-I)

(a) Magnetic field lines do not form closed loops.

(b) Magnetic field lines start from North pole and end at South pole of a magnet.

(c) The tangent at a point on a magnetic field line represents the direction of the magnetic field at that point.

(d) Two magnetic field lines may intersect each other.

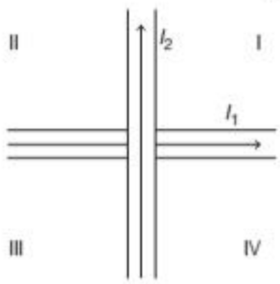
8. The magnetic field at the centre of a current carrying circular loop of radius R is B_1. The magnetic field at a point on its axis at a distance R from the centre of the loop is B_2, then the ratio $\left(\frac{B_1}{B_2}\right)$ is
CBSE 2021 (Term-I)

(a) $2\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 2

9. A current carrying wire kept in a uniform magnetic field will experience a maximum force, when it is **CBSE 2021 (Term-I)**

(a) perpendicular to the magnetic field
(b) parallel to the magnetic field
(c) at an angle of 45° to the magnetic field
(d) at an angle of 60° to the magnetic field

10. Two wires carrying currents I_1 and I_2 lie, one slightly above the other, in a horizontal plane as shown in figure. The region of vertically upward strongest magnetic field is **CBSE 2021 (Term-I)**



(a) I
(b) II
(c) III
(d) IV

11. Two parallel conductors carrying current of 4.0 A and 10.0 A are placed 2.5 cm apart in vacuum. The force per unit length between them is

(a) 6.4×10^{-5} N/m
(b) 6.4×10^{-2} N/m
(c) 4.6×10^{-4} N/m
(d) 3.2×10^{-4} N/m

12. A straight conducting rod of length l and mass m is suspended in a horizontal plane by a pair of flexible strings in a magnetic field of magnitude B. To remove the tension in the supporting strings, the magnitude of the current in the wire is **CBSE 2021 (Term-I)**

(a) $\frac{mgB}{l}$
(b) $\frac{mgl}{B}$
(c) $\frac{mg}{lB}$
(d) $\frac{lB}{mg}$

13. A proton and an alpha particle move in circular orbits in a uniform magnetic field. Their speeds are in the ratio of $9:4$. The ratio of radii of their circular orbits $\left(\frac{r_p}{r_{alpha}}\right)$ is **CBSE 2021 (Term-I)**

(a) $\frac{3}{4}$
(b) $\frac{4}{3}$
(c) $\frac{8}{9}$
(d) $\frac{9}{8}$

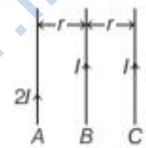
14. The SI unit of magnetic field intensity is **CBSE SQP (Term-I)**

(a) $A\text{-}mN^{-1}$
(b) $NA^{-1}\ m^{-1}$
(c) $NA^{-2}m^{-2}$
(d) $NA^{-1}m^{-2}$

15. The coil of a moving coil galvanometer is wound over a metal frame in order to **[CBSE SQP (Term-I)]**

(a) reduce hysteresis
(b) increase sensitivity
(c) increase moment of inertia
(d) provide electromagnetic damping

16. Three infinitely long parallel straight current carrying wires A, B and C are kept at equal distance from each other as shown in the figure . The wire C experiences net force F .The net force on wire C, when the current in wire A is reversed will be **CBSE SQP (Term-I)**



(a) zero
(b) $F/2$
(c) F
(d) $2F$

17. In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making 10 rps, the magnetic moment associated with the orbital motion of the electron will be **CBSE SQP (Term-I)**

(a) 2.512×10^{-38} A-m^2
(b) 1.256×10^{-38} A-m^2
(c) 0.628×10^{-38} A-m^2
(d) zero

18. The current sensitivity of a galvanometer increases by 20%. If its resistance also increases by 25%, then the voltage sensitivity will **CBSE SQP (Term-I)**

(a) decreased by 1%
(b) increased by 5%
(c) increased by 10%
(d) decreased by 4%

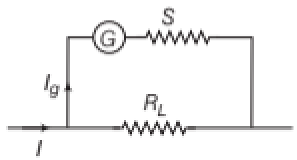
19. Two wires of the same length are shaped into a square of side a and a circle with radius r. If they carry same current, the ratio of their magnetic moment is **CBSE SQP (Term-I)**

(a) $2:\pi$
(b) $\pi:2$
(c) $\pi:4$
(d) $4:\pi$

20. The wire which connects the battery of a car to its starter motor carries current of 300 A during starting. Force per unit length between wires (wires are 0.7 m long and 0.015 m distant apart) is

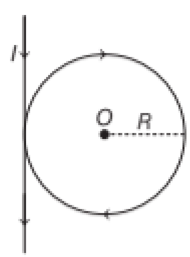
(a) $1.2\ Nm^{-1}$ repulsive
(b) $1.2\ Nm^{-1}$ attractive
(c) $2.4\ Nm^{-1}$ repulsive
(d) $2.4\ Nm^{-1}$ attractive

21. For the voltmeter circuit given,



(a) $\frac{I_g}{I} = \frac{G}{S}$
(b) $\frac{I}{I_g} = \frac{R_L + G}{S}$
(c) $(I - I_g)R_L = I_g(G + S)$
(d) $IR_L = I_g G$

22. A current I flows through a long straight conductor which is bent into a circular loop of radius R in the middle as shown in the figure.



The magnitude of the net magnetic field at point O will be **CBSE 2020 (Term-I)**
(a) zero
(b) $\frac{\mu_0 I}{2R}(1 + \pi)$
(c) $\frac{\mu_0 I}{4\pi R}$
(d) $\frac{\mu_0 I}{2R}\left(1 - \frac{1}{\pi}\right)$

23. A current of 10A is flowing from east to west in a long straight wire kept on a horizontal table. The magnetic field developed at a distance of 10 cm due north on the table is **[CBSE 2020]**

(a) 2×10^{-5} T, acting downwards
(b) 2×10^{-5} T, acting upwards
(c) 4×10^{-5} T, acting downwards
(d) 4×10^{-5} T, acting upwards

24. An electron and a proton are moving along the same direction with the same kinetic energy. They enter a uniform magnetic field acting perpendicular to their velocities. The dependence of radius of their paths on their masses is **CBSE 2020**

(a) $r \propto m$ (b) $r \propto \sqrt{m}$
(c) $r \propto \frac{1}{m}$ (d) $r \propto \frac{1}{\sqrt{m}}$

25. A current of 5 A is flowing from east to west in a long straight wire kept on a horizontal table. The magnetic field developed at a distance of 10 cm due south on the table is **CBSE 2020**

(a) 1×10^{-5} T, acting downwards
(b) 1×10^{-5} T, acting upwards
(c) 2×10^{-5} T, acting downwards
(d) 2×10^{-5} T, acting upwards

26. There are uniform electric and magnetic fields in a region pointing along X-axis. An α-particle is projected along Y-axis with a velocity v. The shape of the trajectory will be **CBSE 2020**

(a) circular in XZ-plane
(b) circular in YZ-plane
(c) helical with its axis parallel to X-axis
(d) helical with its axis parallel to Y-axis

27. The coil of a galvanometer consists of 100 turns and effective area of $1\ cm^2$. The restoring couple is $10^{-8}\ Nm\ rad^{-1}$. The magnetic field between poles is of 5 T. Current sensitivity of this galvanometer is

(a) 5×10^4 rad/μ amp (b) 5×10^6 per amp
(c) 2×10^{-7} per amp (d) 5 rad/μ amp

ASSERTION AND REASON

Directions (Q. Nos. 28-33) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

28. **Assertion** If a proton and an α-particle enter a uniform magnetic field perpendicularly with the same speed, the time period of revolution of α-particle is double than that of proton.

Reason In a magnetic field, the period of revolution of a charged particle is directly proportional to the mass of the particle and inversely proportional to the charge of particle.

29. **Assertion** The magnetic field produced by a current-carrying solenoid is independent of its length and cross-sectional area.

Reason The magnetic field inside the solenoid is uniform.

30. **Assertion** An electron and a proton enter a magnetic field with equal velocities, then the force experienced by the proton will be more than electron.

Reason The mass of proton is 1837 times more than that of electron.

31. **Assertion** A proton and an electron, with same momenta, enter in a magnetic field in a direction at right angles to the lines of the force. The radius of the paths followed by them will be same.

Reason Electron has less mass than the proton.

CBSE SQP (Term1)

32. **Assertion** On increasing the current sensitivity of a galvanometer by increasing the number of turns, may not necessarily increase its voltage sensitivity.

Reason The resistance of the coil of the galvanometer increases on increasing the number of turns. **CBSE SQP (Term1)**

33. **Assertion** When radius of a current carrying loop is doubled, its magnetic moment becomes four times.

Reason The magnetic moment of a current carrying loop is directly proportional to the area of the loop. **CBSE 2021 (Term1)**

CASE BASED QUESTIONS

Directions (Q.Nos. 34-35) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

34. **Electron Moving in Magnetic Field**

An electron with a speed $v_0 << c$ moves in a circle of radius r_0 in a uniform magnetic field. This electron is able to traverse a circular path as magnetic field is perpendicular to the velocity of the electron. The time required for one revolution of the electron is T_0. The speed of the electron is now doubled to $2v_0$.

(i) The radius of the circle will change to

(a) $4r_0$ (b) $2r_0$
(c) r_0 (d) $r_0/2$

(ii) The time required for one revolution of the electron will change to

(a) $4T_0$ (b) $2T_0$
(c) T_0 (d) $T_0/2$

(iii) A charged particle is projected in a magnetic field $\mathbf{B} = (2\hat{i} + 4\hat{j}) \times 10^2$ T. The acceleration of the particle is found to be $\mathbf{a} = (x\hat{i} + 2\hat{j})\,ms^{-2}$. Find the value of x.

(a) $4\ ms^{-2}$ (b) $-4\ ms^{-2}$
(c) $-2\ ms^{-2}$ (d) $2\ ms^{-2}$

(iv) If the given electron has a velocity not perpendicular to **B**, then the trajectory of the electron is

(a) straight line (b) circular
(c) helical (d) zig-zag

(v) If this electron of charge (e) is moving parallel to uniform magnetic field with constant velocity v. The force acting on the electron is

(a) Bev (b) $\frac{Be}{v}$
(c) $\frac{B}{ev}$ (d) zero

35. **Moving Coil Galvanometer**

Moving coil galvanometer operates on Permanent Magnet Moving Coil (PMMC) mechanism and was designed by the scientist D'arsonval.

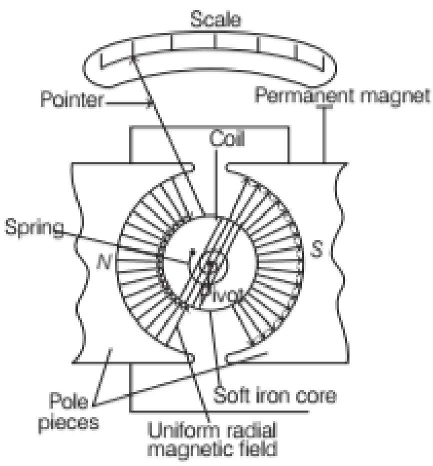
Moving coil galvanometers are of two types

(i) Suspended coil

(ii) Pivoted coil type or tangent galvanometer.

Its working is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.

This torque tends to rotate the coil about its axis of suspension in such a way that the magnetic flux passing through the coil is maximum.



(i) A moving coil galvanometer is an instrument which

(a) is used to measure emf

(b) is used to measure potential difference

(c) is used to measure resistance

(d) is a deflection instrument which gives a deflection when a current flows through its coil

(ii) To make the field radial in a moving coil galvanometer,

(a) number of turns of coil is kept small

(b) magnet is taken in the form of horse-shoe

(c) poles are of very strong magnets

(d) poles are cylindrically cut

(iii) The deflection in a moving coil galvanometer is

(a) directly proportional to torsional constant of spring

(b) directly proportional to the number of turns in the coil

(c) inversely proportional to the area of the coil

(d) inversely proportional to the current in the coil

(iv) In a moving coil galvanometer, having a coil of N-turns of area A and carrying current I is placed in a radial field of strength B.
The torque acting on the coil is

(a) NA^2B^2I (b) $NABI^2$

(c) N^2ABI (d) $NABI$

(v) To increase the current sensitivity of a moving coil galvanometer, we should decrease

(a) strength of magnet

(b) torsional constant of spring

(c) number of turns in coil

(d) area of coil

VERY SHORT ANSWER Type Questions

36. State the rule that is used to find the direction of magnetic field at a point near a current carrying straight conductor.

37. What will be the magnetic field at the centre of a circular coil carrying current, when the current through the coil is doubled and the radius of the coil is halved?

38. What is the force on a charge moving along the direction of the magnetic field?

39. Name the force which is experienced by a moving charged particle in electric and magnetic field.

40. Under what condition does an electron moving through a magnetic field experience maximum force?

41. A charged particle moves through a magnetic field. Is the momentum of the particle affected?

42. An electron beam projected along $+X$-axis experiences a force due to a magnetic field along the $+Y$-axis. What is the direction of the magnetic field?

43. In a certain arrangement, a proton does not get deflected while passing through a magnetic field region. Under what condition is it possible?

44. Write the expression for the force between parallel current carrying conductors.

45. An electron with charge $-e$ and mass m travels at a speed v in a plane perpendicular to a magnetic field of magnitude B. The electron follows a circular path of radius R. In a time t, the electron travels half-way around the circle. What is the amount of work done by the magnetic field?

46. A solenoid with n loops of wire tightly wrapped around an iron core is carrying an electric current I. If the current through this solenoid is reduced to half, then what change would you expect in inductance L of the solenoid.?

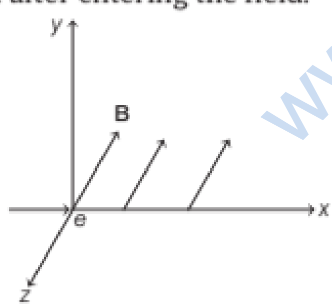
47. A proton is accelerated through a potential difference V, subjected to a uniform magnetic field acting normal to the velocity of the proton. If the potential difference is doubled, how will the radius of the circular path described by the proton in the magnetic field change? **CBSE 2019**

48. Write the relation for the force acting on a charged particle q moving with velocity $\mathbf{v}$ in the presence of a magnetic field $\mathbf{B}$. **CBSE 2019**

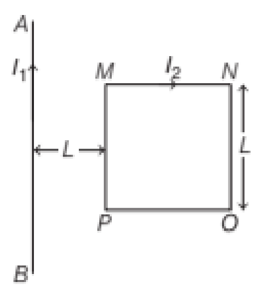
49. When a charge q is moving in the presence of electric (E) and magnetic (B) fields which are perpendicular to each other and also perpendicular to the velocity v of the particle, write the relation expressing v in terms of E and B. **CBSE 2019**

50. Define the term current sensitivity of a moving coil galvanometer. **CBSE 2020**

51. An electron moves along $+x$ direction. It enters into a region of uniform magnetic field $\mathbf{B}$ directed along $-z$ direction as shown in figure. Draw the shape of trajectory followed by the electron after entering the field. **CBSE 2020**



52. A square shaped current carrying loop $MNOP$ is placed near a straight long current carrying wire AB as shown in the figure. The wire and the loop lie in the same plane. If the loop experiences a net force F towards the wire, find the magnitude of the force on the side NO of the loop. **CBSE 2020**



SHORT ANSWER Type Questions

53. A deuteron and an alpha particle having same momentum are in turn allowed to pass through a magnetic field $\mathbf{B}$, acting normal to the direction of motion of the particles. Calculate the ratio of the radii of the circular paths described by them. **CBSE 2019**

54. A charged particle q is moving in the presence of a magnetic field B which is inclined to an angle 30° with the direction of the motion of the particle. Draw the trajectory followed by the particle in the presence of the field and explain how the particle describes this path. **CBSE 2019**

55. An α-particle and a proton of the same kinetic energy are in turn allowed to pass through a magnetic field $\mathbf{B}$, acting normal to the direction of motion of the particles. Calculate the ratio of radii of the circular paths described by them. **CBSE 2019**

56. Two similar coils are placed mutually perpendicular such that their centres coincide. At centre, what will be the ratio of the magnitudes of magnetic fields due to one coil and the resultant magnetic field?

57. In what way, current carrying solenoid behaves like a bar magnet. Find the magnetic field induction at the axis of solenoid due to current flowing through it.

58. What is Lorentz force? Give some important characteristics of this force.

59. Equal currents are flowing through two infinitely long parallel wires in the same direction. What will be the magnetic field at a point mid-way between the two wires?

60. Deduce an expression for the torque on a current carrying loop suspended in a uniform magnetic field.

61. In a moving coil galvanometer having a coil of N turns of area A and carrying current I and is placed in a radial field of strength B. What will be the torque acting on the coil?

62. Is it possible to decrease or increase the range of given voltmeter? Explain.

LONG ANSWER Type I Questions

63. Using Ampere's circuital law, find an expression for the magnetic field at a point situated at a normal distance R from an infinitely long current carrying straight wire.

64. What is the force that a conductor of length dl carrying a current I experiences, when placed in a magnetic field $\mathbf{B}$? What is the direction of this force?

65. An electron being accelerated through a potential difference of V enters a uniform magnetic field of $\mathbf{B}$ perpendicular to the direction of motion. Find the radius of path described by the electron.

66. (a) Derive the expression for the torque acting on a current carrying loop placed in a magnetic field.

(b) Explain the significance of a radial magnetic field when a currrent carrying coil is kept in it. **CBSE 2019**

67. (a) State the underlying principle of a moving coil galvanometer.

(b) Give two reasons to explain why a galvanometer cannot as such be used to measure the value of the current in a given circuit.

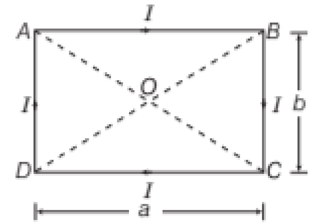
(c) Define the terms (i) voltage sensitivity and (ii) current sensitivity of a galvanometer. **CBSE 2019**

68. Two infinitely long straight wires A_1 and A_2 carrying currents I and $2I$ flowing in the same directions are kept d distance apart. Where should a third straight wire A_3 carrying current 1.5 I be placed between A_1 and A_2, so that it experiences no net force due to A_1 and A_2? Does the net force acting on A_3 depend on the current flowing through it?

LONG ANSWER Type II Questions

69. Three wires of equal lengths are bent into the form of three loops. One of the loops is square-shaped, second loop is triangular-shaped and third loop is circular. These are suspended in a uniform magnetic field and the same current is passed through them. Which loop will experience greater torque? Give reasons.

70. A rectangular current carrying loop of length a and breadth b is shown in the figure. Find the magnetic field at the centre of the loop.



71. Two straight infinitely long wires are fixed in space, so that the current in the left wire is 2 A and directed out of the plane of the page and the current in the right wire is 3 A and directed into the plane of the page. In which region(s) is/are there a point on the X-axis, at which the magnetic field is equal to zero due to these currents carrying wires? **CBSE SQP (Term-I)**
Justify your answer.

72. (a) Show that a current carrying solenoid behaves like a small bar magnet. Obtain the expression for the magnetic field at an external point lying on its axis.

(b) A steady current of 2A flows through a circular coil having 5 turns of radius 7 cm. The coil lies in xy-plane with its centre at the origin. Find the magnitude and direction of the magnetic dipole moment of the coil. **CBSE 2020**

Or

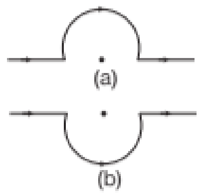
(a) Derive the expression for the force acting between two long parallel current carrying conductors. Hence, define 1A current.

(b) A bar magnet of dipole moment 3 Am^2 rests with its centre on a frictionless pivot. A force F is applied at right angles to the axis of the magnet, 10 cm from the pivot. It is observed that an external magnetic field of 0.25 T is required to hold the magnet in equilibrium at an angle of 30° with the field. Calculated the value of F. How will the equilibrium be effected if F is withdrawn? **CBSE 2020**

NUMERICAL PROBLEMS

73. A copper coil of 100 turns, radius 8×10^{-2} m carries a current of 0.40 A. What will be the magnitude of magnetic field at the centre of coil ?

74. A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in Fig. (a). Consider the magnetic field **B** at the centre of the arc.



(i) What is the magnetic field due to the straight segments?

(ii) In what way the contribution to **B** from the semi-circle differs from that of a circular loop and in what way does it resemble?

(iii) Would your answer be different, if the wire was bent into a semi-circular arc of the same radius but in the opposite way as shown in Fig. (b)?

75. A closely wound solenoid 0.80 m long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8×10^{-2}m. If the current carried is 0.8A, what will be the magnitude of field near the centre?

76. A beam of protons passes undeflected with a horizontal velocity *v*, through a region of electric and magnetic fields, mutually perpendicular to each other and normal to the direction of beam. If the magnitudes of electric and magnetic fields are 100 kV/m and 50 mT respectively, calculate the

(i) velocity of the beam and

(ii) force with which it strikes the target on a screen, if the proton beam current is equal to 0.80 mA.

77. Two concentric circular wire loops of radii 20 cm and 30 cm are located in an *XY*-plane, each carries a clockwise current of 7 A.

(i) Find the magnitude of the net magnetic dipole moment of the system.

(ii) Repeat for reversed current in the inner loop.

78. The coil of galvanometer consists of 100 turns and effective area of 1 cm^2. The restoring couple is 10^{-8} N-m/rad. The magnetic field between poles is of 5 T. What will be the current sensitivity of galvanometer?

79. The current sensitivity of a MCG increases by 20% when its resistance is increased by a factor of 2. Calculate by what factor the voltage sensitivity changes?

80. A galvanometer with a coil of resistance 12.0 Ω shows full scale deflection for a current of 2.5 mA. How will you convert this meter into

(i) an ammeter of range 0 to 7.5 A?

(ii) a voltmeter of range 0 to 10 V? Determine the net resistance of the meter in each case. When an ammeter is put in a circuit, does it read less or more than the actual current in the original circuit? When a voltmeter is put across a part of the circuit, does it read less or more than the required voltage drop? Explain.

81. A galvanometer having 30 divisions has a current sensitivity of 20 μA/ div. It has a resistance of 25 Ω.

(i) How will you convert it into an ammeter of range 0-1 A?

(ii) How will you convert this ammeter into a voltmeter of range 0-1 V?

ANSWERS

1. (c)	2. (c)	3. (c)	4. (c)	5. (d)
6. (d)	7. (c)	8. (a)	9. (a)	10. (b)
11. (d)	12. (c)	13. (d)	14. (b)	15. (d)
16. (a)	17. (b)	18. (d)	19. (c)	20. (a)
21. (c)				

22. (*d*) The magnetic field due to the long straight conductor at *O* is given by

$$B_1 = \frac{\mu_0 I}{2\pi R} \otimes$$

and that due to circular loop of radius *R* is,

$$B_2 = \frac{\mu_0 I}{2R} \bullet$$

As, $$B_2 > B_1$$

∴ The magnitude of net magnetic field at point O is

$$B_{net} = B_2 - B_1 = \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1 - \frac{1}{\pi}\right) \bullet$$

23. (*a*) Refer to example-1 given on Page-179

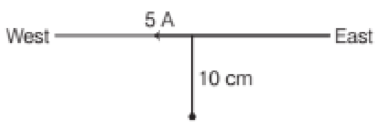
24. (*b*) Since, the angle between their velocities and uniform magnetic field is 90°. So, the path followed by them will be circular in nature.

The radius of circular path followed by charged particle in a uniform magnetic field,

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

As kinetic energy K is same, so $r \propto \sqrt{m}$.

25. (*b*) The situation is as shown,



Magnetic field due to long straight wire is

$$B = \frac{\mu_0 I}{2\pi r}$$

Here, $I = 5$A and $r = 10$ cm $= 0.1$ m

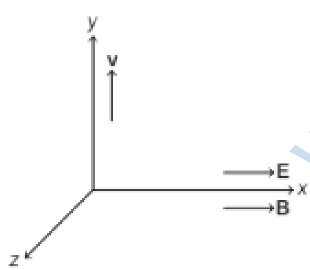
$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.1} = 1 \times 10^{-5} \text{ T}$$

The direction of magnetic field is given by right hand thumb rule. So, the direction of magnetic field of point 10 cm is upward due south on the table.

26. (*a*) The net force acting on α-particle is given by

$$F = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

The direction of E, v and **B** are shown as,



$$\therefore \quad \mathbf{F} = q[E\hat{i} + (v\hat{j} \times B\hat{i})]$$

$$= q\,[E\hat{i} + vB(-\hat{k})]$$

So, the shape of trajectory will be circular in xz- plane.

27. (*b*) **28.** (*a*) **29.** (*b*) **30.** (*d*)

31. (*b*) **32.** (*a*) **33.** (*a*)

34. (i) (*b*) As, $r_0 = \frac{mv}{qB} \Rightarrow r' = \frac{m(2v_0)}{qB} = 2r_0$

(ii) (c) As, $T = \frac{2\pi m}{qB}$

Thus, it remains same as it is independent of velocity.

(iii)(*b*) As, $F \perp B$

Hence, $a \perp B$

$$\therefore \quad \mathbf{a} \cdot \mathbf{B} = 0$$

$$\Rightarrow (x\hat{i} + 2\hat{j}) \cdot (2\hat{i} + 4\hat{j}) = 0$$

$$2x + 8 = 0$$

$$\Rightarrow \quad x = -4 \text{ ms}^{-2}$$

(iv) (*c*) If the charged particle has a velocity not perpendicular to **B**, then component of velocity along **B** remains unchanged as the motion along the **B** will not be affected by **B**.

Then, the motion of the particle in a plane perpendicular to **B** is as before circular one. Thereby, producing helical motion.

(v) (*d*) The force on electron, $F = qvB\sin\theta$

The electron is moving parallel to the **B**, so $\theta = 0°$.

$$\Rightarrow \quad F = qvB\sin 0° = 0$$

35. (i) (*d*) A moving coil galvanometer is a sensitive instrument which is used to measure a deflection when a current flows through its coil.

(ii) (*d*) Uniform field is made radial by cutting pole pieces cylindrically.

(iii) (*b*) The deflection in a moving coil galvanometer, $\phi = \frac{NAB}{k} \cdot I$ or $\phi \propto N$, where N is number of turns in a coil, B is magnetic field and A is area of cross-section.

(iv) (*d*) The deflecting torque acting on the coil,

$$\tau_{deflection} = NIAB$$

(v) (*b*) Current sensitivity of galvanometer

$$\frac{\phi}{I} = S_i = \frac{NBA}{k}$$

Hence, to increase (current sensitivity) S_i, (torsional constant of spring) k must be decreased.

36. Right hand thumb rule states that, if we imagine a linear wire conductor to be need in the grip of the right hand such that the thumb points in the direction of current, then the curvature of the fingers around the conductor will give the direction of magnetic field lines.

37. Magnetic field at the centre of the coil is given by

$$B = \frac{\mu_0 I}{2R}$$

$$B' = \frac{\mu_0(2I)}{2(R/2)} = 4B$$

38. Force on a moving charge in magnetic field is given as,

$$F = qvB\sin\theta$$

Here, $\theta = 0° \Rightarrow F = 0$

39. Lorentz force

40. Magnetic forec, $\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = qvB\sin\theta$

Maximum force, $F_{max} = qvB$

When, $\sin\theta = 1$ or $\theta = 90°$

41. No, its momentum does not get affected.

42. Direction of magnetic field is in Z-axis direction.

43. When it is along the magnetic field.

44. Force between the parallel current carrying conductors is $F = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{r}$

45. When a charged particle move in a plane, perpendicular to the direction of magnetic field, then a magnetic force acts on it. The direction of this force is perpendicular to the velocity of the charged particle.

So, no work is done, i.e. work done is zero.

46. The inductance (L) of a solenoid does not depend on the current flowing through it, but depends on the magnetic field and permeability of material of iron core. So, on reducing the current to half, the L remains same.

47. The kinetic energy of proton due to potential V is given by

$$K = eV$$

where, e = charge on proton.

The radius of circular path of proton in a magnetic field is

$$r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2meV}}{qB}$$

If potential is doubled, i.e. $V' = 2V$, then

$$r' = \frac{\sqrt{2me \times 2V}}{qB} = \sqrt{2}r$$

Thus, radius becomes $\sqrt{2}$ times of previous value.

48. When a charged particle q moves with velocity $\mathbf{v}$ in a uniform magnetic field $\mathbf{B}$, then the force acting on it is given by

$$\mathbf{F} = q\,(\mathrm{v} \times \mathrm{B})$$

49. $F_{\text{lorentz}} = F_{\text{electric}} + F_{\text{magnetic}}$

$$= q\mathbf{E} + q\,(\mathbf{v} \times \mathbf{B}) = q[\mathrm{E} + (\mathbf{v} \times \mathbf{B})]$$

50. Current sensitivity of the galvanometer is the deflection per unit current flowing through it.

It is given as, $I_C = \frac{\theta}{I} = \frac{NAB}{R}$

where, R is the restoring torque per unit twist of phosphor bronze strip.

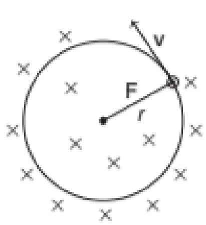
51. The magnetic force acting on the electron is given as,

$$\mathrm{F} = e(\mathrm{v} \times \mathrm{B}) = evB\sin\theta$$

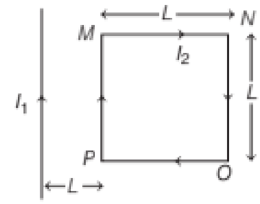
If the electron moves along $+x$ direction and B is directed along $-z$ direction, then $\theta = 90°$.

$\Rightarrow \quad F = evB$

So, the trajectory followed by the electron after entering the field will be circular as shown below



52. The given loop can be shown below as



The force acting on the arms MN and PO of the given loop are equal, mutually opposite and collinear. Hence, they balance each other.

Force on arm PM,

$$F_1 = \frac{\mu_0 I_1 I_2 L}{2\pi L}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \text{ attractive in nature} \quad \text{... (i)}$$

Force on arm NO,

$$F_2 = \frac{\mu_0 I_1 I_2 L}{2\pi(2L)}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}, \text{ repulsive in nature} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$F_{\text{net}} = F = F_1 - F_2 = \frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}, \text{ attractive in nature} \quad \text{...(iii)}$$

So, from Eqs. (ii) and (iii), we can conclude that the magnitude of the force on side NO of the loop is $F = \left(\frac{\mu_0 I_1 I_2}{4\pi}\right)$ when the net force F is towards the wire.

53. Mass on deuteron, $m_d = 2\,m$

Charge on deuteron, $q_d = e$

Mass on α-particle, $m_\alpha = 4\,m$

Charge on α-particle, $q_\alpha = 2e$

The radius of circular path is given by

$$r = \frac{mv}{qB}$$

Momentum of particle, $p = mv$

$\therefore \quad r \propto \frac{1}{qB} \quad$ [$\because$ momentum is same]

So, $\quad \frac{r_d}{r_\alpha} = \frac{q_\alpha}{q_d} = \frac{2e}{e} = \frac{2}{1}$

or $\quad r_d : r_\alpha = 2:1$

54. When an charged particle q enters a uniform magnetic field at an angle of 30°, then its path becomes helix of radius

$$r = \frac{mv\sin 30°}{eB} = \frac{mv}{2eB}$$

For diagram and discription Refer to text on pages 181 and 182 (Force on a Moving Charge in a Uniform Magnetic Field)

55. Refer to sol. of Example 4 on page 182.

56. Ratio $= \dfrac{B_1}{\sqrt{B_1^2 + B_2^2}} = \dfrac{1}{\sqrt{2}}$

57. Refer to text on pages 180 and 181.

58. Refer to text on page 183.

59. Zero

60. Refer to text on pages 193 and 194.

61. Refer to text on pages 194 and 195.

62. Refer to text on page 196.

63. Refer to text on pages 179 and 180.

64. Refer to text on page 191.

65. $K = \dfrac{1}{2}mv^2 = eV$

$$r = \frac{mv}{qB} = \frac{mv}{eB} = \sqrt{\frac{2mV}{eB^2}}$$

66. (a) Refer to text on pages 193 and 194 (Torque Experienced by a Current Loop in Uniform Magnetic Field).

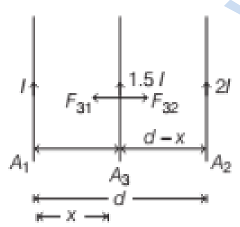
(b) In a radial magnetic field, the magnetic torque remains maximum for all positions of the coils.

67. (a) Refer to text on page 194 [Moving Coil Galvanometer (Principle)].

(b) Refer to page 195.

(c) Refer to text on page 195 (Working of Moving Coil Galvanometer)

68. Let the current in the third wire A_3 be in same direction as that of A_1 and A_2, so it will experience attractive force due to both.



The force on A_3 due to A_1 is $F_{31} = \dfrac{\mu_0}{2\pi} \cdot \dfrac{I \times 1.5\, I \times l}{x}$

where, l = unit length of conductor wire A_2 and x = distance between A_1 and A_3.

Similarly, force on A_3 due to A_2 is

$$F_{32} = \frac{\mu_0}{2\pi} \cdot \frac{1.5I \times 2I \times l}{(d - x)}$$

According to question, $F_{31} = F_{32}$

$$\Rightarrow \quad \frac{\mu_0}{2\pi} \cdot \frac{1.5I^2 l}{x} = \frac{\mu_0}{2\pi} \frac{3I^2}{(d - x)} l \Rightarrow \frac{1.5}{x} = \frac{3}{d - x}$$

$$\Rightarrow \quad d - x = 2x \text{ or } x = \frac{d}{3}$$

Yes, the net force acting on A_3 depends on the current flowing through it.

69. For magnetic moment refer Q. 17 on page 198.

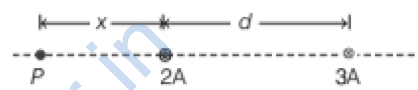
Now, apply the formula, $\tau = MB$

Square will experience maximum torque.

70. Refer to Example 2 on page 170.

$$\left[\textbf{Ans. } B = \frac{2\mu_0 I \sqrt{a^2 + b^2}}{\pi ab}\right]$$

71. Let d be the distance between two current carrying wires, then the magnetic field in the region I at a point P at a distance x can be calculated using figure given below.

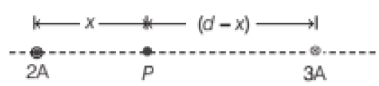


Due to 2A, $B_1 = \dfrac{\mu_0 \times I_1}{2\pi x} = \dfrac{\mu_0 \times 2}{2\pi x}$, downward

Due to 3A, $B_2 = \dfrac{\mu_0 \times I_2}{2\pi(x + d)} = \dfrac{\mu_0 \times 3}{2\pi(x + d)}$, upward

$\therefore$ Net magnetic field, $B_P = \dfrac{\mu_0}{2\pi}\left(\dfrac{2}{x} - \dfrac{3}{x + d}\right)$, downward ...(i)

The magnetic field in region II is

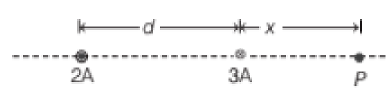


Due to 2A, $B_1 = \dfrac{\mu_0 \times 2}{2\pi x}$, upward

Due to 3A, $B_2 = \dfrac{\mu_0 \times 3}{2\pi(d - x)}$, upward

$\therefore$ Net magnetic field, $B_P = \dfrac{\mu_0}{2\pi}\left(\dfrac{2}{x} + \dfrac{3}{d - x}\right)$, upward

The magnetic field in region III is



Due to 2A, $B_1 = \dfrac{\mu_0 \times 2}{2\pi(x + d)}$, upward

Due to 3A, $B_2 = \dfrac{\mu_0 \times 3}{2\pi d}$, downward

$\therefore$ Net magnetic field, $B_P = \dfrac{\mu_0}{2\pi}\left(\dfrac{3}{d} - \dfrac{2}{x + d}\right)$, downward

As, the current and hence the magnetic field, due to 2A is less than that due to 3A.

So, for zero magnetic field, $\frac{\mu_0}{2\pi}\left(\frac{2}{x}-\frac{3}{x+d}\right)=0$

$\Rightarrow \qquad 2x + 2d = 3x$ or $x = 2d$

So, the point lies in region I.

72. (a) Refer to text pages 180 and 181.

(b) Given, $I = 2\text{A}$ and $N = 5$

and $r = 7$ cm $= 0.07$ m

Magnetic dipole moment of a coil,

$M = NIA = 5 \times 2 \times \pi\,(0.07)^2 = 0.154\ \text{Am}^2$

The direction of magnetic dipole moment is perpendicular to the plane of coil, i.e. it is along Z-axis.

Or

(a) Refer to text on page 192.

(b) Given, $M = 3\text{Am}^2$, $d = 10$ cm $= 0.1$ m

$B = 0.25$ T and $\theta = 30°$

The torque acting on the bar magnet,

$\tau = MB \sin\theta\hat{\mathbf{n}} = \mathbf{M}\times\mathbf{B}$

Also, $\tau = F\cdot d \Rightarrow F\cdot d = MB\sin\theta$

$$\Rightarrow \quad F = \frac{MB\sin\theta}{d} = \frac{3\times 0.25\times \sin 30°}{0.1} = \frac{0.75}{2\times 0.1} = 3.75\ \text{N}$$

If F is withdrawn from the bar magnet, then it will rotate, due to the torque (τ) and align itself along the field direction.

73. Refer to Example 3 on page 171.

[**Ans.** $B = \frac{\mu_0 I}{2r} = 3.1\times 10^{-4}$ T]

74. (i) Zero, magnetic field due to a semi-circular wire at its centre is half of magnetic field due to a circular loop.

(ii) Now, refer to text on page 170.

$$B_{\text{semi-circle}} = \frac{\mu_0 I}{4r} = 37.68\times 10^{-5}\ \text{T}$$

(iii) The magnitude of the magnetic field remains same but the direction will be opposite.

75. Refer to Example 3 on page 181.

[**Ans.** $B = \mu_0 nI = 2.5\times 10^{-2}$ T]

76. For undeflected beam, $v = \frac{E}{B}$

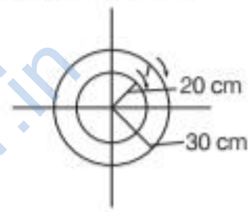
(i) 2×10^6 m/s

(ii) $\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = 1.675\times 10^{-5}$ N

77. (i) $M_1 = N_1 I_1 A_1 \otimes$

$M_2 = N_2 I_2 A_2 \otimes$

$\therefore M = M_1 + M_2 = 2.86$ A-m^2



(ii) $M = |M_1 - M_2| = 1.10$ A-m^2

78. Current sensitivity, $I_s = \frac{NAB}{K} = \frac{100\times 1\times 10^{-4}\times 5}{10^{-8}}$

$= 5\times 10^6\ \text{A}^{-1}$

79. Refer to Example 8 on page 195.

[**Ans.** Decreased by a factor 0.4]

80. Refer to Example 9 and 10 on pages 195 and 196.

[**Ans.** (i) Resistance of ammeter $4\times 10^{-3}\,\Omega$

(ii) Resistance of voltmeter $= 4000\ \Omega$

81. Refer to Example 9 on pages 195 and 196.

[**Ans.** (i) Shunt $= 0.815\Omega$

(ii) Resistance in series 0.985Ω