05

A naturally occurring ore of iron, magnetite attracts small pieces of iron towards it. The phenomenon of attraction of small bits of iron, steel, cobalt, nickel, etc., towards the ore, is called **magnetism**.

MAGNETISM AND MATTER

|TOPIC 1|
Bar Magnet and Magnetic Dipole

A **magnet** is a material or an object that produces a magnetic field. The magnetic field is invisible but is responsible for most notable property of a magnet.

Magnets are of two types

(i) Natural magnets (ii) Artificial magnets

Natural magnets are generally irregular in shape and weak in strength. On the other hand, **artificial magnets** have desired shape and desired strength. A bar magnet, a horse shoe magnet, compass needle, etc., all are the examples of artificial magnets.

Some commonly known ideas about magnetism are given below

(i) The earth behaves as a magnet with the magnetic field pointing approximately from the geographic South to North.

(ii) When a bar magnet is suspended freely, it points in the North-South direction. The tip of the magnet which points to the geographic North is called the **North pole** and the tip which points to the geographic South is called the **South pole** of the magnet.

(iii) There is a **repulsive force**, when North poles (or South poles) of two magnets are brought close together. Conversely, there is an **attractive force** between the North pole of one magnet and the South pole of the other.

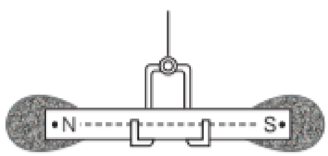
CHAPTER CHECKLIST
- Bar Magnet and Magnetic Dipole
- Magnetic Properties of Materials

(iv) It is possible to make magnets out of iron and its alloys. When a piece of a substance, such as soft iron, steel, cobalt, nickel, etc., is placed near a magnet, it acquires magnetism.

THE BAR MAGNET

The bar magnet has two poles similar to the positive and negative charges of an electric dipole. One pole is designated as **North pole** (N) and the other as **South pole** (S). When a bar magnet is suspended freely, these poles point approximately towards the geographic North and South poles, respectively.

Like magnetic poles repel each other and **unlike magnetic poles attract** each other. If a bar magnet is dropped into a pile of iron-filings, then the maximum amount of filings get deposited near the ends of the magnet and almost nil in the middle.

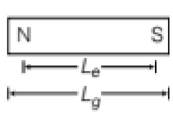


The pattern suggests that attraction is maximum at the two ends of the bar magnet. These ends are called **poles** of the magnet. The poles of a magnet can never be separated.

Magnetic Length of a Bar Magnet

The distance between two poles of a bar magnet is known as **magnetic length** of a magnet. Its direction is from S-pole of the magnet to N-pole and is represented by $2l$. It is sometimes also known as effective length (L_e) of the magnet and is less than its geometric length (L_g).

For a bar magnet, $L_e = \left(\frac{5}{6}\right) L_g$.



Bar magnet

EXAMPLE |1| Consider a short magnetic dipole of magnetic length 20 cm. Find its geometric length.

Sol. Geometric length of a magnet is $\frac{6}{5}$ times its magnetic length.

$\therefore$ Geometric length $= \frac{6}{5} \times 20$

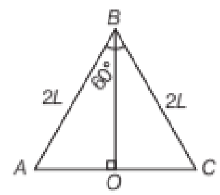
$= 24 \text{ cm}$

EXAMPLE |2| A thin bar magnet of length $4L$ is bent at the mid-point, so that the angle between them is 60°. Find the new length of the bar magnet.

Sol. On bending the bar magnet, the length of the bar magnet,

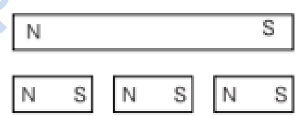
$$AC = AO + OC = 2L\sin\left(\frac{60°}{2}\right) + 2L\sin\left(\frac{60°}{2}\right)$$

$$= 4L\sin 30° = 4L \times \frac{1}{2} = 2L$$



The Non-existance of Magnetic Monopole

We cannot isolate the North or South pole of a magnet, i.e. magnetic poles exist in pairs. If a bar magnet is broken into two halves, we get two similar bar magnets with somewhat weaker properties. Unlike electric charges, isolated magnetic North and South poles are known as magnetic monopoles which do not exist.



If a bar magnet is broken, each piece behaves as a small magnet

Pole Strength

It is defined as the strength of a magnetic pole to attract magnetic materials towards itself. It is a scalar quantity and its SI unit is ampere-metre (A-m). The strength of N and S-pole of a magnet is conventionally represented by $+m$ and $-m$, respectively. It depends on the nature of material and area of cross-section of the magnet.

Strength of N and S-pole of a magnet is always equal and opposite ($+m$ and $-m$).

Force between Two Magnetic Poles

The force of attraction or repulsion F between two magnetic poles of strengths m_1 and m_2 separated by a distance r is directly proportional to the product of pole strengths and inversely proportional to the square of the distance between their centres, i.e.

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = K\frac{m_1 m_2}{r^2}$$

where, K is magnetic force constant.

In SI unit,

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\ \text{Wb A}^{-1}\ \text{m}^{-1}$$

where, μ_0 is absolute magnetic permeability of free space (air/vacuum).

$$\boxed{F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2}} \quad \ldots \text{(i)}$$

This is called **Coulomb's law** of magnetic force. SI unit of magnetic pole strength is ampere-metre.

Suppose $\quad m_1 = m_2 = m$ (say), $r = 1$ m

and $\quad F = 10^{-7}$ N

From Eq. (i), we have

$$10^{-7} = 10^{-7} \times \frac{(m)(m)}{1^2}$$

or $\quad m^2 = 1 \quad$ or $\quad m = \pm 1$ A-m

Therefore, strength of a magnetic pole is said to be **one ampere-metre**, if it repels an equal and similar pole, when placed in vacuum (or air) at a distance of one metre from it, with a force of 10^{-7} N.

EXAMPLE |3| Two poles one of which is 5 times as strong as the other, exert on each other a force equal to 0.8×10^{-3} kg-wt, when placed 10 cm apart in air. Find the strength of each pole.

Sol. Let m and $5m$ be the pole strength of the two poles.

Here, $\quad F = 0.8 \times 10^{-3}$ kg-wt $= 0.8 \times 10^{-3} \times 9.8$ N,

$r = 10$ cm $= 0.1$ m

$$\therefore \quad F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2}$$

$$\Rightarrow 0.8 \times 10^{-3} \times 9.8 = \frac{10^{-7} \times m \times 5m}{(0.1)^2}$$

$\Rightarrow \quad m = 12.52$ A-m

and $\quad 5m = 5 \times 12.52$ A-m $= 62.6$ A-m

EXAMPLE |4| Two identical magnets with a length 100 cm are arranged freely with their like poles facing in a vertical glass tube. The upper magnet hangs in air above the lower one so that the distance between the nearest poles of the magnet is 3 mm. If the pole strength of the pole of these magnets is 6.64 A-m, then determine the force between the two magnets.

Sol. Given, pole strength, $m = 6.64$ A-m

$$r = 3\text{mm} = 3 \times 10^{-3}\ \text{m}$$

Since, force, $F = \frac{\mu_0}{4\pi} \times \frac{m_1 m_2}{r^2}$

$$\therefore \quad F = \frac{\mu_0}{4\pi} \times \frac{m^2}{r^2} \quad [\because m_1 = m_2 = m]$$

$$= \frac{\mu_0}{4\pi} \times \frac{(6.64)^2}{(3 \times 10^{-3})^2}$$

$$= \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{44.0896}{9 \times 10^{-6}} = 10^{-1} \times 4.8988$$

$$= 0.49\ \text{N}$$

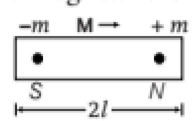
MAGNETIC DIPOLE

A **magnetic dipole** is an arrangement of two magnetic poles of equal and opposite strengths $(-m, +m)$ separated by a small distance. e.g. A bar magnet, a compass needle, etc., are magnetic dipoles.

The two poles of a magnetic dipole (or a magnet), called North pole and South pole, are always of equal strength and of opposite nature. Further, such two magnetic poles always exist in pair.

Magnetic Dipole Moment

It is the product of the strength of either pole and the magnetic length of the magnet. It is represented by **M**.



The direction of magnetic dipole moment is same as that of 2l. Therefore,

$$\boxed{\mathbf{M} = m(2\mathbf{l})}$$

SI unit of magnetic dipole moment is ampere-metre^2 (A-m^2)

EXAMPLE |5| A magnetic wire of dipole moment 4π A-m^2 is bent in the form of semicircle. Find the new magnetic moment.

Sol. If length of wire is $2l$, then magnetic moment

$$M = m \times 2l = 4\pi\ \text{A-m}^2 \quad \text{[given]}$$

As wire is bent in the form of semicircle, effective distance between the ends is $2r$.

So, new dipole moment

$$M' = m \times 2r = m \times 2 \times \frac{2l}{\pi} = \frac{2}{\pi}(m \times 2l) \quad [\because \pi r = 2l]$$

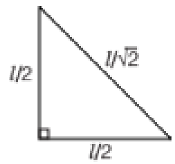
$$= \frac{2}{\pi} M = \frac{2}{\pi} 4\pi$$

$$= 8\ \text{A-m}^2$$

EXAMPLE |6| The length of a magnetised steel wire is l and its magnetic moment is M. It is bent into the shape of L with two sides equal. What will be the new magnetic moment?

Sol. If m is strength of each pole, then magnetic moment $M = m \times l$

When the wire is bent into L shape, effective distance between the poles



$$= \sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = \frac{l}{\sqrt{2}}$$

$\therefore$ New magnetic moment,

$$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}} \quad [m \text{ will remain unchanged}]$$

Magnetic Field Lines

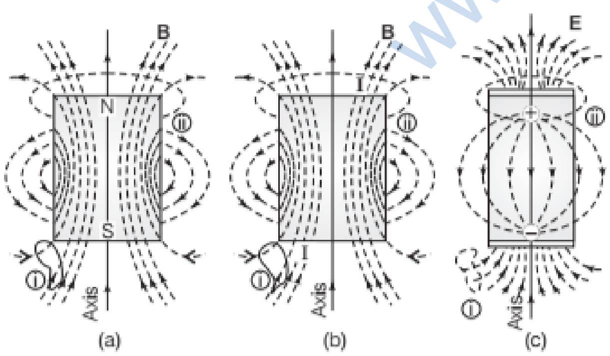
The magnetic field lines are a visual and intuitive realisation of the magnetic field. The magnetic field lines in a magnetic field are those imaginary lines which continuously represent the direction of the magnetic field. The tangent drawn at any point on magnetic field line shows the direction of magnetic field at that point.

Properties of Magnetic Field Lines

Important properties of magnetic field lines are given below

(i) The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole, where these field lines begin from a positive charge and end on the negative charge or escape to infinity.

(ii) The tangent to the field line at a given point represents the direction of the net magnetic field **B** at that point.



The field lines of (a) a bar magnet, (b) a current carrying finite solenoid and (c) an electric dipole. At large distances, their field lines are very similar. The curves labelled (i) and (ii) are closed to Gaussian surfaces.

(iii) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field **B**.

(iv) The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

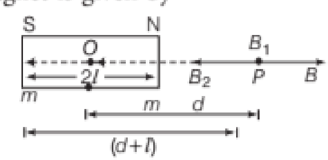
MAGNETIC FIELD STRENGTH AT A POINT DUE TO A BAR MAGNET

The strength of magnetic field at any point is defined as, the force experienced by a hypothetical unit North pole placed at that point. It is a **vector** quantity. The direction of magnetic field **B** is the direction along which hypothetical unit North pole would tend to move, if free to do so.

We have used the word hypothetical unit North pole in the above discussion because an isolated magnetic pole does not exist.

When Point Lies on Axial Line of a Bar Magnet

Let $2l$ be the magnetic length of a bar magnet with centre O. The magnetic dipole moment of the magnet is M, where $M = m \times 2l$, $OP = d$, is the distance of the point P on the axial line from the centre of the magnet. If m is the strength of each pole, then magnetic field strength at P due to N-pole of magnet is given by



$$B_1 = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{(NP)^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d-l)^2}, \text{ along } NP \text{ produced.}$$

Magnetic field strength at P due to S-pole of magnet is given by

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{(SP)^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d+l)^2}, \text{ along } PS \text{ produced.}$$

$\therefore$ Magnetic field strength at P due to the bar magnet

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d-l)^2} - \frac{\mu_0}{4\pi} \cdot \frac{m}{(d+l)^2}$$

$$= \frac{\mu_0 m}{4\pi}\left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right]$$

$$= \frac{\mu_0 m}{4\pi}\left[\frac{(d+l)^2 - (d-l)^2}{(d^2 - l^2)^2}\right] \quad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

$$= \frac{\mu_0 m \cdot 4ld}{4\pi (d^2 - l^2)^2} = \frac{\mu_0 (m \times 2l) 2d}{4\pi (d^2 - l^2)^2}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2Md}{(d^2 - l^2)^2} \qquad [\because M = m \times 2l]$$

When the magnet is short, $l^2 << d^2$, such that l^2 is neglected.

$$\therefore \qquad B = \frac{\mu_0}{4\pi} \cdot \frac{2Md}{d^4}$$

$$\Rightarrow \qquad \boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}}$$

The direction of **B** is along NP produced.

EXAMPLE |7| What is the magnitude of the axial fields due to a bar magnet of length 5 cm at a distance of 50 cm from its mid-point? The magnetic moment of the bar magnet is 0.40 A -m^2.

Sol. Given, magnetic length of bar magnet, $2l = 5$ cm

$\Rightarrow \qquad l = 2.5 \text{ cm} = 2.5 \times 10^{-2}$ m

Distance, $d = 50 \text{cm} = 0.5 \text{m}$

Magnetic moment, $M = 0.40 \text{A-m}^2$

$$\therefore \qquad B = \frac{\mu_0}{4\pi} \cdot \frac{2Md}{(d^2 - l^2)^2}$$

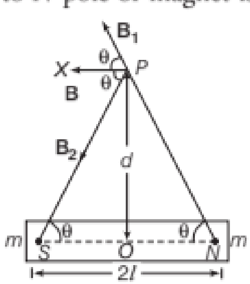
$$= \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3} \qquad [\because l << d]$$

$$= \frac{10^{-7} \times 2 \times 0.40}{(0.5)^3}$$

$$= 6.4 \times 10^{-7} \text{T}$$

When Point Lies on Equatorial Line of a Bar Magnet

In the given figure, the point P is shown on equatorial line of the same bar magnet, where $OP = d$. Magnetic field strength at P due to N-pole of magnet is



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{m \times 1}{(NP)^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d^2 + l^2)}, \text{ along } NP \text{ produced.}$$

Magnetic field strength at P due to S-pole of magnet is

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m \times 1}{(SP)^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d^2 + l^2)}, \text{ along } PS \text{ produced.}$$

As $B_1 = B_2$ in magnitude, their components $B_1 \sin\theta$ along OP produced and $B_2 \sin\theta$ along PO will cancel out. However, components along PX parallel to NS will add. Therefore, magnetic field strength at P due to the bar magnet, $B = B_1 \cos\theta + B_2 \cos\theta = 2 B_1 \cos\theta$, along PX.

$$B = 2 \frac{\mu_0}{4\pi} \cdot \frac{m}{(d^2 + l^2)} \times \frac{l}{\sqrt{d^2 + l^2}} \qquad \left[\because \cos\theta = \frac{l}{\sqrt{d^2 + l^2}}\right]$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{m \times 2l}{(d^2 + l^2)^{3/2}}$$

$$\Rightarrow \qquad \boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2 + l^2)^{3/2}}}$$

If the magnet is short, $l^2 << d^2$, such that l^2 is neglected.

$$\therefore \qquad B = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$$

$$\Rightarrow \qquad \boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}}$$

The direction of B is along PX, *a* line parallel to NS.

Note From the formulae of magnetic field due to a bar magnet at a point in axial position and at a point in equatorial position, it is clear that magnetic field due to a short bar magnet at any point on the axial line of magnet is twice the magnetic field at a point at the same distance on the equatorial line of the magnet.

EXAMPLE |8| Determine the magnitude of the equatorial fields due to a bar magnet of length 6 cm at a distance of 60 cm from its mid-point. The magnetic moment of the bar magnet is 0.60 A - m^2.

Sol. Given, magnetic length of bar magnet, $2l = 6$ cm

$\Rightarrow \qquad l = 3 \text{ cm} = 3 \times 10^{-2}$ m

Distance, $d = 60 \text{cm} = 0.6$ m

Magnetic moment, $M = 0.60 \text{ A-m}^2$

$$\therefore \text{ Magnetic field, } B = \frac{\mu_0}{4\pi} \times \frac{M}{(d^2 + l^2)^{3/2}}$$

$$= \frac{\mu_0 M}{4\pi d^3} \qquad [\because l << d]$$

$$= \frac{4\pi \times 10^{-7} \times 0.60}{4\pi \times (0.6)^3} = 2.7 \times 10^{-7} \text{T}$$

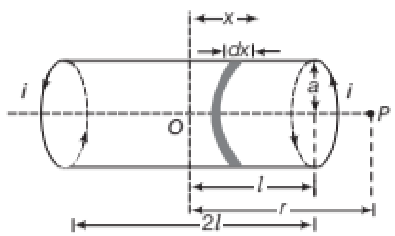
Bar Magnet as an Equivalent Solenoid

The magnetic field lines for a bar magnet and a current carrying solenoid resemble very closely. Therefore, a bar magnet can be thought as a large number of circulating currents in analogy with a solenoid.

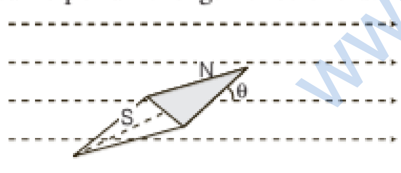
Cutting a bar magnet in half is like cutting a solenoid. We get two smaller solenoids with weaker magnetic properties. The field lines remain continuous, emerging from one face of the solenoid and entering into other face.

One can test this analogy by moving a small compass needle in the neighbourhood of a bar magnet and a current carrying finite solenoid and noting that the deflections of the needle are similar in both the cases.

To prove mathematically that magnetic field produced by a solenoid on any point on the axial line is same as that of a bar magnet. This analogy between bar magnet and solenoid can be shown by calculating the magnetic field at an axial point of solenoid which resembles to that of a bar magnet.



Let i be the current passing through a solenoid, a be the radius of solenoid, $2l$ be the length of solenoid and n be the number of turns per unit length of solenoid.



Let P be the point at distance r from centre at which magnetic field is to be calculated. Consider a small element of thickness dx of the solenoid at a distance (x) from the centre O.

Number of turns in the element $= n\,dx$

The magnitude of the field at point P due to the circular element is given by

$$dB = \frac{\mu_0\, i\, a^2 (n\,dx)}{2[(r-x)^2 + a^2]^{3/2}} \qquad \ldots\text{(i)}$$

If P lies at a very large distance from O, i.e. $r >>> a$ and $r >>> x$, then $[(r-x)^2 + a^2]^{3/2} \approx r^3$

$$dB = \frac{\mu_0\, i\, a^2 n\,dx}{2r^3} \qquad \ldots\text{(ii)}$$

Total magnetic field at point P due to current carrying solenoid.

$$B = \frac{\mu_0 n\, i\, a^2}{2r^3}\int_{-l}^{+l} dx$$

[∵ range of variation of x is from $-l$ to $+l$]

$$= \frac{\mu_0 ni\, a^2}{2r^3}[x]_{-l}^{+l} = \frac{\mu_0\, ni\, a^2}{2r^3}(2l)$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2n\,(2l)\, i\pi\, a^2}{r^3} \qquad \ldots\text{(iii)}$$

If M is the magnetic moment of the solenoid, then

M = Total number of turns × Current × Area of cross-section

$M = n\,(2l) \times i \times \pi\, a^2$

$$\Rightarrow \quad \boxed{B = \frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3}}$$

This is the expression for magnetic field on the axial line of a short bar magnet.

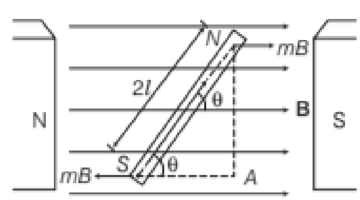
The magnetic moment of bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

TORQUE ON A MAGNETIC DIPOLE IN A UNIFORM MAGNETIC FIELD

When a bar magnet is placed in a uniform magnetic field, torque acts on the magnet. Also, magnetic potential energy is associated with the magnet due to its orientation as discussed below.

In the figure below, a uniform magnetic field **B** is represented by equidistant parallel lines. NS is a bar magnet of length $2l$ and strength of each pole is m.

The magnet is held at $\angle\theta$ with the direction of **B**.



Bar magnet in a uniform magnetic field

Force on N-pole $= mB$, along **B**

Force on S-pole $= mB$, opposite to **B** where, m = strength of each pole and

and B = strength of magnetic field.

These equal and unlike forces form a couple which tend to rotate the magnet clockwise, so as to align it along **B**.

Torque acting on the bar magnet,

$$\tau = \text{Force} \times \text{Perpendicular distance}$$

$$\Rightarrow \quad \tau = mB \times NA$$

In ΔNAS, $\sin\theta = \frac{NA}{NS} = \frac{NA}{2l}$

$$\therefore \quad NA = 2l\sin\theta$$

Now, $\tau = mB \times 2l\sin\theta = B \times (m2l)\sin\theta$

$$\Rightarrow \quad \boxed{\tau = MB\sin\theta}$$

In vector form, $\boxed{\boldsymbol{\tau} = \mathbf{M} \times \mathbf{B}}$

The direction of τ is perpendicular to the plane containing **M** and **B** and is given by right handed screw rule.

When $B = 1$ and $\theta = 90°$,

then $\tau = M \times 1\sin 90° = M$

Hence, we may define **magnetic dipole moment** as the torque acting on a dipole held perpendicular to a uniform magnetic field of unit strength.

Unit of M is unit of τ divided by unit of B. Therefore, SI unit of M is joule per tesla (J T^{-1}).

EXAMPLE |9| A straight solenoid of length 50 cm has 1000 turns per metre and a mean cross-sectional area of 2×10^{-4} m^2. It is placed with its axis at 30°, with a uniform magnetic field of 0.32 T. Find the torque acting on the solenoid when a current of 2 A is passed through it.

Sol. Given, $l = 50$ cm

Number of turns per metre = 1000

$\therefore$ Total number of turns $(N) = 1000 \times \frac{1}{2} = 500$

Area, $A = 2 \times 10^{-4}\,\text{m}^2$

Current, $I = 2$A

Magnetic field, $B = 0.32$T

$\therefore$ Torque, $\tau = MB\sin\theta = (NIA)B\sin\theta$

$$= 500 \times 2 \times (2 \times 10^{-4}) \times 0.32 \times \frac{1}{2}$$

$$= 0.032 \text{ N-m}$$

EXAMPLE |10| A bar magnet when suspended horizontally and perpendicular to the earth's magnetic field experiences a torque of 3×10^{-4} N-m. What is the magnetic moment of the magnet? Horizontal component of earth's magnetic field at that place is 0.4×10^{-4} T.

Sol. Given, $\theta = 90°$, $\tau = 3 \times 10^{-4}$ N-m

and $B = 0.4 \times 10^{-4}$ T

Since, torque $\tau = MB\sin\theta$

$$\therefore \quad M = \frac{\tau}{B\sin\theta} = \frac{3 \times 10^{-4}}{0.4 \times 10^{-4}\sin 90°}$$

$$= 7.5 \text{ J T}^{-1}$$

Oscillations of a Freely Suspended Magnet

The torque acting on the bar magnet makes it oscillate in the uniform magnetic field.

Since, in equilibrium, $\tau = -MB\sin\theta$

Torque acting on a body, $\tau = I\alpha$

where, I = moment of inertia

and α = angular acceleration.

Now, $I\alpha = -MB\sin\theta$ $\quad [\because \tau = I\alpha]$

$$\Rightarrow \quad I\frac{d^2\theta}{dt^2} = -MB\sin\theta \quad \left[\because \alpha = \frac{d^2\theta}{dt^2}\right]$$

In the above equation, negative sign with $MB\sin\theta$ indicates that restoring torque is in the opposite direction to the deflecting torque. For small values of θ in radians, $\sin\theta \approx \theta$.

$$\Rightarrow \quad I\frac{d^2\theta}{dt^2} = -MB\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = \frac{-MB\theta}{I} \quad \ldots\text{(i)}$$

On comparing the Eq.(i) with equation of SHM, i.e. $\frac{d^2x}{dt^2} = -\omega^2 x$ or in angular terms, $\frac{d^2\theta}{dt^2} = -\omega^2\theta$

We can say that, the oscillations of a freely suspended magnet (magnetic dipole) in a uniform magnetic field are simple harmonic.

$$\therefore \quad \omega^2 = \frac{MB}{I} \quad \text{or} \quad \omega = \sqrt{\frac{MB}{I}}$$

So, the time period of oscillations, $T = \frac{2\pi}{\omega}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{MB}}$$

or

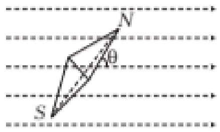
$$\boxed{B = \frac{4\pi^2 I}{MT^2}}$$

where, I = moment of inertia about the axis of the magnet

M = magnetic moment

and B = magnetic field intensity.

EXAMPLE |11| A magnetic needle is free to oscillate in a uniform magnetic field as shown in figure. The magnetic moment of magnetic needle is 7.2 A-m^2 and moment of inertia $I = 6.5 \times 10^{-6}$ kg-m^2. The number of oscillations performed in 5s is 10. Calculate the magnitude of magnetic field.



Sol. Here, $T = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{5}{10} = 0.5$ s,

$M = 7.2$ A-m^2, $I = 6.5 \times 10^{-6}$ kg-m^2

As, $T = 2\pi\sqrt{\frac{I}{MB}}$ or $T^2 = 4\pi^2 \frac{I}{MB}$

The magnitude of the magnetic field is

$$B = \frac{4\pi^2 I}{MT^2} = \frac{4 \times (3.14)^2 \times 6.5 \times 10^{-6}}{7.2 \times (0.5)^2}$$

$$= 1.42 \times 10^{-4} \text{ T}$$

POTENTIAL ENERGY OF A MAGNETIC DIPOLE IN A UNIFORM MAGNETIC FIELD

When a magnetic dipole of moment **M** is held at an angle θ with the direction of a uniform magnetic field **B**, the magnitude of the torque acting on the dipole is given by

$$\tau = MB\sin\theta$$

This torque tends to align the dipole in the direction of the field. Work has to be done in rotating the dipole against the action of the magnetic torque. This work done is stored as potential energy of the dipole.

Now, a small amount of work done in rotating the dipole through a small angle $d\theta$,

$$dW = \tau\, d\theta = MB\sin\theta \cdot d\theta$$

Total work done in rotating the dipole from $\theta = \theta_0$ to $\theta = \theta$,

$$W = \int_{\theta_0}^{\theta} dW = \int_{\theta_0}^{\theta} MB\sin\theta d\theta = MB[-\cos\theta]_{\theta_0}^{\theta}$$

$\Rightarrow$ $W = -MB[\cos\theta - \cos\theta_0]$

$\therefore$ Potential energy of the dipole,

$$U = W = -MB(\cos\theta - \cos\theta_0)$$

Let us assume that, $\theta_0 = 90°$

$$U = W = -MB(\cos\theta - \cos 90°)$$

Therefore, $\boxed{U = -MB\cos\theta}$

In vector notation, we may rewrite this equation as

$$\boxed{U = -\mathbf{M} \cdot \mathbf{B}}$$

Particular Cases

(i) When $\theta = 90°$,

$$U = -MB\cos\theta$$
$$= -MB\cos 90° = 0$$

i.e. when the dipole is perpendicular to magnetic field, its potential energy is zero.

(ii) When $\theta = 0°$, $U = -MB\cos\theta$

$$= -MB\cos 0° = -MB$$

i.e. when the magnetic dipole is aligned along the magnetic field, it is in **stable equilibrium** having minimum potential energy.

(iii) When $\theta = 180°$, $U = -MB\cos\theta$

$$= -MB\cos 180° = MB$$

which is maximum. This is the position of **unstable equilibrium**.

EXAMPLE |12| A circular coil of 100 turns and have an effective radius of 5 cm carries a current of 0.1 A. How much work is required to turn it in an external magnetic field of 1.5 Wb/m^2 through 180° about an axis perpendicular to the magnetic field? The plane of the coil is initially perpendicular to the magnetic field.

Sol. Given, number of turns, $N = 100$

Radius, $r = 5\text{cm} = 0.05$ m

Current, $I = 0.1$ A

Magnetic field, $B = 1.5$ Wb / m^2

$\theta_1 = 0°, \theta_2 = 180°$

Area, $A = \pi r^2 = 3.14(0.05)^2$ m^2

$\therefore$ Required work done, $W = -MB(\cos\theta_2 - \cos\theta_1)$

$$= -(NIA)B(\cos\theta_2 - \cos\theta_1)$$
$$= -100 \times 0.1 \times 3.14\,(0.05)^2 \times 1.5 \times (\cos 180° - \cos 0°)$$
$$= -10 \times 3.14(0.05)^2 \times 1.5(-1-1) = 0.24 \text{ J}$$

THE ELECTROSTATIC ANALOG

The magnetic dipole moment of a bar magnet is given by

$$\mathbf{M} = m\,(2\mathbf{l})$$

where, m = strength of each pole
and $2\mathbf{l}$ = length of the dipole.

The magnetic dipole is analogous to an electric dipole consisting of two equal charges of opposite signs $(\pm q)$ separated by a certain distance $(2\mathbf{a})$. It has an electric dipole moment, i.e. $\mathbf{p} = q\,(2\mathbf{a})$

The equations for magnetic field **B** due to a magnetic dipole can be obtained from the equations of electric field **E** due to an electric dipole by making the following changes,

$\mathbf{E} \to \mathbf{B}$, $\mathbf{p} \to \mathbf{M}$

$$\frac{1}{4\pi\varepsilon_0} \rightarrow \frac{\mu_0}{4\pi} \Rightarrow \frac{1}{\varepsilon_0} \rightarrow \mu_0$$

Thus, for any point on axial line of a bar magnet at a distance $d\,(d >>> l)$ from the centre of magnet.

$$B_A = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$$

Similarly, for any point on equatorial line of a bar magnet at a distance, for $d >>> l$

$$B_E = \frac{\mu_0 M}{4\pi d^3}$$

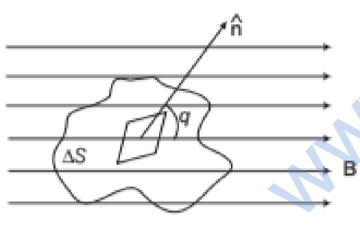
The table below shows the analogy between electric and magnetic dipoles.

The Dipole Analogy

	Electrostatics	Magnetism
Dipole moment	p	M
Equatorial field for a short dipole	$-p/4\pi\varepsilon_0 r^3$	$-\mu_0 M/4\pi r^3$
Axial field for a short dipole	$2p/4\pi\varepsilon_0 r^3$	$\mu_0 2M/4\pi r^3$
External field : Torque	$p \times E$	$M \times B$
External field : Energy	$-p \cdot E$	$-M \cdot B$

MAGNETISM AND GAUSS' LAW

The net magnetic flux (ϕ_B) through any closed surface is always zero.



This law suggests that the number of magnetic field lines leaving any closed surface is always equal to the number of magnetic field lines entering it. Suppose a closed surface S is held in a uniform magnetic field **B**. Consider a small vector area element ΔS of this surface.Magnetic flux through this area element is defined as, $\Delta\phi_B = \mathbf{B} \cdot \Delta \mathbf{S}$

Considering all small area elements of the surface, we obtain net magnetic flux through the surface as,

obtain net magnetic flux through the surface as,

$$\phi_B = \sum_{\text{all}} \Delta\phi_B = \sum_{\text{all}} \mathbf{B} \cdot \Delta \mathbf{S} = 0$$

Comparing this with Gauss' law in electrostatics,

$$\phi_E = \oint_S \mathbf{E} \cdot \Delta \mathbf{S} = \frac{q}{\varepsilon_0}$$

The difference between the Gauss's law of magnetism and electrostatics is that isolated magnetic poles (also called **monopoles**) does not exist.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Two magnets have the same length and the same pole strength. But one of the magnets has a small hole at its centre. Then,

(a) both have equal magnetic moment
(b) one with hole has small magnetic moment
(c) one with hole has large magnetic moment
(d) one with hole loses magnetism through the hole

2. A large magnet is broken into two pieces so that their lengths are in the ratio 2 : 1. The pole strengths of the two pieces will have ratio

(a) 2 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 1

3. The intensity of magnetic field at a point X on the axis of a small magnet is equal to the field intensity at another point Y on equatorial axis. The ratio of distance of X and Y from the centre of the magnet will be

(a) $(2)^{-3}$ (b) $(2)^{-1/3}$ (c) 2^3 (d) $2^{1/3}$

4. Work done in rotating a bar magnet from 0 to angle 120° is

(a) $\frac{1}{2}MB$ (b) $\frac{3}{2}MB$ (c) MB (d) $\frac{2}{3}MB$

5. Gauss's law for magnetism is

(a) the net magnetic flux through any closed surface is $B \cdot \Delta S$
(b) the net magnetic flux through any closed surface is $E \cdot \Delta S$
(c) the net magnetic flux through any closed surface is zero
(d) Both (a) and (c)

VERY SHORT ANSWER Type Questions

6. On what factors does the pole strength of a magnet depend?

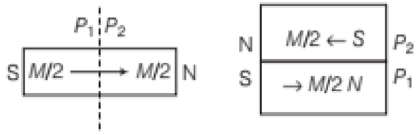
magnet depend?

7. What is Coulomb's law of magnetic force?

8. Define magnetic dipole moment. Also, write its SI unit.

9. A bar magnet is cut into two equal parts as shown in the Fig. (a). One part is now kept over the other such that, the P_2 is above P_1 as shown in the Fig. (b).

If M is the magnetic moment of the original magnet, what would be the magnetic moment of new combination of magnets so formed?



(a) (b)

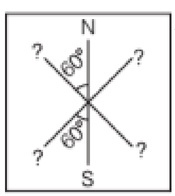
10. A coil of N turns and radius R carries a current I. It is unwound and rewound to make a square coil of side a having same number of turns N.

Keeping the current I same, find the ratio of the magnetic moments of the square coil and the circular coil. **Delhi 2013, Delhi 2013C**

11. Why do magnetic lines of force form continuous closed loops?

SHORT ANSWER Type Questions

12. Three identical bar magnets are rivetted together at centre in the same plane as shown in the figure. This system is placed at rest in a slowly varying magnetic field. It is found that the system of magnets does not show any motion. The North-South poles of one magnet is shown in the figure. Determine the poles of the remaining two. **NCERT Exemplar**



13. What happens to a bar magnet if it is cut into two pieces

(i) transverse to its length?

(ii) along its length?

14. State Gauss's law in magnetism and compare it with Gauss's law in electrostatics.

15. State whether the given statement is correct or incorrect and explain it.

"The magnetic field lines of a magnet form continuous closed loops unlike electric field lines."

16. A circular coil of closely wound N turns and radius r carries a current I. Write the expressions for the following:

(i) The magnetic field at its centre.

(ii) The magnetic moment of this coil.

All India 2012

17. A short bar magnet placed with its axis making an angle θ with a uniform external field B, experiences a torque τ.

(i) What is the magnetic moment of the magnet?

(ii) Write the condition of stable equilibrium.

18. A small compass needle of magnetic moment M and moment of inertia I is free to oscillate in a magnetic field B. It is slightly disturbed from its equilibrium position and then released. Show that it executes simple harmonic motion. Hence, write the expression for its time period. **Delhi 2013, 2011C**

19. Two bar magnets having same geometry with magnetic moments M and $2M$ are placed in such a way that their similar poles are on the same side, then its time period of oscillation is T_1. Now, if the polarity of one of the magnets is reversed, then time period of oscillation is T_2, then find the relation between T_1 and T_2.

20. Suppose we want to verify the analogy between electrostatic and magnetostatic by an explicit experiment. Consider the motion of

(i) electric dipole **p** in an electrostatic field **E** and

(ii) magnetic dipole **M** in a magnetic field **B**.

Write down a set of conditions on **E**, **B**, **p**, **M**, so that the two motions are verified to be identical. (Assume identical initial conditions). **NCERT Exemplar**

21. Answer the following

(i) Is it possibe to have a magnetic field configuration with three poles?

(ii) If magnetic monopoles existed, how would Gauss's law of magnetism be modified?

LONG ANSWER Type I Questions

22. An observer to the left of a solenoid of N turns each of cross-section area A observes that a steady current I in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment $m = NIA$. **All India 2015**

23. A uniform conducting wire of length $12a$ and resistance R is wound up as a current carrying coil in the shape of
- (i) an equilateral triangle of side a,
- (ii) a square of sides a and
- (iii) a regular hexagon of side a. The coil is connected to a voltage source V_0. Find the magnetic moment of the coils in each case.

NCERT Exemplar

24. An electron of mass m_e revolves around a nucleus of charge $+Ze$. Show that it behaves like a tiny magnetic dipole. Hence, prove that the magnetic moment associated with it is expressed as $\mu = -\frac{e}{2m_e}\mathbf{L}$, where $\mathbf{L}$ is the orbital angular momentum of the electron. Give the significance of negative sign. **Delhi 2017**

25. Verify the Gauss' law for magnetic field of a point dipole of dipole moment $\mathbf{M}$ at the origin for the surface which is a sphere of radius R.

NCERT Exemplar

LONG ANSWER Type II Questions

26. Derive the expression for
- (i) magnetic field at a point lies on axial line of a bar magnet.
- (ii) magnetic field at a point lies on equatorial line of a bar magnet.

  Also, find the ratio of magnetic fields at the axial and equatorial points.

27. (i) Derive the expression for potential energy of a magnetic dipole in a magnetic field.
(ii) Compare the magnetic fields of a bar magnet and a solenoid.

NUMERICAL PROBLEMS

28. A short bar magnet has a magnetic moment of 0.48 J/T. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on
- (i) the axis,
- (ii) the equatorial lines (normal bisector) of the magnet. **NCERT**

29. A short bar magnet of magnetic moment 5.25×10^{-2} J/T is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with the earth's field on (i) its normal bisector and (ii) its axis. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved. **NCERT**

30. A closely wound solenoid of 800 turns and area of cross-section 2.5×10^{-4} m^2 carries a current of 3.0 A. If it can be treated as a bar magnet, then find its magnetic moment. **(2 M)**

31. If the solenoid is treated as a magnet of moment (= 0.6 J/T) is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of 30° with the direction of applied field? **NCERT**

32. A closely wound solenoid of 2000 turns and area of cross-section 1.6×10^{-4} m^2, carrying a current of 4 A, is suspended through its centre allowing it to turn in a horizontal plane. If the solenoid is treated as magnet, then
- (i) What is the magnetic moment associated with the solenoid?
- (ii) What are the force and torque on the solenoid, if a uniform horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30° with the axis of the solenoid? **NCERT**

33. A short magnet oscillates with a time period 0.1 s at a place, where horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire 20 cm East of the magnet. What will be the new time period of the oscillator?

34. A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A, rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of $2.0\ s^{-1}$. What is the moment of inertia of the coil about its axis of rotation? **NCERT**

35. If two magnets having magnetic moments M and $M\sqrt{3}$ are joined to form a cross (i.e. ×). The combination is suspended freely in a uniform magnetic field. In equilibrium position, the magnet having magnetic moment M makes an angle θ with the field. Calculate the value of θ.

36. A short bar magnet of magnetic moment $M = 0.32$ J/T is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (i) stable and (ii) unstable equilibrium? What is the potential energy of the magnet in each case? **NCERT**

37. A bar magnet of magnetic moment 1.5 J/T lies aligned with the direction of a uniform magnetic field of 0.22 T.

(i) What is the amount of work required by an external torque to turn the magnet, so as to align its magnetic moment (a) normal to the field direction, (b) opposite to the field direction?

(ii) What is the torque on the magnet in cases (a) and (b)? **NCERT**

HINTS AND SOLUTIONS

1. (b) As, we know, magnetic dipole moment $\mathbf{m} = m(2\mathbf{l})$, so hole reduces the effective length of the magnet and hence magnetic moment reduces.

2. (d) Pole strength does not depend on length. So, strength of the two pieces will remain same.

3. (d) If d_1 is distance of point X one axial line and d_2 is distance of point Y on equatorial line.

Then, $B_A = \frac{\mu_0}{4\pi}\frac{2m}{d_1^3}$, $B_E = \frac{\mu_0}{4\pi}\cdot\frac{m}{d_2^3}$

As, $B_A = B_E$

$$\therefore \frac{\mu_0 2m}{4\pi d_1^3} = \frac{\mu_0 m}{4\pi d_2^3} \Rightarrow d_1^3 = 2d_2^3 \Rightarrow d_1/d_2 = 2^{1/3}$$

4. (b) Work done in rotating a magnet (from angle 0 to 120°) is given by

$$W = \int_0^{120^\circ} \tau\, d\theta$$
$$= MB\int_0^{120^\circ} \sin\theta\, d\theta = MB(-\cos\theta)_0^{120^\circ}$$
$$= MB(-\cos 120^\circ + \cos 0^\circ) \Rightarrow MB(1+\frac{1}{2}) = \frac{3}{2}MB$$

5. (c) Gauss's law for magnetism is the net magnetic flux through any closed surface is zero.

6. The pole strength of a magnet may depend on its cross-section, nature of material.

7. Coulomb's law of magnetic force is inversely proportional to the squared distance between the magnetic poles and directly proportional to the product of magnetic poles.

8. The magnetic moment of a magnet is a quantity that determines the torque, it will experience in an external magnetic field. Its SI unit is A-m^2.

9. When the bar magnet is cut into two equal parts, as shown in the Fig. (a), P_1 behaves as N and P_2 behaves as S and magnetic moment of each part of magnet becomes $M/2$. When pole P_2 is placed over pole P_1 as shown in Fig. (b), the net magnetic moment of the combination is zero, i.e. $\frac{M}{2} - \frac{M}{2} = 0$.

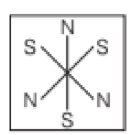
10. Ratio of the magnetic moments,

$$\frac{M_s}{M_c} = \frac{2INA_s}{I\,NA_c} = \frac{2\left(\frac{R}{2}\right)^2}{(R)^2} = \frac{1}{2}$$

11. Magnetic lines of force come out from North pole and enter into the South pole outside the magnet and travels from South pole to North pole inside the magnet. So, magnetic lines of force form closed loop, magnetic monopoles do not exist.

Note When South pole of the magnet is viewed with the frame of reference inside the magnet would appear as North pole and similarly, North pole as South pole. Therefore, magnetic lines of force traversed from South pole to North pole inside the magnet.

12. The system of magnets will be in stable equilibrium, if the net force on the system is zero and net torque on it is also zero. It will be possible, if the poles of the remaining two magnets is as shown in the figure.



13. In both the cases (i) and (ii), we get two magnets, each with a North and South pole.

14. Refer to text on page 227.

15. The statement is correct. The number of magnetic field lines leaving a surface is balanced by the number of lines entering it. The net magnetic flux is zero.

16. (i) Magnetic field at centre due to circular current carrying coil, $B = \frac{\mu_0 NI}{2r}$

(ii) Magnetic moment, $M = NIA = NI(\pi r^2)$

$$M = \pi N I r^2$$

where, r is the radius of circular coil, μ_0 is permeability of free space and N is number of turns.

17. (i) Refer to text on pages 224 and 225.

(ii) Refer to text on page 226.

18. Refer to text on page 225.

19. Using the formula for time period for magnetic system

$$T = 2\pi\sqrt{\left(\frac{I}{MH}\right)} \Rightarrow T \propto \frac{1}{\sqrt{M}} \qquad \text{...(i)}$$

When similar poles placed at same side, then

$$M_1 = M + 2M = 3M$$

So, from Eq. (i), $T_1 \propto \frac{1}{\sqrt{3M}}$ ...(ii)

When the polarity of a magnet is reversed, then

$$M_2 = 2M - M = M$$

So, from Eq . (i),

$$T_2 \propto \frac{1}{\sqrt{M}} \qquad \text{...(iii)}$$

Now, on dividing Eq. (ii) by Eq. (iii), we get

$$\frac{T_1}{T_2} = \frac{\sqrt{M}}{\sqrt{3M}} = \frac{1}{\sqrt{3}} \Rightarrow T_2 = \sqrt{3}T_1$$

Hence, $T_1 < T_2$

20. Now, suppose that the angle between M and B is θ.

Torque on magnetic dipole moment M in magnetic field B,

$$\tau' = MB\sin\theta$$

Two motions will be identical, if

$$pE\sin\theta = MB\sin\theta$$

$$\Rightarrow \qquad pE = MB \qquad \text{...(i)}$$

But $\qquad E = cB$

Putting this value in Eq. (i), we get

$$pcB = MB \Rightarrow p = \frac{M}{c}$$

21. (i) Yes

(ii) $\Sigma \mathbf{B} \cdot \Delta \mathbf{S} = mM_0$

22. Since, it is given that the current flows in the clockwise direction for an observer on the left side of the solenoid.

It means that the left face of the solenoid acts as South pole and right face acts as North pole. Inside a bar, the magnetic field lines are directed from South to North.

Therefore, the magnetic field lines are directed from left to right in the solenoid.

Magnetic moment of a single current carrying loop is given by, $m' = IA$.

So, magnetic moment of the whole solenoid is given by

$$m = Nm' = N(IA)$$

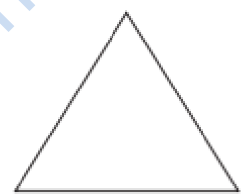
23. We know that magnetic moment of the coil $M = NIA$. Since, the same wire is used in three cases with same potentials, therefore, same current flows in three cases.

> **Hints:** The different shapes form figures of different area and hence, their magnetic moments vary.

(i) For an equilateral triangle of side a,

$N = 4$, as the total wire of length $= 12a$

Magnetic moment of the coil,



$N=4$

$$M = NIA = 4I\left(\frac{\sqrt{3}}{4}a^2\right)$$

$$\Rightarrow \qquad M = Ia^2\sqrt{3}$$

(ii) For a square of side a, $A = a^2$

$N = 3$, as the total wire of length $= 12a$

Magnetic moment of the coil,

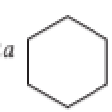
$N=3$

$$M = NIA = 3I(a^2) = 3Ia^2$$

(iii) For a regular hexagon of sides a,

$N = 2$, as the total wire of length $= 12a$

Magnetic moment of the coils,



$N=2$

$$M = NIA = 2I\left(\frac{6\sqrt{3}}{4}a^2\right) = 3\sqrt{3}a^2 I$$

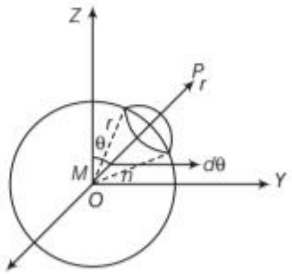
$\therefore$ M is in a geometric series.

24. Refer to text on page 219.

In vector form, $\mu = \frac{-e\mathbf{L}}{2m_e}$

Here, negative sign indicates that μ directs away from $\mathbf{L}$.

25. Let us draw the figure for given situation,



We have to prove that $\oint \mathbf{B} \cdot d\mathbf{S} = 0$. This is called Gauss's law in magnetisation.

According to the question,

Magnetic moment of dipole at origin O is $\mathbf{M} = M\hat{\mathbf{k}}$

Let P be a point at distance r from O and OP makes an angle θ with Z-axis. Component of $\mathbf{M}$ along $OP = M\cos\theta$.

Now, the magnetic field induction at P due to dipole of moment $M\cos\theta$ is $\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{2M\cos\theta}{r^3}\hat{\mathbf{r}}$

From the diagram, r is the radius of sphere with centre at O lying in YZ-plane. Take an elementary area $d\mathbf{S}$ of the surface at P. Then,

$$d\mathbf{S} = r(r\sin\theta\ d\theta)\hat{\mathbf{r}} = r^2\sin\theta\ d\theta\hat{\mathbf{r}}$$

$$\therefore \oint \mathbf{B} \cdot d\mathbf{S} = \oint \frac{\mu_0}{4\pi} \cdot \frac{2M\cos\theta}{r^3}\hat{\mathbf{r}}(r^2\sin\theta\ d\theta\hat{\mathbf{r}})$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{M}{r}\int_0^{2\pi} 2\sin\theta.\cos\theta\ d\theta$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{M}{r}\int_0^{2\pi} \sin 2\theta d\theta = \frac{\mu_0}{4\pi} \cdot \frac{M}{r}\left(\frac{-\cos 2\theta}{2}\right)_0^{2\pi}$$

$$= -\frac{\mu_0}{4\pi} \cdot \frac{M}{2r}[\cos 4\pi - \cos 0]$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{M}{2r}[1-1] = 0$$

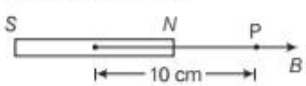
26. (i) Refer to text on pages 222 and 223.

(ii) Refer to text on page 223.

27. (i) Refer to text on page 226.

(ii) Refer to text on page 224.

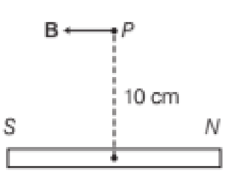
28. (i)



Refer to Example 7 on page 223.

The direction of magnetic field is from S to N-pole of magnet.

(ii)



Refer to Example 8 on page 223.

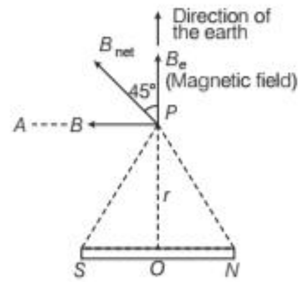
The direction of magnetic field is from N to S-pole of magnet.

29. Given, magnetic moment,

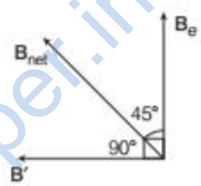
$$m = 5.25 \times 10^{-2}\ \text{J/T}$$

Let the resultant magnetic field be B_{net}.

It makes an angle of 45° with B_e.



According to the vector analysis,



$$\Rightarrow \quad \tan 45° = \frac{B\sin 90°}{B\cos 90° + B_e}$$

$$\Rightarrow \quad 1 = \frac{B}{B_e} \text{ or } B = B_e \quad \text{...(i)}$$

From Eq. (i), we get

$$\Rightarrow 0.42 \times 10^{-4} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

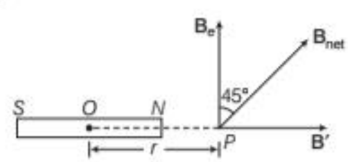
$$0.42 \times 10^{-4} = \frac{10^{-7} \times 5.25 \times 10^{-2}}{r^3}$$

$$r^3 = \frac{5.25 \times 10^{-9}}{0.42 \times 10^{-4}} = 12.5 \times 10^{-5}$$

$$r = 0.05\ \text{m}$$

(ii) **When Point Lies on Axial Line**

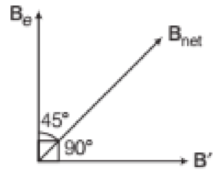
Let the resultant magnetic field B_{net} makes an angle 45° from B_e. The magnetic field on the axial line of the magnet at a distance of r from the centre of magnet.



$$B' = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} \quad \text{...(i)}$$

Direction of magnetic field is from S to N.

According to the vector analysis.



$$\tan 45° = \frac{B' \sin 90°}{B' \cos 90° + B_e}$$

$$\Rightarrow \quad 1 = \frac{B'}{B_e} \Rightarrow B_e = B'$$

From Eq. (i), we get

$$0.42 \times 10^{-4} = \frac{\mu_0}{4\pi} \times \frac{2m}{r^3}$$

$$\Rightarrow \quad 0.42 \times 10^{-4} = \frac{10^{-7} \times 2 \times 5.25 \times 10^{-2}}{r^3}$$

$$\Rightarrow \quad r^3 = \frac{10^{-9} \times 2 \times 5.25}{0.42 \times 10^{-4}} = 25.0 \times 10^{-5}$$

$$\therefore \quad r = 0.063 \text{ m or } 6.3 \text{ cm}$$

30. Given, number of turns, $n = 800$

Area of cross-section of solenoid, $A = 2.5 \times 10^{-4}$ m^2

Current through solenoid, $I = 3$ A

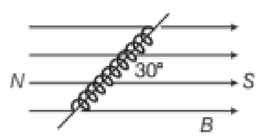
Magnetic moment of bar magnet,

$M = nIA = 800 \times 3 \times 2.5 \times 10^{-4}$

$= 0.6$ J/T along the axis of the solenoid.

31. Given, magnetic field, $B = 0.25$ T

Angle between magnetic moment and the magnetic field, $\theta = 30°$



Magnetic moment, $M = 0.6$ J/T

Torque acting on the solenoid, when it is placed at an angle θ with the magnetic field.

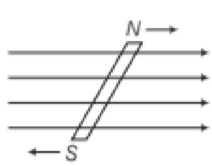
$\tau = MB \sin\theta = 0.6 \times 0.25 \sin 30°$

$= 0.6 \times 0.25 \times \frac{1}{2} = 0.075$ N-m

Thus, the magnitude of torque on the solenoid is 0.075 N-m.

32. (i) $M = 1.28$ J/T;

(ii) The force (net) on the solenoid is zero because two equal and opposite forces (on each of its poles) are acting but their lines of action are parallel, so they form a couple thus a torque (not force) is applied on it.



$\tau = 4.8 \times 10^{-2}$ N-m; refer to Q. 33 on page 240.

33. Initially,

$$T = 2\pi\sqrt{\frac{I}{mB'}} \text{ and finally, } T' = 2\pi\sqrt{\frac{I}{m(B + B')}}$$

where, B' = horizontal magnetic field $= 24\mu$T

and B = magnetic field due to downward conductor

$$= \frac{\mu_0}{4\pi} \cdot \frac{2i}{a} = 18\mu\text{T}$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{B'}{B + B'}}$$

$$\Rightarrow \quad \frac{T'}{0.1} = \sqrt{\frac{24}{18 + 24}}$$

$$\Rightarrow \quad T' = 0.076\text{s}$$

34. Given, number of turns of circular coil, $n = 16$

Radius of circular coil, $r = 10$ cm $= 0.1$ m

Current, $I = 0.75$ A, frequency, $f = 2$ s^{-1}

Magnetic field, $B = 5.0 \times 10^{-2}$ T

Magnetic moment of the coil,

$M = nIA = 16 \times 0.75 \times \pi \, (0.1)^2$

$= 16 \times 0.75 \times 3.14 \times (0.1)^2$

$= 0.377$ J/T

Frequency of oscillation of the coil, $f = \frac{1}{2\pi}\sqrt{\frac{M \times B}{I}}$

where, I = moment of inertia of the coil.

Squaring on both sides, we get

$$f^2 = \frac{1}{4\pi^2} \cdot \frac{MB}{I} \Rightarrow I = \frac{MB}{4\pi^2 f^2}$$

$$= \frac{0.377 \times 5 \times 10^{-2}}{4 \times 3.14 \times 3.14 \times 2 \times 2}$$

$$= 1.2 \times 10^{-4} \text{ kg-m}^2$$

Thus, the moment of inertia of the coil

$= 1.2 \times 10^{-4}$ kg-m^2

35. If magnet of magnetic moment M makes an angle θ with the field, then other magnet of magnetic moment $M\sqrt{3}$ makes an angle $(90° - \theta)$ with the field.

In equilibrium, $\tau_1 = \tau_2$

$$\Rightarrow \quad MB\sin\theta = M\sqrt{3}\, B\cos\theta$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60°$$

36. Given, magnetic moment of magnet, $M = 0.32$ J/T

Magnitude of magnetic field, $B = 0.15$ T

(i) For stable equilibrium, the angle between magnetic moment **M** and magnetic field **B** is $\theta = 0°$.

[$\because$ In this position, it will be in a direction parallel to the magnetic field, thus no torque will act on it.]

$\because$ The potential energy of the magnet,

$U = -\mathbf{M}\cdot\mathbf{B} = -MB\cos\theta$

$[\because \mathbf{M}\cdot\mathbf{B} = MB\cos\theta]$

$= -0.32 \times 0.15 \cos 0°$

$= -4.8 \times 10^{-2}$ J

Thus, for the stable equilibrium the potential energy is -4.8×10^{-2} J.

(ii) For unstable equilibrium, the angle between the magnetic moment and magnetic field is 180°.

($\because$ At $\theta = 180°$, although torque is zero but if it is displaced by small angle $d\theta$, then resulting torque would not restore it to the original position).

Potential energy of the magnet,

$U = -MB\cos 180°$

$= -0.32 \times 0.15\,(-1)$

$= 4.8 \times 10^{-2}$ J

Thus, for the unstable equilibrium, the potential energy is 4.8×10^{-2} J.

37. Given, magnetic moment of magnet, $M = 1.5$ J/T

Uniform magnetic field, $B = 0.22$ T

(i) (a) Angle, $\theta_1 = 0°$ [$\because$ the magnet lies aligned in the direction of field]

and $\theta_2 = 90°$ [$\because$ the magnet is to be aligned normal to the field direction]

Work done in rotating the magnet from angle θ_1 to θ_2,

$W = -MB(\cos\theta_2 - \cos\theta_1)$

$= -1.5 \times 0.22\,(\cos 90° - \cos 0°)$

$= 0.33$ J

(b) Angle, $\theta_1 = 0°$ and $\theta_2 = 180°$

[$\because$ Magnet is to be aligned opposite to the direction of field]

Work done $= -MB(\cos\theta_2 - \cos\theta_1)$

$= -1.5 \times 0.22\,(\cos 180° - \cos 0°) = 0.66$ J

(ii) Using the formula of torque,

$\tau = MB\sin\theta$

(a) When magnetic moment is normal to the field, $\theta = 90°$

$\tau = 1.5 \times 0.22 \sin 90° = 0.33$ N-m

(b) When magnetic moment is opposite to the field, $\theta = 180°$

$\tau = 1.5 \times 0.22 \sin 180° = 0$

|TOPIC 2|

Magnetic Properties of Materials

VARIOUS TERMS RELATED TO MAGNETISM

Various terms related to the magnetism are given below.

Magnetic Intensity (H)

The capability of magnetic field to magnetise the substance is measured in terms of magnetic intensity of the field. The magnitude of magnetic intensity may be defined as the number of ampere turns flowing round the unit length of toroid to produce the magnetic induction B_0, in the toroid. It is denoted by H.

$$H = \frac{B_0}{\mu_0} \quad \ldots(i)$$

where, $B_0 =$ magnetic field inside vacuum

and $\mu_0 = 4\pi \times 10^{-7}$ T-m A^{-1}

Its SI unit is Am^{-1}.

Magnetic intensity is also known as **magnetising force** and **magnetic field strength**.

Intensity of Magnetisation (I)

The intensity of magnetisation of a magnetised substance represents the degree to which the substance is magnetised. It is defined as the net magnetic moment M developed per unit volume V, when a magnetic specimen is subjected to magnetising field. It is denoted by I.

$$I = \frac{M}{V}$$

Its SI unit is Am^{-1}. Its dimension is $[L^{-1}A]$ and it is a vector quantity.

Magnetic Induction (B)

It is defined as the number of magnetic lines of induction crossing per unit area normally through the magnetic substance. It is denoted by B. Magnetic induction B is the sum of the magnetic field B_0 and the magnetic field $\mu_0 I$ produced due to the magnetisation of the substance.

Thus, $B = B_0 + \mu_0 I = \mu_0 H + \mu_0 I$

$$B = \mu_0 (H + I) \quad \ldots(ii)$$

Magnetic induction is also known as **magnetic flux density** or simply magnetic field.

Its SI unit is T or Wb m^{-2}.

Magnetic Susceptibility (χ_m)

It is a measure of how easily a substance is magnetised in a magnetising field. The magnetic susceptibility of a magnetic substance is defined as the ratio of the intensity of magnetisation to the magnetic intensity. It is denoted by χ_m.

$$\boxed{\chi_m = \frac{I}{H}} \quad \ldots\text{(iii)}$$

As units of H and I are same (A m^{-1}), therefore it has no unit.

Magnetic Permeability (μ)

It is a measure of conduction of magnetic field lines through a substance. The magnetic permeability of a magnetic substance is defined as the ratio of the magnetic induction to the magnetic intensity. It is denoted by μ.

$$\boxed{\mu = \frac{B}{H}}$$

Its SI unit is T m A^{-1}.

Relative Magnetic Permeability (μ_r)

It is the ratio of the magnetic permeability μ of the substance to the permeability of free space.

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

It is a dimensionless quantity and is equal to 1 for vacuum.

The **relative magnetic permeability of a substance** is defined as the ratio of magnetic flux density B in that substance and flux density B_0 in vacuum in the same field.

$$\boxed{\mu_r = \frac{B}{B_0}}$$

Relation between Relative Magnetic Permeability (μ_r) and Magnetic Susceptibility (χ_m)

When a substance is placed in a magnetising field, it becomes magnetised. The total magnetic flux density B within the substance is the flux density that would have been produced by the magnetising field in vacuum plus the flux density due to the magnetisation of the substance. If I be the intensity of magnetisation of the substance, then by definition, the magnetic intensity of the magnetising field is given by

$$H = \frac{B}{\mu_0} - I \quad \text{or} \quad B = \mu_0 (H + I)$$

But $\quad I = \chi_m H$

where, χ_m is the susceptibility of the substance.

$\therefore \quad B = \mu_0 H (1 + \chi_m)$

Again $B = \mu H$, where μ is the permeability of the substance.

$$\therefore \quad \mu = \mu_0 (1 + \chi_m) \quad \text{or} \quad \frac{\mu}{\mu_0} = 1 + \chi_m$$

$\frac{\mu}{\mu_0}$ is the relative permeability μ_r.

Thus, $\quad \boxed{\mu_r = 1 + \chi_m}$

The quantity $(1 + \chi_m) = \mu_r$ is the analog of dielectric constant in electrostatics and is known as relative magnetic permeability. It is a dimensionless quantity.

The value of magnetic susceptibility is small and positive for paramagnetic materials and small and negative for diamagnetic materials.

EXAMPLE |1| The magnetic field B and the magnetic intensity H in a material are found to be 1.6 T and 1000 Am^{-1}, respectively. Determine the relative permeability μ_r and the susceptibility χ_m of the material.

Sol. Magnetic permeability, $\mu = \frac{B}{H} = \frac{1.6}{1000}$

$$= 1.6 \times 10^{-3} \text{ TmA}^{-1}$$

Since, relative magnetic permeability,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.6 \times 10^{-3}}{4\pi \times 10^{-7}} = 0.127 \times 10^4$$

Therefore, susceptibility, $\chi_m = \mu_r - 1 = 1.27 \times 10^3 - 1$

$\Rightarrow \quad \chi_m = 1.27 \times 10^3$

EXAMPLE |2| A solenoid of 600 turns per metre is carrying a current of 4 A. Its core is made of iron with relative permeability of 5000. Calculate the magnitudes of magnetic intensity, intensity of magnetisation and magnetic field inside the core.

Sol. Given, current, $I = 4$ A

Number of turns per unit length, $n = 600$

Relative permeability, $\mu_r = 5000$

Since, magnetic intensity, $H = nI = 600 \times 4 = 2400 \text{ Am}^{-1}$

Since, $\quad \mu_r = 1 + \chi_m$

$\Rightarrow \quad \chi_m = \mu_r - 1$

$$= 5000 - 1 = 4999 = 5000$$

Here, χ_m = magnetic susceptibility.

Intensity of magnetisation can be given as

$$I = \chi_m H = 5000 \times 2400$$
$$= 1.2 \times 10^7 \ \text{Am}^{-1}$$

Therefore, magnetic field, $B = \mu_r \mu_0 H$

$$= 5000 \times (4\pi \times 10^{-7}) \times 2400$$
$$= 15 \ \text{T}$$

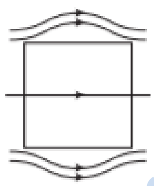
MAGNETIC PROPERTIES OF MATERIALS

Materials can be classified as diamagnetic, paramagnetic and ferromagnetic on the basis of susceptibility (χ_m).

Diamagnetic	Paramagnetic	Ferromagnetic
$-1 \le \chi_m < 0$	$0 < \chi_m < \varepsilon$	$\chi_m >> 1$
$0 \le \mu_r < 1$	$1 < \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

Here, ε is a small positive number introduced to quantify paramagnetic materials.

Diamagnetism



Diamagnetic substances are those substances which have a tendency to move from stronger to the weaker part of the external magnetic field.

When a bar of diamagnetic material is placed in an external magnetic field, the field lines are repelled or expelled and the field inside the material is reduced.

Explanation of Diamagnetism

Diamagnetic substances are those substances in which resultant magnetic moment in an atom is zero.

When magnetic field is applied, those electrons having orbital magnetic moment in the same direction slow down and those in opposite directions speed up. This happens due to induced current in accordance with Lenz's law. Thus, the substance develops a net magnetic moment in the direction opposite to that of the applied field and hence, repels. e.g. Bismuth, copper, lead, silicon, nitrogen (at STP), water and sodium chloride.

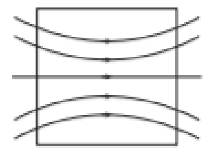
Meissner Effect

Superconductors exhibit perfect diamagnetism. A superconductor repels a magnet and (by Newton's third law) is repelled by the magnet. This phenomenon of perfect diamagnetism in superconductors is called the **Meissner effect**. Superconducting magnets have been used for running magnetically levitated superfast trains.

Paramagnetism

The substances which get weakly magnetised in the direction of external field, when placed in an external magnetic field are called paramagnetic substances. These substances have the tendency to move from a region of weak magnetic field to strong magnetic field, i.e. they get weakly attracted to a magnet.

Explanation of Paramagnetism



The atoms of a paramagnetic material possess a permanent magnetic dipole moment of their own. On account of the ceaseless random motion of the atoms, no net magnetisation is seen. But in the presence of an external field B_0, which is strong enough and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as B_0.

The field lines get concentrated inside the material and the field inside is enhanced.

When placed in a non-uniform magnetic field, the paramagnetic material tends to move from weaker part of the field to the stronger part.

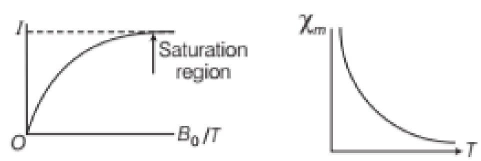
e.g. Aluminium, sodium, calcium, oxygen (at STP) and a copper chloride.

Curie's Law

Magnetisation of a paramagnetic material is inversely proportional to the absolute temperature (T),

$$I = C\frac{B_0}{T} \quad \text{or equivalently} \quad \chi_m = \frac{C\mu_0}{T}$$

where, C is called Curie's constant.



The variation of intensity of magnetisations with B_0/T is shown in the figure. At a particular stage, all the atomic dipoles present in the specimen align in the direction of the external field and this leads to saturation region.

Ferromagnetism

The substances which get strongly magnetised when placed in an external magnetic field are called ferromagnetic substances. They have strong tendency to move from a region of weak magnetic field to strong magnetic field, i.e. they get strongly attracted to a magnet.

Curie-Weiss Law

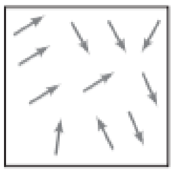
This describes the magnetic susceptibility χ_m of a ferromagnet in the paramagnetic region above the Curie point. It is expressed as

$$\chi_m = \frac{C}{T - T_C} \qquad [\because T > T_C]$$

where, C is called Curie's constant, T is absolute temperature in kelvin and T_C is Curie temperature.

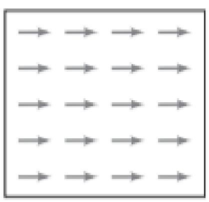
Explanation of Ferromagnetism

The atoms in a ferromagnetic material possess a dipole moment aligned in a common direction over a macroscopic volume called **domain**. Each domain has net magnetisation.



Randomly oriented domains

When we apply an external magnetic field B_0, the domains orient themselves in the direction of B_0 and simultaneously the domains grow in size.



Aligned domains

In some ferromagnetic materials, the magnetisation persists on removal of external magnetic field. Such materials are called **hard magnetic materials** or **hard ferromagnets**, e.g. alnico (an alloy of iron, aluminium, nickel, cobalt and copper) is one such material which forms permanent magnets to be used among other things as a compass needle. There are some ferromagnetic materials in which the magnetisation disappears on removal of external magnetic field, e.g. soft iron. Such materials are called **soft magnetic materials or soft ferromagnets.** e.g. Iron, cobalt, nickel, gadolinium, etc. The relative magnetic permeability of these substances is greater than 1000.

The ferromagnetic property depends on the temperature. At high temperature, a ferromagnet becomes a paramagnet.

The transition of temperature from ferromagnetism to paramagnetism is called the **Curie temperature** (T_C). The susceptibility in the paramagnetic phase is described by

$$\chi_m = \frac{C}{T - T_C} \qquad [\because T > T_C]$$

Curie Temperature T_C of Some Ferromagnetic Materials

Material	T_C (K)
Cobalt	1394
Iron	1043
Ferric oxide	893
Nickel	631
Gadolinium	317

EXAMPLE |3| A solenoid having 5000 turns/m carries a current of 2A. An aluminium ring at temperature 300K inside the solenoid provides the core.

(a) If the magnetisation I is 2×10^{-2} A/m, find the susceptibility of aluminium at 300 K.

(b) If temperature of the aluminium ring is 320 K, what will be the magnetisation?

Sol. (a) Here, $H = nI = 5000 \times 2 = 10^4$ A/m

and $I = \chi H$

$\therefore \chi = \frac{I}{H}$

$= \frac{2\times 10^{-2}}{10^4} = 2\times 10^{-6}$

(b) According to Curie law,

$$\chi = \frac{C}{T}$$

$$\Rightarrow \frac{\chi_2}{\chi_1} = \frac{T_2}{T_1}$$

$$\chi_2 = \frac{T_2}{T_1}\chi_1 = \frac{320}{300}\times 2\times 10^{-6}$$

$$= 2.13\times 10^{-6}$$

$\therefore$ Magnetisation at 320 K,

$$I = \chi_2 H = 2.13\times 10^{-6}\times 10^4$$

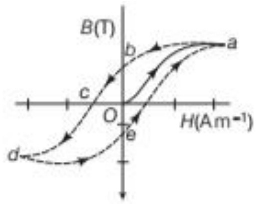
$$= 2.13\times 10^{-2}\text{A/m}$$

Hysteresis Curve

The hysteresis curve represents the relation between the magnetic induction B or intensity of magnetisation I of a ferromagnetic material with magnetic intensity H. The graph shows the behaviour of the material as we take it through one cycle of magnetisation shown in the figure.

Formation of Hysteresis Curve

An unmagnetised sample is placed in a solenoid and current through the solenoid is increased. The magnetic field B in the material rises and saturates as depicted in the curve Oa. Next, if H is decreased and reduces to zero.



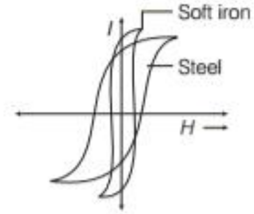
Then, at $H = 0, B \neq 0$ (curve ab)

The value of B at $H = 0$ is called **retentivity**.

Now, the current in the solenoid is reversed and slowly increased, we again obtain saturation in the reverse direction at d. The value of H at c is called **coercivity**.

Now, the current is reduced (curve de), increased, reversed (curve ea). The cycle repeats itself. For a given value of H, B is not unique, but depends on previous history of the sample. This phenomenon is called **hysteresis**.

It is found that the area of hysteresis loop is proportional to the net energy absorbed per unit volume by the material, as it is taken over a complete cycle of magnetisation. The energy so, absorbed by the specimen appears in the form of heat energy. Hysteresis loop for soft iron is large and narrow, whereas the hysteresis loop for steel is short and wide as shown in the figure.



From the hysteresis loops of soft iron and steel, we say

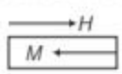
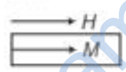
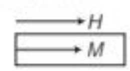
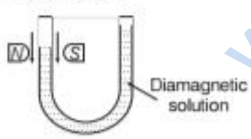
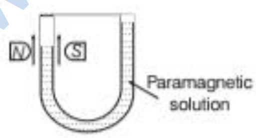
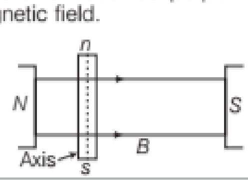
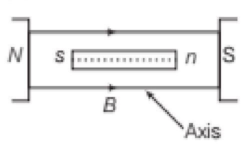
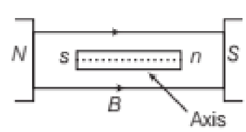
(i) Retentivity of soft iron is greater than that of steel.

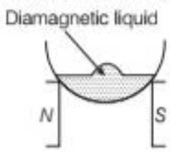
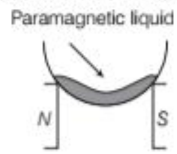
(ii) Soft iron is more strongly magnetised than steel.

(iii) Coercivity of soft iron is less than that of steel.

(iv) As area of hysteresis loop for soft iron is smaller than that for steel, therefore, hysteresis loss in case of soft iron is smaller than that in case of steel.

Comparative Study of Magnetic Materials

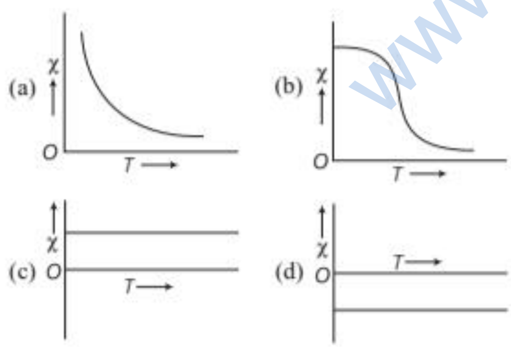
Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
These substances when placed in a magnetic field, acquire feeble magnetism opposite to the direction of the magnetic field. 	These substances when placed in a magnetic field, acquire feeble magnetism in the direction of the magnetic field. 	These substances when placed in a magnetic field are strongly magnetised in the direction of the field. 
These substances are feebly repelled by a magnet.	These substances are feebly attracted by a magnet.	These substances are strongly attracted by a magnet.
When a diamagnetic solution is poured into a U-tube and one arm is placed between the poles of strong magnet, the level of solution in that arm is lowered. 	The level of the paramagnetic solution in that arm rises. 	No liquid is ferromagnetic.
If a rod of diamagnetic material is suspended freely between two magnetic poles, its axis becomes perpendicular to the magnetic field. 	Paramagnetic rod becomes parallel to the magnetic field. 	Ferromagnetic rod also becomes parallel to the magnetic field. 
In non-uniform magnetic field, the diamagnetic substances are attracted towards the weaker fields, i.e. they move from stronger to weaker magnetic field.	In non-uniform magnetic field, paramagnetic substances move from weaker to stronger part of the magnetic field slowly.	In non-uniform magnetic field, ferromagnetic substances move from weaker to stronger magnetic field rapidly.

Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
Their permeability is less than one ($\mu < 1$).	Their permeability is slightly greater than one ($\mu > 1$).	Their permeability is much greater than one ($\mu >> 1$).
Their susceptibility is small and negative. Their susceptibility is independent of temperature.	Their susceptibility is small and positive. Their susceptibility is inversely proportional to absolute temperature, which is Curie's law. i.e. $\chi_m \propto \frac{1}{T}$.	Their susceptibility is large and positive. They follow Curie-Weiss law, when heated above Curie's temperature (T_C). i.e. $\chi_m \propto \frac{1}{T - T_C}$. At Curie temperature, ferromagnetic substances change into paramagnetic substances.
Shape of diamagnetic liquid in a glass crucible and kept over two magnetic poles. 	Shape of paramagnetic liquid in a glass crucible and kept over two magnetic poles. 	No liquid is ferromagnetic.
In these substances, the magnetic field lines are farther than in air.	In these substances, the magnetic field lines are closer than in air.	In these substances, the magnetic field lines are much closer than in air.
The resultant magnetic moment of these substances is zero.	These substances have a permanent magnetic moment.	These substances also have a permanent magnetic moment.

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. The variation of magnetic susceptibility (χ) with temperature for a diamagnetic substance is best represented by figure



2. The relative permeability of a substance X is slightly less than unity and that of substance Y is slightly more than unity, then

(a) X is paramagnetic and Y is ferromagnetic
(b) X is diamagnetic and Y is ferromagnetic
(c) X and Y both are paramagnetic
(d) X is diamagnetic and Y is paramagnetic

VERY SHORT ANSWER Type Question

3. In what way, the behaviour of a diamagnetic material is different from that of a paramagnetic, when kept in an external magnetic field? **All India 2016**

SHORT ANSWER Type Questions

4. From molecular view point, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism. **NCERT Exemplar**

5. Show diagrammatically the behaviour of magnetic field lines in the presence of

(i) paramagnetic and **All India 2014**
(ii) diamagnetic substances. How does one explain this distinguishing feature?

6. Out of the two magnetic materials, A has relative permeability slightly greater than unity while B has less than unity. Identify the nature of the materials A and B. Will their susceptibilities be positive or negative? **Delhi 2014**

7. A ball of superconducting material is dipped in liquid nitrogen and placed near a bar magnet.

(i) In which direction will it move?
(ii) What will be the direction of its magnetic moment? **NCERT Exemplar**

8. Explain quantitatively the order of magnitude difference between the diamagnetic susceptibility of N_2 ($\sim 5\times10^{-9}$) (at STP) and Cu ($\sim 10^{-5}$). **NCERT Exemplar**

9. (i) How does a diamagnetic material behave when it is cooled at very low temperature?
(ii) Why does a paramagnetic sample display greater magnetisation when cooled? Explain. **Delhi 2012**

10. The susceptibility of a magnetic material is 0.9853. Identify the type of magnetic material. Draw the modification of the field pattern on keeping a piece of this material in a uniform magnetic field. **CBSE 2018**

11. Out of the following, identify the materials which can be classified as
(i) paramagnetic
(ii) diamagnetic
(a) Aluminium (b) Bismuth
(c) Copper (d) Sodium

LONG ANSWER Type I Questions

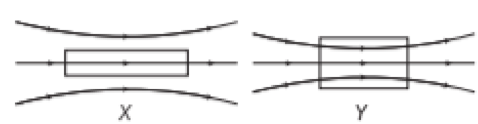
12. Three identical specimens of a magnetic material, nickel, antimony, aluminium are kept in a non-uniform magnetic field. Draw the modification in the field lines in each case. Justify your answer. **Delhi 2011**

13. Answer the following questions:
(i) Why does a paramagnetic sample display greater magnetisation (for the same magnetising field) when cooled?
(ii) If a toroid uses bismuth for its core, then will the field in the core be (slightly) greater or (slightly) less than when the core is empty?
(iii) Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?
(iv) Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point.) Why?
(v) Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetisation of a ferromagnet? **NCERT**

14. Answer the following questions.
(i) Explain qualitatively on the basis of domain picture the irreversibility in the magnetisation curve of a ferromagnet.
(ii) The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy?
(iii) A system displaying a hysteresis loop such as a ferromagnet, is a device for storing memory. Explain the meaning of this statement.
(iv) What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player or for building 'memory stores' in a modern computer?
(v) A certain region of space is to be shielded from magnetic fields. Suggest a method. **NCERT**

15. A bar magnet of magnetic moment 6 J/T is aligned at 60° with a uniform external magnetic field of 0.44 T. Calculate (a) the work done in turning the magnet to align its magnetic moment (i) normal to the magnetic field, (ii) opposite to the magnetic field, and (b) the torque on the magnet in the final orientation in case (ii). **CBSE 2018**

16. When two materials are placed in an external magnetic field, the behaviour of magnetic field lines is as shown in the figure. Identify the magnetic nature of each of these two materials. **Delhi 2009C**



17. A sample of paramagnetic salt contains 2×10^{24} atomic dipoles, each of dipole moment 1.5×10^{-23} J/T. The sample is placed under a homogenous magnetic field of 0.84 T and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What will be the total dipole moment of the sample for a magnetic field of 0.98 T and at a temperature of 2.8 K? **NCERT**

LONG ANSWER Type II Question

18. (i) Discuss briefly electron theory of magnetism for diamagnetic and paramagnetic materials.
(ii) Give two methods to destroy the magnetism of a magnet.

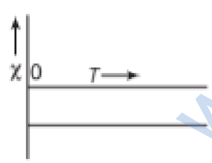
NUMERICAL PROBLEMS

19. If the bar magnet in Q. 14 is turned around by 180°, where will the new null points be located? **NCERT**

20. A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic North-South direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 gauss and the angle of dip is zero.
What is the total magnetic field on the normal bisector of the magnet at the same distance as the null point (i.e. 14 cm) from the centre of the magnet? (At null points, field due to a magnet is equal and opposite to the horizontal component of the earth's magnetic field.) **NCERT**

HINTS AND SOLUTIONS

1. (d) For diamagnetic substances, the magnetic susceptibility is negative, and it is independent of temperature. Therefore, choice (d) is correct in figure.



2. (d) As $\mu_r < 1$ for substance X, it must be diamagnetic and $\mu_r > 1$ for substance Y, is must be paramagnetic.
3. When paramagnetic materials are placed in external magnetic field, these are feebly magnetised in the direction of the applied external magnetic field whereas in case of diamagnetic materials, these are feebly magnetised opposite to that of applied external magnetic field.
4. Susceptibility of magnetic material $\chi = \frac{I}{H}$, where I is the intensity of magnetisation induced in the material and H is the magnetising force.
Diamagnetism is due to orbital motion of electrons in an atom developing magnetic moments opposite to applied field. Thus, the resultant magnetic moment of the diamagnetic material is zero and hence, the susceptibility χ of diamagnetic material is not much affected by temperature.
Paramagnetism and ferromagnetism is due to alignment of atomic magnetic moments in the direction of the applied field. As temperature is raised, the alignment is disturbed, resulting decrease in susceptibility of both with increase in temperature.
5. Refer to page 236 for diagram.
Magnetic permeability of paramagnetic substance is more than air, so it allows more lines to pass through it while permeability of diamagnetic substance is less than air, so it does not allow lines to pass through it. Thus, diamagnetic substances expel magnetic field lines, while paramagnetic substances attract them.
6. The nature of the material A is paramagnetic and its susceptibility χ_m is positive.
The nature of the material B is diamagnetic and its susceptibility χ_m is negative.
7. Both a superconducting material and nitrogen are diamagnetic in nature. When a ball of superconducting material is dipped in liquid nitrogen, it behaves as a diamagnetic material. When placed near a bar magnet, it will be feebly magnetised opposite to the direction of magnetising field.
Because of this, (i) it will be repelled (i.e. move away from magnet) (ii) the direction of magnetic moment will be opposite to the direction of magnetic field of bar magnet.
8. Here, $\chi_{m(N_2)} = 5 \times 10^{-9}$ and $\chi_{m(Cu)} = 10^{-5}$

$$\therefore \quad \frac{\chi_{m(N_2)}}{\chi_{m(Cu)}} = \frac{5 \times 10^{-9}}{10^{-5}} = 5 \times 10^{-4}$$

As, $$\chi_m = \frac{I}{H} = \frac{M/V}{H} = \frac{M}{HV} = \frac{M\rho}{Hm}$$

where, M = magnetic moment
V = volume, m = mass and ρ = density

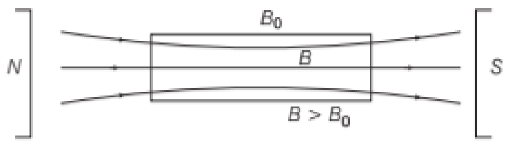
$$\therefore \quad \chi_m \propto \rho, \text{ for given value of } \frac{M}{Hm}$$

Thus, $$\frac{\chi_{m(N_2)}}{\chi_{m(Cu)}} = \frac{\rho_{N_2}}{\rho_{Cu}} = \frac{28 \text{ g}/22400 \text{ cc}}{8 \text{ g/cc}} = 1.6 \times 10^{-4}$$

9. (i) As, the resistance (electrical of metal decreases with decrease in temperature.
But for diamagnetic substances, the variation of susceptibility is very small $(0 < \chi_m < \varepsilon)$, i.e. diamagnetic materials are unaffected by the change in temperature (except bismuth).
(ii) Paramagnetic materials when cooled due to thermal agitation tendency alignment of magnetic dipoles decreases. Hence, they shows greater magnetisation.
10. Given, susceptibility, $\chi_m = 0.9853$
As the susceptibility of material is positive but small.
∴ The material is paramagnetic in nature. For paramagnetic material, magnetic lines of external

magnetic field will passes through the material without much deviation, when it is placed in between magnetic poles.

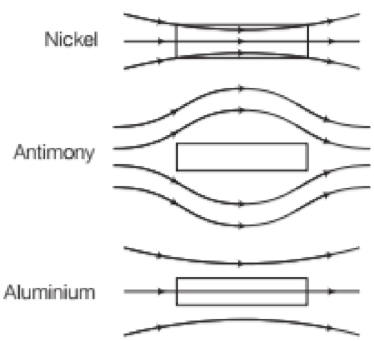
The modification of the field pattern is shown in the following figure.



11. (i) **Paramagnetic substance** Aluminium, sodium

(ii) **Diamagnetic substance** Bismuth, copper, the susceptibility of the diamagnetic materials is small and negative, i.e. $-1 < \chi_m < 0$, whereas for paramagnetic substance the susceptibility is small and positive, i.e. $0 < \chi_m < a$, where a is a small number.

12.



The modification in the field lines shown in the figure are as such because

(i) nickel is a ferromagnetic substance.

(ii) antimony is a diamagnetic substance.

(iii) aluminium is a paramagnetic substance.

Refer to text on pages 236 and 237.

13. (i) A paramagnetic sample displays greater magnetisation when cooled because at the lower temperatures, the tendency to disrupt the alignment of magnetic dipoles decreases due to the reduced random thermal motion of atoms or molecules.

(ii) Bismuth is a diamagnetic element, so the magnetic field in the core will be slightly less than when the core is empty, because the diamagnetic substances are feebly magnetised in the opposite direction of magnetic field.

(iii) No, the permeability of a ferromagnetic material is not independent of the magnetic fields. By observing the hysteresis curve, the value of permeability is greater for lower fields.

(iv) The magnetic field lines are always nearly normal to the surface of a ferromagnet at every point because the value of permeability for ferromagnetic substance is always greater than 1 ($\mu >> 1$). It is based on the conditions of B and H at the interface of two media in the hysteresis curve.

(v) Yes, the maximum possible magnetisation of a paramagnetic sample will be of the same order of magnitude as the magnetisation of a ferromagnet. Although, the condition of saturation for paramagnets, requires very high magnetising fields which cannot be achieved.

14. (i) To explain qualitatively the domain picture of the irreversibility in the magnetisation curve of a ferromagnet, we draw the hysteresis curve for ferromagnetic substance. We can observe that the magnetisation persists even when the external field is removed. This gives the idea of irreversibility of a ferromagnet.

(ii) As we know that, in hysteresis curve, the energy dissipated per cycle is directly proportional to the area of hysteresis loop. So, according to the question, the area of hysteresis loop is more for carbon steel, thus carbon steel piece will dissipate greater heat energy.

(iii) The magnetisation of a ferromagnet depends not only on the magnetising field, but also on the history of magnetisation (i.e. how many times it was already magnetised in the past). Thus, the value of magnetisation of a specimen is a record of memory of the cycles of magnetisation, it had undergone. The system displaying such a hysteresis loop can thus act as a device for storing memory.

(iv) The ferromagnetic materials which are used for coating magnetic tapes in a cassette player or for building memory stores in the modern computer are ferrites. The most commonly ferrites used are $MnFe_2O_4$, $FeFe_2O_4$, $CoFe_2O_4$, $NiFe_2O_4$, etc.

(v) To shield any space from magnetic field, surround the space with soft iron ring. As the magnetic field lines will be drawn into the ring, the enclosed region will become free of magnetic field.

15. (a) Given, magnetic moment, $M = 6$ J/T

Aligned angle, $\theta_1 = 60°$

External magnetic field, $B = 0.44$ T

(i) When the bar magnet is align normal to the magnetic field, i.e. $\theta_2 = 90°$

$\therefore$ Amount of work done in turning the magnet,

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$
$$= -6 \times 0.44\,(\cos 90° - \cos 60°)$$
$$= +6 \times 0.44 \times \frac{1}{2} \quad \begin{pmatrix}\because \cos 90° = 0 \\ \text{and } \cos 60° = 1/2\end{pmatrix}$$
$$= 1.32 \text{ J}$$

(ii) When the bar magnet align opposite to the magnetic field, i.e. $\theta_2 = 180°$

$$\therefore\ W = -MB(\cos 180° - \cos 60°)$$
$$= -6 \times 0.44 \left(-1 - \frac{1}{2}\right) (\because \cos 180° = -1)$$
$$= 6 \times 0.44 \times \frac{3}{2} = 3.96 \text{ J}$$

(b) We know that, torque,

$$\tau = \mathbf{M} \times \mathbf{B} = MB\sin\theta$$

For case (ii), $\theta = 180°$

$\therefore \quad \tau = MB\sin 180° \quad (\because \sin 180° = 0)$
$= 0$

$\therefore$ Amount of torque is zero for case (ii).

16. (i) Material X is paramagnetic substance. When a specimen of a paramagnetic substance is placed in a magnetising field, the lines of force prefer to pass through the specimen rather than through air. Thus, magnetic induction inside the sample is more than the magnetic intensity.

(ii) Material Y is ferromagnetic substance. These are the substances in which a strong magnetism is produced in the same direction as the applied magnetic field, these are strongly attracted by a magnet, exhibits highly concentrated lines of force.

17. According to Curie's law, $\chi_m = \dfrac{C}{T}$

As magnetic susceptibility, $\chi_m = \dfrac{I}{H}$

$$\Rightarrow \quad I = \frac{M}{V} \text{ and } H = \frac{B}{\mu}$$

$$\Rightarrow \quad \frac{M/V}{B/\mu} = \frac{C}{T}$$

$$\Rightarrow \quad M = \frac{CV}{\mu}\left(\frac{B}{T}\right)$$

For a given sample, $CV/\mu =$ constant

Thus, $M = \left(\dfrac{B}{T}\right)$

or $\dfrac{M_1}{M_2} = \dfrac{B_1/T_1}{B_2/T_2}$

Given, $B_1 = 0.84$ T, $B_2 = 0.98$ T
$T_1 = 4.2$ K, $T_2 = 2.8$ K

Thus, $\dfrac{M_1}{M_2} = \dfrac{0.84/4.2}{0.98/2.8} = \dfrac{4}{7}$

or $M_2 = \left(\dfrac{7}{4}\right) M_1$

Initial total magnetic moment of the sample,

$$M_1 = 15\% \text{ of } (2 \times 10^{24})(1.5 \times 10^{-23}) = 4.5 \text{ J/T}$$

Thus, $M_2 = \left(\dfrac{7}{4}\right) 4.5 = 7.9$ J/T

18. (i) Refer to text on page 236.

(ii) We can destroy the magnetism of a magnet
(a) by heating it.
(b) by applying magnetic field across it in reverse direction.

19. When a bar magnet is turned by $180°$, then the null points are obtained on the equatorial line.

So, magnetic field on the equatorial line at a distance d' is given by

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{M}{d'^3}$$

This magnetic field is equal to the horizontal component of the earth's magnetic field,

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{M}{d'^3} = H \quad \text{...(i)}$$

As we know that,

Magnetic field, $B_1 = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2M}{d^3} = H$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{d'^3} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$$

or $\dfrac{1}{d'^3} = \dfrac{2}{d^3}$

or $d'^3 = \dfrac{d^3}{2} = \dfrac{(14)^3}{2}$ $[\because d = 14 \text{ cm}]$

or $d' = \dfrac{14}{(2)^{1/3}} = 11.1$ cm

Thus, the null points are located on the equatorial line at a distance of 11.1 cm.

20. Distance of the null point from the centre of magnet,

$$d = 14 \text{ cm} = 0.14 \text{ m}$$

The earth's magnetic field, where the angle of dip is zero, is the horizontal component of the earth's magnetic field.

i.e. $H = 0.36$ gauss

Initially, the null points are on the axis of the magnet. We use the formula of magnetic field on axial line (consider that the magnet is short in length).

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$$

This magnetic field is equal to the horizontal component of the earth's magnetic field.

i.e. $B_1 = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2M}{d^3} = H$ ...(i)

On the equatorial line of magnet at same distance d magnetic field due to the magnet,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3} = \frac{B_1}{2} = \frac{H}{2} \quad \text{...(ii)}$$

The total magnetic field on equatorial line at this point (as given in question),

$$B = B_2 + B_1 = \frac{H}{2} + H$$
$$= \frac{3}{2}H = \frac{3}{2} \times 0.36$$
$$= 0.54 \text{ gauss}$$

The direction of magnetic field is in the direction of earth's field.

SUMMARY

- The phenomenon of attraction of small bits of iron, steel, cobalt, nickel, etc., towards the ore is called magnetism.
- Magnetic materials tend to point in the North-South direction. Like magnetic poles repel and unlike poles attract each other.
- Cutting a bar magnet creates two smaller magnets. Therefore, magnetic poles cannot be separated, i.e. magnetic monopole does not exist.
- **Force between two magnetic poles** is given by, $F = \frac{km_1m_2}{r^2}$

  where k is magnetic force constant and is given by
  $$k = \frac{M_0}{4\pi} = 10^{-7}$$
- The magnetic dipole moment of a magnetic dipole is given by
  $$\mathbf{M} = m \times 2l$$
  where, m is pole strength and $2l$ is dipole length directed from S to N. The SI unit of magnetic dipole moment is A-m^2 or JT^{-1}. It is a vector quantity and its direction is from South pole to North pole.
- **Magnetic Dipole** is defined as two magnetic poles of equal and opposite strengths separated by a small distance, e.g. bar magnet, compass needle, etc.
- **The Magnetic Field Lines** These are the imaginary lines which continuously represent the direction of magnetic field.
- **Magnetic Field Strength at a Point due to Bar Magnet** The force experienced by a hypothetical unit North pole placed at that point.

  (i) **When Point Lies on Axial Line of Bar Magnet** In this case, $B = \frac{\mu_0 2Md}{4\pi(d^2 - l^2)^2}$

  (ii) **When Point Lies on Equatorial Line of a Bar Magnet** In this case, $B = \frac{\mu_0 M}{4\pi(d^2 + l^2)^{3/2}}$
- Torque on a bar magnet in a uniform magnetic field is
  $$\tau = MB\sin\theta = \mathbf{M} \times \mathbf{B}$$
- **Oscillation of a Freely Suspended Magnet** The oscillations of a freely suspended magnet (magnetic dipole) in a uniform magnetic field are SHM.

  The time period of oscillation, $T = 2\pi\sqrt{\frac{I}{MB}}$
- **Potential energy** of a magnetic dipole in a magnetic field is given by $U = -MB\cos\theta = -\mathbf{M}\cdot\mathbf{B}$

  where, θ is the angle between **M** and **B**.
- **Magnetism and Gauss' Law** The number of magnetic field lines leaving any closed surface is always equal to the number of magnetic field lines entering it.
- **Magnetic Intensity** i.e. $H = \frac{B_0}{\mu_0}$
- **Intensity of Magnetisation** i.e. $I = \frac{M}{V}$
- **Magnetic Induction** (B) i.e. $B = \mu_0(H + I)$
- **Magnetic Susceptibility** i.e. $\chi_m = I/H$
- **Magnetic Permeability** i.e. $\mu = (B/H)$
- **Relative Magnetic Permeability** i.e. $\mu_r = (\mu/\mu_0)$
- **Relation between Relative Permeability (μ_r) and Magnetic Susceptibility (χ_m)** It is given by, $\mu_r = 1 + \chi_m$
- Magnetic materials are broadly classified as diamagnetic, paramagnetic and ferromagnetic. For diamagnetic materials, χ is negative and small and for paramagnetic materials, it is positive and small.
- The magnetic susceptibility of a ferromagnetic materials varies as $\chi_m \propto \frac{1}{(T - T_c)}$ or $\chi_m = \frac{C}{(T - T_c)}$

  where, C is a constant. It is known as **Curie-Weiss** law and T_c is called Curie temperature.

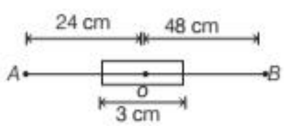
CHAPTER PRACTICE

OBJECTIVE Type Questions

1. Cutting a bar magnet in half is like cutting a solenoid, such that we get two smaller solenoids with

(a) weaker magnetic properties
(b) strong magnetic properties
(c) constant magnetic properties
(d) Both (a) and (b)

2. A bar magnet of length 3 cm has points A and B along axis at a distance of 24 cm and 48 cm on the opposite ends. Ratio of magnetic fields at these points will be



(a) 8 (b) 3 (c) 4 (d) $1/2\sqrt{2}$

3. A short bar magnet placed with its axis at 30° with an external field of 800 G experiences a torque of 0.016 Nm. The magnetic moment of the magnet is

(a) 4 Am^2 (b) 0.5 Am^2
(c) 2 Am^2 (d) 0.40 Am^2

4. A bar magnet has magnetic dipole moment **M**. Its initial position is parallel to the direction of uniform magnetic field **B**. In this position, the magnitudes of torque and force acting on it, respectively are **CBSE 2021 Term-I**

(a) 0 and **MB** (b) **MB** and **MB**
(c) 0 and 0 (d) $|\mathbf{M} \times \mathbf{B}|$ and 0

5. If a diamagnetic substance is brought near the North or the South-pole of a bar magnet, then it is

(a) attracted by the both poles
(b) repelled by both the poles
(c) repelled by the North-pole and attracted by the South-pole
(d) attracted by the North-pole and repelled by the South-pole

6. Ferromagnetism show their properties due to

(a) filled inner subshells
(b) vacant inner subshells
(c) partially filled inner subshells
(d) all the subshells equally filled

7. The relative permeability of a substance is 0.9999. The nature of substance will be

(a) diamagnetic
(b) paramagnetic
(c) magnetic moment
(d) intensity of magnetic field

8. Hysteresis loss is minimised by using

(a) alloy of steel
(b) shell type of core
(c) thick wire which has low resistance
(d) metal

ASSERTION AND REASON

Directions (Q. Nos. 9-19) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

9. **Assertion** The true geographic North direction cannot be found by using a compass needle.
Reason The magnetic meridian of the earth is along the axis of rotation of the earth.

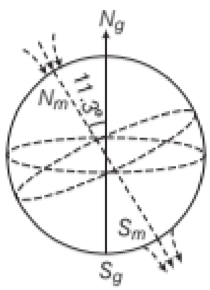
10. **Assertion** The axis of the dipole does not coincide with the axis of rotation of the earth but is presently tilted by approximately 11.3° with respect to the later.

Reason The magnetic poles are located where the magnetic field lines due to the dipole enter or leave the earth.

11. Consider the figure,

Assertion Unlike in the case of bar magnet, the field lines go into the earth at the North magnetic pole (N_m) and come out from the South magnetic pole (S_m).

Reason The magnetic North was the direction to which the North-pole of a magnetic needle pointed; the North-pole of a magnet was so named as it was the North seeking pole.



12. **Assertion** A magnetic needle, which is free to swing horizontally, would lie in the magnetic meridian and the North-pole of the needle would point towards the magnetic North-pole.

Reason The line joining the magnetic poles is tilted with respect to the geographic axis of the earth, the magnetic meridian at a point makes angle with the geographic meridian.

13. **Assertion** When magnetic field is applied to a diamagnetic substance, those electrons having orbital magnetic moment in the same direction slow down and those in the opposite direction speed up.

Reason This happens due to induced current in accordance with Lenz's law and the substance develops a net magnetic moment in direction opposite to that of the applied field and hence repulsion.

14. **Assertion** Susceptibility is defined or the ratio of intensity of magnetisation I to magnetic intensity H.

Reason Greater the value of susceptibility smaller value of intensity magnetisation I.

15. **Assertion** The pole of magnet cannot be separated by breaking into two pieces.

Reason The magnetic moment will be reduced to half when a magnet is broken into two equal pieces.

16. **Assertion** Substances which at room temperature retain their ferromagnetic property for a long period of time are called permanent magnets.

Reason Permanent magnet can be made by placing a ferromagnetic rod in a solenoid and passing current through it.

17. **Assertion** Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field.

Reason The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material.

18. **Assertion** A paramagnetic sample displays greater magnetisation (for the same magnetising field) when cooled.

Reason The magnetisation does not depends on temperature.

19. **Assertion** The poles of a bar magnet cannot be separated.

Reason Magnetic monopoles do not exist.

CASE BASED QUESTIONS

Directions (Q.No. 20) *This question is case study based question. Attempt any 4 sub-parts from this question. Each question carries 1 mark.*

20. **Dipole in Magnetic Fields**

To determine the magnitude of **B** accurately, a small compass needle of known magnetic moment m and moment of inertia I is allowed to oscillate in the magnetic field. This arrangement is shown in figure.

(i) The torque on the needle is

(a) $\tau = 2\mathbf{m} \times \mathbf{B}$ (b) $\tau = \mathbf{m} \times \mathbf{B}$

(c) $\tau = \mathbf{m} \times \mathbf{B}/2$ (d) $\tau = \mathbf{m} \times 2\mathbf{B}$

(ii) Which of the following represents a simple harmonic motion?

(a) $\frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta$

(b) $\frac{d\theta}{dt^2} = -\frac{mB}{I}\theta$

(c) $\frac{d^2\theta}{dt} = \frac{mB}{I}\theta$

(d) $\frac{d^2\theta}{dt^2} = \frac{mB}{I}\theta$

(iii) The time period of oscillation of the dipole is

(a) $2\pi\sqrt{\frac{2I}{mB}}$ (b) $2\pi\sqrt{\frac{I}{2mB}}$

(c) $4\pi\sqrt{\frac{I}{mB}}$ (d) $2\pi\sqrt{\frac{I}{mB}}$

(iv) The magnitude of the magnetic field if time period is T is

(a) $B=\frac{4\pi^2 I}{mT^2}$ (b) $B=\frac{2\pi^2 I}{mT^2}$

(c) $B=\frac{\pi^2 I}{2mT^2}$ (d) $B=\frac{3\pi^2 I}{2\,mT^2}$

(v) The magnetic potential energy U_m is given by

(a) $U_m = -\mathbf{m}\cdot\mathbf{B}$

(b) $U_m = \mathbf{m}\cdot\mathbf{B}$

(c) $U_m = 2\mathbf{m}\cdot\mathbf{B}$

(d) $U_m = -2\mathbf{m}\cdot\mathbf{B}$

VERY SHORT ANSWER Type Questions

21. Name the physical quantity having unit J/T. **CBSE Sample Paper**

22. What is the basic difference between magnetic and electric field lines?

23. In a submarine, a compass becomes ineffective. Why?

24. Why is diamagnetism almost independent of temperature?

25. If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?

26. One cannot write the proportionality $B=\mu H$ for the ferromagnets. Comment.

27. "Alkali halides are diamagnetic rather than paramagnetic." Explain why?

28. The magnetic susceptibility of magnesium at 300K is 1.2×10^{5}. At what temperature will its magnetic susceptibility become 1.44×10^{5}? **[CBSE 2019]**

29. The magnetic susceptibility of χ of a given material is -0.5. Identify the magnetic material. **[CSBE 2019]**

SHORT ANSWER Type Questions

30. What is the net magnetic moment of two identical magnets each of magnetic moment m_0 inclined at 60° with each other?

31. What are the magnetic field lines? State their properties. Why two such lines do not intersect each other?

32. A wire of length L is bent in the form of a circle of radius R and carries current I. What is its magnetic moment?

33. Derive an expression for the torque acting on a bar magnet placed in the uniform magnetic field.

34. Suppose you have two bars of identical dimensions, one made of paramagnetic substance and the another of diamagnetic substance. If you place these bars along a uniform magnetic field, show diagrammatically, what modifications in the field pattern would take place in each case?

LONG ANSWER Type I Questions

35. (a) State Gauss's law for magnetism. Explain its significance.

(b) Write the four important properties of the magnetic field lines due to a bar magnet. **[CBSE 2019]**

36. Write three points of differences between para-, dia- and ferro- magnetic materials, giving one example for each. **[CBSE 2019]**

37. Define magnetic susceptibility of a material. Name two elements, one having positive susceptibility and the other having negative susceptibility. What does negative susceptibility signify?

38. Draw a plot showing the variation of intensity of magnetisation with the applied magnetic field intensity for bismuth. Under what condition does a diamagnetic material exhibit perfect conductivity and perfect diamagnetism?

39. Explain the following:

(i) Diamagnetism is the result of induced magnetic dipole moments.

(ii) Hysteresis associated with a loss in electromagnetic energy.

40. Explain the phenomenon of hysteresis in magnetic materials. Draw a hysteresis loop showing remanence and coercive force.

LONG ANSWER Type II Question

41. (i) A bar magnet of magnetic moment M is aligned parallel to the direction of a uniform magnetic field B. What is the work done, to turn the magnets, so as to align its magnetic moment
(a) opposite to field direction and
(b) normal to field direction?
(ii) Steel is preferred for making permanent magnets, whereas soft iron is preferred for making electromagnets. Give one reason.

NUMERICAL PROBLEMS

42. Calculate the magnetic induction at a point 4 cm from the centre and along the equator of a bar magnet of length 6 cm and magnetic moment 0.26 A-m^2.

43. A bar magnet of length 0.1 m and a pole strength 10^{-4} A-m is placed in a magnetic field of 30 Wb/m^2 at an angle 30°. Determine the couple acting on it.

ANSWERS

1. (a) 2. (a) 3. (d) 4. (d) 5. (b)
6. (c) 7. (a) 8. (d)

9. (c) A compass is simply a needle shaped magnet that mounted so that it can rotate freely about a vertical axis. When it is held in a horizontal plane, the North-pole end of the needle points, generally, towards the geomagnetic North-pole (really a South magnetic pole). Thus, true geographic North direction cannot be found by using a compass needle. Now, vertical plane passing through the magnetic axis of earth's magnet is called magnetic meridian.

10. (b)

11. (a) If one looks at the magnetic field lines of the earth one sees that unlike in the case of a bar magnet, the field lines go into the earth at the North magnetic pole (N_m) and come out from the South magnetic pole (S_m).
The convention arose because the magnetic North was the direction to which the North-pole of a magnetic needle pointed; the North-pole of a magnet was so named as it was the North seeking pole.
Thus, in reality, the North magnetic pole behaves like the South-pole of a bar magnet inside the earth and *vice-versa*.

12. (b) A magnetic needle which is free to swing horizontally would, lie in the magnetic meridian and the North-pole of the needle would point towards the magnetic North-pole.
The line joining the magnetic poles is tilted with respect to the geographic axis of the Earth, the magnetic meridian at a point makes angle with the geographic meridian.

13. (c)

14. (c) From the relation, susceptibility of the material is

$$\chi_m = \frac{I}{H} \Rightarrow I = \chi_m H$$

Thus, it is obvious that greater the value of susceptibility of a material greater will be the value of intensity of magnetisation *i.e.*, more easily it can be magnetised.

15. (b) 16. (b)
17. (c) 18. (b) 19. (a)

20. (i) (b) The torque on the needle is

$$\tau = \mathbf{m} \times \mathbf{B}$$

Magnitude of torque, $\tau = mB \sin\theta$

(ii) (a) Torque on the needle

$$\tau = mB \sin\theta$$

Here, τ is restoring torque and θ is the angle between $\mathbf{m}$ and $\mathbf{B}$.

Therefore, in equilibrium $I\dfrac{d^2\theta}{dt^2} = -mB\sin\theta$

Negative sign with $mB\sin\theta$ implies that restoring torque is in opposition to deflecting torque. For small values of θ in radians, we approximate $\sin\theta = \theta$ and we get

$$I\frac{d^2\theta}{dt^2} = -mB\theta \quad \text{or} \quad \frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta$$

This represents a simple harmonic motion.

(iii) (d) The square of the angular frequency is $\omega^2 = mB/II$ and the time period,

$$T = 2\pi\sqrt{\frac{I}{mB}} \quad \text{or} \quad B = \frac{4\pi^2 I}{mT^2}$$

(iv) (a) The magnitude of the magnetic field is $B = \dfrac{4\pi^2 I}{mT^2}$.

(v) (a) An expression for magnetic potential energy can also be obtained on lines similar to electrostatic potential energy.
The magnetic potential energy U_m is given by

$$U_m = \int \tau(\theta)\, d\theta = \int mB\sin\theta\, d\theta$$

$$= (-mB\cos\theta) = -\mathbf{m}\cdot\mathbf{B}$$

21. The torque acting on a bar magnet placed in a uniform magnetic field is

$$\tau = \mathbf{M} \times \mathbf{B}$$

where, M is the magnetic dipole moment and B is the magnetic field.

$$\Rightarrow \quad \mathbf{M} = \frac{\tau}{\mathbf{B}} = \text{ Joule (J)/Tesla(T)}$$

$\therefore$ Magnetic dipole moment $(\mathbf{M})$ has unit of J/T.

22. The magnetic field lines form continuous closed loops, whereas the electric field lines begin from a positive charge and end on the negative charge or escape to infinity.

23. The body of a submarine is made of steel and other magnetic substances which causes the compass needle to deviate from the magnetic meridian.

24. Diamagnetism is independent of temperature because the value of susceptibility (a measure of relative amount of induced magnetism) is always negative.

25. Bismuth is a diamagnetic material. So, when it is kept in an external magnetic field the field lines are repelled and the field inside the material is reduced.

26. For ferromagnets, we cannot write $B = \mu H$ because the relation between B and H is not linear and it depends on the magnetic history of the sample. This phenomenon is called hysterisis.

27. The alkali halides are all diamagnetic because of the absence of unpaired electrons. So, they do not show paramagnetism.

28. The susceptibility of magnetic material is inversely proportional to temperature, i.e.

$$\chi_m \propto \frac{1}{T}$$

$$\therefore \quad \frac{\chi_m(T)}{\chi_m\,(300\,\text{K})} = \frac{300}{T} \Rightarrow T = \frac{300 \times 1.2 \times 10^5}{1.44 \times 10^5} = 250\,\text{K}$$

29. Substance having (small) negative value (-0.5) of magnetic susceptibility χ_m are diamagnetic.

30. $M = \sqrt{M_1^2 + M_2^2 + 2M_1M_2 \cos\theta}$

31. Refer to text on page 222.

32. As, $L = 2\pi R$

$$\Rightarrow \quad R = \frac{L}{2\pi}$$

$$\Rightarrow \quad M = IA = I \times \pi R^2$$

$$= I\pi \times \frac{L^2}{4\pi^2} = \frac{IL^2}{4\pi}$$

33. Refer to text on pages 224 and 225.

34. Refer to text on pages 236 and 237.

35. (a) Refer to text on page 227. (Magnetism and Gauss' Law)

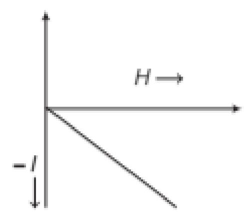
(b) Refer to text on page 222. (Properties of Magnetic Field Lines)

36. **Difference between para-, dia- and ferro-magnetic materials**

Refer to text on pages 238 and 239. (Comparative Study of Magnetic Materials)

37. Refer to text on page 235.

38. Here, intensity of magnetisation varies inversely with magnetic field strength i.e., $-I \propto H$ as shown in figure.



Refer to text on page 236.

39. (i) Refer to text on page 236.

(ii) Refer to text on page 238.

40. Refer to text on pages 237 and 238.

41. (i) $W = -MB\,(\cos\theta - \cos\theta_0)$

(ii) Refer to text on page 238.

42. Refer to Q. 29 on page 229.

43. Torque, $\tau = MB\sin\theta$