Chapter - III

Set

Learning Objectives:

After the completion of this section, the students will be able to:

- Define sets and type of sets
- Draw venn diagrams and do operation on sets

Concept Map

What is a Set?

Representation:
- Roster form
- Tabular form

Types of Sets:
- Empty Set
- Finite Set
- Infinity Set
- Subset
- Proper Set
- Universal Set

Intervals as Subsets of:
- Open interval
- Closed interval
- Semi open/ closed interval

Operations on sets:
- Union of Sets
- Intersection of sets
- Complement of a Act
- De Mogan's Law

Venn Diagrams:
- Practical Problems
- Solved using
- Venn Diagrams

3.1 Introduction

In everyday life we speak of collections - whether it be collection of books written by great poet Munshi Prem Chand, or it be collection of movies featuring great actors like Amitabh Bachchan or it be collection of melodious songs by Nightingale of India, Lata Mangeshkar or collection of months in a year and the list is endless.

Similarly, most of us listen to music but not everyone has same taste of songs. Rock songs are separated from classical or any other genre, where particular songs belong to that particular genre.

Consider one more real life situation. You may have noticed that every school has some set of rules that need to be followed by every student and employee of the school. These may include disciplinary rules, timing rules, rules for leave etc. Hence all different types of rules are separated from each other, thus forming different categories or sets.

In Mathematics too, we come across collection of numbers, functions etc., for example, natural numbers, integers, trigonometric functions, algebraic functions and so on.

To learn sets we often talk about such collection of objects. The concept of sets is used in the foundation of various topics in Mathematics.

Now, the above collection of objects or numbers or functions is well-defined as we can identify and decide whether a particular object belongs to the collection or not.

3.1 What is a Set?

Definition 1 : A set is a **well-defined** collection of **distinct** objects.

The word 'well-defined' refers to a specific property or some definite rule on the basis of which it is easy to identify and decide whether the given object belongs to the set or not. The word `distinct' implies that the objects of the set must be all different.

For example, consider the collection of all the students in your class whose names begin with letter 'S'. Then this collection represents a set as, the rule of the names beginning with 'S' has been clearly specified.

However, the collection of all the intelligent students in your class does not represent a set because the word 'intelligent' is relative and degree of intelligence of a student may be measured in different spheres. Whosoever may appear intelligent to one person may not appear the same to another person.

Illustration 1: Which of the following collections is not a set? Give reason for your answers.

(i) The collection of all boys of your class.

(ii) The collection of all monuments in Delhi made by Mughal emperor Akbar.

(iii) The collection of all the rivers in Delhi and Uttar Pradesh.

(iv) The collection of most talented singers of India.

(v) The collection of measures of central tendency of a given data.

(vi) The solution of the equation $x^2 - 6x + 8 = 0$

Solution: (iv)

This is because it is not a well-defined collection as the criteria for determining the talented singer may vary from person to person.

In (i), (ii), (iii), (v) and (vi) we can list the objects belonging to the given collection.

For example, in part (vi), $x = 4$ and $x = 2$ belong to the collection which satisfy the given equation $x^2 - 6x + 8 = 0$,

In fact, no real number except $x = 2, 4$ belongs to this collection.

Note:

1. The objects that belongs to a particular collection or set are also known as its members or elements.
2. Sets are usually denoted by capital letters, say, A, B, C, X, Y etc.

3.2 Representation of Sets

There are two methods of representing a set:

1. Roster form or Tabular form
2. Set builder form

Let us discuss both of them in detail.

1. Roster Form or Tabular Form

In Roster form, all the elements of a set are listed, separated by commas and enclosed within curly braces { }.

For example, the set of vowels of English Alphabet may be described in roster form as:

$$V = \{a, e, i, o, u\}$$

Since 'a' is an element of a set A, we say that '-"a belongs to A"' and the Greek symbol '$\in$' (epsilon) is used to denote the phrase 'belongs to'.

Thus, mathematically, we may write the above statement as:

$$a \in A$$

Similarly, $e \in A$, $i \in A$, $o \in A$, and $u \in A$.

Also, since 'b' is not an element of set A, we write, $b \notin A$ and is read as 'b does not belong to A'.

Let us consider some more examples of representing a set in roster form:

(a) The set of all even natural numbers less than 10 may be written as:

$A = \{2, 4, 6, 8\}$

(b) The set of letters forming the word 'NUMBERS' is $B = \{N, U, M, B, E, R, S\}$

Note 1: While writing the set in roster form, the element is not repeated, that is, all the elements in the set are distinct. Please note that the repetition of the elements of a set does not alter the set.

For example, the set of letters of the word 'FOLLOW' is written as:

$$A = \{F, O, L, W\}$$

Interestingly, when we consider the set of letters of the word 'WOLF', it is given by $B = \{W, O, L, F\}$. Please note that the elements of set A are exactly the same as set B. Such are called equal sets, which would be discussed later. Moreover, a set of particular alphabets may form different words, as noticed above.

Similarly, the set of alphabets of the word, 'ABLE' is given by X = {A, B, L, E} and the set of alphabets of the word 'LABEL' is also Y = {A, B, L, E}.

So, we may understand from this now that the set comprising of different alphabets may form different words, depending on the repetition of alphabets in the words. Moreover, the elements in the set may be written in any order. So, X = {A,B,L,E} is equal to the set Z = {B,E,A,L}.

2. Set-Builder Form

In this form, a set is described by a characterising property of its elements.

For example, in the set A = {1, 2, 3, 4, 5}, all the elements possess a common property, that is, each of them is a natural number less than 6. So, this set can be written as:

A = {x : x is a natural number and $x < 6$} and is read as "the set of all 'x' such that x is a natural number and x is less than 6."

Hence, the numbers 1, 2, 3, 4 and 5 are the elements of the set A.

Illustrations 2: Write the following sets in roster form:

(i) A = {x : x is a letter in the word 'MATHEMATICS'}

(ii) B = {x : x is a letter in the word 'STATISTICS'}

(iii) C = {x : x is a two digit number such that the product of its digits is 12}

(iv) D = {x : x is a solution of the equations $x^2 - 4 = 0$}

Solution:

(i) A = {M, A, T, H, E, I, C, S}

(ii) B = {S, T, A, I, C}

(iii) C = {26, 34, 43, 62}

(iv) D = {–2, 2}

Illustrations 3: Write the following sets in the set-builder form:

(i) A = {1, 4, 9, 16}

(ii) B = {2, 3, 5, 7, 11}

(iii) C = {–2, 0, 2}

Solution:

(i) A = {x : x = n^2, where n = 1, 2, 3, 4}

(ii) B = {x : x is a prime number less than 12}

(iii) C = {x : x is a solution of the equation $x^3 - 4x = 0$}

Note: The symbols for special sets used particularly in mathematics and referred to throughout, are given as follow:

N : the set of all natural numbers

Z : the set of all integers

W : the set of all whole numbers

Q : the set of all rational numbers

R : the set of all real numbers

Z^+ : the set of all positive integers

Q^+ : the set of all positive rational numbers

R^+ : the set of positive real numbers

Check your progress 3.1

Q.1 Which of the following are sets?

(i) The collection of most talented authors of India.

(ii) The collection of all months of a year beginning with letter M.

(iii) The collection of all integers from –2 to 20.

(iv) The collection of all even natural numbers.

(v) The collection of best tennis players of the world.

Q.2 Write the following in set-builder form:

(i) A = {4, 8, 12, 16, 20}

(ii) B = {2, 3, 5, 7, 11, 13, 17, 19, ...}

(iii) C = {b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z}

(iv) D = {–1, 1}

(v) E = {41, 43, 47}

(vi) F = $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\right\}$

(vii) G = $\left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, ...\right\}$

Q.3 Write the following in roster form:

(i) A = {x : x is a whole number less than 5}

(ii) B = {x : x is an integer and $-4 < x \leq 6$}

(iii) C = The set of all letters of the word FOLLOW.

(iv) D = {x : x is a two-digit number such that the sum of its digits is 6}

(v) E = the set of all letters of the word 'ARITHMETIC'

Q.4 Match each of the sets on the left expressed in roster form with the same set described in the set builder form on the right:

(i) {1, 2, 3, 6, 12}	(a) {x : x is a day of the week beginning with T}
(ii) {2, 3, 5}	(b) { x : x = n^2 - 1, n∈N and n ≤ 4}
(iii) {Tuesday, Thursday}	(c) {x : x is a factor of 12}
(iv) {0, 3, 8, 15}	(d) {x : x is the smallest natural number}
(v) {1}	(e) {x : x is a prime number less than 7}

Q.5 Let A = {1, 2, 3, 4, 5}, B = {x : x is a factor of 4}, C = {1, 4, 9}. Insert the correct symbol '∈' or '∉' in each of the following to make the statement true

(i) 4 _______ A

(ii) 3 _______ B

(iii) 9 _______ C

(iv) 1 _______ B

(v) 3 _______ C

3.3 Types of Sets

Consider the following sets:

(i) A = {x : x is an even prime number}

(ii) B = {x : x is an even prime number greater than 2}

(iii) C = {x : x is a prime number less than 50}

(iv) D = {x : x is a prime number}

Now, in case (i), we observe that there exists only one even prime number which is 2. Therefore, the set A contains only one element. Such a set which consists of only single element is called a singleton set.

Definition 2: Singleton Set

A set consisting of a single element is called a singleton set.

Examples:

(a) Let X = {x : 1 < x < 3 and x is a natural number}. This is a singleton set as the only element of set B is '2'.

(b) Let Q = {x : x is the day of the week starting with alphabet 'M'}. This is also a singleton set as the only day of the week starting with alphabet 'M' is Monday.

Consider case (ii). We observe that there does not exist any prime number which is greater than 2. Therefore, this set B does not contain any element. Such a set is called an empty set or a void set.

Definition 3: Empty Set

A set which does not contain any element is called the empty set or null set or the void set and is denoted by the symbol ø or { }.

Examples:

(a) Let A = {x : x is a month of the year starting with alphabet B}. This is an empty set as no month of the year in English calendar starts with alphabet 'B'.

(b) Let A = {x : x is a real solution of the equation $x^2 + 2 = 0$}. This set is an empty set as there is no real solution of the quadratic equation $x^2 + 2 = 0$.

In case (iii), we observe that there are fifteen elements, which is finite in number. Such a type of set is called a finite set.

Definition 4: Finite Set

A set which is either a null set or whose elements can be numbered from 1 to n for some positive integer n is called a finite set.

In other words, a finite set is either an empty set or has definite number of elements.

The number 'n' is called the cardinal number or order of the finite set and is denoted by n (A).

Note: The cardinality of an empty set is zero, i.e., n (ϕ) = 0.

Examples:

(a) Let X = {x : x is month of the year beginning with letter J}

= {January, June, July}

There are three elements is the set x, given above. Therefore, the set X is a finite set and n (x) = 3.

(b) Let Y = {x : $-1 \leq x \leq 4$ and x is an integer}.

= {−1, 0, 1, 2, 3, 4}

There are six elements is this set Y. Therefore, set Y is a finite set and n (Y) = 6.

Now we consider case (iv). The number of elements in this set is infinite and moreover, the elements of this set cannot be listed by positive integers. Such a set is called an infinite set.

Examples:

(a) The set of natural numbers, N = {1, 2, 3, 4, ...} is an infinite set.

(b) The set of even natural numbers, say, E = {2, 4, 6, 8, ...} is an infinite set.

Note: All infinite sets cannot be described in roster form. For example, the set of real numbers cannot be described in this form as between any two given real numbers, there exists infinite number of real numbers. So, it is not possible to list them in roster form.

Illustration 4: State which of one following sets are finite and which are infinite.

(i) $\{x : x \in Z$ and satisfies the equation $(x+1)(x+2) = 0\}$

(ii) $\{x : x \in N$ and $x^2 - 1 = 3\}$

(iii) $\{x : x$ is an odd natural number$\}$

(iv) $\{x : x$ is an integer and $-1 < x < 0\}$

(v) $\{x : x$ is an integer less than $0\}$

(vi) $\{x : x$ satisfies the identity $\cos^2x + \sin^2x = 1\}$

Solution:

(i) $A = \{-1, -2\}$, is a finite set as it consists of only two elements.

(ii) $B = \{2\}$, is a finite set having only one element.

(iii) $C = \{1, 3, 5, 7, 9, ...\}$ is an infinite set as it doesn't have a definite number of elements.

(iv) ϕ, which is a finite set.

(v) $D = \{....., -3, -2, -1\}$ is an infinite set as all its elements cannot be listed.

(vi) There are infinite real numbers that satisfy the identity $\cos^2x + \sin^2x = 1$. Therefore, it is an infinite set.

3.4 Subsets

Consider the following sets:

E = set of all students in your school.

F = set of all students in your school who have taken up commerce as their subject stream.

We note that every element (student) in set F is a student of your school itself, which is described by set E.

In other words, every element of F is also an element of E. The set F is said to be a subset of E and in symbols it is expressed as $F \subseteq E$. The symbol ' $\subseteq$ ' stands for 'is a subset of' or 'is contained in'.

Definition 5 : Subset

Let A and B be two sets. If every element of set A is an element of set B also, then A is called a subset of B and is written as $A \subseteq B$.

In other words, $A \subseteq B$, if whenever $a \in A$, then $a \in B$. Using the symbol '$\Rightarrow$' which means 'implies', we can write the definition of subset as:

$A \subseteq B$ if $a \in A \Rightarrow a \in B$

If A is not a subset of B, we write $A \not\subseteq B$

Note:

1. The empty set is a subset of every set.
2. Every set is a subset of itself.
3. The symbol ' $\subseteq$ ' suggests that either $A \subseteq B$ or $A = B$. $A \subset B$ implies that A is a proper subset of B, that is,

   A has lesser number of elements than B.

   $A = B$ implies that A and B are equal sets, that is, they have exactly the same elements.

   In other words, two sets are said to be equal if every element of A is a member of B and every element of B is an element of A.

   That is, to say, $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.

Illustration 5: Consider the sets ϕ; $A = \{1, 2, 3\}$; $B = \{1, 4, 5\}$; $C = \{1, 2, 3, 4\ 5\}$; $D = \{x : x \in N \text{ and } x < 4\}$.

Insert the symbol ' $\subseteq$ ', ' $\not\subseteq$ ' or '=' between the following pair of sets:

(i) $\varnothing$ __ B

(ii) A ______ B

(iii) B ______ C

(iv) A ______ D

Solutions:

(i) $\phi \subseteq B$ as ϕ, the empty set is a subset of every set.

(ii) $A \not\subseteq B$ as $2 \in A$ but $2 \notin B$.

(iii) $B \subseteq C$ as $1, 4, 5 \in B$ and they also belong to C.

(iv) A = D as sets A and D have exactly the same elements which are 1, 2 and 3.

Illustration 6: Let A {1, 2, 3, {4}, 5}. Which of the following are incorrect? Give reasons.

(i) $1 \in A$

(ii) $\{1, 2\} \subseteq A$

(iii) $\{1, 2, 4\} \subseteq A$

(iv) $\{4\} \in A$

(v) $\phi \subseteq A$

(vi) $\{4\} \subseteq A$

Solutions:

(i) Correct. This is because 1 is an element of A. Therefore, $1 \in A$.

(ii) Correct. Every element belonging to the set {1, 2} also belongs to A. Therefore, $\{1, 2\} \subseteq A$.

(iii) Incorrect. This is because, 4 is not an element of A but {4} is an element of A.

$\therefore \{1, 2, \{4\}\} \subseteq A$ but $\{1, 2, 4\} \not\subseteq A$

(iv) Correct. {4} is one of the elements of A. Therefore, $\{4\} \in A$.

(v) Correct. Empty set is a subset of every set.

(vi) Incorrect. This is because {4} is an element of set A. Therefore $\{\{4\}\} \subseteq A$ but $\{4\} \not\subseteq A$.

3.5 Power Set

Consider the set A = {1, 2, 3}

Now $\subseteq \phi$ A as empty set is a subset of every set. Also, A $\subseteq$ A, as every set is a subset of itself.

The other subsets of A are: {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}.

The set of all these subsets of set A is called the power set of A.

Definition 6: Let A be set. Then, the set of all the subsets of A is called the power set of A and is denoted by P(A).

Thus, in the above example, if A = {1, 2, 3}, then,

P(A) = {ϕ {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}}

Note: n (P(A)) = 2^3 = 8

Therefore, n (P(A)) = 2^n, where 'n' is the number of elements in A. In other words, the number of subsets of any set A are given by 2^n, where, 'n' is the number of elements in A.

Also, the number of proper subsets of A having 'n' elements is given by 2^n–1, which excludes the set itself.

3.6 Universal Set

A set that contains all the objects or elements in a given context is called the universal set and is denoted by U.

For example, let A = set of all even natural numbers and B = set of odd natural numbers. Then the universal set in this context, may be the set which contains all the elements, that is,

U = set of natural numbers

Universal set in this case can also be taken as whole numbers or integers or real numbers.

Note:

1. Universal set is the superset of all the sets under consideration.
2. $N \subseteq Z \subseteq Q \subseteq R$. This relation explains that every natural number is an integer, every integer can be put is the form of p/q, where $q \neq 0$, hence every integer is a rational number and all rational numbers are real numbers.

3.7 Intervals as Subset of the set of Real Numbers R

Consider the solution set of the equation $x^2 - 5x + 6 = 0$. In tabular form it is written as A = {2, 3}

On the real number line it is represented as:

Let us consider another case.

Represent all the integers from –3, to 2, both in the roster form and graphically on the number line.

Now, in set notation form, it is given by A = {–3, –2, –1, 0, 1, 2} and plotting on the number line, we get,

Let's consider the following example of representing the real numbers, both as set notation and on the number line from –3 to 2.

The set of real numbers from –3 to 2 is denoted by $B = \{x : x \in R, -3 \leq x \leq 2\}$.

In this set it is impossible to list the real numbers, as between any two real numbers lie infinite real numbers, as real number cannot be separated by commas and listed. Therefore, in order to denote the above set we introduce the concept of intervals.

What is an Interval?

An interval is a set of real numbers between a given pair of real numbers, excluding or including one or both these real numbers.

It can be considered as a segment of the real number line, where in, the end points of the interval mark the end points of that segment. Let us now understand different types of intervals.

3.8 Types of Intervals

1. Closed Interval

Let a, b $\in$ R and a < b. Then the set of all real numbers from a to b, in the set builder form is written as A = $\{x : x \in R, a \leq x \leq b\}$.

In interval form, this set is represented as (a, b), indicating that it includes all real numbers form a to b, including the end points 'a' and 'b'. This type of interval which includes the end points in the set of real numbers between a given pair of real numbers is called a closed interval. The inclusion of the end points is indicated by square brackets ( ) in the interval notation.

Illustration 7: The interval notation for all real numbers from –3 to 2 is written as:

$(-3, 2) = \{x : x \in R, -3 \leq x \leq 2\}$

As a segment of the real number line, the representation is given by:

2. Open Interval

Let's now consider the set of real numbers between 'a' and 'b', where $a, b \in R$ and $a < b$.

This set of real numbers in set-builder form is written as:

$$B = \{x : a < x < b\}$$

In interval form, it is expressed as (a, b), indicating that it includes all real numbers between 'a' and 'b', excluding the end points 'a' and 'b'.

This type of interval which does not includes the end points in the set of real numbers is called an open interval and exclusion of the points is indicated by round brackets ( ) in the interval notation.

Illustration 8: The interval of real numbers between –3 and 2 is written as:

$$(-3, 2) = \{x : x \in R, -3 < x < 2\}$$

As a segment of the real number line, the representation is given by:

where the exclusion of end points –3 and 2 is illustrated by an open dot.

3. Semi-Open/Closed Interval

In the above cases, we had intervals where either both the end points are included or both excluded.

But there are intervals where either of the end points is included or excluded.

Consider the following set of real numbers: $A = \{x : a < x \leq b;\ a, b \in R \text{ and } a < b\}$

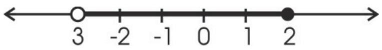
This set consists of all real numbers between 'a' and 'b' but 'b' is included in the set and 'a' is excluded from it. So, in interval notation, it is expressed as:

$$(a, b) = \{x : x \in R, a < x \leq b\}$$

Illustration 9: Consider the interval $(-3,2]$.

Now, $(-3,2]$ = {x : x ∈ R,–3 < x ≤ 2} represents the set of real numbers between –3 and 2 where '2' is included in the interval but–3 is excluded from it.

As a segment on the real number line, the representation is given by:



Now consider the set of real numbers:

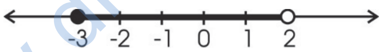
{x : a ≤ x < b; a , b ∈ R, a < b}

This set consists of all real numbers between 'a' and 'b' but 'a' is included in the set and 'b' is excluded from it. So in interval rotation, it is expressed as:

$[a,b)$ = {x : x ∈ R, a ≤ x < b}

Example: $(-3,2]$= {x : x ∈ R, –3 ≤ x < 2} represents the set of real numbers between –3 and 2 where, –3 is included and 2 is excluded from the set.

As a segment of the real number line, the representation is given by:

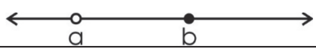


Note: The number (b – a) is called the length of the interval (a, b), (a, b), (a, b) or (a, b).

Illustration 10: The length of the interval (2, 7) is 5 and the length of the interval (–3, 4) is (4 – (–3)) = 7.

Let us consider brief summary of the context discussed pertaining to intervals.

Table 1.1.1

S. No.	Interval	Description	Graphical Representation
1.	Closed interval: (a, b)	{x : x ∈ R, a ≤ x ≤ b}	
2.	Semi-open closed interval		 

	(a, b)	{x : x ∈ R,a ≤ x < b}	
	(a, b)	{x : x ∈ R,a < x ≤ b}	
3.	Open Interval: (a, b)	{x : x ∈ R,a < x < b}	
4.	(a, ∞)	{x : x ∈ R, x > a}	
5.	(–∞ ,b)	{x : x ∈ R, x < b}	
6.	(a, ∞)	{x : x ∈ R, x ≥ a}	
7.	(–∞, b)	{x : x ∈ R, x ≤ b}	
8.	(–∞, ∞)	R (real numbers)	

Check your progress 3.2

Q.1 Which of the following sets are finite and which are infinite? In case of finite sets, write its cardinality.

(i) {x : x is a natural number less than 100}.

(ii) The set of all prime numbers.

(iii) The set of the days of the week.

(iv) {x : x = n^2, where n is a natural number}.

(v) The set of all lines in a plane parallel to the line 2y = 3x + 7.

(vi) {x : x is a real number and 0 < x < 1}.

Q.2 Which of the following sets are empty and which are singleton sets?

(i) {x : x is an even prime number}.

(ii) {x : x is a natural number and –1 < x < 1}.

(iii) {x : x is an integer and –1 < x < 1}.

(iv) {x : x is a vowel in the word 'EYE'}.

(v) {x : x + 10 = 0; x ∈ N}.

Q.3 Which of the following pairs of sets are equal? Give reasons.

(i) A = {–2, 3}, B = {x is a solution, of $x^2 - x - 6 = 0$}

(ii) A = {x : x is a letter of the word 'FOLLOW'}

B = {y : y is a letter of the word 'WOLF'}

(iii) A = {x : x is a letter of the word 'ASSET'}

B = {y : y is a letter of the word 'EAST'}

(iv) A = {–1, 1}; B = {x : x is a real number satisfying the equation $x^2 + 1 = 0$}

(v) A = {1, 4, 9}; B = {x : x = n^2 where 'n' is a natural number less than 5}

Q.4 Let A = {1, 2, {3, 4}, 5}. Put the correct symbol in each of the following.

(i) {3, 4} _______ A

(ii) {1} _______ A

(iii) {3} _______ A

(iv) {{3, 4}} _______ A

(v) {1, 3} _______ A

(vi) {1, 5} _______ A

(vii) ϕ _______ A

Q.5 Write the following in set-builder form.

(i) (1, 3)

(ii) (–1, 3)

(iii) (–4, 0)

(iv) (–1, 1)

(v) (0, ∞)

Q.6 Let A = ϕ. Find P (P(A))

Q.7 Find the number of subsets of the set B = {a,b,c,d}.

3.9 Venn Diagrams

Venn diagrams are the pictorial representation of the relationships between sets. They are named after the English logician John Venn.

In Venn diagrams, the universal set is usually denoted by a rectangle and its subsets by circles or ellipses. It is denoted by U.

Illustration 10: Let U = {1, 2, 3, 4, 5} and A = {2, 4}. Represent the relationship between U and A using Venn diagram.

Solution: Set U = {1, 2, 3, 4, 5} is the universal set of which A = {2, 4} is a subset.

Therefore, in set notation we can express the relationship between A and U as $A \subseteq U$.

Using, Venn diagram, we can pictorially represent this relationship as given in Fig. 1.1.

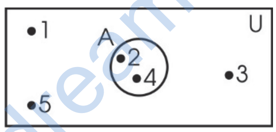


Fig. 1.1

Illustration 11: Let U = {1, 2, 3, 4, 5}, A = {1, 2}, B = {3, 5}. Represent the relationship between U, A and B using Venn diagrams.

Solution: Set U = {1, 2, 3, 4, 5} is the universal set. Sets A and B are subsets of u and we may also observe that A and B do not share any common element. Therefore, in set notation, we can express the relationship between A, B and U as: $A \subseteq U$ and $B \subseteq U$.

Therefore, pictorially, it can be represent as in Fig. 1.2.

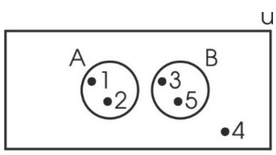


Fig. 1.2

Illustration 12: Let U = {1, 2, 3, 4, 5}, A = {1, 2, 3} and B = {1, 2}. Represent the relationship between U, A and B using Venn diagrams.

Solution: Set U = {1, 2, 3, 4, 5} is the universal set of which A = {1, 2, 3} and B = {1, 2} are subsets. Moreover, in this case we also observe further that $B \subseteq A$.

Pictorially, we represent this as given in Fig. 1.3.

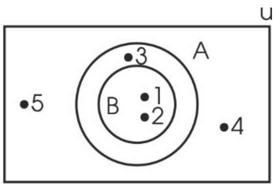


Fig. 1.3

We will further see the extensive use of Venn diagrams when we discuss operations on sets, which are union, intersection and difference of sets.

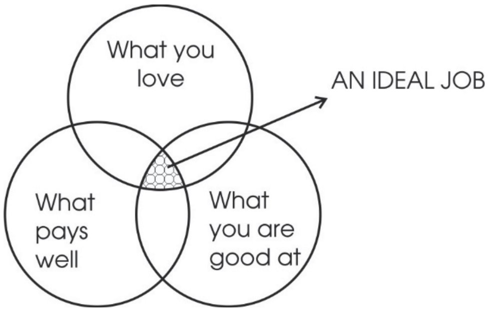
3.10 Operations on Sets

In previous classes we have applied the basic operation of addition, subtraction, multiplication and division on real numbers. Each of these operations on an pair of numbers gives us another number.

For example, when we perform the operation of multiplication on two numbers say 8 and 11, we get a number 88. Now, when we apply the operation of addition on these numbers 8 and 11, we get a number 19.

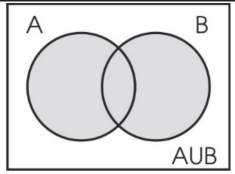
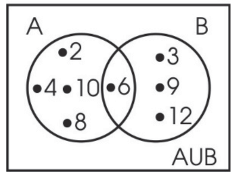
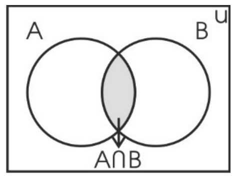
Similarly, there are some operations which when performed on two or more given sets gives rise to another set.

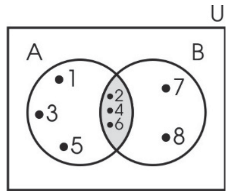
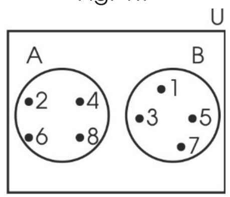
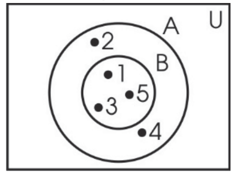
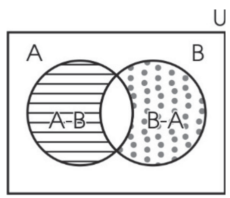
Life's most complicated question can be answered through Venn diagrams and that is choosing an ideal job. A simple Venn diagram can simplify this thought process to a great extent. But, first we need to select the factors which matter in choosing the above, such as, doing what you love, doing something that pays you well and doing what you are good at so, a job which meets all these three criteria, would be a dream job for anyone. Whatever the priority, because you already have listed down the criteria, making the decision becomes easier.

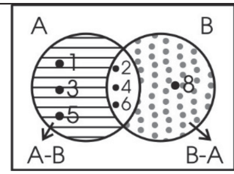
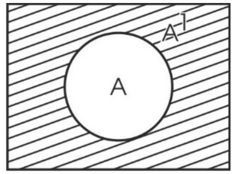
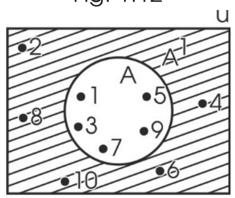


Let us now understand these operations, examine their properties, express them pictorially as Venn diagrams and briefing them through examples.

Table 1.1.2

S. No.	Operations	Symbol	Read as	Definition	Examples	Venn Diagram The shaded portions indicated represent operations	Properties
1.	Union of sets	$A \cup B$ (Fig. 1.4)	A union B	The union of two sets A and B is the set which consists of all those elements which are either in A or in B, including those which are in both, taken once. That is, to say, $A \cup B = \{x : x \in A \text{ or } x \in B\}$	**1.** Let $A = \{2, 4, 6, 8, 10\}$ and $B = \{3, 6, 9, 12\}$. Then $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$ (see Fig. 1.5) **2.** Let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 3\}$. Then, $A \cup B = \{1, 2, 3, 4, 5\}$ **Note:** If $B \subseteq A$, then, $A \cup B = A$	 Fig. 1.4  Fig. 1.5	(i) $A \cup B = B \cup A$ (Commutative law) (ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law) (iii) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of U) (iv) $A \cup A = A$ (identity law) (v) $U \cup A = U$ (law of U)
2.	Intersection of sets	$A \cap B$ (Fig. 1.6)	Intersection of sets A and B	The intersection of two sets A and B is the set of all those elements which belong to both A and B. That is, to say, $A \cap B = \{x : x \in A \text{ and } x \in B\}$ **Note:** If A and B are two sets such that $A \cap B = \phi$, that is, there are no elements	**3.** Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 7, 8\}$. Then $A \cap B = \{2, 4, 6\}$. (See Fig. 1.7) **4.** Let $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$. Then $A \cap B = \phi$. (See Fig. 1.8) **5.** Let $A = \{1, 2, 3, 4, 5\}$	 Fig. 1.6	(i) $A \cap B = B \cap A$ (Commutative law) (ii) $\phi \cap A = \phi$ and $U \cap A = A$ (Law of ϕ and U) (iii) $A \cap A = A$ (identity law) (iv) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law) (v) $A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$ (Distributive law

				which are common to A and B, then A and B are called **disjoint sets**.	and B = {1, 3, 5}. Then A∩B = B. (See Fig. 1.9) **Note:** If B⊆A, then A∩B = B.	 Fig. 1.7  Fig. 1.8  Fig. 1.9	which states that '∩' distributes over '∪')
3.	Difference of sets	A – B (Fig. 1.10)	A minus B	The difference of sets A and B in this order is the set of elements which belong to A but not to B. That is, A – B = {x : x ∈ A and x ∉B}.	**6.** Let A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8}. Then, A – B = {1, 3, 5} since the elements 1, 3 and 5 belong to A but not B. B – A = {8} because '8' belongs to B and not to A. (See Fig. 1.11)	 Fig. 1.10	(i) A – B ≠ B – A (ii) (A–B) ∪ (A∩B) ∪ (B–A) = A∪B (iii) A – B, B – A and A∩B are disjoint sets.

						 Fig. 1.11	
4.	Complement of a set	A' (Fig.1.12)	Complem-ent of A.	Let U be the universal set and A be the subset of U. Then the complement of A is the set of all elements of U which are not the elements of A with respect to U. That is, to say, $A' = \{x : x \in U \text{ and } x \notin A\}$.	**7.** Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9}. Then, $A' = \{2, 4, 6, 8, 10\}$ (See Fig. 1.13)	 Fig. 1.12  Fig. 1.13	(i) $(A')' = A$ (Law of double complement) (ii) $A \cup A' = U$ $A \cap A' = \phi$ (Complement laws) (iii) De Morgan's law: $(A \cup B)' = A' \cap B'$ $(A \cap B)' = A' \cup B'$ (iv) Laws of empty set and universal set: $\phi' = U$ and $U' = \phi$

Check your progress 3.3

Q.1 For the following sets, find their union and intersection.

(i) A = {x : x is the letter of the word 'MATHEMATICS'}.

B = {x : x is the letter of the word 'TRIGONOMETRY'}.

(ii) A = {x : x is a natural number less than 6}.

B = {x : x is a multiple of 2 from 1 to 10.}

(iii) $A = \{x : x = 2n - 1, n \in N\}$ and $B = \{x : x = 2n, n \in N\}$

(iv) A = {2, 4, 6, 8, 10} and B = {2, 4}

(v) $A = \{x : x = \sin\theta \text{ where } 0 \leq \theta \leq \pi/2\}$

$B = \{x : x = \cos\theta \text{ where } 0 \leq \theta \leq \pi/2\}$

Q.2 Let U = {1, 2, 3, 4, 5, 6, 7, 8}; A = {1, 2, 3, 4}; B = {3, 4, 6}; C = {5, 6, 7, 8}, find:

(i) $A - (B \cup C)$

(ii) $A \cap C'$

(iii) $B' \cap C'$

(iv) $B' \cup A'$

(v) $A - (B \cup C)'$

Q.3 Let U = {1, 2, 3, 4, 5, 6, 7, 8}; A = {1, 2, 3}; B = {2, 4, 6}, C = {3, 6, 9, 12}. Verify.

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

(iii) $A' \cap (B \cup C) = (A' \cap B) \cup (A' \cap C)$

(iv) $A' \cup B = (A \cap B')'$

(v) $A' - B' = B - A$

Q.4 Draw suitable Venn diagrams for each of the following.

(i) $(A \cup B)'$

(ii) $(A \cap B)'$

(iii) $A' \cap B'$

(iv) $A' \cup B'$

Q.5 Two finite sets have 'm' and 'n' elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of 'm' and 'n'.

Q.6 Let A = {a, b, c} and B = {a, b, c, d}. Is $A \subseteq B$? What is $A \cup B$? What is $A \cap B$?

Q.7 Fill in the blanks.

(i) $A' \cap \phi$ = ___________

(ii) $A \cap \phi'$ = ___________

(iii) $A \cup U'$ = ___________

(iv) $A \cap U'$ = ___________

(v) $A \cap \phi'$ = ___________

(vi) $U' \cap \phi$ = ___________

(vii) $U' \cap \phi'$ = ___________

(viii) $U' \cup \phi$ = ___________

(ix) $U' \cup \phi'$ = ___________

(x) $U \cup \phi'$ = ___________

Q.8 If A = {1, 2, 3, 4}, then the number of subsets of set A containing element 3, is:

(i) 24

(ii) 28

(iii) 8

(iv) 16

Q.9 If A = {1, 2, 3, 4, 5}, B = {2, 4, 6} and C = {3, 4, 6}, then $(A \cup B) \cap C$ is

(i) {3, 4, 6}

(ii) {1, 2, 3}

(iii) {1, 4, 3}

(iv) None of these

3.11 Practical Problems on Operations on Sets

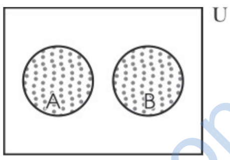
In this section, we will now apply the concept of sets to our daily life situations and solve some problems.

Let A, B and C be finite sets.

(i) $n(A \cup B) = n(A) + n(B)$; if $A \cap B = \phi$ i.e. A and B are disjoint.

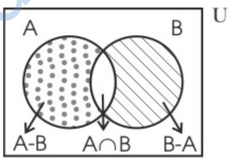
(ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ in general

Now, case (i) follows immediately as the elements in $A \cup B$ are either in A or in B but not in both because $A \cap B = \phi$.



Let's move ahead with case (ii)

Now, $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$, where, $A - B$, $B - A$ and $A \cap B$ are mutually disjoint.



Therefore, $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

$$= n(A - B) + n(A \cap B) + n(B - A) + n(A \cap B) - n(A \cap B)$$

$$= n(A) + n(B) - n(A \cap B)$$

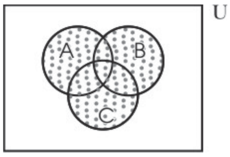
(iii) If A, B and C are finite sets, then,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Now, proceeding with the proof of how we obtain the formula for case (iii).

$$n(A \cup B \cup C) = n(A \cup (B \cup C))$$

$= n(A) + n(B \cup C) - n(A \cap (B \cup C))$ (from case ii)



$= n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap (B \cup C))$

Since, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\therefore \quad n(A \cap (B \cup C)) = n((A \cap B) \cup (A \cap C))$

$= n(A \cap B) + n(A \cap C) - n((A \cap B) \cap (A \cap C))$

$= n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)$

$\therefore \quad n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap C) - n(A \cap B) + n(A \cap B \cap C)$

Illustration 13: Out of 20 members in a family, 12 like tea and 15 like coffee. Assume that each one likes atleast one of the two drinks, how many like.

(i) Both coffee and tea.

(ii) Only tea and not coffee

(iii) Only coffee and not tea

Solution:

(i) Let T denote the set of people who like tea and C be the set of people who like coffee.

Therefore, $n(T) = 12$, $n(C) = 15$ and $n(T \cup C) = 20$

Using the formula, $n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$20 = 12 + 15 - n(T \cap C)$

$\therefore \quad n(T \cap C) = 12 + 15 - 20 = 7$

Hence 7 people like both tea and coffee.

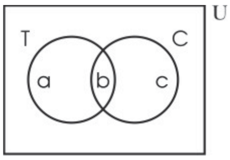
(ii) $n(T \cap C') = n(T) - n(T \cap C) = 12 - 7 = 5$

Therefore, 5 people like to take tea but not coffee.

(iii) Now, $n(C \cap T') = n(C) - n(T \cap C) = 15 - 7 = 8$

Therefore, 8 people like to take coffee and not tea.

Alternatively: Using Venn diagram, we can understand the problem as:



(a) Let a be the number of people who like tea but not coffee.

(b) Let b be the number of people who like both tea and coffee.

(c) Let c be the number of people who like coffee but not tea.

According to the question,

$a + b + c = 20$ (i)

$a + b = 12$ (ii)

$b + c = 15$ (iii)

Form (i) and (ii), $c = 8$.

Substituting $c = 8$ in (iii), we get $b = 7$ and substituting $b = 7$ in (ii) we get $a = 5$. Therefore,

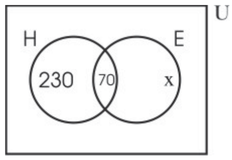
(i) $n(C \cap T) = b = 7$

(ii) $n(T \cap C') = a = 5$

(iii) $n(C \cap T') = c = 8$

Illustration 14: In a group of 400 people, who speak either Hindi or English or both, if 230 speak Hindi only and 70 speak both English and Hindi, then how many of them speak English only?

Solution:



According to the question, 230 + 70 + x = 400

$\Rightarrow$ x = 400 – 300 = 100

$\Rightarrow$ x = 100

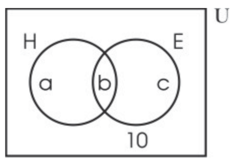
$\therefore$ The number of people who speak English only is 100.

Illustration 15: In a group of 70 people, 45 speak Hindi language and 33 speak English language and 10 speak neither Hindi nor English. How many can speak both English as well as Hindi language? How many can speak only English language?

Let H = the set of people who speak Hindi

E = the set of people who speak English

Solution: According to the question, out of 70 people, 10 speak neither Hindi nor English. Therefore, n $(H \cup E)$ = 70 – 10 = 60.



Let a be the number of people who speak Hindi but not English, b be the number of people who speak both Hindi and English and c be the number of people who speak only English.

Now, according to the question,

a + b + c = 60 (i)

a + b = 45 (ii)

b + c = 33 (iii)

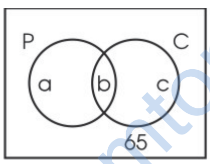
Therefore, from (i) and (ii), the number of people speaking English language only is c = 15. Now, substituting c = 15 in (iii) we get number of people speaking English and Hindi is b = 18.

Illustration 16: In a certain town, 25% of the families own a phone. 15% own a car and, 65% families own neither a phone nor a car. 2000 families own both a car and a phone. Find how much percentage of families own either a car or a phone. Also, find how many families live in the town.

Let P= the set of families who own a phone

C= the set of families who own a car

Solution: Let the number of families in the town be 100.



According to the questions,

$a + b = 25$ (i)

$b + c = 15$ (ii)

$a + b + c = 100 - 65 = 35$ (iii)

Solving, (i), (ii) and (iii), we get,

$a = 20, b = 5$ and $c = 10$

$\therefore$ Percentage of families, owning either a car or a phone is 35% as $a + b + c = 20 + 5 + 10 = 35$

Now, let 'x' be the number of families living in the town.

Therefore, $\frac{5}{100} \times x = 2000$ (given)

$\therefore \quad x = 40{,}000$

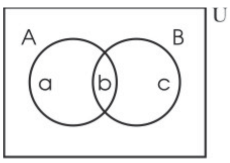
$\therefore$ The number of families in the town are 40,000

Illustration 17: Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is the data correct?

Let A= the set of Car owners of Car A

B= the set of Car owners of Car B

Solution:



$n(U) = 500$

$a + b = 400$

$b + c = 200$

$b = 50$

$\Rightarrow$ $a = 350$ and $c = 150$

$\therefore$ $a + b + c = 350 + 150 + 50 = 550 \neq 500$

Now, according to the above solution and given question,

$n(A \cup B) > n(U)$, which is not possible as $n(AUB) \neq n(U)$

Therefore, the above data is incorrect.

Illustration 18: Let U be the universal set for sets A and B such that $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$.

Then, $n(A' \cap B') = 300$, provided n (u) is equal to:

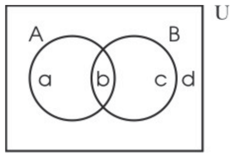
(i) 600

(ii) 700

(c) 800

(d) 900

Solution:



According to the question,

$a + b = 200 \quad \text{(i)}$

$b + c = 300 \quad \text{(ii)}$

$b = 100$

$d = 300$

Adding (i) and (ii),

$a + b + b + c = 500$

$\Rightarrow \quad (a + b + c) + b = 500$

$\Rightarrow \quad a + b + c = 500 - b = 500 - 100 = 400$

$\therefore \quad n(A \cup B) = 400$

Now, $n(U) = n(A \cup B) + n((A \cup B)')$

$= n(A \cup B) + n(A' \cap B')$

$= 400 + 300$

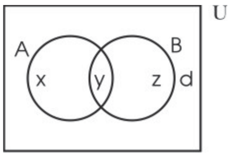
$= 700$

Illustration 19: A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B. What is the least number that must have liked both the products?

Let A = the set of consumers who like product A

B = the set of consumers who like product B

Solution:



n (U) = 1000, let n (A) = x + y, n (b) = y + z and n ($A \cap B$) = y

$\therefore$ $x + y = 720$ (i)

$y + z = 450$ (ii)

Now, n ($A \cup B$) $\leq$ n (U)

$\therefore$ $x + y + z \leq 1000$ (iii)

Adding (i) and (ii) we get

$(x + y + z) + y = 1170$

Now from (iii), as $x + y + z \leq 1000$

$\therefore$ $(x + y + z) + y \leq 1000 + y$ (adding y on both sides)

$\therefore$ $1170 \leq 1000 + y$

$\therefore$ $y \geq 170$

Therefore, the least number that liked both the products is 170.

Illustration 20: In a survey of 60 people, it was found that 25 people read newspaper H; 26 read newspaper T, 26 read newspapers I, 9 read both H and I, 11 read both H and T, 8 read both T and I and, 3 read all three newspapers. Find:

(i) the number of people who read atleast one of the newspapers.

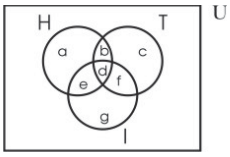
(ii) the number of people who read exactly one newspaper.

Let H = the set of people who read newspaper H

T = the set of people who read newspaper T

I = the set of people who read newspaper I

Solution:



According to the question, n (u) = 60

$a + b + d + e = 25$ (i)

$b + c + d + f = 26$ (ii)

$e + d + f + g = 26$ (iii)

$e + d = 9$ (iv)

$b + d = 11$ (v)

$d + f = 8$ (vi)

$d = 3$ (vii)

Substituting (vii) in (iv), (v) and (vi), we get,

$f = 5, b = 8, e = 6$

From, (i), (ii) and (iii), $g = 12$, $c = 10$ and $a = 8$

(i) the number of people reading atleast one newspaper is

$a + b + c + d + e + f + g = 8 + 8 + 10 + 3 + 6 + 5 + 12 = 52$

(ii) the number of people who read exactly one newspaper are

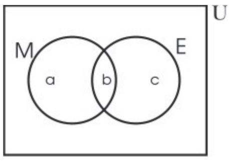
$a + c + g = 8 + 10 + 12 = 30$

Illustration 21: In a class of 35 students, 17 have taken Mathematics, 10 have taken Mathematics but not Economics. If each student has taken either of the two subjects, then find the number of students who have taken Economics but not Mathematics.

Let M = the set of students who have taken Maths

E = the set of students who have taken Economics

Solution:



According to the question,

$a + b + c = 35$ (i)

$a + b = 17$ (ii)

$a = 10$

We need to find the number of students who have taken Economics but not Mathematics i.e. we need to find 'c' Subtracting (ii) from (i), we get c = 18.

Illustration 22: With corona virus threating to run riot in India, prevention appears to be the best cure available so far. It is crucial for people to have awareness and knowledge about the virus and need to take proper precautions to discourage its spread. A survey was conducted on 25 people to see if proper precautions were being taken by people and following points were observed:

(a) 15 people used face masks

(b) 14 consciously maintained social distancing

(c) 5 used face masks and washed their hand regularly

(d) 9 maintained social distancing and used face masks.

(e) 3 were practicing all the three measures.

(f) 4 maintained social distancing and washing hands regularly.

(g) 4 practised only social distancing norms.

Assuming that everyone took at least one of the precautionary measures, find:

(i) How many exercised only washing hands as precautionary measure.

(ii) How many pratised social distancing and washing hands but not wearing masks.

(iii) How many exercised only wearing masks.

Solution:

Consider the Venn diagram as in Fig. 1.14. Let S denote the set of people maintaining social distancing M denote the set of people wearing fall masks and W denote the people washing hands regularly.

Let a, b, c, d, e, f, g denote the number of people in the respective regions.

n (maintaining social distancing) = n (S) = a + b + d + e = 14 (i)

n (wearing face mask = n (M) = d + e + f + g = 15 (ii)

n (washed hands and wear face masks) = n ($W \cap M$) = e + f = 5 (iii)

n (who took all the these measures) = n ($S \cap W \cap M$) = e = 3 (iv)

n (maintaining distancing & washing hands) = n ($S \cap W$) = b + e = 4 (v)

n (maintained only social distancing = n ($S \cap W' \cap M'$) = a = 4 (vi)

n (maintained social distancing and wear face masks

= n ($S \cap M$) = e + d = 9 (vii)

Now, a = 4, e = 3. Showing the above questions:

n ($S \cap W$) = b + e = 4 $\Rightarrow$ b = 1

n ($S \cap M$) = e + d = 9 $\Rightarrow$ d = 6

n ($W \cap M$) = e + f = 5 $\Rightarrow$ f = 2

n (M) = d + e + f + g = 15 $\Rightarrow$ g = 4

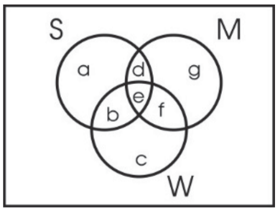
Now, a + b + c + d + e + f + g = 25

$\Rightarrow$ c = 5

$\therefore$ (i) Number of people who exercised only washing hands = c = 5

(ii) Number of people who practised social distancing and washing hands but not wearing masks = b = 1

(iii) Number of people who exercised only wearing masks = g = 4



Check your progress 3.4

Q.1 In a group of 70 people, 37 like coffee, 52 like tea and each person likes atleast one of the two drinks. How many people like both coffee and tea.

Q.2 In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only? How many like tennis.

Q.3 In a survey of 600 students in a school, 150 students were found to the taking apple juice, 225 taking orange juice and 100 were taking both apple and orange juice. Find how many were taking neither apple juice nor orange juice.

Q.4 In an election, two contestants A and B contested. y% of the total votes voted for A and (y + 30)% for B. If 20% of the voters did not vote, then y =

(i) 30

(ii) 25

(iii) 40

(iv) 35

Q.5 In a class, 70 students wrote two tests, test-I and test-II. 50% of the students failed in test-I and 40% of the students in test-II. How many students passed in both the tests?

(i) 21

(ii) 7

(iii) 28

(iv) 14

Solution: Check Your Progress 3.1

1. (ii), (iii), (iv) are sets.

2. (i) A = {x : x = 4n, where n = 1, 2, 3, 4, 5}

 (ii) B = {x : x is a prime number}

 (iii) C = {x : x is a consonant}

 (iv) D = {x : x is a solution of the equation $x^2 - 1 = 0$}

 (v) E = {x : x is a prime number between 40 and 50}

 (vi) F={x:x=$\frac{1}{n}$, where n $\in$ N

 (vii) G = {x : x = $\frac{1}{n^2}$ where n $\in$ N

3. (i) A = {1, 2, 3, 4}

 (ii) B = {–3, –2, –1, 0, 1, 2, 3, 4, 5, 6}

 (iii) C = {F, O, L, W}

 (iv) D = {15, 24, 33, 42, 51, 60}

 (v) E = {A, R, I, T, H, M, E, C}

4. (i) c

 (ii) e

 (iii) a

 (iv) b

 (v) d

5. (i) $\in$

 (ii) $\notin$

 (iii) $\in$

 (iv) $\in$

(v) $\not\in$

Solution: Check Your Progress 3.2

1. (i) Finite; cordiality : 99

 (ii) Infinite

 (iii) Finite; cordiality : 7

 (iv) Infinite

 (v) Infinite

 (vi) Infinite

2. (i) Singleton

 (ii) Empty set

 (iii) Singleton

 (iv) Singleton

 (v) Empty

3. (i), (ii), (iii)

4. (i) $\in$

 (ii) $\subset$

 (iii) $\not\in$

 (iv) $\subset$

 (v) $\not\in$

 (vi) $\subset$

 (vii) $\subset$

5. (i) $\{x : x \in R, 1 < x < 3\}$

 (ii) $\{x : x \in R, -1 \leq x \leq 3\}$

(iii) $\{x : x \in R, -4 < x \leq 0\}$

(iv) $\{x : x \in R, -1 \leq x < 1\}$

(v) $\{x : x \in R, 0 \leq x < \infty\}$

6. $P(P(A)) = 2$

7. 16

Solution: Check Your Progress 3.3

1. (i) $A \cap B = \{M, T, E, I\}$; $A \cup B = \{M, A, T, H, E, I, C, S, R, G, O, N, Y\}$

   (ii) $A \cap B = \{2, 4\}$; $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$

   (iii) $A \cap B = \phi$; $A \cup B = N$

   (iv) $A \cap B = \{2, 4\}$; $A \cup B = \{2, 4, 6, 8, 10\}$

   (v) $A \cap B = \{0, 1\}$; $A \cup B = \{0, 1\}$

2. (i) $A - (B \cup C) = \{1, 2\}$

   (ii) $A \cap C) = \{1, 2, 3, 4\}$

   (iii) $B' \cap C' = \{1, 2\}$

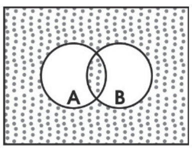
   (iv) $B' \cup A' = \{1, 2, 5, 6, 7, 8\}$

   (v) $A - (B \cup C)' = \{3, 4\}$

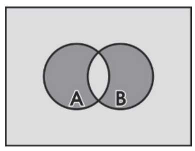
4. (i) $(A \cup B)'$:

(ii) $(A \cap B)'$:

(iii) $A' \cap B'$:



(iv) $A' \cup B'$



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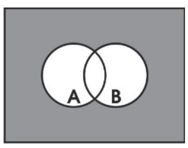

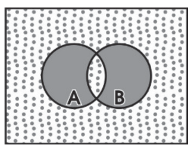

5. m = 6, n = 3

6. Yes; B; A

7.	(i)	ϕ	(ii)	A
	(iii)	A	(iv)	ϕ
	(v)	A	(vi)	ϕ
	(vii)	ϕ	(viii)	ϕ
	(ix)	U	(x)	U
8.	(iii)	8		
9.	(i)	{3, 4, 6}		

Solution: Check Your Progress 1.4

1. 19
2. 25; 35
3. 325
4. (iii) 25
5. (ii) 7

Summary

This chapter deals with basic definitions, operations and practical problems involving sets. These are summarised below:

- A set is a well-defined collection of distinct objects.
- Representation of sets in Roster or Tabular form and set builder form.
- Types of sets:

    (a) A set which does not contain any element is called an empty set.

    (b) A set which contains only one element is called a singleton set.

    (c) A set which contains definite number of elements is called a finite set, otherwise it is called an infinite set.

    (d) Two sets are said to be equal if they have exactly the same elements.

    (e) A set 'A' is said to be a subset of a set 'B', if every element of A is also an element of B.

    (f) Power set of a set A is the set of all the subsets of A and is denoted by P(A).

    (g) A set that contains all the elements in a given context is called a universal set and is denoted by U.

    (h) Intervals are also subsets of R, set of real numbers.

- Venn diagrams
- Operations on set

    (a) Union of two sets A and B is the set of all those elements which are either in A or in B.

    (b) Intersection of two sets A and B is the set of all elements which are common to both A and B.

(c) Difference of sets A and B in this order is the set of elements which belong to A but not to B.

(d) The complement of a set A, which is a subset of U, the universal set, is the set of all elements of U which are not the elements of A.

- De Morgan's Law: For any two sets A & B, $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$.
- If A and B are finite sets, then if:

(a) $A \cap B = \phi$, then $n(A \cup B) = n(A) + n(B)$

(b) $A \cap B \neq \phi$, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$