Chapter 5

Measures of Central Tendency

Central Tendency
A central tendency refers to a central value or a representative value of a statistical series.
According to Clark, "An average is a figure that represents the whole group".

Types of Statistical Averages
Averages are broadly classified into two categories

- Mathematical Averages
- Positional Averages

Arithmetic Mean

Arithmetic Mean is the number which is obtained by adding the values of all the items of a series and dividing the total by the number of items.
Arithmetic Mean is generally written as X. It may be expressed in the form of following formula
$X^{---}=x1+x2+x3+.....xNN$ or $\Sigma X^{---}N$

Types of Arithmetic Mean

- Simple Arithmetic Mean
- Weighted Arithmetic Mean

Methods of Calculating Simple Arithmetic Mean

(i) Individual Series In the case of individual series, Arithmetic Mean may be calculated by two methods

- Direct Method According to this method, we find the Arithmetic mean from the following formula
  $X^{---}=\Sigma XN$ or $X^{---}=$ Total value of the item Number of items
- Short-cut Method By short cut method, we find the Arithmetic Mean from the following formula

$\bar{X}=A+\Sigma dN$

Here, $\bar{X}$ = Arithmetic Mean, A = Assumed average of Ed = Net sum of the deviations of the different values from the assumed average; and N = Number of items in the series,

(ii) Discrete Series There are three methods of calculating mean of the discrete series

- Direct Method Direct method of estimating mean of the discrete frequency series uses the formula
  $\bar{X}=\Sigma fX\Sigma f$
- Short-cut Method Short cut method of estimating mean of the discrete frequency series uses the following formula
  $\bar{X}=A+\Sigma fd\Sigma f$
- Step-deviation Method This method is a variant of short-cut method. It is adopted when deviations from the assumed mean have some common factor
  $\bar{X}=A+\Sigma fd\Sigma f\times c$

(iii) Frequency Distribution
There are three methods of calculating mean in frequency distribution
(a) Direct Method Direct method of estimating mean of the discrete frequency series uses the formula
$\bar{X}=\Sigma fm\Sigma f$
m = mid-value, mid-value = L1+L22
L_1 = lower limit of the class
L_2 = upper limit of the class
(b) Short-cut Method Short cut method of estimating mean of the frequency distribution uses the formula
$\bar{X}=A+\Sigma fd\Sigma f$
(c) Step Deviation Method According to this method, we find the Arithmetic Mean by the following formula
$\bar{X}=A+\Sigma fd'\Sigma f\times c$
(d) Weighted Arithmetic Mean It is the mean of weighted items of the series. Different items are accorded different weights depending on their relative importance. The weighted sum of the items is divided by the sum of the weights.

Calculation of Weighted Mean

According to this way, we find weighted mean from the following information
$\bar{X}W=\Sigma WX\Sigma W$
(i) Merits

- Simplicity
- Certainty
- Based on all items
- Algebraic treatment
- Stability
- Basis of comparison
- Accuracy test

(ii) Demerits

- Effect of extreme value
- Mean value may not figure in the series at all
- Laughable conclusions
- Unsuitability
- Misleading conclusions

Median

"The Median is that value of the variable which divides the group into two equal parts, one part comprising all values greater than the Median value and the other part comprising all the values smaller than the Median value".
(i) Calculation of Median
(a) Individual Series Calculation of Median in individual series involves the following formula
M = Size of (N+12)th item
When N of the series is an even number, Median is estimated using the following formula

$$M = \frac{\text{Size of } \left(\frac{N}{2}\right)\text{th item} + \text{Size of } \left(\frac{N}{2}+1\right)\text{th item}}{2}$$

(b) Discrete Series Calculation of Median in case of discrete series or frequency array involves the following formula
M = Size of (N+12)th item
(c) Frequency Distribution Series
The following formula is applied to determine the Median Value

$$M = \text{Size of } \left(\frac{N}{2}\right)\text{th item}$$

$$\text{or} \quad M = L_1 + \frac{\frac{N}{2} - cf}{F} x_i \quad \text{or} \quad L_1 + \frac{i}{f}(m - c)$$

Quartiles

If a statistical series is divided in to four equal parts, the end value of each part is called a Quartile.

(i) Calculation of Quartiles Quartile values (Q_1 and Q_3) are estimated differently for different sets of series,

(a) Individual and Discrete Series

$$Q_1 = \text{Size of } \left[\frac{N+1}{4}\right] \text{th item of the series}$$

$$Q_3 = \text{Size of } 3\left[\frac{N+1}{4}\right] \text{th item of the series}$$

(b) Frequency Distribution Series In frequency distribution series, the class interval of Q_1 and Q_3 are first identified as under

$$Q_1 = \text{Size of } \left(\frac{N}{4}\right) \text{th item}$$

$$Q = L_1 + \frac{\left[\frac{3}{4} - cf\right]}{f} x_i \quad \text{or} \quad Q_1 = L_1 + \frac{i}{f}(Q_1 - c)$$

$$Q_3 = \text{Size of } 3\left(\frac{N}{4}\right) \text{th item}$$

$$Q_3 = L_1 + \frac{\left[\frac{3N}{4} - (cf)\right]}{f} x_i$$

or $$Q_3 = L_1 + \frac{i}{f}(Q_3 - c)$$

Percentiles

Percentiles divide the series into 100 equal parts, and is generally expressed as P. Percentiles are estimated for different types of series as under

(i) Individual and Discrete Series

$$P_1 = \text{Size of } \left(\frac{N+1}{100}\right) \text{th item}$$

or $$P_5 = \text{Size of } 4\left(\frac{N+1}{100}\right) \text{th item}$$

or $$P_{99} = \text{Size of } 99\left(\frac{N+1}{100}\right) \text{th item}$$

(ii) Frequency Distribution Series

$$P_1 = \text{Size of } \left(\frac{N}{100}\right)\text{th item, } P_1 = L_1 + \left(\frac{\frac{N}{100} - cf}{f}\right) x_i$$

$$P_4 = \text{Size of } 4\left(\frac{N}{100}\right)\text{th item, } P_4 = L_1 + \left(\frac{4\frac{N}{100} - c}{f}\right) x_i$$

$$P_{99} = \text{Size of } 99\left(\frac{N}{100}\right)\text{th item, } P_{99} = L_1 + 99\left(\frac{\frac{N}{100} - c}{100}\right) \times i$$

Mode

The value of the variable which occurs most frequently in a distribution is called the mode.
According to Croxton and Cowden, " The mode may be regarded as the most typical of a series of value".
(i) Calculation of Mode

- Individual Series There are two ways of calculating Mode in individual series
    - By inspection
    - By converting individual series into discrete series
- Discrete Series There are two methods for calculation of mode indiscrete frequency series
    - Inspection Method
    - Grouping Method
- Frequency Distribution Series The exact value of Mode can be calculated with the following formula
  $Z=L1+f1-f02f1-f0-f2xi$

Relative Position of Arithmetic Mean, Median and Mode Suppose we express,
Arithmetic Mean = M_e
Median = M_i
Mode = M_o
The relative magnitude of the three are $M_e > M_i > M_o$ or $M_e < M_i < M_o$ The Median is always between the Arithmetic Mean and the Mode.