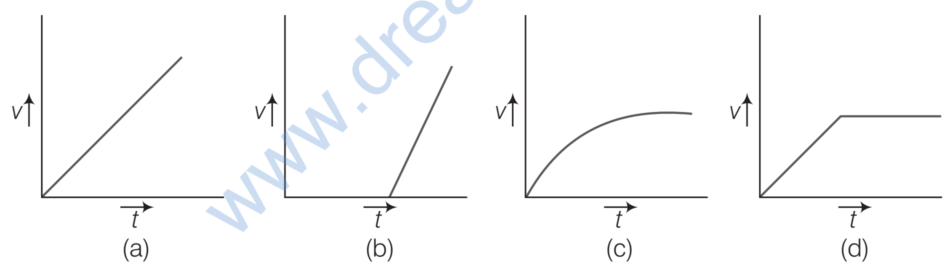
9

Mechanical Properties of Fluids

Multiple Choice Questions (MCQs)

Q. 1 A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero initial velocity. From the plot shown in figure, indicate the one that represents the velocity (v) of the pebble as a function of time (t) .



Thinking Process

When the pebble is dropped from the top, a variable force called viscous force will act which increases with increase in speed. And at equilibrium this velocity becomes constant.

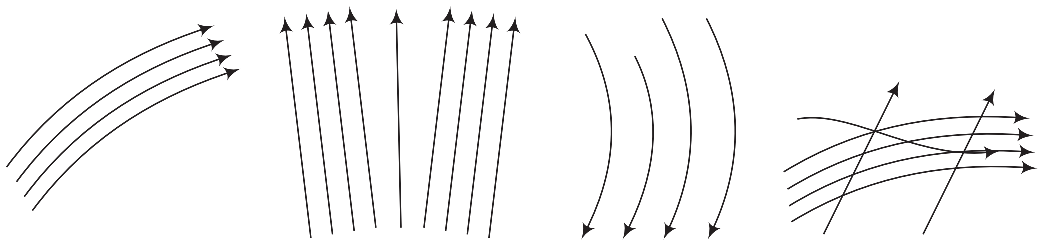
Ans. *(c)* When the pebble is falling through the viscous oil the viscous force is

$$F = 6\pi\eta r v$$

where r is radius of the pebble, v is instantaneous speed, η is coefficient of viscosity.

As the force is variable, hence acceleration is also variable so v-t graph will not be straight line.First velocity increases and then becomes constant known as terminal velocity.

Q. 2 Which of the following diagrams does not represent a streamline flow?



(a) (b) (c) (d)

Ans. ***(d)*** In a streamline flow at any given point, the velocity of each passing fluid particles remaines constant. If we consider a cross-sectional area, then a point on the area cannot have different velocities at the same time, hence two streamlines of flow cannot cross each other.

Q. 3 Along a streamline,

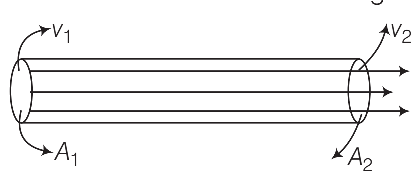
(a) the velocity of a fluid particle remains constant
(b) the velocity of all fluid particles crossing a given position is constant
(c) the velocity of all fluid particles at a given instant is constant
(d) the speed of a fluid particle remains constant

Ans. ***(b)*** As we know for a streamline flow of a liquid velocity of each particle at a particular cross-section is constant, because Av = constant (law of continuity) between two cross-section of a tube of flow.

Q. 4 An ideal fluid flows through a pipe of circular cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in the two pipes is

(a) 9 : 4 (b) 3 : 2 (c) $\sqrt{3}:\sqrt{2}$ (d) $\sqrt{2}:\sqrt{3}$

Ans. ***(a)*** Consider the diagram where an ideal fluid is flowing through a pipe.



As given

d_1 = Diameter at 1st point is 2.5.

d_2 = Diameter at 2nd point is 3.75.

Applying equation of continuity for cross-sections A_1 and A_2.

$$\Rightarrow \quad A_1 v_1 = A_2 v_2$$

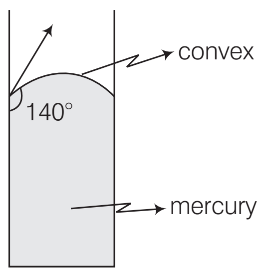
$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi(r_2^2)}{\pi(r_1^2)} = \left(\frac{r_2}{r_1}\right)^2$$

$$= \left(\frac{\frac{3.75}{2}}{\frac{2.5}{2}}\right)^2 = \left(\frac{3.75}{2.5}\right)^2 = \frac{9}{4} \quad \begin{bmatrix} r_2 = \frac{d_2}{2} \\ r_1 = \frac{d_1}{2} \end{bmatrix}$$

Q. 5 The angle of contact at the interface of water-glass is 0°, ethyl alcohol-glass is 0°, mercury-glass is 140° and methyliodide-glass is 30°. A glass capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is

(a) water (b) ethylalcohol (c) mercury (d) methyliodide

Ans. *(c)* According to the question, the observed meniscus is of convex figure shape. Which is only possible when angle of contact is obtuse. Hence, the combination will be of mercury-glass (140°)



Multiple Choice Questioins (More Than One Options)

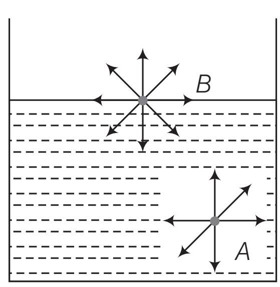
Q. 6 For a surface molecule,

(a) the net force on it is zero
(b) there is a net downward force
(c) the potential energy is less than that of a molecule inside
(d) the potential energy is more than that of a molecule inside

Ans. *(b, d)*

Consider the diagram where two molecules of a liquid are shown. One is well inside the liquid and other is on the surface. The molecule (*A*) which is well inside experiences equal forces from all directions, hence net force on it will be zero.

And molecules on the liquid surface have some extra energy as it surrounded surraind by only lower half side of liquid molecules.



Q. 7 Pressure is a scalar quantity, because

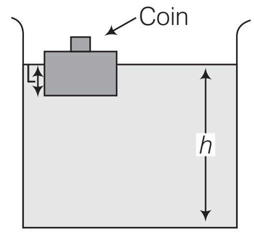
(a) it is the ratio of force to area and both force and area are vectors
(b) it is the ratio of the magnitude of the force to area
(c) it is the ratio of the component of the force normal to the area
(d) it does not depend on the size of the area chosen

Ans. *(b, c)*

Pressure is defined as the ratio of magnitude of component of the force normal to the area and the area under consideration.

As magnitude of component is considered, hence, it will not have any direction. So, pressure is a scalar quantity.

Q. 8 A wooden block with a coin placed on its top, floats in water as shown in figure.



The distance l and h are shown in the figure. After sometime, the coin falls into the water. Then,

(a) l decreases (b) h decreases (c) l increases (d) h increases

Thinking Process

When any body floats in a liquid, the upthrust force acting on the body due to the displaced liquid is balanced by its weight.

Ans. *(a,b)*

When the coin falls into the water, weight of the (block + coin) system decreases, which was balanced by the upthrust force earlier. As weight of the system decreases, hence upthrust force will also decrease which is only possible when l decreases.

As l decreases volume of water displaced by the block decreases, hence h decreases.

Note *As the coin falls into water, it displaces some volume of water which is very less hence, we neglect volume of the coin.*

Q. 9 With increase in temperature, the viscosity of

(a) gases decreases (b) liquids increases
(c) gases increases (d) liquids decreases

Ans. *(c, d)*

For liquids coefficient of viscosity, $\eta \propto \frac{1}{\sqrt{T}}$

i.e., with increase in temperature η decreases.

For gases coefficient of viscosity, $\eta \propto \sqrt{T}$

i.e., with increase in temperature η increases.

Q. 10 Streamline flow is more likely for liquids with

(a) high density (b) high viscosity
(c) low density (d) low viscosity

Ans. *(b, c)*

Streamline flow is more likely for liquids having low density. We know that greater the coefficient of viscosity of a liquid more will be velocity gradient hence each line of flow can be easily differentiated. Also higher the coefficient of viscosity lower will be Reynolds number, hence flow more like to be streamline.

Very Short Answer Type Questions

Q. 11 Is viscosity a vector?

Ans. Viscosity is a property of liquid it does not have any direction, hence it is a scalar quantity.

Q. 12 Is surface tension a vector?

Ans. *No*, surface tension is a scalar quantity.

Surface tension $= \frac{\text{Work done}}{\text{Surface area}}$, where work done and surface area both are scalar quantities.

Q. 13 Iceberg floats in water with part of it submerged. What is the fraction of the volume of iceberg submerged, if the density of ice is $\rho_i = 0.917 \text{ g cm}^{-3}$?

Ans. Given, density of ice $(\rho_{ice}) = 0.917 \text{g /cm}^3$

Density of water $(\rho_w) = 1 \text{ g/cm}^3$

Let V be the total volume of the iceberg and V' of its volume be submerged in water.
In floating condition.
Weight of the iceberg = Weight of the water displaced by the submerged part by ice

$$V\rho_{ice} g = V' \rho_w g$$

or $$\frac{V'}{V} = \frac{\rho_{ice}}{\rho_w} = \frac{0.917}{1} = 0.917 \qquad (\because \text{ Weight} = mg = v\rho g)$$

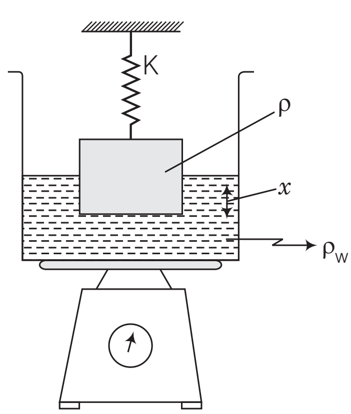
Q. 14 A vessel filled with water is kept on a weighing pan and the scale adjusted to zero. A block of mass M and density ρ is suspended by a massless spring of spring constant k. This block is submerged inside into the water in the vessel. What is the reading of the scale?

Ans. Consider the diagram,
The scale is adjusted to zero, therefore, when the block suspended to a spring is immersed in water, then the reading of the scale will be equal to the thrust on the block due to water.
Thrust = weight of water displaced

$= V\rho_w g$ (where V is volume of the block and ρ_w is density of water)

$$= \frac{m}{\rho}\rho_w g = \left(\frac{\rho_w}{\rho}\right) mg$$

$$(\because \text{Density of the block } \rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{V})$$

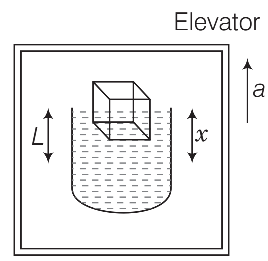


Q. 15 A cubical block of density ρ is floating on the surface of water. Out of its height L, fraction x is submerged in water. The vessel is in an elevator accelerating upward with acceleration a. What is the fraction immersed?

Thinking Process

As the elevator is accelerating upward the net acceleration of the block with respect to the elevator can be calculate by the concept of pseudo force.

Ans. Consider the diagram.



Let the density of water be ρ_w and a cubical block of ice of side L be floating in water with x of its height (L) submerged in water.

$$\text{Volume of the block } (V) = L^3$$

$$\text{Mass of the block } (m) = V\rho = L^3\rho$$

$$\text{Weight of the block } = mg = L^3\rho g$$

1st case

Volume of the water displaced by the submerged part of the block $= xL^2$

∴ Weight of the water displaced by the block

In floating condition, $xL^2\rho_w g$

Weight of the block = Weight of the water displaced by the block

$$L^3\rho g = xL^2\rho_w g$$

or $$\frac{x}{L} = \frac{\rho}{\rho_w} = x$$

2nd case

When elevator is accelerating upward with an acceleration a, then effective acceleration

$$= (g + a) \quad (\because \text{ Pseudo force is downward})$$

Then, weight of the block $= m(g + a)$

$$= L^3\rho(g + a)$$

Let x_1 fraction be submerged in water when elevator is accelerating upwards.

Now, in the floating condition, weight of the block = weight of the displaced water

$$L^3\rho(g + a) = (x_1L^2)\rho_w(g + a)$$

or $$\frac{x_1}{L} = \frac{\rho}{\rho_w} = x$$

From 1st and 2nd case,

We see that,the fraction of the block submerged in water is independent of the acceleration of the elevator.

Note *We should not confuse with the concept of pseudo force, i.e., pseudo force is downward, hence fraction will change due to increased force.*

Short Answer Type Questions

Q. 16 The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius $r = 2.5 \times 10^{-5}$ m. The surface tension of sap is $T = 7.28 \times 10^{-2}$ Nm^{-1} and the angle of contact is 0°. Does surface tension alone account for the supply of water to the top of all trees?

Ans. Given, radius $(r) = 2.5 \times 10^{-5}$ m

Surface tension $(S) = 7.28 \times 10^{-2}$ N/m

Angle of contact $(\theta) = 0°$

The maximum height to which sap can rise in trees through capillarity action is given by

$$h = \frac{2S\cos\theta}{r\rho g} \text{ where } S = \text{Surface tension}, \rho = \text{Density}, r = \text{Radius}$$

$$= \frac{2 \times 7.28 \times 10^{-2} \times \cos 0°}{2.5 \times 10^{-5} \times 1 \times 10^{-3} \times 9.8} = 0.6\,\text{m}$$

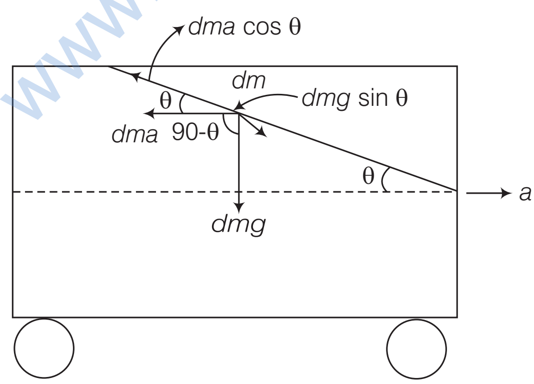
This is the maximum height to which the sap can rise due to surface tension. Since, many trees have heights much more than this, capillary action alone cannot account for the rise of water in all trees.

Q. 17 The free surface of oil in a tanker, at rest, is horizontal. If the tanker starts accelerating the free surface will be titled by an angle θ. If the acceleration is a ms^{-2}, what will be the slope of the free surface?

Thinking Process

As the tanker starts accelerating, free surface of the tanker will not be horizontal because pseudo force acts.

Ans. Consider the diagram where a tanker is accelerating with acceleration a.



Consider an elementary particle of the fluid of mass dm.

The acting forces on the particle with respect to the tanker are shown above .

Now, balancing forces (as the particle is in equilibrium) along the inclined direction component of weight = component of pseudo force $dmg\sin\theta = dma\cos\theta$ (we have assumed that the surface is inclined at an angle θ) where, dma is pseudo force

$$\Rightarrow \quad g\sin\theta = a\cos\theta$$

$$\Rightarrow \quad a = g\tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{a}{g} = \text{slope}$$

Q. 18 Two mercury droplets of radii 0.1 cm and 0.2 cm collapse into one single drop. What amount of energy is released? The surface tension of mercury $T = 435.5 \times 10^{-3}$ Nm^{-1}.

Thinking Process

In this process, conservation of mass will be applied. Before and after collapse, mass and hence, volume of the system remains conserved.

Ans. Consider the diagram.

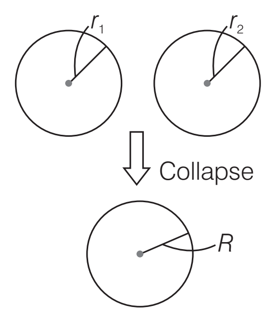
Radii of mercury droplets $r_1 = 0.1 \text{cm} = 1 \times 10^{-3}$ m

$r_2 = 0.2 \text{ cm} = 2 \times 10^{-3}$ m

Surface tension $(T) = 435.5 \times 10^{-3}$ N/m

Let the radius of the big drop formed by collapsing be R.

$\therefore$ Volume of big drop = Volume of small droplets



$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^2$$

or $R^3 = r_1^3 + r_2^3$

$= (0.1)^3 + (0.2)^3$

$= 0.001 + 0.008$

$= 0.009$

or $R = 0.21 \text{cm} = 2.1 \times 10^{-3}$ m

$\therefore$ Change in surface area $\Delta A = 4\pi R^2 - (4\pi r_1^2 + 4\pi r_2^2)$

$= 4\pi[R^2 - (r_1^2 + r_2^2)]$

$\therefore$ Energy released $= T \cdot \Delta A$ (where T is surface tension of mercury)

$= T \times 4\pi[R^2 - (r_1^2 + r_2^2)]$

$= 435.5 \times 10^{-3} \times 4 \times 3.14[(2.1 \times 10^{-3})^2 - (1 \times 10^{-6} + 4 \times 10^{-6})]$

$= 435.5 \times 4 \times 3.14[4.41 - 5] \times 10^{-6} \times 10^{-3}$

$= -32.23 \times 10^{-7}$ (Negative sign shows absorption)

Therefore, 3.22×10^{-6} J energy will be absorbed.

Note *In this process, energy is not conserved. Energy is lst due to the collapsing, in the form of radiations.*

Q. 19 If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of the droplets. Let a drop of radius R, break into N small droplets each of radius r. Estimate the drop in temperature.

Ans. When a big drop of radius R, breaks into N droplets each of radius r, the volume remains constant.

$\therefore$ Volume of big drop $= N \times$Volume of each small drop

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3$$

or $R^3 = Nr^3$

or $N = \frac{R^3}{r^3}$

Now, change in surface area $= 4\pi R^2 - N4\pi r^2$

$= 4\pi(R^2 - Nr^2)$

Energy released $= T \times \Delta A$ $= S \times 4\pi(R^2 - Nr^2)$ [T = Surface tension]

Due to releasing of this energy, the temperature is lowered.
If ρ is the density and s is specific heat of liquid and its temperature is lowered by $\Delta\theta$, then energy released = $ms\Delta\theta$ [s = specific heat $\Delta\theta$ = change in temperature]

$$T \times 4\pi(R^2 - Nr^2) = \left(\frac{4}{3} \times R^3 \times \rho\right) s\Delta\theta \qquad [\because m = v\rho = \frac{4}{3}\pi R^3 \rho]$$

$$\Rightarrow \quad \Delta\theta = \frac{T \times 4\pi(R^2 - Nr^2)}{\frac{4}{3}\pi R^3 \rho \times s}$$

$$= \frac{3T}{\rho s}\left[\frac{R^2}{R^3} - \frac{Nr^2}{R^3}\right]$$

$$= \frac{3T}{\rho s}\left[\frac{1}{R} - \frac{(R^3/r^3) \times r^2}{R^3}\right]$$

$$= \frac{3T}{\rho s}\left[\frac{1}{R} - \frac{1}{r}\right]$$

Q. 20 The surface tension and vapour pressure of water at 20°C is 7.28×10^{-2} Nm^{-1} and 2.33×10^3 Pa, respectively. What is the radius of the smallest spherical water droplet which can form without evaporating at 20°C?

Ans. Given, surface tension of water

$$(S) = 7.28 \times 10^{-2} \text{ N/m}$$

Vapour pressure $(p) = 2.33 \times 10^3$ Pa

The drop will evaporate, if the water pressure is greater than the vapour pressure.
Let a water droplet or radius R can be formed without evaporating.
Vapour pressure = Excess pressure in drop.

$$\therefore \quad p = \frac{2S}{R}$$

$$\text{or} \quad R = \frac{2S}{p} = \frac{2 \times 7.28 \times 10^{-2}}{2.33 \times 10^3}$$

$$= 6.25 \times 10^{-5} \text{ m}$$

Long Answer Type Questions

Q. 21 (a) Pressure decreases as one ascends the atmosphere. If the density of air is ρ, what is the change in pressure dp over a differential height dh?

(b) Considering the pressure p to be proportional to the density, find the pressure p at a height h if the pressure on the surface of the earth is p_0.

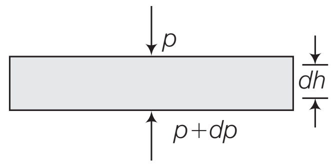
(c) If $p_0 = 1.03 \times 10^5$ Nm^{-2}, $\rho_0 = 1.29$ kg m^{-3} and $g = 9.8$ ms^{-2}, at what height will be pressure drop to (1/10) the value at the surface of the earth?

(d) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model.

Thinking Process

As we are going up in the atmosphere, thickness of the gases above us decreases hence, pressure also decreases.

Ans. (a) Consider a horizontal parcel of air with cross-section A and height dh.



Let the pressure on the top surface and bottom surface be p and $p + dp$. If the parcel is in equilibrium, then the net upward force must be balanced by the weight.

i.e., $(p + dp)A - pA = -\rho gAdh$ ($\because$ Weight = Density × Volume × g)

$$= -\rho \times Adh \times g$$

$\Rightarrow$ $dp = -\rho gdh$. (ρ = density of air)

Negative sign shows that pressure decreases with height.

(b) Let ρ_o be the density of air on the surface of the earth.

As per question, pressure $\propto$ density

$$\Rightarrow \quad \frac{p}{p_o} = \frac{\rho}{\rho_o}$$

$$\Rightarrow \quad \rho = \frac{\rho_o}{p_o} p$$

$$\therefore \quad dp = -\frac{\rho_o g}{p_o} pdh \qquad [\because dp = -\rho gdh]$$

$$\Rightarrow \quad \frac{dp}{p} = -\frac{\rho_o g}{p_o} dh$$

$$\Rightarrow \quad \int_{p_o}^{p} \frac{dp}{p} = -\frac{\rho_o g}{p_o} \int_0^h dh \qquad \begin{bmatrix} \because \text{at } h = o, \ r = p_o \\ \text{and} \quad \text{at } h = h, p = p \end{bmatrix}$$

$$\Rightarrow \quad \ln \frac{p}{p_o} = -\frac{\rho_o g}{p_o} h$$

By removing log, $$p = p_o e^{\left(-\frac{\rho_o gh}{p_o}\right)}$$

(c) As $p = p_o e^{-\frac{\rho_o gh}{p_o}}$,

$$\Rightarrow \quad \ln \frac{p}{p_o} = -\frac{\rho_o gh}{P_o}$$

By question, $$p = \frac{1}{10} p_o$$

$$\Rightarrow \quad \ln \left(\frac{\frac{1}{10} p_o}{p_o} \right) = -\frac{\rho_o g}{p_o} h$$

$$\Rightarrow \quad \ln \frac{1}{10} = -\frac{\rho_o g}{p_o} h\rho_o$$

$\therefore \quad h = -\frac{p_0}{\rho_0 g} \ln \frac{1}{10} = -\frac{p_0}{\rho_0 g} \ln (10)^{-1} = \frac{p_0}{\rho_0 g} \ln 10$

$= \frac{p_0}{\rho_0 g} \times 2.303 \quad [\because \ln (x) = 2.303 \log_{10}(x)]$

$= \frac{1.013 \times 10^5}{1.22 \times 9.8} \times 2.303 = 0.16 \times 10^5 \text{m}$

$= 16 \times 10^3 \text{ m}$

(d) We know that $p \propto \rho$ (when T = constant *i.e.,* isothermal pressure)

Temperature (T) remains constant only near the surface of the earth, not at greater heights.

Q. 22 Surface tension is exhibited by liquids due to force of attraction between molecules of the liquid. The surface tension decreases with increase in temperature and vanishes at boiling point. Given that the latent heat of vaporisation for water $L_v = 540$ k cal kg^{-1}, the mechanical equivalent of heat $J = 4.2$ J cal^{-1}, density of water $\rho_w = 10^3$ kg l^{-1}, Avagadro's number $N_A = 6.0 \times 10^{26}$ k mole^{-1} and the molecular weight of water $M_A = 10$ kg for 1 k mole.

(a) Estimate the energy required for one molecule of water to evaporate.

(b) Show that the inter-molecular distance for water is $d = \left[\frac{M_A}{N_A} \times \frac{1}{\rho_w}\right]^{1/3}$ and find its value.

(c) 1 g of water in the vapour state at 1 atm occupies 1601cm^3. Estimate the inter-molecular distance at boiling point, in the vapour state.

(d) During vaporisation a molecule overcomes a force F, assumed constant, to go from an inter-molecular distance d to d'. Estimate the value of F.

(e) Calculate F / d, which is a measure of the surface tension.

Ans. (a) Given , $L_v = 540 \text{ kcal kg}^{-1}$

$= 540 \times 10^3 \text{ cal kg}^{-1} = 540 \times 10^3 \times 4.2 \text{ J kg}^{-1}$

$\because$ Energy required to evaporate 1 kg of water = L_v kcal

$\therefore$ M_A kg of water requires $M_A L_v$ kcal $[\because Q = mL]$

Since, there are N_A molecules in M_A kg of water the energy required for 1 molecule to evaporate is

$U = \frac{M_A L_v}{N_A}$ J [where $N_A = 6 \times 10^{26}$ = Avogadro number]

$= \frac{18 \times 540 \times 4.2 \times 10^3}{6 \times 10^{26}}$ J

$= 90 \times 18 \times 4.2 \times 10^{-23}$ J

$= 6.8 \times 10^{-20}$J

(b) Let the water molecules to be points and are separated at distance d from each other.

Volume of N_A molecule of water $= \frac{M_A}{\rho_w}$ $\left[\because V = \frac{M}{\rho}\right]$

Thus, the volume around one molecule is $= \frac{M_A}{N_A \rho_w}$

The volume around one molecule is

$$d^3 = (M_A / N_A \rho_w)$$

$$\therefore \quad d = \left(\frac{M_A}{N_A \rho_w}\right)^{1/3} = \left(\frac{18}{6 \times 10^{26} \times 10^3}\right)^{1/3}$$

$$(30 \times 10^{-30})^{1/3} \text{ m} \approx 3.1 \times 10^{-10} \text{ m}$$

(c) $\because$ 1 kg of vapour occupies volume $= 1601 \times 10^{-3} \text{m}^3$

$\therefore$ 18 kg of vapour occupies $18 \times 1601 \times 10^{-3} \text{m}^3$

6×10^{26} molecules occupies $18 \times 1601 \times 10^{-3} \text{m}^3$

$\therefore$ 1 molecule occupies $\frac{18 \times 1601 \times 10^{-3}}{6 \times 10^{26}} \text{m}^3$

If d is the inter- molecular distance, then

$$d_1^3 = (3 \times 1601 \times 10^{-29}) \text{ m}^3$$

$$\therefore \quad d_1 = (30 \times 1601)^{1/3} \times 10^{-10} \text{m}$$

$$= 36.3 \times 10^{-10} \text{m}$$

(d) Work done to change the distance from d to d_1 is $= F(d_1 - d)$

This work done is equal to energy required to evaporate 1 molecule.

$$\therefore \quad F(d_1 - d) = 6.8 \times 10^{-20}$$

or

$$F = \frac{6.8 \times 10^{-20}}{d_1 - d}$$

$$= \frac{6.8 \times 10^{-20}}{(36.3 \times 10^{-10} - 3.1 \times 10^{-10})}$$

$$= 2.05 \times 10^{-11} \text{N}$$

(e) Surface tension $= \frac{F}{d} = \frac{2.05 \times 10^{-11}}{3.1 \times 10^{-10}} = 6.6 \times 10^{-2}$ N/m.

Q. 23 A hot air balloon is a sphere of radius 8 m. The air inside is at a temperature of 60°C. How large a mass can the balloon lift when the outside temperature is 20°C? Assume air in an ideal gas, $R = 8.314$ J mole^{-1}K^{-1}, 1 atm $= 1.013 \times 10^5$ Pa, the membrane tension is 5 Nm^{-1}.

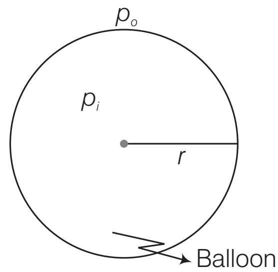
Thinking Process

Pressure inside the curved surface will be greater than of outside pressure.

Ans. Let the pressure inside the balloon be p_i and the outside pressure be p_o, then excess pressure is $p_i - p_o = \frac{2S}{r}$.

where, S = Surface tension

r = radius of balloon



Considering the air to be an ideal gas $p_i V = n_i RT_i$ where, V is the volume of the air inside the balloon, n_i is the number of moles inside and T_i is the temperature inside, and $p_o V = n_o RT_o$ where V is the volume of the air displaced and n_o is the number of moles displaced and T_o is the temperature outside.

So. $$n_i = \frac{p_i V}{RT_i} = \frac{M_i}{M_A}$$

where, M_i is the mass of air inside and M_A is the molar mass of air

and $$n_o = \frac{p_o V}{RT_o} = \frac{M_o}{M_A}$$

where, M_o is the mass of air outside that has been displaced. If w is the load it can raise, then $w + M_i g = M_o g$

$$\Rightarrow \quad w = M_o g - M_i g$$

As in atmosphere 21% O_2 and 79% N_2-is present

$\therefore$ Molar mass of air

$$M_i = 0.21 \times 32 + 0.79 \times 28 = 28.84\,\text{g}.$$

$\therefore$ Weight raised by the balloon

$$w = (M_o - M_i)g$$

$$\Rightarrow \quad w = \frac{M_A V}{R}\left(\frac{p_o}{T_o} - \frac{p_i}{T_i}\right)g$$

$$= \frac{0.02884 \times \frac{4}{3}\pi \times 8^3 \times 9.8}{8.314} \left(\frac{1.013 \times 10^5}{293} - \frac{1.013 \times 10^5}{333} - \frac{2 \times 5}{8 \times 313}\right)$$

$$= \frac{0.02884 \; \frac{4}{3}\pi \times 8^3}{8.314} \times 1.013 \times 10^5 \left(\frac{1}{293} - \frac{1}{333}\right) \times 9.8$$

$$= 3044.2\ \text{N}$$

$$\therefore \quad \text{Mass lifted by the balloon} = \frac{w}{g} = \frac{3044.2}{10} \approx 304.42\ \text{kg}.$$

$$\approx 305\ \text{kg}.$$