5

Work, Energy and Power

Multiple Choice Questions (MCQs)

Q. 1 An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is, because

(a) the two magnetic forces are equal and opposite, so they produce no net effect
(b) the magnetic forces do not work on each particle
(c) the magnetic forces do equal and opposite (but non-zero) work on each particle
(d) the magnetic forces are necessarily negligible

Thinking Process

In this problem as the electron and proton are moving under the influence of mutual forces, they will perform circular motion about their centre (i.e., about middle point of the line joining them).

Ans. *(b)* When electron and proton are moving under influence of their mutual forces, the magnetic forces will be perpendicular to their motion hence no work is done by these forces.

Q. 2 A proton is kept at rest. A positively charged particle is released from rest at a distance *d* in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In the same time *t*, the work done on the two moving charged particles is

(a) same as the same force law is involved in the two experiments
(b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens
(c) more for the case of a positron, as the positron moves away a larger distance
(d) same as the work done by charged particle on the stationary proton

Ans. *(c)* Force between two protons is same as that of between proton and a positron.

As positron is much lighter than proton, it moves away through much larger distance compared to proton.

We know that work done = force × distance. As forces are same in case of proton and positron but distance moved by positron is larger, hence, work done will be more.

Q. 3 A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is

(a) constant and equal to mg in magnitude
(b) constant and greater than mg in magnitude
(c) variable but always greater than mg
(d) at first greater than mg and later becomes equal to mg

Ans. *(d)* When the man is squatting on the ground he is tilted somewhat, hence he also has to balance frictional force besides his weight in this case.

$$R = \text{reactional force} = \text{friction} + mg$$

$$\Rightarrow \qquad R > mg$$

When the man gets straight up in that case friction ≈ 0

$$\Rightarrow \qquad \text{Reactional force} \approx mg$$

Q. 4 A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is

(a) + 2000J (b) – 200J (c) zero (d) – 20,000J

Thinking Process

In this problem energy will be lost due to dissipation by friction.

Ans. *(c)* Here, work is done by the frictional force on the cycle and is equal to $-200 \times 10 = -2000$ J.

As the road is not moving, hence, work done by the cycle on the road = zero.

***Note** We should be aware that here the energy of bicyclist is lost during the motion, but it is lost due to friction in the form of heat.*

Q. 5 A body is falling freely under the action of gravity alone in vaccum. Which of the following quantities remain constant during the fall?

(a) Kinetic energy (b) Potential energy
(c) Total mechanical energy (d) Total linear momentum

Ans. *(c)* As the body is falling freely under gravity, the potential energy decreases and kinetic energy increases but total mechanical energy (PE + KE) of the body and earth system will be constant as external force on the system is zero.

Q. 6 During inelastic collision between two bodies, which of the following quantities always remain conserved?

(a) Total kinetic energy (b) Total mechanical energy
(c) Total linear momentum (d) Speed of each body

Thinking Process

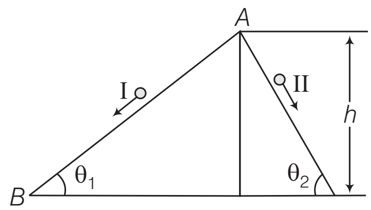
In an inelastic collision between two bodies due to some deformation, energy may be lost in the form of heat and sound etc.

Ans. *(c)* When we are considering the two bodies as system the total external force on the system will be zero.

Hence, total linear momentum of the system remain conserved.

Q. 7 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure.

Which of the following statement is correct?



(a) Both the stones reach the bottom at the same time but not with the same speed
(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II
(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I
(d) Both the stones reach the bottom at different times and with different speeds

Ans. ***(c)*** As the given tracks are frictionless, hence, mechanical energy will be conserved. As both the tracks having common height, h.

From conservation of mechanical energy,

$$\frac{1}{2}mv^2 = mgh \qquad \text{(for both tracks I and II)}$$

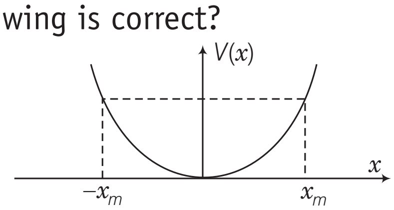
$$v = \sqrt{2gh}$$

Hence, speed is same for both stones. For stone I, a_1 = acceleration along inclined plane $= g \sin \theta_1$

Similarly, for stone II $a_2 = g \sin \theta_2$ as $\theta_2 > \theta_1$ hence, $a_2 > a_1$.

And both length for track II is also less hence, stone II reaches earlier than stone I.

Q. 8 The potential energy function for a particle executing linear SHM is given by $V(x) = \frac{1}{2}kx^2$ where k is the force constant of the oscillator (Fig). For $k = 0.5$ N/m, the graph of $V(x)$ *versus* x is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$. If V and K indicate the PE and KE, respectively of the particle at $x = +x_m$, then which of the following is correct?



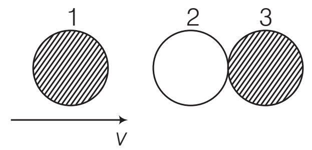
(a) $V = O,\quad K = E$
(b) $V = E,\quad K = O$
(c) $V < E,\quad K = O$
(d) $V = O,\quad K < E$

Ans. ***(b)*** Total energy is $E = \text{PE} + \text{KE}$ ...(i)

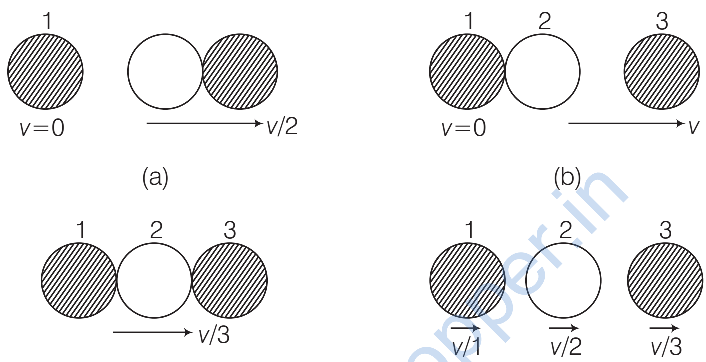
When particle is at $x = x_m$ *i.e.*, at extreme position, returns back. Hence, at $x = x_m$; $x = 0$; K E = 0

From Eq. (i) $\quad E = \text{PE} + 0 = \text{PE} = V(x_m) = \frac{1}{2}kx_m^2$

Q. 9 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v as shown in figure.



If the collision is elastic, which of the following (figure) is a possible result after collision?



(a) (b) (c) (d)

Ans. *(b)* When two bodies of equal masses collides elastically, their velocities are interchanged.

When ball 1 collides with ball-2, then velocity of ball-1, v_1 becomes zero and velocity of ball-2, v_2 becomes v, *i.e.*, similarly.

$$v_1 = 0 \Rightarrow v_2 = v$$

when ball 2 collides will ball 3 $\quad v_2 = 0, v_3 = v$

Q. 10 A body of mass 0.5 kg travels in a straight line with velocity $v = a\,x^{3/2}$ where $a = 5\text{ m}^{-1/2}\text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is

(a) 1.5 J (b) 50 J (c) 10 J (d) 100 J

Ans. Given, $v = ax^{3/2}$

$$m = 0.5\,\text{kg},\ a = 5\,\text{m}^{-1/2}\text{s}^{-1},\ \text{work done } (W) = ?$$

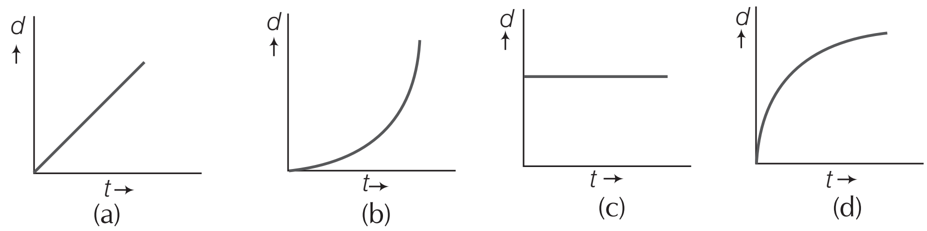
We know that

Acceleration
$$a_0 = \frac{dv}{dt} = v\frac{dv}{dx} = ax^{3/2}\frac{d}{dx}(ax^{3/2})$$
$$= ax^{3/2} \times a \times \frac{3}{2} \times x^{1/2} = \frac{3}{2}a^2x^2$$

Now,
$$\text{Force} = ma_0 = m\frac{3}{2}a^2x^2$$

$$\text{Work done} = \int_{x=0}^{x=2} F\,dx = \int_0^2 \frac{3}{2}ma^2x^2\,dx$$
$$= \frac{3}{2}ma^2 \times (x^3/3)_0^2$$
$$= \frac{1}{2}ma^2 \times 8 = \frac{1}{2} \times (0.5) \times (25) \times 8 = 50\,\text{J}$$

Q. 11 A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams shown in figure correctly shown the displacement-time curve for its motion?



Ans. ***(b)*** Given, power = constant

We know that power (P)

$$P = \frac{dW}{dt} = \frac{\mathbf{F.ds}}{dt} = \frac{F\,ds}{dt} \qquad (\because \text{body is moving unidirectionally})$$

Hence, $F.ds = Fds \cos 0°$

$$P = \frac{Fds}{dt} = \text{constant} \qquad (\because P = \text{constant by question})$$

Now, writing dimensions

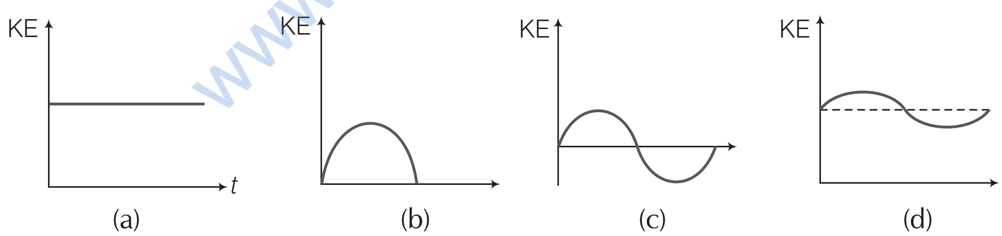
$$[\text{F}]\ [\text{v}] = \text{constant}$$

$\Rightarrow$ $[\text{MLT}^{-2}]\ [\text{LT}^{-1}] = \text{constant}$

$\Rightarrow$ $\text{L}^2\ \text{T}^{-3} = \text{constant}$ $(\because \text{mass is constant})$

$\Rightarrow$ $L \propto T^{3/2} \Rightarrow \text{Displacement } (d) \propto t^{3/2}$

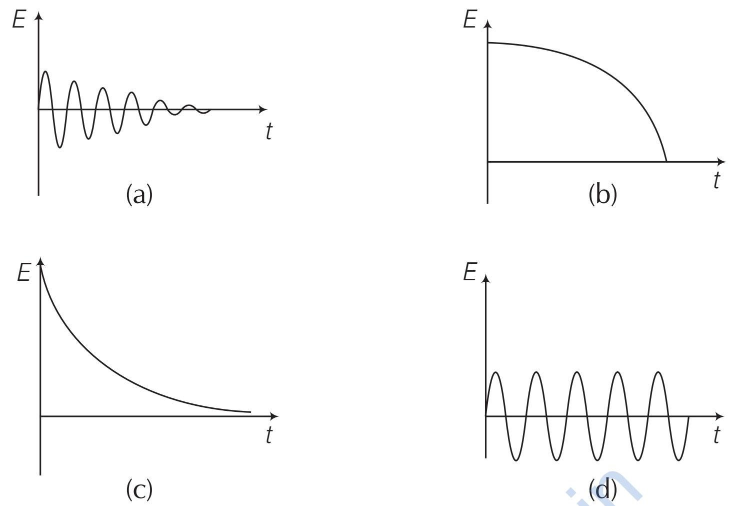
Q. 12 Which of the diagrams shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit?



Ans. ***(d)*** When the earth is closest to the sun, speed of the earth is maximum, hence, KE is maximum. When the earth is farthest from the sun speed is minimum hence, KE is minimum but never zero and negative.

This variation is correctly represented by option(d).

Q. 13 Which of the diagrams shown in figure represents variation of total mechanical energy of a pendulum oscillating in air as function of time?



Ans. ***(c)*** When a pendulum oscillates in air, it will lose energy continuously in overcoming resistance due to air. Therefore, total mechanical energy of the pendulum decreases continuously with time.

The variation is correctly represented by curve (c).

Q. 14 A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 rev/min, its kinetic energy would be

(a) $250\pi^2$ (b) $100\pi^2$

(c) $5\pi^2$ (d) 0

Ans. ***(a)*** Given, mass $= m = 5$ kg

Radius $= 1\text{ m} = R$

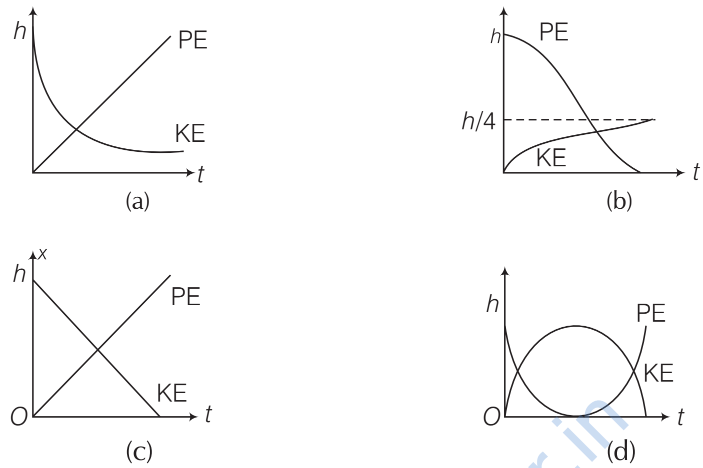
Revolution per minute $\omega = 300$ rev/min

$$= (300 \times 2\pi)\text{ rad/min}$$
$$= (300 \times 2 \times 3.14)\text{ rad/60 s}$$
$$= \frac{300 \times 2 \times 3.14}{60}\text{ rad/s} = 10\,\pi\text{ rad/s}$$

$\Rightarrow$ linear speed $= v = \omega R$

$$= \left(\frac{300 \times 2\pi}{60}\right)(1)$$
$$= 10\pi\text{ m/s}$$
$$\text{KE} = \frac{1}{2} mv^2$$
$$= \frac{1}{2} \times 5 \times (10\pi)^2$$
$$= 100\pi^2 \times 5 \times \frac{1}{2}$$
$$= 250\,\pi^2\text{ J}$$

Q. 15 A raindrop falling from a height h above ground, attains a near terminal velocity when it has fallen through a height $(3/4)h$. Which of the diagrams shown in figure correctly shows the change in kinetic and potential energy of the drop during its fall up to the ground?



Thinking Process

During fall of a raindrop first velocity of the drop increases and then become constant after sometime.

Ans. *(b)* When drop falls first velocity increases, hence, first KE also increases. After sometime speed (velocity) is constant this is called terminal velocity, hence, KE also become constant. PE decreases continuously as the drop is falling continuously.

The variation in PE and KE is best represented by (b).

Q. 16 In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1 m s^{-1} at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m s^{-2}, the kinetic energy of the shotput when it just reaches the ground will be

(a) 2.5 J (b) 5.0 J (c) 52.5 J (d) 155.0 J

Thinking Process

As air resistance is negligible, total mechanical energy of the system will remain constant.

Ans. *(d)* Given, $h = 1.5\,\text{m}$, $v = 1\,\text{m/s}$, $m = 10\,\text{kg}$, $g = 10\,\text{ms}^{-2}$

From conservation of mechanical energy.

$$(\text{PE})\,i + (\text{KE})\,i = (\text{PE})\,f + (\text{KE})\,f$$

$$\Rightarrow \quad mgh + \frac{1}{2}mv^2 = 0 + (\text{KE})\,f$$

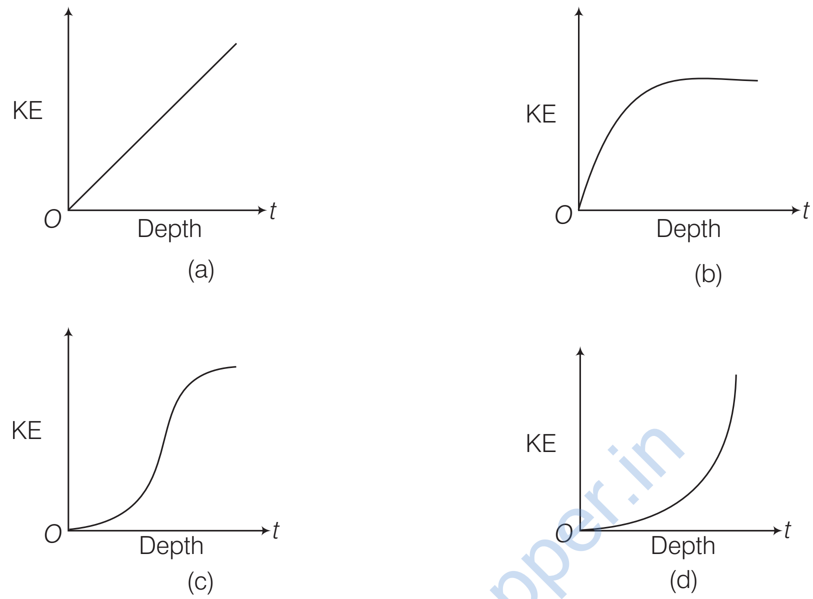
$$\Rightarrow \quad (\text{KE})\,f = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow \quad (\text{KE})\,f = 10 \times 10 \times 1.5 + \frac{1}{2} \times 10 \times (1)^2$$

$$= 150 + 5 = 155\,\text{J}$$

Note *We should be careful about the reference taken for PE, it may or may not be the ground.*

Q. 17 Which of the diagrams in figure correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it a terminal velocity?



Ans. *(b)* First velocity of the iron sphere increases and after sometime becomes constant, called terminal velocity. Hence, accordingly first KE increases and then becomes constant which is best represented by (b).

Q. 18 A cricket ball of mass 150 g moving with a speed of 126 km/h hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.001s, the force that the batsman had to apply to hold the bat firmly at its place would be

(a) 10.5 N (b) 21 N (c) 1.05×10^4 N (d) 2.1×10^4 N

Ans. *(c)* Given, $m = 150\,\text{g} = \frac{150}{1000}\,\text{kg} = \frac{3}{20}\,\text{kg}$

$$\Delta t = \text{time of contact} = 0.001\,\text{s}$$

$$u = 126\ \text{km/h} = \frac{126 \times 1000}{60 \times 60}\ \text{m/s} = 35\ \text{m/s}$$

$$v = -126\ \text{km/h} = -35\ \text{m/s}$$

Change in momentum of the ball

$$\Delta p = m(v - u) = \frac{3}{20}(-35 - 35)\ \text{kg-m/s}$$

$$= \frac{3}{20}(-70) = -\frac{21}{2}$$

We know that force $F = \frac{\Delta p}{\Delta t}$

$$= \frac{-21/2}{0.001}\ \text{N} = -1.05 \times 10^4\ \text{N}$$

Here, – ve sign shown that force will be opposite to the direction of movement of the ball before hitting.

Multiple Choice Questions (More Than One Options)

Q. 19 A man of mass m, standing at the bottom of the staircase, of height L climbs it and stands at its top.

(a) Work done by all forces on man is equal to the rise in potential energy mgL
(b) Work done by all forces on man is zero
(c) Work done by the gravitational force on man is mgL
(d) The reaction force from a step does not do work because the point of application of the force does not move while the force exists

Ans. *(b, d)*

When a man of mass m climbs up the staircase of height L, work done by the gravitational force on the man is-mgl work done by internal muscular forces will be mgL as the change in kinetic energy is almost zero.

Hence, total work done $= - mgL + mgL = 0$

As the point of application of the contact forces does not move hence work done by reaction forces will be zero.

Note *Here work done by friction will also be zero as there is no dissipation or rubbing is involved.*

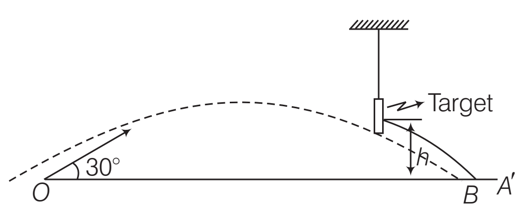
Q. 20 A bullet of mass m fired at 30° to the horizontal leaves the barrel of the gun with a velocity v. The bullet hits a soft target at a height h above the ground while it is moving downward and emerge out with half the kinetic energy it had before hitting the target.

Which of the following statements are correct in respect of bullet after it emerges out of the target?

(a) The velocity of the bullet will be reduced to half its initial value
(b) The velocity of the bullet will be more than half of its earlier velocity
(c) The bullet will continue to move along the same parabolic path
(d) The bullet will move in a different parabolic path
(e) The bullet will fall vertically downward after hitting the target
(f) The internal energy of the particles of the target will increase

Ans. *(b, d, f)*

Consider the adjacent diagram for the given situation in the question.



(b) Conserving energy between "O" and "A"

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow \quad 0 + \frac{1}{2}mv^2 = mgh + \frac{1}{2}mv'$$

$$\Rightarrow \quad \frac{(v')^2}{2} = \frac{v^2}{2} = -gh$$

$$\Rightarrow \quad (v')^2 = v^2 - 2gh \Rightarrow v' = \sqrt{v^2 - 2gh} \qquad \text{...(i)}$$

where v' is speed of the bullet just before hitting the target. Let speed after emerging from the target is v'' then,

By question, $$= \frac{1}{2}(mv'')^2 = \frac{1}{2}\left[\frac{1}{2}m(v')^2\right]$$

$$\frac{1}{2}m(v'')^2 = \frac{1}{4}m(v')^2 = \frac{1}{4}m[v^2 - 2gh]$$

$$\Rightarrow \quad (v'')^2 = \frac{v^2 - 2gh}{2} = \frac{v^2}{2} - gh$$

$$\Rightarrow \quad v'' = \sqrt{\frac{v^2}{2} - gh} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$\frac{v'}{v''} = \frac{\sqrt{v^2 - 2gh}}{\frac{\sqrt{v^2 - 2gh}}{\sqrt{2}}} = \sqrt{2}$$

$$\Rightarrow \quad v'' = \frac{v'}{\sqrt{2}} = v^2\left(\frac{v'}{2}\right)$$

$$\Rightarrow \quad \frac{v''}{\frac{v'}{2}} = \sqrt{2} = 1.414 > 1$$

$$\Rightarrow \quad v'' > \frac{v'}{2}$$

Hence, after emerging from the target velocity of the bullet (v'') is more than half of its earlier velocity v' (velocity before emerging into the target).

(d) As the velocity of the bullet changes to v' which is less than v^1 hence, path, followed will change and the bullet reaches at point B instead of A', as shown in the figure.

(f) As the bullet is passing through the target the loss in energy of the bullet is transferred to particles of the target. Therefore, their internal energy increases.

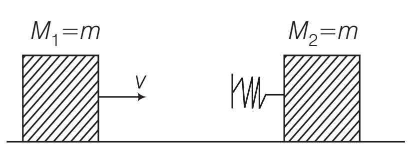
Q. 21 Two blocks M_1 and M_2 having equal mass are free to move on a horizontal frictionless surface. M_2 is attached to a massless spring as shown in figure. Initially M_2 is at rest and M_1 is moving toward M_2 with speed v and collides head-on with M_2.

(a) While spring is fully compressed all the KE of M_1 is stored as PE of spring

(b) While spring is fully compressed the system momentum is not conserved, though final momentum is equal to initial momentum

(c) If spring is massless, the final state of the M_1 is state of rest

(d) If the surface on which blocks are moving has friction, then collision cannot be elastic



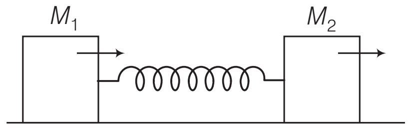
Ans. *(c)* Consider the adjacent diagram when M_1 comes in contact with the spring, M_1 is retarded by the spring force and M_2 is accelerated by the spring force.

(a) The spring will continue to compress until the two blocks acquire common velocity.

(b) As surfaces are frictionalless momentum of the system will be conserved.

(c) If spring is massless whole energy of M_1 will be imparted to M_2 and M_1 will be at rest, then

(d) Collision is inelastic, even if friction is not involved.

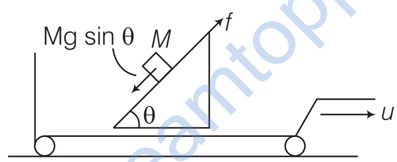


Very Short Answer Type Questions

Q. 22 A rough inclined plane is placed on a cart moving with a constant velocity u on horizontal ground. A block of mass M rests on the incline. Is any work done by force of friction between the block and incline? Is there then a dissipation of energy?

Ans. Consider the adjacent diagram. As the block M is at rest.

Hence, f = frictional force = $Mg \sin \theta$



The force of friction acting between the block and incline opposes the tendency of sliding of the block. Since, block is not in motion, therefore, no work is done by the force of friction. Hence, no dissipation of energy takes place.

Q. 23 Why is electrical power required at all when the elevator is descending? Why should there be a limit on the number of passengers in this case?

Ans. When the elevator is descending, then electric power is required to prevent it from falling freely under gravity.

Also, as the weight inside the elevator increases, its speed of descending increases, therefore, there should be a limit on the number of passengers in the elevator to prevent the elevator from descending with large velocity.

Q. 24 A body is being raised to a height h from the surface of earth. What is the sign of work done by

(a) applied force and

(b) gravitational force?

Ans. (a) Force is applied on the body to lift it in upward direction and displacement of the body is also in upward direction, therefore, angle between the applied force and displacement is $\theta = 0°$

$\therefore$ Work done by the applied force

$$W = Fs\cos\theta = Fs\cos 0° = Fs \qquad (\because \cos 0° = 1)$$

i.e., W = Positive

(b) The gravitational force acts in downward direction and displacement in upward direction, therefore, angle between them is $\theta = 180°$.

$\therefore$ Work done by the gravitational force

$$W = Fs\cos 180° = -Fs \qquad (\because \cos 180° = 1)$$

Q. 25 Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2 m.

Ans. Force of gravity acts on the car vertically downward while car is moving along horizontal road, *i.e.,* angle between them is 90°.

Work done by the car against gravity

$$W = Fs\cos 90° = 0 \qquad (\because \cos 90° = 0)$$

Q. 26 A body falls towards earth in air. Will its total mechanical energy be conserved during the fall? Justify.

Ans. *No*, total mechanical energy of the body falling freely under gravity is not conserved, because a small part of its energy is utilised against resistive force of air, which is non-conservative force. In this condition, gain in KE < loss in PE.

Q. 27 A body is moved along a closed loop. Is the work done in moving the body necessarily zero? If not, state the condition under which work done over a closed path is always zero.

Ans. *No*, work done in moving along a closed loop is not necessarily zero. It is zero only when all the forces are conservative forces.

Q. 28 In an elastic collision of two billiard balls, which of the following quantities remain conserved during the short time of collision of the balls (*i.e.,* when they are in contact)?

(a) Kinetic energy.

(b) Total linear momentum.

Give reason for your answer in each case.

Ans. Total linear momentum of the system of two balls is always conserved. While balls are in contact, there may be deformation which means elastic PE which came from part of KE Therefore, KE may not be conserved.

Q. 29 Calculate the power of a crane in watts, which lifts a mass of 100 kg to a height of 10 m in 20s.

Ans. Given, mass $= m = 100$ kg

height $= h = 10$ m, time duration $t = 20$s

$$\text{power} = \text{Rate of work done}$$

$$= \frac{\text{change of PE}}{\text{time}} = \frac{mgh}{t}$$

$$= \frac{100 \times 9.8 \times 10}{20}$$

$$= 5 \times 98 = 490 \text{ W}$$

Q. 30 The average work done by a human heart while it beats once is 0.5 J. Calculate the power used by heart if it beats 72 times in a minute.

Ans. Given, average work done by a human heart per beat = 0.5 J

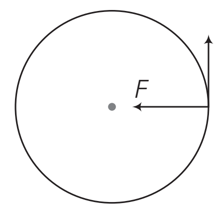
Total work done during 72 beats

$$= 72 \times 0.5\,\text{J} = 36\,\text{J}$$

$$\text{Power} = \frac{\text{Work done}}{\text{Time}} = \frac{36\ \text{J}}{60\ \text{s}} = 0.6\ \text{W}$$

Q. 31 Give example of a situation in which an applied force does not result in a change in kinetic energy.

Ans. When a charged particle moves in a uniform normal magnetic field, the path of the particle is circular, as given field is uniform hence, radius of the circular path is also constant.



As the force is central and movement is tangential work done by the force is zero. As speed is also constant we can say that $\Delta K = 0$.

Q. 32 Two bodies of unequal mass are moving in the same direction with equal kinetic energy. The two bodies are brought to rest by applying retarding force of same magnitude. How would the distance moved by them before coming to rest compare?

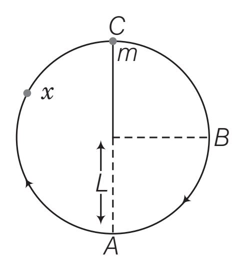
Ans. According to work-energy theorem,

$$\text{Change in KE} = \text{Work done by the retarding force}$$

$$\text{KE of the body} = \text{Retarding force} \times \text{Displacement}$$

As KE of the bodies and retarding forces applied on them are same, therefore, both bodies will travel equal distances before coming to rest.

Q. 33 A bob of mass m suspended by a light string of length L is whirled into a vertical circle as shown in figure. What will be the trajectory of the particle, if the string is cut at



(a) point B ?

(b) point C?

(c) point X?

Thinking Process

In a uniform circular motion, velocity is always tangential in the direction of motion at any point.

Ans. When bob is whirled into a vertical circle, the required centripetal force is obtained from the tension in the string. When string is cut, tension in string becomes zero and centripetal force is not provided, hence, bob start to move in a straight line path along the direction of its velocity.

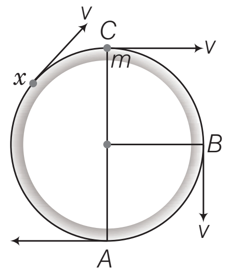
(a) At point B, the velocity of B is vertically downward, therefore, when string is cut at B, bob moves vertically downward.

(b) At point C, the velocity is along the horizontal towards right, therefore, when string is cut at C, bob moves horizontally towards right.

Also, the bob moves under gravity simultaneously with horizontal uniform speed. So, it traversed on a parabolic path with vertex at C.

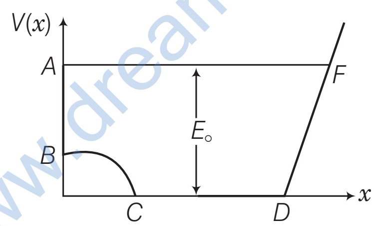
(c) At point X, the velocity of the bob is along the tangent drawn at point X, therefore when string is cut at point C, bob moves along the tangent at that point X.

Also, the bob move under gravity simultaneously with horizontal uniform speed . So, it traversed on a parabolic path with vertex higher than C.



Short Answer Type Questions

Q. 34 A graph of potential energy $V(x)$ *versus* x is shown in figure. A particle of energy E_0 is executing motion in it. Draw graph of velocity and kinetic energy *versus* x for one complete cycle AFA.



Thinking Process

We will assume total mechanical energy of the system to be constant.

Ans. **KE *versus* x graph**

We know that $\quad$ Total ME = KE + PE

$\Rightarrow \quad E_0 = \text{KE} + V(x)$

$\Rightarrow \quad \text{KE} = E_0 - V(x)$

at A_1 $x = 0$, $V(x) = E_0$

$\Rightarrow \quad \text{KE} = E_0 - E_0 = 0$

at B_1 $V(x) < E_0$

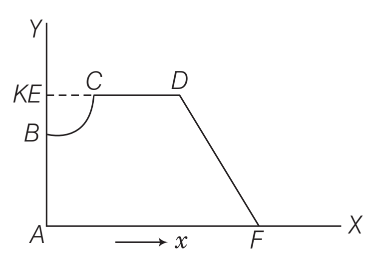
$\Rightarrow \quad \text{KE} > 0 \quad$ (positive)

at C and D_1 $V(x) = 0$

$\Rightarrow$ KE is maximum at F_1 $V(x) = E_o$

Hence, KE = 0

The variation is shown in adjacent diagram.



Velocity *versus* x graph

As $\quad \text{KE} = \dfrac{1}{2}mv^2$

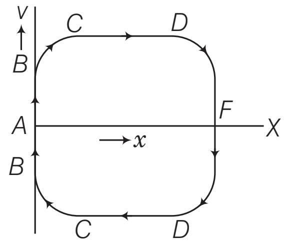
$\therefore$ At A and F, where KE = 0, $v = 0$.

At C and D, KE is maximum. Therefore, v is $\pm$ max.

At B, KE is positive but not maximum.

Therefore, v is $\pm$ some value (< max.)

The variation is shown in the diagram.



Q. 35 A ball of mass m, moving with a speed $2v_0$, collides inelastically ($e > 0$) with an identical ball at rest. Show that

(a) For head-on collision, both the balls move forward.

(b) For a general collision, the angle between the two velocities of scattered balls is less than 90°.

Ans. (a) Let v_1 and v_2 are velocities of the two balls after collision.

Now, by the principle of conservation of linear momentum,

$$2mv_0 = mv_1 + mv_2$$

or $$2v_0 = v_1 + v_2$$

and $$e = \frac{v_2 - v_1}{2v_0}$$

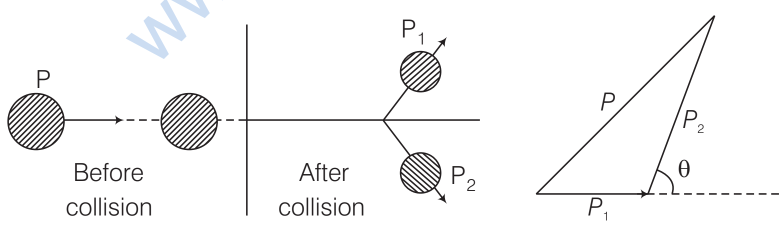
$\Rightarrow$ $$v_2 = v_1 + 2v_0 e$$

$\therefore$ $$2v_1 = 2v_0 - 2ev_0$$

$\therefore$ $$v_1 = v_0(1 - e)$$

Since, $e < 1 \Rightarrow v_1$ has the same sign as v_0, therefore, the ball moves on after collision.

(b) Consider the diagram below for a general collision.



By principle of conservation of linear momentum,

$$\mathbf{P} = \mathbf{P_1} + \mathbf{P_2}$$

For inelastic collision some KE is lost, hence $\frac{p^2}{2m} > \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$

$\therefore$ $$p^2 > p_1^2 + p_2^2$$

Thus, $\mathbf{p}$, $\mathbf{p}_1$ and $\mathbf{p}_2$ are related as shown in the figure.

θ is acute (less than 90°) ($p^2 = p_1^2 + p_2^2$ would given $\theta = 90°$)

Q. 36 Consider a one-dimensional motion of a particle with total energy **E.** There are four regions *A, B, C* and *D* in which the relation between potential energy *V*, kinetic energy (*K*) and total energy *E* is as given below

Region A : $V > E$ **Region B :** $V < E$

Region C : $K < E$ **Region D :** $V > E$

State with reason in each case whether a particle can be found in the given region or not.

Thinking Process

A particle cannot be found in the given region when KE < 0.

Ans. We know that

$$\text{Total energy } E = \text{PE} + \text{KE}$$

$$\Rightarrow \quad E = V + K \quad \ldots\text{(i)}$$

For region A Given, $V > E$, From Eq. (i)

$$K = E - V$$

as $$V > E \Rightarrow E - V < 0$$

Hence, $K < 0$, this is not possible.

For region B Given, $V < E \Rightarrow E - V > 0$

This is possible because total energy can be greater than PE *(V)*.

For region C Given, $K > E \Rightarrow K - E > 0$

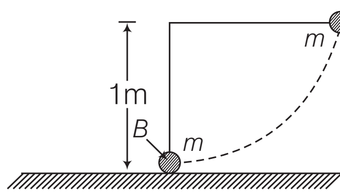
from Eq. (i) **PE** $= V = E - K < 0$

Which is possible, because PE can be negative.

For region D Given, $V > K$

This is possible because for a system PE (*V*) may be greater than KE (K).

Q. 37 The bob *A* of a pendulum released from horizontal to the vertical hits another bob *B* of the same mass at rest on a table as shown in figure.



If the length of the pendulum is 1m, calculate

(a) the height to which bob A will rise after collision.

(b) the speed with which bob B starts moving.

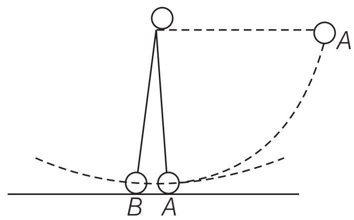
Neglect the size of the bobs and assume the collision to be elastic.

Thinking Process

When two bodies of equal masses collides elastically momentum is interchanged. At the bottom point bob A is having almost horizontal velocity.

Ans. When ball *A* reaches bottom point its velocity is horizontal, hence, we can apply conservation of linear momentum in the horizontal direction.

(a) Two balls have same mass and the collision between them is elastic, therefore, ball A transfers its entire linear momentum to ball B. Hence, ball A will come to at rest after collision and does not rise at all.



(b) Speed with which bob B starts moving

$$= \text{Speed with which bob } A \text{ hits bob } B$$
$$= \sqrt{2gh}$$
$$= \sqrt{2 \times 9.8 \times 1}$$
$$= \sqrt{19.6}$$
$$= 4.42 \text{ m/s}$$

Note *When the bob A is at the bottommost point, its velocity is horizontal and tension is the external force on the bob but still momentum can be considered to be conserved in horizontal direction, because the tension has no effect in horizontal direction at the bottommost point.*

Q. 38 A raindrop of mass 1.00 g falling from a height of 1 km hits the ground with a speed of 50 m s^{-1}. Calculate

(a) the loss of PE of the drop.

(b) the gain in KE of the drop.

(c) Is the gain in KE equal to loss of PE? If not why?

Take, $g = 10 \text{ ms}^{-2}$.

Ans. Given, mass of the rain drop $(m) = 1.00$ g

$$= 1 \times 10^{-3} \text{ kg}$$

Height of falling $(h) = 1 \text{ km} = 10^3$ m

$$g = 10 \text{ m/s}^2$$

Speed of the rain drop $(v) = 50$ m/s

(a) Loss of PE of the drop $= mgh$

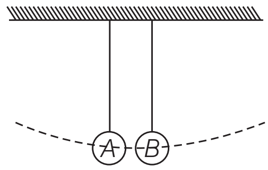
$$= 1 \times 10^{-3} \times 10 \times 10^3 = 10 \text{ J}$$

(b) Gain in KE of the drop $= \frac{1}{2} mv^2$

$$= \frac{1}{2} \times 1 \times 10^{-3} \times (50)^2$$
$$= \frac{1}{2} \times 10^{-3} \times 2500$$
$$= 1.250 \text{ J}$$

(c) *No*, gain in KE is not equal to the loss in its PE, because a part of PE is utilised in doing work against the viscous drag of air.

Q. 39 Two pendulums with identical bobs and lengths are suspended from a common support such that in rest position the two bobs are in contact (figure). One of the bobs is released after being displaced by $10°$ so that it collides elastically head-on with the other bob.



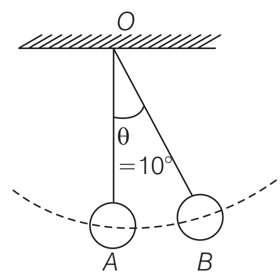
(a) Describe the motion of two bobs.

(b) Draw a graph showing variation in energy of either pendulum with time, for $0 \le t \le 2T$. where T is the period of each pendulum.

Thinking Process

As collision is elastic, mechanical energy of the system is conserved. We have to apply energy conservation principle to describe the motion of the two bobs.

Ans. (a) Consider the adjacent diagram in which the bob B is displaced through an angle θ and released.



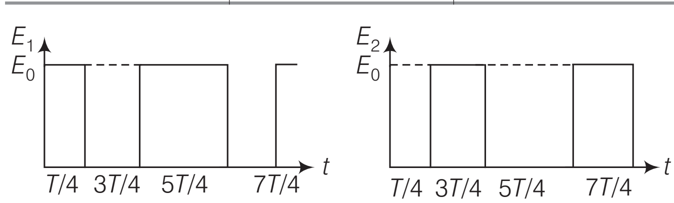
At $t = 0$, suppose bob B is displaced by $\theta = 10°$ to the right. It is given potential energy $E_1 = E$. Energy of A, $E_2 = 0$.

When B is released, it strikes A at $t = T/4$. In the head-on elastic collision between B and A comes to rest and A gets velocity of B. Therefore, $E_1 = 0$ and $E_2 = E$. At $t = 2T/4$, B reaches its extreme right position when KE of A is converted into PE $= E_2 = E$. Energy of B, $E_1 = 0$.

At $t = 3T/4$, A reaches its mean position, when its PE is converted into KE $= E_2 = E$. It collides elastically with B and transfers whole of its energy to B. Thus, $E_2 = 0$ and $E_1 = E$. The entire process is repeated.

(b) The values of energies of B and A at different time intervals are tabulated here. The plot of energy with time $0 \le t \le 2T$ is shown separately for B and A in the figure below.

Time (t)	**Energy of A** (E_1)	**Energy of B** (E_2)
0	E	0
$T/4$	0	E
$2T/4$	0	E
$3T/4$	E	0
$4T/4$	E	0
$5T/4$	0	E
$6T/4$	0	E
$7T/4$	E	0
$8T/4$	E	0



Q. 40 Suppose the average mass of raindrops is 3.0×10^{-5} kg and their average terminal velocity 9 m s^{-1}. Calculate the energy transferred by rain to each square metre of the surface at a place which receives 100cm of rain in a year.

Ans. Given, average mass of rain drop

$$(m) = 3.0 \times 10^{-5} \text{ kg}$$

Average terminal velocity $= (V) = 9$ m/s.

Height $(h) = 100\,\text{cm} = 1\,\text{m}$

Density of water $(\rho) = 10^3$ kg/m^3

Area of the surface $(A) = 1\,\text{m}^2$

Volume of the water due to rain (V) = Area $\times$ height

$$= A \times h$$
$$= 1 \times 1 = 1\,\text{m}^3$$

Mass of the water due to rain (M) = Volume $\times$ density

$$= V \times \rho$$
$$= 1 \times 10^3$$
$$= 10^3 \text{ kg}$$

$\therefore$ Energy transferred to the surface $= \frac{1}{2} mv^2$

$$= \frac{1}{2} \times 10^3 \times (9)^2$$
$$= 40.5 \times 10^3 \text{ J} = 4.05 \times 10^4 \text{ J}$$

Q. 41 An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 50,000 kg is moving with a speed of 36 kmh^{-1} when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, calculate the spring constant.

Ans. Given, mass of the system $(m) = 50{,}000$ kg

Speed of the system $(v) = 36$ km/h

$$= \frac{36 \times 1000}{60 \times 60} = 10\,\text{m/s}$$

Compression of the spring $(x) = 1.0$ m

KE of the system $= \frac{1}{2} mv^2$

$$= \frac{1}{2} \times 50000 \times (10)^2$$
$$= 25000 \times 100 \text{ J} = 2.5 \times 10^6 \text{ J}$$

Since, 90% of KE of the system is lost due to friction, therefore, energy transferred to shock absorber, is given by

$$\Delta E = \frac{1}{2} kx^2 = 10\% \text{ of total KE of the system}$$

$$= \frac{10}{100} \times 2.5 \times 10^6 \text{ J} \quad \text{or } k = \frac{2 \times 2.5 \times 10^6}{10 \times (1)^2}$$

$$= 5.0 \times 10^5 \text{ N/m}$$

Q. 42 An adult weighting 600 N raises the centre of gravity of his body by 0.25m while taking each step of 1 m length in jogging. If he jogs for 6 km, calculate the energy utilised by him in jogging assuming that there is no energy loss due to friction of ground and air. Assuming that the body of the adult is capable of converting 10% of energy intake in the form of food, calculate the energy equivalents of food that would be required to compensate energy utilised for jogging.

Thinking Process

Here, shift in centre of gravity of his body is equal to the height of each step.

Ans. Given, weight of the adult $(w) = mg = 600$ N

Height of each step $= h = 0.25$ m

Length of each step $= 1$ m

Total distance travelled $= 6$ km $= 6000$ m

$\therefore$ Total number of steps $= \dfrac{6000}{1} = 6000$

Total energy utilised in jogging $= n \times mgh$

$= 6000 \times 600 \times 0.25 \text{ J} = 9 \times 10^5$ J

Since, 10% of intake energy is utilised in jogging.

$\therefore$ Total intake energy $= 10 \times 9 \times 10^5 \text{ J} = 9 \times 10^6$ J.

Q. 43 On complete combustion a litre of petrol gives off heat equivalent to 3×10^7 J. In a test drive, a car weighing 1200 kg including the mass of driver, runs 15 km per litre while moving with a uniform speed on a straight track. Assuming that friction offered by the road surface and air to be uniform, calculate the force of friction acting on the car during the test drive, if the efficiency of the car engine were 0.5.

Ans. Energy is given by the petrol in the form of heat of combustion.

Thus, by question,

Energy given by 1 litre of petrol $= 3 \times 10^7$ J

Efficiency of the car engine $= 0.5$

$\therefore$ Energy used by the car $= 0.5 \times 3 \times 10^7$ J

$E = 1.5 \times 10^7$ J

Total distance travelled $(s) = 15$ km $= 15 \times 10^3$ m

If f is the force of friction then,

$$E = f \times s \quad (\because \text{Energy is utilised in working against friction})$$

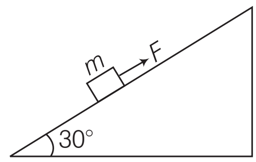
$$1.5 \times 10^7 = f \times 15 \times 10^3$$

$$\Rightarrow \quad f = \frac{1.5 \times 10^7}{15 \times 10^3} = 10^3 \text{ N}$$

$$f = 1000 \text{ N}$$

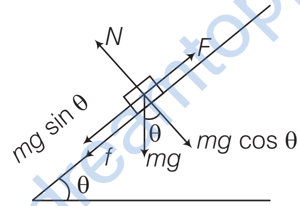
Long Answer Type Questions

Q. 44 A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 30° by a force of 10 N parallel to the inclined surface (figure). The coefficient of friction between block and the incline is 0.1. If the block is pushed up by 10 m along the incline, calculate



(a) work done against gravity
(b) work done against force of friction
(c) increases in potential energy
(d) increase in kinetic energy
(e) work done by applied force

Ans. Consider the adjacent diagram the block is pushed up by applying a force F.



Normal reaction (N) and frictional force (f) is shown.

Given, mass $= m = 1$ kg, $\theta = 30°$

$F = 10$ N, $\mu = 0.1$ and s = distance moved by the block along the inclined plane = 10 m

(a) Work done against gravity = Increase in PE of the block

$$= mg \times \text{Vertical distance travelled}$$
$$= mg \times s(\sin\theta) = (mgs)\sin\theta$$
$$= 1 \times 10 \times 10 \times \sin 30° = 50 \text{ J} \qquad (\because g \leq 10 \text{ m/s}^2)$$

(b) Work done against friction

$$wf = f \times s = \mu N \times s = \mu\, mg\cos\theta \times s$$
$$= 0.1 \times 1 \times 10 \times \cos 30° \times 10$$
$$= 10 \times 0.866 = 8.66 \text{ J}$$

(c) Increase in PE $= mgh = mg\,(s\sin\theta)$

$$= 1 \times 10 \times 10 \times \sin 30°$$
$$= 100 \times \frac{1}{2} = 50 \text{ J}$$

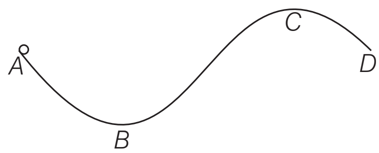
(d) By work-energy theorem, we know that work done by all the forces = change in KE

$$(W) = \Delta K$$
$$\Delta k = W_g + W_f + W_f$$
$$\Rightarrow \quad = -mgh - fs + FS$$
$$= -50 - 8.66 + 10 \times 10$$
$$= 50 - 8.66 = 41.34 \text{ J}$$

(e) Work done by applied force, $F = FS$

$$= (10)(10) = 100 \text{ J}$$

Q. 45 A curved surface is shown in figure. The portion BCD is free of friction. There are three spherical balls of identical radii and masses. Balls are released from rest one by one from A which is at a slightly greater height than C.



With the surface AB, ball 1 has large enough friction to cause rolling down without slipping; ball 2 has a small friction and ball 3 has a negligible friction.

(a) For which balls is total mechanical energy conserved?

(b) Which ball (s) can reach D?

(c) For balls which do not reach D, which of the balls can reach back A?

Ans. **(a)** As ball 1 is rolling down without slipping there is no dissipation of energy hence, total mechanical energy is conserved.

Ball 3 is having negligible friction hence, there is no loss of energy.

(b) Ball 1 acquires rotational energy, ball 2 loses energy by friction. They cannot cross at C. Ball 3 can cross over.

(c) Ball 1, 2 turn back before reaching C. Because of loss of energy, ball 2 cannot reach back to A. Ball 1 has a rotational motion in "wrong" sense when it reaches B. It cannot roll back to A, because of kinetic friction.

Q. 46 A rocket accelerates straight up by ejecting gas downwards. In a small time interval Δt, it ejects a gas of mass Δm at a relative speed u. Calculate KE of the entire system at $t + \Delta t$ and t and show that the device that ejects gas does work $= (1/2)\,\Delta m u^2$ in this time interval (negative gravity).

Thinking Process

As the gas is ejected, the rocket gets propelled in forward direction due to upward thrust.

Ans. Let M be the mass of rocket at any time t and v_1 the velocity of rocket at the same time t.

Let Δm = mass of gas ejected in time interval Δt.

Relative speed of gas ejected $= u$.

Consider at time $t + \Delta t$

$$(\text{KE})_t + \Delta t = \text{KE of rocket} + \text{KE of gas}$$

$$= \frac{1}{2}(M - \Delta m)(v + \Delta v)^2 + \frac{1}{2}\Delta m (v - u)^2$$

$$= \frac{1}{2}Mv^2 + Mv\Delta v - \Delta mvu + \frac{1}{2}\Delta mu^2$$

$$(\text{KE})_t = \text{KE of the rocket at time } t = \frac{1}{2}Mv^2$$

$$\Delta K = (\text{KE})_t + \Delta t - (\text{KE})_t$$

$$= (M\Delta v - \Delta mu)\,v + \frac{1}{2}\Delta mu^2$$

Since, action-reaction forces are equal.

Hence, $$M\frac{dv}{dt} = \frac{dm}{dt}|u|$$

$\Rightarrow$ $$M\Delta v = \Delta mu$$

$$\Delta K = \frac{1}{2}\Delta mu^2$$

Now, by work-energy theorem,

$$\Delta K = \Delta W$$

$\Rightarrow$ $$\Delta W = \frac{1}{2}\Delta mu^2$$

Q. 47 Two identical steel cubes (masses 50g, side 1 cm) collide head-on face to face with a space of 10 cm/s each. Find the maximum compression of each. Young's modulus for steel $= Y = 2 \times 10^{11}$ N/m^2.

Ans. Let $$m = 50g = 50 \times 10^{-3} \text{ kg}$$

$$\text{Side } = L = 1 \text{ cm} = 0.01 \text{ m}$$

$$\text{Speed } = v = 10 \text{ cm/s} = 0.1 \text{ m/s}$$

$$\text{Young's modulus} = Y = 2 \times 10^{11} \text{ N/m}^2$$

$$\text{Maximum compression } \Delta L = ?$$

In this case, all KE will be converted to PE

By Hooke's law , $$\frac{F}{A} = Y\frac{\Delta L}{L}$$

where A is the surface area and L is length of the side of the cube. If k is spring or compression constant, then

$$\text{force } F = k\,\Delta L$$

$\therefore$ $$k = Y\frac{A}{L} = YL$$

$$\text{Initial KE} = 2 \times \frac{1}{2}mv^2 = 5 \times 10^{-4} \text{ J}$$

$$\text{Final PE} = 2 \times \frac{1}{2}k(\Delta L)^2$$

$\therefore$ $$\Delta L = \sqrt{\frac{\text{KE}}{k}} = \sqrt{\frac{\text{KE}}{YL}} = \sqrt{\frac{5 \times 10^{-4}}{2 \times 10^{11} \times 0.1}} = 1.58 \times 10^{-7} \text{ m} \qquad [\because \text{PE} = \text{KE}]$$

Q. 48 A balloon filled with helium rises against gravity increasing its potential energy. The speed of the baloon also increases as it rises. How do you reconcile this with the law of conservation of mechanical energy? You can neglect viscous drag of air and assume that density of air is constant.

Thinking Process

In this problem, as viscous drag of air is neglected, hence there is no dissipation of energy.

Ans. Let $m =$ Mass of balloon

$$V = \text{Volume of balloon}$$

$$\rho_{He} = \text{Density of helium}$$

$$\rho_{air} = \text{Density of air}$$

Volume V of balloon displaces volume V of air.

So, $$V(\rho_{air} - \rho_{He})g = ma = m\frac{dv}{dt} = \text{up thrust} \qquad \text{...(i)}$$

Integrating with respect to t,

$$V(\rho_{air} - \rho_{He})\, gt = mv \quad \text{...(ii)}$$

$$\Rightarrow \quad \frac{1}{2} mv^2 = \frac{1}{2} m \frac{V^2}{m^2} (\rho_{air} - \rho_{He})^2 g^2 t^2$$

$$= \frac{1}{2m} V^2 (\rho_{air} - \rho_{He})^2 g^2 t^2$$

If the balloon rises to a height h, from $s = ut + \frac{1}{2} at^2$,

$$\text{We get } h = \frac{1}{2} at^2 = \frac{1}{2} \frac{V(\rho_{air} - \rho_{He})}{m} gt^2 \quad \text{...(iii)}$$

From Eqs. (iii) and (ii),

$$\frac{1}{2} mv^2 = [V(\rho_a - \rho_{He})\, g] \left[\frac{1}{2m} V(\rho_{air} - \rho_{He})\, gt^2\right]$$

$$= V(\rho_a - \rho_{He})\, gh$$

Rearranging the terms,

$$\Rightarrow \quad \frac{1}{2} mv^2 + V\rho_{He}\, gh = V_{\rho_{air}}\, hg$$

$$\Rightarrow \quad KE_{balloon} + PE_{balloon} = \text{Change in PE of air.}$$

So, as the balloon goes up, an equal volume of air comes down, increase in PE and KE of the balloon is at the cost of PE of air [which comes down].