# Chapter 3 Matrices

## EXERCISE 3.1

**Question 1:**

In the matrix $A=\begin{pmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{pmatrix}$, write:

(i) The order of the matrix

(ii) The number of elements

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

**Solution:**

(i) Since, in the given matrix, the number of rows is $3$ and the number of columns is $4$, the order of the matrix is $3\times 4$.

(ii) Since the order of the matrix is $3\times 4$, there are $3\times 4=12$ elements.

(iii) Here,

$$a_{13}=19$$
$$a_{21}=35$$
$$a_{33}=-5$$
$$a_{24}=12$$
$$a_{23}=\frac{5}{2}$$

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**Question 2:**

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

**Solution:**

We know that if a matrix is of the order $m\times n$, it has $mn$ elements. Thus, to find all the possible orders of a matrix having $24$ elements, we have to find all the ordered pairs of natural numbers whose product is $24$.

The ordered pairs are: $(1,24),(24,1),(2,12),(12,2),(3,8),(8,3),(4,6)$ and $(6,4)$.

Hence, the possible orders of a matrix having $24$ elements are:

$$(1\times 24),(24\times 1),(2\times 12),(12\times 2),(3\times 8),(8\times 3),(4\times 6) \text{ and } (6\times 4).$$

$(1,13)$ and $(13,1)$ are the ordered pairs of natural numbers whose product is $13$.

Hence, the possible orders of a matrix having $13$ elements are $(1\times13)$ and $(13\times1)$.

**Question 3:**

If a matrix has 18 elements, what are the possible order it can have? What, if it has 5 elements?

**Solution:**

We know that if a matrix is of the order $m\times n$, it has $mn$ elements. Thus, to find all the possible orders of a matrix having $18$ elements, we have to find all the ordered pairs of natural numbers whose product is $18$.

The ordered pairs are: $(1,18),(18,1),(2,9),(9,2),(3,6)$ and $(6,3)$.

Hence, the possible orders of a matrix having 18 elements are:

$$(1\times18),(18\times1),(2\times9),(9\times2),(3\times6) \text{ and } (6\times3).$$

$(1\times5)$ and $(5\times1)$ are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are $(1\times5)$ and $(5\times1)$.

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**Question 4:**

Construct a $2\times2$ matrix, $A=\left[a_{ij}\right]$, whose elements are given by:

(i) $a_{ij}=\dfrac{(i+j)^2}{2}$

(ii) $a_{ij}=\dfrac{i}{j}$

(iii) $a_{ij}=\dfrac{(i+2j)^2}{2}$

**Solution:**

In general, a $2\times2$ matrix is given by $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

(i) $a_{ij}=\dfrac{(i+j)^2}{2};\quad i,j=1,2$

Therefore,

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

Thus, the required matrix is $A = \begin{pmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{pmatrix}$

(ii) $a_{ij} = \frac{i}{j};\quad i, j = 1, 2$

Therefore,

$$a_{11} = \frac{1}{1} = 1$$

$$a_{12} = \frac{1}{2}$$

$$a_{21} = \frac{2}{1} = 2$$

$$a_{22} = \frac{2}{2} = 1$$

Thus, the required matrix is $A = \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$

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(iii) $a_{ij} = \frac{(i+2j)^2}{2};\quad i, j = 1, 2$

Therefore,

$$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2)^2}{2} = 8$$

$$a_{22} = \frac{(2+4)^2}{2} = 18$$

Thus, the required matrix is $A = \begin{pmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{pmatrix}$

**Question 5:**

In general, a $3\times 4$ matrix whose elements are given by

(i) $a_{ij} = \frac{1}{2}\left|-3i + j\right|$

(ii) $a_{ij} = 2i - j$

**Solution:**

In general, a $3\times 4$ matrix is given by $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$

(i) Given $a_{ij} = \frac{1}{2}\left|-3i + j\right|;\ i = 1,2,3\ \ j = 1,2,3,4$

$$a_{11} = \frac{1}{2}\left|-3(1)+1\right| = \frac{1}{2}\left|-3+1\right| = \frac{1}{2}\left|-2\right| = \frac{2}{2} = 1$$

$$a_{21} = \frac{1}{2}\left|-3(2)+1\right| = \frac{1}{2}\left|-6+1\right| = \frac{1}{2}\left|-5\right| = \frac{5}{2}$$

$$a_{31} = \frac{1}{2}\left|-3(3)+1\right| = \frac{1}{2}\left|-9+1\right| = \frac{1}{2}\left|-8\right| = \frac{8}{2} = 4$$

$$a_{12} = \frac{1}{2}\left|-3(1)+2\right| = \frac{1}{2}\left|-3+2\right| = \frac{1}{2}\left|-1\right| = \frac{1}{2}$$

$$a_{22} = \frac{1}{2}\left|-3(2)+2\right| = \frac{1}{2}\left|-6+2\right| = \frac{1}{2}\left|-4\right| = \frac{4}{2} = 2$$

$$a_{32} = \frac{1}{2}\left|-3(3)+2\right| = \frac{1}{2}\left|-9+2\right| = \frac{1}{2}\left|-7\right| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}\left|-3(1)+3\right| = \frac{1}{2}\left|-3+3\right| = 0$$

$$a_{23} = \frac{1}{2}\left|-3(2)+3\right| = \frac{1}{2}\left|-6+3\right| = \frac{1}{2}\left|-3\right| = \frac{3}{2}$$

$$a_{33} = \frac{1}{2}\left|-3(3)+3\right| = \frac{1}{2}\left|-9+3\right| = \frac{1}{2}\left|-6\right| = \frac{6}{2} = 3$$

$$a_{14} = \frac{1}{2}\left|-3(1)+4\right| = \frac{1}{2}\left|-3+4\right| = \frac{1}{2}\left|1\right| = \frac{1}{2}$$

$$a_{24} = \frac{1}{2}\left|-3(2)+4\right| = \frac{1}{2}\left|-6+4\right| = \frac{1}{2}\left|-2\right| = \frac{2}{2} = 1$$

$$a_{34} = \frac{1}{2}\left|-3(3)+4\right| = \frac{1}{2}\left|-9+4\right| = \frac{1}{2}\left|-5\right| = \frac{5}{2}$$

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Thus, the required matrix is $A=\begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{pmatrix}$

(ii) $a_{ij}=2i-j;\ i=1,2,3\ \ j=1,2,3,4$

$$a_{11}=2(1)-1=2-1=1$$
$$a_{21}=2(2)-1=4-1=3$$
$$a_{31}=2(3)-1=6-1=5$$

$$a_{12}=2(1)-2=2-2=0$$
$$a_{22}=2(2)-2=4-2=2$$
$$a_{32}=2(3)-2=6-2=4$$

$$a_{13}=2(1)-3=2-3=-1$$
$$a_{23}=2(2)-3=4-3=1$$
$$a_{33}=2(3)-3=6-3=3$$

$$a_{14}=2(1)-4=2-4=-2$$
$$a_{24}=2(2)-4=4-4=0$$
$$a_{34}=2(3)-4=6-4=2$$

Thus, the required matrix is $A=\begin{pmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{pmatrix}$

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## Question 6:

Find the value of $x$, $y$ and $z$ from the following equation:

(i) $\begin{pmatrix} 4 & 3 \\ x & 5 \end{pmatrix}=\begin{pmatrix} y & z \\ 1 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix}=\begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

(iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix}=\begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

**Solution:**

(i) $\begin{pmatrix} 4 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 1 & 5 \end{pmatrix}$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$x = 1, y = 4$ and $z = 3$

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$$x + y = 6$$
$$xy = 8$$
$$5 + z = 5$$

Hence,

$$\Rightarrow 5 + z = 5$$
$$\Rightarrow z = 0$$

We know that $(a-b)^2 = (a+b)^2 - 4ab$

$$\Rightarrow (x-y)^2 = (6)^2 - 8\times 4$$
$$\Rightarrow (x-y)^2 = 36 - 32$$
$$\Rightarrow (x-y)^2 = 4$$
$$\Rightarrow (x-y) = \pm 2$$

Equating $x - y = 2$ and $x + y = 6$, we get $x = 4, y = 2$

Similarly, Equating $x - y = -2$ and $x + y = 6$, we get $x = 2, y = 4$

Thus, $x = 4, y = 2, z = 0$ or $x = 2, y = 4, z = 0$

(iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$$x + y + z = 9 \qquad \ldots(1)$$
$$x + z = 5 \qquad \ldots(2)$$
$$y + z = 7 \qquad \ldots(3)$$

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From $(1)$ and $(2)$, we have

$$\Rightarrow y+5=9$$
$$\Rightarrow y=4$$

From $(3)$, we have

$$\Rightarrow 4+z=7$$
$$\Rightarrow z=3$$

Therefore,

$$\Rightarrow x+z=5$$
$$\Rightarrow x+3=5$$
$$\Rightarrow x=2$$

Thus, $x=2, y=4, z=3$

**Question 7:**

Find the value of $a,b,c$ and $d$ from the equation:

$$\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix}=\begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$$

**Solution:**

$$\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix}=\begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$$a-b=-1 \qquad \ldots(1)$$
$$2a-b=0 \qquad \ldots(2)$$
$$2a+c=5 \qquad \ldots(3)$$
$$3c+d=13 \qquad \ldots(4)$$

From $(2)$,

$$b=2a$$

Putting this value in $(1)$,

$$\Rightarrow a-2a=-1$$
$$\Rightarrow a=1$$

Hence,

$$\Rightarrow b=2$$

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Putting $a=1$ in $(3)$,

$$\Rightarrow 2(1)+c=5$$
$$\Rightarrow c=3$$

Putting $c=3$ in $(4)$,

$$\Rightarrow 3(3)+d=13$$
$$\Rightarrow d=4$$

Thus, $a=1, b=2, c=3$ and $d=4$.

## Question 8:

$A=\left[a_{ij}\right]_{m\times n}$ is a square matrix, if

(A) $m<n$ (B) $m>n$ (C) $m=n$ (D) None of these

**Solution:**

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore, $A=\left[a_{ij}\right]_{m\times n}$ is a square matrix, if $m=n$.

Thus, the correct option is C.

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## Question 9:

Which of the given values of $x$ and $y$ make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x=\frac{-1}{3}, y=7$ (B) Not possible to find (C) $y=7, x=\frac{-2}{3}$ (D) $x=\frac{-1}{3}, y=\frac{-2}{3}$

**Solution:**

The given matrices are $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$ and $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements, we get:

$$3x+7=0 \Rightarrow x=\frac{-7}{3}$$
$$y-2=5 \Rightarrow y=7$$
$$y+1=8 \Rightarrow y=7$$
$$2-3x=4 \Rightarrow x=\frac{-2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of $x$, which is not possible.

Hence, it is not possible to find the values of $x$ and $y$ for which the given matrices are equal.

Thus, the correct option is B.

**Question 10:**

The number of all possible matrices of order $3\times3$ with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512

**Solution:**

The given matrix of the order $3\times3$ has $9$ elements and each of these elements can be either 0 or 1.

Now, each of the $9$ elements can be filled in two possible ways.

Hence, by the multiplication principle, the required number of possible matrices is $2^9=512$.

Thus, the correct option is D.

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# EXERCISE 3.2

## Question 1:

Let $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$. Find each of the following:

(i) $A+B$

(ii) $A-B$

(iii) $3A-C$

(iv) $AB$

(v) $BA$

## Solution:

(i) $A+B$

$$\Rightarrow \begin{pmatrix}2 & 4\\ 3 & 2\end{pmatrix}+\begin{pmatrix}1 & 3\\ -2 & 5\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2+1 & 4+3\\ 3-2 & 2+5\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}3 & 7\\ 1 & 7\end{pmatrix}$$

(ii) $A-B$

$$\Rightarrow \begin{pmatrix}2 & 4\\ 3 & 2\end{pmatrix}-\begin{pmatrix}1 & 3\\ -2 & 5\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2-1 & 4-3\\ 3+2 & 2-5\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}1 & 1\\ 5 & -3\end{pmatrix}$$

(iii) $3A-C$

$$\Rightarrow 3\begin{pmatrix}2 & 4\\ 3 & 2\end{pmatrix}-\begin{pmatrix}-2 & 5\\ 3 & 4\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}3\times 2 & 3\times 4\\ 3\times 3 & 3\times 2\end{pmatrix}-\begin{pmatrix}-2 & 5\\ 3 & 4\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}6+2 & 12-5\\ 9-3 & 6-4\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}8 & 7\\ 6 & 2\end{pmatrix}$$

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(iv) $AB$

$$\Rightarrow \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -6 & 26 \\ -1 & 19 \end{pmatrix}$$

(v) $BA$

$$\Rightarrow \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1(2)+3(3) & 1(4)+3(2) \\ -2(2)+5(3) & -2(4)+5(2) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 11 & 10 \\ 11 & 2 \end{pmatrix}$$

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### Question 2:

Compute the following:

(i) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}+\begin{pmatrix} a & b \\ b & a \end{pmatrix}$

(ii) $\begin{pmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{pmatrix}+\begin{pmatrix} 2ab & 2bc \\ -2ac & -2ab \end{pmatrix}$

(iii) $\begin{pmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{pmatrix}+\begin{pmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{pmatrix}$

(iv) $\begin{pmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{pmatrix}+\begin{pmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{pmatrix}$

**Solution:**

(i) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} a & b \\ b & a \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a+a & b+b \\ -b+b & a+a \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2a & 2b \\ 0 & 2a \end{pmatrix}$$

(ii) $\begin{pmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{pmatrix} + \begin{pmatrix} 2ab & 2bc \\ -2ac & -2ab \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{pmatrix}$$

(iii) $\begin{pmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{pmatrix} + \begin{pmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{pmatrix}$$

(iv) $\begin{pmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{pmatrix} + \begin{pmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} \cos^2 x+\sin^2 x & \sin^2 x+\cos^2 x \\ \sin^2 x+\cos^2 x & \cos^2 x+\sin^2 x \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

**Question 3:**

Compute the indicated products:

(i) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

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(ii) $\begin{pmatrix}1\\2\\3\end{pmatrix}\begin{pmatrix}2 & 3 & 4\end{pmatrix}$

(iii) $\begin{pmatrix}1 & -2\\2 & 3\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\2 & 3 & 1\end{pmatrix}$

(iv) $\begin{pmatrix}2 & 3 & 4\\3 & 4 & 5\\4 & 5 & 6\end{pmatrix}\begin{pmatrix}1 & -3 & 5\\0 & 2 & 4\\3 & 0 & 5\end{pmatrix}$

(v) $\begin{pmatrix}2 & 1\\3 & 2\\-1 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 1\\-1 & 2 & 1\end{pmatrix}$

(vi) $\begin{pmatrix}3 & -1 & 3\\-1 & 0 & 2\end{pmatrix}\begin{pmatrix}2 & -3\\1 & 0\\3 & 1\end{pmatrix}$

**Solution:**

(i) $\begin{pmatrix}a & b\\-b & a\end{pmatrix}\begin{pmatrix}a & -b\\b & a\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}a(a)+b(b) & a(-b)+b(a)\\-b(a)+a(b) & -b(-b)+a(a)\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}a^2+b^2 & -ab+ab\\-ab+ab & b^2+a^2\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{pmatrix}$$

(ii) $\begin{pmatrix}1\\2\\3\end{pmatrix}\begin{pmatrix}2 & 3 & 4\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}1(2) & 1(3) & 1(4)\\2(2) & 2(3) & 2(4)\\3(2) & 3(3) & 3(4)\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2 & 3 & 4\\4 & 6 & 8\\6 & 9 & 12\end{pmatrix}$$

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(iii) $\begin{pmatrix}1 & -2\\ 2 & 3\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}$

$$\Rightarrow\begin{pmatrix}1(1)-2(2) & 1(2)-2(3) & 1(3)-2(1)\\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1)\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}-3 & -4 & 1\\ 8 & 13 & 9\end{pmatrix}$$

(iv) $\begin{pmatrix}2 & 3 & 4\\ 3 & 4 & 5\\ 4 & 5 & 6\end{pmatrix}\begin{pmatrix}1 & -3 & 5\\ 0 & 2 & 4\\ 3 & 0 & 5\end{pmatrix}$

$$\Rightarrow\begin{pmatrix}2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5)\\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5)\\ 4(1)+5(0)+6(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5)\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}14 & 0 & 42\\ 18 & -1 & 56\\ 22 & -2 & 70\end{pmatrix}$$

(v) $\begin{pmatrix}2 & 1\\ 3 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 1\\ -1 & 2 & 1\end{pmatrix}$

$$\Rightarrow\begin{pmatrix}2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1)\\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1)\\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1)\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}1 & 2 & 3\\ 1 & 4 & 5\\ -2 & 2 & 0\end{pmatrix}$$

(vi) $\begin{pmatrix}3 & -1 & 3\\ -1 & 0 & 2\end{pmatrix}\begin{pmatrix}2 & -3\\ 1 & 0\\ 3 & 1\end{pmatrix}$

$$\Rightarrow\begin{pmatrix}3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1)\\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1)\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}14 & -6\\ 4 & 5\end{pmatrix}$$

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**Question 4:**

If $A=\begin{pmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{pmatrix}, B=\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}, C=\begin{pmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{pmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C)=(A+B)-C$.

**Solution:**

$$(A+B)=\begin{pmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{pmatrix}+\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}$$

$$=\begin{pmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{pmatrix}$$

$$(B-C)=\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}-\begin{pmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{pmatrix}$$

$$=\begin{pmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{pmatrix}$$

Now,

$$A+(B-C)=\begin{pmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{pmatrix}+\begin{pmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{pmatrix}$$

$$=\begin{pmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{pmatrix}$$

$$(A+B)-C=\begin{pmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{pmatrix}-\begin{pmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{pmatrix}$$

$$=\begin{pmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{pmatrix}$$

Hence, $A+(B-C)=(A+B)-C$.

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**Question 5:**

If $A=\begin{pmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{pmatrix}$ and $B=\begin{pmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{pmatrix}$, then compute $3A-5B$.

**Solution:**

$$3A-5B=3\begin{pmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{pmatrix}-5\begin{pmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{pmatrix}$$

$$=\begin{pmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{pmatrix}-\begin{pmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{pmatrix}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

**Question 6:**

Simplify $\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{pmatrix}$.

**Solution:**

$$\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{pmatrix}+\begin{pmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}\cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

**Question 7:**

Find $X$ and $Y$, if

(i) $X+Y=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix}$ and $X-Y=\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$

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(ii) $2X+3Y=\begin{pmatrix}2 & 3\\ 4 & 0\end{pmatrix}$ and $3X+2Y=\begin{pmatrix}2 & -2\\ -1 & 5\end{pmatrix}$

**Solution:**

(i)
$$X+Y=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix} \qquad \ldots(1)$$
$$X-Y=\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix} \qquad \ldots(2)$$

Adding equations $(1)$ and $(2)$,

$$2X=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix}+\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$$
$$=\begin{pmatrix}10 & 0\\ 2 & 8\end{pmatrix}$$
$$X=\frac{1}{2}\begin{pmatrix}10 & 0\\ 2 & 8\end{pmatrix}$$
$$=\begin{pmatrix}5 & 0\\ 1 & 4\end{pmatrix}$$

Now,

$$\Rightarrow X+Y=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix}$$
$$\Rightarrow Y=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix}-\begin{pmatrix}5 & 0\\ 1 & 4\end{pmatrix}$$
$$\Rightarrow Y=\begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix}$$

(ii)
$$2X+3Y=\begin{pmatrix}2 & 3\\ 4 & 0\end{pmatrix} \qquad \ldots(1)$$
$$3X+2Y=\begin{pmatrix}2 & -2\\ -1 & 5\end{pmatrix} \qquad \ldots(2)$$

Multiplying equation $(1)$ by 2,

$$2(2X+3Y)=2\begin{pmatrix}2 & 3\\ 4 & 0\end{pmatrix}$$
$$4X+6Y=\begin{pmatrix}4 & 6\\ 8 & 0\end{pmatrix} \qquad \ldots(3)$$

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Multiplying equation $(2)$ by 3,

$$3(3X+2Y)=3\begin{pmatrix}2 & -2\\ -1 & 5\end{pmatrix}$$

$$9X+6Y=\begin{pmatrix}6 & -6\\ -3 & 15\end{pmatrix} \qquad \ldots(4)$$

From $(3)$ and $(4)$,

$$(4X+6Y)-(9X+6Y)=\begin{pmatrix}4 & 6\\ 8 & 0\end{pmatrix}-\begin{pmatrix}6 & -6\\ -3 & 15\end{pmatrix}$$

$$-5X=\begin{pmatrix}4-6 & 6+6\\ 8+3 & 0-15\end{pmatrix}$$

$$-5X=\begin{pmatrix}-2 & 12\\ 11 & -15\end{pmatrix}$$

$$X=\frac{-1}{5}\begin{pmatrix}-2 & 12\\ 11 & -15\end{pmatrix}$$

$$=\begin{pmatrix}\frac{2}{5} & \frac{-12}{5}\\ \frac{-11}{5} & 3\end{pmatrix}$$

Now

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$$\Rightarrow 2X+3Y=\begin{pmatrix}2&3\\4&0\end{pmatrix}$$

$$\Rightarrow 2\begin{pmatrix}\frac{2}{5}&\frac{-12}{5}\\\frac{-11}{5}&3\end{pmatrix}+3Y=\begin{pmatrix}2&3\\4&0\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}\frac{4}{5}&\frac{-24}{5}\\\frac{-22}{5}&6\end{pmatrix}+3Y=\begin{pmatrix}2&3\\4&0\end{pmatrix}$$

$$\Rightarrow 3Y=\begin{pmatrix}2&3\\4&0\end{pmatrix}-\begin{pmatrix}\frac{4}{5}&\frac{-24}{5}\\\frac{-22}{5}&6\end{pmatrix}$$

$$\Rightarrow 3Y=\begin{pmatrix}\frac{6}{5}&\frac{39}{5}\\\frac{42}{5}&-6\end{pmatrix}$$

$$\Rightarrow Y=\frac{1}{3}\begin{pmatrix}\frac{6}{5}&\frac{39}{5}\\\frac{42}{5}&-6\end{pmatrix}$$

$$\Rightarrow Y=\begin{pmatrix}\frac{2}{5}&\frac{13}{5}\\\frac{14}{5}&-2\end{pmatrix}$$

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### Question 8:

Find $X$, if $Y=\begin{pmatrix}3&2\\1&4\end{pmatrix}$ and $2X+Y=\begin{pmatrix}1&0\\-3&2\end{pmatrix}$.

**Solution:**

$$2X+Y=\begin{pmatrix}1 & 0\\ -3 & 2\end{pmatrix}$$

$$\Rightarrow 2X+\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}=\begin{pmatrix}1 & 0\\ -3 & 2\end{pmatrix}$$

$$\Rightarrow 2X=\begin{pmatrix}1 & 0\\ -3 & 2\end{pmatrix}-\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}$$

$$\Rightarrow 2X=\begin{pmatrix}-2 & -2\\ -4 & -2\end{pmatrix}$$

$$\Rightarrow X=\frac{1}{2}\begin{pmatrix}-2 & -2\\ -4 & -2\end{pmatrix}$$

$$\Rightarrow X=\begin{pmatrix}-1 & -1\\ -2 & -1\end{pmatrix}$$

## Question 9:

Find $x$ and $y$, if $2\begin{pmatrix}1 & 3\\ 0 & x\end{pmatrix}+\begin{pmatrix}y & 0\\ 1 & 2\end{pmatrix}=\begin{pmatrix}5 & 6\\ 1 & 8\end{pmatrix}$.

**Solution:**

$$\Rightarrow 2\begin{pmatrix}1 & 3\\ 0 & x\end{pmatrix}+\begin{pmatrix}y & 0\\ 1 & 2\end{pmatrix}=\begin{pmatrix}5 & 6\\ 1 & 8\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2 & 6\\ 0 & 2x\end{pmatrix}+\begin{pmatrix}y & 0\\ 1 & 2\end{pmatrix}=\begin{pmatrix}5 & 6\\ 1 & 8\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2+y & 6\\ 1 & 2x+2\end{pmatrix}=\begin{pmatrix}5 & 6\\ 1 & 8\end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$2+y=5$$
$$\Rightarrow y=3$$

$$2x+2=8$$
$$\Rightarrow x=3$$

Therefore, $x=3$ and $y=3$.

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## Question 10:

Solve the equation for $x, y, z$ and $t$ if $2\begin{pmatrix} x & z \\ y & t \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$.

## Solution:

$$\Rightarrow 2\begin{pmatrix} x & z \\ y & t \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x & 2z \\ 2y & 2t \end{pmatrix} + \begin{pmatrix} 3 & -3 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 9 & 15 \\ 12 & 18 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{pmatrix} = \begin{pmatrix} 9 & 15 \\ 12 & 18 \end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$2x + 3 = 9$$
$$\Rightarrow 2x = 6$$
$$\Rightarrow x = 3$$

$$2y = 12$$
$$\Rightarrow y = 6$$

$$2z - 3 = 15$$
$$\Rightarrow 2z = 18$$
$$\Rightarrow z = 9$$

$$2t + 6 = 18$$
$$\Rightarrow 2t = 12$$
$$\Rightarrow t = 6$$

Therefore, $x = 3, y = 6, z = 9$ and $t = 6$.

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## Question 11:

If $x\begin{pmatrix} 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$, find values of $x$ and $y$.

**Solution:**

$$\Rightarrow x\begin{pmatrix}2\\3\end{pmatrix}+y\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}10\\5\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2x\\3x\end{pmatrix}+\begin{pmatrix}-y\\y\end{pmatrix}=\begin{pmatrix}10\\5\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2x-y\\3x+y\end{pmatrix}=\begin{pmatrix}10\\5\end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$2x-y=10 \qquad \ldots(1)$$
$$3x+y=5 \qquad \ldots(2)$$

By adding these two equations, we get

$$5x=15$$
$$\Rightarrow x=3$$

Now, putting this value in $(2)$

$$\Rightarrow 3x+y=5$$
$$\Rightarrow y=5-3x$$
$$\Rightarrow y=5-3(3)$$
$$\Rightarrow y=5-9$$
$$\Rightarrow y=-4$$

Therefore, $x=3$ and $y=4$.

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**Question 12:**

Given $3\begin{pmatrix}x & y\\z & w\end{pmatrix}=\begin{pmatrix}x & 6\\-1 & 2w\end{pmatrix}+\begin{pmatrix}4 & x+y\\z+w & 3\end{pmatrix}$, find values of $w, x, y$ and $z$.

**Solution:**

$$\Rightarrow 3\begin{pmatrix}x & y\\z & w\end{pmatrix}=\begin{pmatrix}x & 6\\-1 & 2w\end{pmatrix}+\begin{pmatrix}4 & x+y\\z+w & 3\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}3x & 3y\\3z & 3w\end{pmatrix}=\begin{pmatrix}x+4 & 6+x+y\\-1+z+w & 2w+3\end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$\Rightarrow 3x = x+4$$
$$\Rightarrow 2x = 4$$
$$\Rightarrow x = 2$$

$$\Rightarrow 3y = 6+x+y$$
$$\Rightarrow 2y = 6+x$$
$$\Rightarrow 2y = 6+2$$
$$\Rightarrow 2y = 8$$
$$\Rightarrow y = 4$$

$$\Rightarrow 3w = 2w+3$$
$$\Rightarrow w = 3$$

$$\Rightarrow 3z = -1+z+w$$
$$\Rightarrow 2z = w-1$$
$$\Rightarrow 2z = 3-1$$
$$\Rightarrow 2z == 2$$
$$\Rightarrow z = 1$$

Therefore, $x=2, y=4, z=1$ and $w=3$

## Question 13:

If $F(x)=\begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$, show that $F(x)F(y)=F(x+y)$.

## Solution:

It is given that $F(x)=\begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Then, $F(y)=\begin{pmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Now,

$$F(x+y)=\begin{pmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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$$F(x)F(y)=\begin{pmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}\cos x\cos y-\sin x\sin y+0 & -\cos x\sin y-\sin x\cos y+0 & 0\\ \sin x\cos y+\cos x\sin y+0 & -\sin x\sin y+\cos x\cos y+0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$=\begin{pmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$=F(x+y)$$

Therefore, $F(x)F(y)=F(x+y)$

**Question 14:**

Show that

(i) $\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\neq\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}$

(ii) $\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{pmatrix}\begin{pmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{pmatrix}\neq\begin{pmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{pmatrix}$

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**Solution:**

(i) $\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\neq\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}$

$$\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}=\begin{pmatrix}5(2)-1(3) & 5(1)-1(4)\\ 6(2)+7(3) & 6(1)+7(4)\end{pmatrix}$$

$$=\begin{pmatrix}10-3 & 5-4\\ 12+21 & 6+28\end{pmatrix}$$

$$=\begin{pmatrix}7 & 1\\ 33 & 34\end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} = \begin{pmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{pmatrix}$$
$$= \begin{pmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{pmatrix}$$
$$= \begin{pmatrix} 16 & 5 \\ 39 & 25 \end{pmatrix}$$

Thus, $\begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \neq \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \neq \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{pmatrix}$$
$$= \begin{pmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) & 0(3)+-1(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{pmatrix}$$
$$= \begin{pmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{pmatrix}$$

Thus, $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \neq \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$

## Question 15:

Find $A^2 - 5A + 6I$, if $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$

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**Solution:**

$A^2 = A.A$

$$= \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+1(1) & 2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)(3)+0(0) \end{pmatrix}$$

$$= \begin{pmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$

Therefore,

$$A^2 - 5A + 6I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - 5\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} + 6\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{pmatrix}$$

**Question 16:**

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.

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**Solution:**

$$A^2 = A.A$$

$$= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$$

Now,

$$A^3 = A^2.A$$

$$= \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{pmatrix}$$

$$= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$$

Therefore,

$$A^3 - 6A^2 + 7A + 2I = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - 6\begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix}$$

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$$=\begin{pmatrix}30 & 0 & 48\\ 12 & 24 & 30\\ 48 & 0 & 78\end{pmatrix}-\begin{pmatrix}30 & 0 & 48\\ 12 & 24 & 30\\ 48 & 0 & 78\end{pmatrix}$$

$$=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}=0$$

Hence, $A^3-6A^2+7A+2I=0$.

## Question 17:

If $A=\begin{pmatrix}3 & -2\\ 4 & -2\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, find $k$ so that $A^2=kA-2I$.

## Solution:

$A^2=A.A$

$$=\begin{pmatrix}3 & -2\\ 4 & -2\end{pmatrix}\begin{pmatrix}3 & -2\\ 4 & -2\end{pmatrix}$$

$$=\begin{pmatrix}3(3)+(-2)(4) & 3(-2)+(-2)(-2)\\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2)\end{pmatrix}$$

$$=\begin{pmatrix}1 & -2\\ 4 & -4\end{pmatrix}$$

Now,

$$\Rightarrow A^2=kA-2I$$

$$\Rightarrow \begin{pmatrix}1 & -2\\ 4 & -4\end{pmatrix}=k\begin{pmatrix}3 & -2\\ 4 & -2\end{pmatrix}-2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}1 & -2\\ 4 & -4\end{pmatrix}=\begin{pmatrix}3k & -2k\\ 4k & -2k\end{pmatrix}-\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}1 & -2\\ 4 & -4\end{pmatrix}=\begin{pmatrix}3k-2 & -2k\\ 4k & -2k-2\end{pmatrix}$$

Comparing the corresponding elements, we have:

$$3k-2=1$$

$$\Rightarrow 3k=3$$

$$\Rightarrow k=1$$

Therefore, the value of $k=1$.

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**Question 18:**

If $A=\begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$ and $I$ is the identity matrix of order $2$, show that $I+A=(I-A)\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

**Solution:**

$LHS = I + A$

$$=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}+\begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{pmatrix} \qquad \ldots(1)$$

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$$RHS=(I-A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$$

$$=\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}-\begin{pmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{pmatrix}\right)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$$

$$=\begin{pmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$$

$$=\begin{pmatrix}\cos\alpha+\sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha+\cos\alpha\tan\frac{\alpha}{2}\\ -\cos\alpha\tan\frac{\alpha}{2}+\sin\alpha & \sin\alpha\tan\frac{\alpha}{2}+\cos\alpha\end{pmatrix}$$

$$=\begin{pmatrix}1-2\sin^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2}\\ -\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2}\end{pmatrix}$$

$$=\begin{pmatrix}1-2\sin^2\frac{\alpha}{2}+2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}-\tan\frac{\alpha}{2}\\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2}\end{pmatrix}$$

$$=\begin{pmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix}\quad\ldots(2)$$

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Thus, from $(1)$ and $(2)$, we get

$$I+A=(I-A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$$

### Question 19:

A trust fund has ₹30000 that must be invested in two different types of bonds. The first bond pays $5\%$ interest per year, and the second bond pays $7\%$ interest per year. Using matrix multiplication, determine how to divide ₹30000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(i) ₹ 1800

(ii) ₹ 2000

**Solution:**

(i) Let ₹$x$ be invested in the first bond. Then, the sum of money invested in the second bond will be ₹$(30000-x)$.

It is given that the first bond pays $5\%$ interest per year and the second bond pays $7\%$ interest per year.

Therefore, in order to obtain an annual total interest of ₹1800, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}=1800 \qquad \left[\text{S.I for 1 year}=\frac{\text{Principal}\times\text{Rate}}{100}\right]$$

$$\Rightarrow \frac{5x}{100}+\frac{7(30000-x)}{100}=1800$$

$$\Rightarrow 5x+210000-7x=180000$$

$$\Rightarrow 210000-2x=180000$$

$$\Rightarrow 2x=210000-180000$$

$$\Rightarrow 2x=30000$$

$$\Rightarrow x=15000$$

Thus, in order to obtain an annual total interest of ₹1800, the trust fund should invest ₹15000 in the first bond and the remaining ₹15000 in the second bond.

(ii) Let ₹$x$ be invested in the first bond. Then, the sum of money invested in the second bond will be ₹$(30000-x)$.

Therefore, in order to obtain an annual total interest of ₹2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}=2000$$

$$\Rightarrow \frac{5x}{100}+\frac{7(30000-x)}{100}=2000$$

$$\Rightarrow 5x+210000-7x=200000$$

$$\Rightarrow 210000-2x=200000$$

$$\Rightarrow 2x=210000-200000$$

$$\Rightarrow 2x=10000$$

$$\Rightarrow x=5000$$

Thus, in order to obtain an annual total interest of ₹1800, the trust fund should invest ₹5000 in the first bond and the remaining ₹25000 in the second bond.

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## Question 20:

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹$80$, ₹$60$ and ₹$40$ each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

## Solution:

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$\begin{aligned}12\begin{bmatrix}10 & 8 & 10\end{bmatrix}\begin{bmatrix}80\\60\\40\end{bmatrix} &= 12\left[10(80)+8(60)+10(40)\right]\\ &= 12(800+480+400)\\ &= 12(1680)\\ &= 20160\end{aligned}$$

Thus, the bookshop will receive ₹$20160$ from the sale of all these books.

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## Question 21:

Assume $X, Y, Z, W$ and $P$ are matrices of order $2\times n, 3\times k, 2\times p, n\times 3$ and $p\times k$ respectively. The restriction on $n, k$ and $p$ so that $PY+WY$ will be defined are:

(A) $k=3, p=n$

(B) $k$ is arbitrary, $p=2$

(C) $p$ is arbitrary, $k=3$

(D) $k=2, p=3$

## Solution:

Matrices $P$ and $Y$ are of the orders $p\times k$ and $3\times k$ respectively.
Therefore, matrix $PY$ will be defined if $k=3$.
Consequently, $PY$ will be of the order $p\times k$.
Matrices $W$ and $Y$ are of the orders $n\times 3$ and $3\times k$ respectively.

Since the number of columns in $W$ is equal to the number of rows in $Y$, matrix $WY$ is well-defined and is of the order $n\times k$.

Matrices $PY$ and $WY$ can be added only when their orders are the same.

However, $PY$ is of the order $p\times k$ and $WY$ is of the order $n\times k$.
Therefore, we must have $p=n$.

Thus, $k=3$ and $p=n$ are the restrictions on $n, k$ and $p$ so that $PY+WY$ will be defined.

The correct option is A.

**Question 22:**

Assume $X, Y, Z, W$ and $P$ are matrices of order $2\times n, 3\times k, 2\times p, n\times 3$ and $p\times k$ respectively. If $n = p$, then the order of the matrix $7X - 5Z$ is:

(A) $p\times 2$ (B) $2\times n$

(C) $n\times 3$ (D) $p\times n$

**Solution:**

Matrix $X$ is of the order $2\times n$.

Therefore, matrix $7X$ is also of the same order.

Matrix $Z$ is of the order $2\times p$, i.e., $2\times n$ [Since $n = p$]

Therefore, matrix $5Z$ is also of the same order.

Now, both the matrices $7X$ and 5Z are of the order $2\times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of the order $2\times n$.

The correct option is B.

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# EXERCISE 3.3

**Question 1:**

Find the transpose of each of the following matrices:

(i) $\begin{pmatrix} 5 \\ \frac{1}{2} \\ -1 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{pmatrix}$

**Solution:**

(i) Let $A = \begin{pmatrix} 5 \\ \frac{1}{2} \\ -1 \end{pmatrix}$

Then $A^T = \begin{pmatrix} 5 & \frac{1}{2} & -1 \end{pmatrix}$

(ii) Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

Then $A^T = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

(iii) Let $A = \begin{pmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{pmatrix}$

Then $A^T = \begin{pmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{pmatrix}$

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**Question 2:**

If $A=\begin{pmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{pmatrix}$, then verify that

(i) $(A+B)'=A'+B'$

(ii) $(A-B)'=A'-B'$

**Solution:**

It is given that $A=\begin{pmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{pmatrix}$

Hence, we have $A'=\begin{pmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{pmatrix}$ and $B'=\begin{pmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{pmatrix}$

(i) $$(A+B)=\begin{pmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{pmatrix}+\begin{pmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{pmatrix}=\begin{pmatrix}-5 & 3 & -2\\ 6 & 9 & 9\\ -1 & 4 & 2\end{pmatrix}$$

Hence,

$$(A+B)'=\begin{pmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{pmatrix}$$

Now,

$$A'+B'=\begin{pmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{pmatrix}+\begin{pmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{pmatrix}$$

Thus, $(A+B)'=A'+B'$.

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(ii) $$(A-B)=\begin{pmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{pmatrix}-\begin{pmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{pmatrix}=\begin{pmatrix}3 & 1 & 8\\ 4 & 5 & 9\\ -3 & -2 & 0\end{pmatrix}$$

Hence,

$$(A-B)'=\begin{pmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{pmatrix}$$

Now,

$$A'-B'=\begin{pmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{pmatrix}-\begin{pmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{pmatrix}$$
$$=\begin{pmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{pmatrix}$$

Thus, $(A-B)'=A'-B'$.

**Question 3:**

If $A'=\begin{pmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{pmatrix}$, then verify that

(i) $(A+B)'=A'+B'$

(ii) $(A-B)'=A'-B'$

**Solution:**

It is known that $A=(A')'$

Hence,

$$A=\begin{pmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{pmatrix} \text{ and } B'=\begin{pmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{pmatrix}$$

(i) $$A+B=\begin{pmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{pmatrix}+\begin{pmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{pmatrix}=\begin{pmatrix}2 & 1 & 1\\ 5 & 4 & 4\end{pmatrix}$$

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Therefore,

$$(A+B)' = \begin{pmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{pmatrix}$$

Now,

$$A'+B' = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{pmatrix}$$

Hence, $(A+B)' = A' + B'$.

(ii) $$A - B = \begin{pmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{pmatrix}$$

Therefore,

$$(A-B)' = \begin{pmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{pmatrix}$$

Now,

$$A' - B' = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{pmatrix}$$

Hence, $(A-B)' = A' - B'$.

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## Question 4:

If $A' = \begin{pmatrix} -2 & 3 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix}$, then find $(A+2B)'$.

## Solution:

It is known that $A = (A')'$.

Therefore,

$$A = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$$

Now,

$$A+2B=\begin{pmatrix}-2 & 1\\ 3 & 2\end{pmatrix}+2\begin{pmatrix}-1 & 0\\ 1 & 2\end{pmatrix}$$
$$=\begin{pmatrix}-2 & 1\\ 3 & 2\end{pmatrix}+\begin{pmatrix}-2 & 0\\ 2 & 4\end{pmatrix}$$
$$=\begin{pmatrix}-4 & 1\\ 5 & 6\end{pmatrix}$$

## Question 5:

For the matrices $A$ and $B$, verify that $(AB)'=B'A'$ where

(i) $A=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}, B=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$

(ii) $A=\begin{bmatrix}0\\ 1\\ 2\end{bmatrix}, B=\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

## Solution:

(i) It is given that $A=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$

Hence,

$$AB=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$$
$$=\begin{bmatrix}-1 & 2 & 1\\ 4 & -8 & -4\\ -3 & 6 & 3\end{bmatrix}$$

Therefore,

$$(AB)'=\begin{bmatrix}-1 & 4 & -3\\ 2 & -8 & 6\\ 1 & -4 & 3\end{bmatrix}$$

Now,

$A'=\begin{bmatrix}1 & -4 & 3\end{bmatrix}$ and $B'=\begin{bmatrix}-1\\ 2\\ 1\end{bmatrix}$

Hence,

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$$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Thus, $(AB)' = B'A'$

(ii) It is given that $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Hence,

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

Therefore,

$$(AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Now,

$A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$ and $B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

Therefore,

$$B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

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Thus, $(AB)' = B'A'$.

**Question 6:**

If

(i) $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, then verify $A'A = I$

(ii) $A = \begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix}$, then verify $A'A = I$

**Solution:**

(i) It is given that $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

Therefore,

$$A' = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Now,

$$A'A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha\cos\alpha + (-\sin\alpha)(-\sin\alpha) & \sin\alpha\cos\alpha + (-\sin\alpha)\cos\alpha \\ \sin\alpha\cos\alpha + \cos\alpha(-\sin\alpha) & \sin\alpha\sin\alpha + \cos\alpha\cos\alpha \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I$$

Thus, $A'A = I$

(ii) It is given that $A = \begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix}$

Therefore,

$$A' = \begin{pmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix}$$

Now,

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$$A'A=\begin{pmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{pmatrix}\begin{pmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{pmatrix}$$
$$=\begin{pmatrix}\sin\alpha\sin\alpha+(-\cos\alpha)(-\cos\alpha) & \sin\alpha\cos\alpha+(-\cos\alpha)\sin\alpha\\ \sin\alpha\cos\alpha+\sin\alpha(-\cos\alpha) & \sin\alpha\sin\alpha+\cos\alpha\cos\alpha\end{pmatrix}$$
$$=\begin{pmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha\end{pmatrix}$$
$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$
$$=I$$

Thus, $A'A=I$

## Question 7:

(i) Show that the matrix $A=\begin{pmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{pmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A=\begin{pmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}$ is a skew symmetric matrix.

## Solution:

(i) $A=\begin{pmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{pmatrix}$

Now,

$$A'=\begin{pmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{pmatrix}$$
$$=A$$

Hence, A is a symmetric matrix.

(ii) $A=\begin{pmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}$

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$$A' = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$
$$= -\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$
$$= -A$$

Hence, A is a skew symmetric matrix.

**Question 8:**

For the matrix $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix.

(ii) $(A - A')$ is a skew symmetric matrix.

**Solution:**

It is given that $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$

Hence, $A' = \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix}$

(i) $(A + A') = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix}$

Therefore,

$$(A + A')' = \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix}$$
$$= (A + A')$$

Thus, $(A + A')$ is a symmetric matrix.

(ii) $(A - A') = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix} - \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Therefore,

$$(A - A')' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
$$= -\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$
$$= -(A - A')$$

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Thus, $(A-A')$ is a skew symmetric matrix.

**Question 9:**

Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when $A=\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$.

**Solution:**

It is given that $A=\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$

Hence,

$$A'=\begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}$$

Now,

$$(A+A')=\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}+\begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}$$
$$=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Therefore,

$$\frac{1}{2}(A+A')=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now,

$$(A-A')=\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}-\begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}$$
$$=\begin{pmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{pmatrix}$$

Thus,

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$$\frac{1}{2}(A-A')=\begin{pmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{pmatrix}$$
$$=\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$$

## Question 10:

Express the following as the sum of a symmetric and skew symmetric matrix:

(i) $\begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$

(ii) $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

(iv) $\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}$

## Solution:

(i) Let $A=\begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$

Hence,

$$A'=\begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$$

Now,

$$(A+A')=\begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}+\begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$$
$$=\begin{pmatrix} 6 & 6 \\ 6 & -2 \end{pmatrix}$$

Let

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$$P=\frac{1}{2}(A+A')$$
$$=\frac{1}{2}\begin{pmatrix}6 & 6\\ 6 & -2\end{pmatrix}$$
$$=\begin{pmatrix}3 & 3\\ 3 & -1\end{pmatrix}$$

Now,

$$P'=\begin{pmatrix}3 & 3\\ 3 & -1\end{pmatrix}$$
$$=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

Now,

$$(A-A')=\begin{pmatrix}3 & 5\\ 1 & -1\end{pmatrix}-\begin{pmatrix}3 & 1\\ 5 & -1\end{pmatrix}$$
$$=\begin{pmatrix}0 & 4\\ -4 & 0\end{pmatrix}$$

Let

$$Q=\frac{1}{2}(A-A')$$
$$=\frac{1}{2}\begin{pmatrix}0 & 4\\ -4 & 0\end{pmatrix}$$
$$=\begin{pmatrix}0 & 2\\ -2 & 0\end{pmatrix}$$

Now,

$$Q'=\begin{pmatrix}0 & -2\\ 2 & 0\end{pmatrix}$$
$$=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing $A$ as the sum of $P$ and $Q$:

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$$P+Q=\begin{pmatrix}3 & 3\\ 3 & -1\end{pmatrix}+\begin{pmatrix}0 & 2\\ -2 & 0\end{pmatrix}$$

$$=\begin{pmatrix}3 & 5\\ 1 & -1\end{pmatrix}$$

$$=A$$

(ii) Let $A=\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$

Hence,

$$A'=\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$$

Now,

$$(A+A')=\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}+\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}=\begin{pmatrix}12 & -4 & 4\\ -4 & 6 & -2\\ 4 & -2 & 6\end{pmatrix}$$

Let

$$P=\frac{1}{2}(A+A')$$

$$=\frac{1}{2}\begin{pmatrix}12 & -4 & 4\\ -4 & 6 & -2\\ 4 & -2 & 6\end{pmatrix}$$

$$=\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$$

Now,

$$P'=\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$$

$$=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

Now,

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$$(A - A') = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} - \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Let

$$Q = \frac{1}{2}(A - A')$$
$$= \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now,

$$Q' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$= -Q$$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Representing $A$ as the sum of $P$ and $Q$:

$$P + Q = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$
$$= A$$

(iii) Let $A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

Hence,

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$$A' = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$$

Now,

$$(A + A') = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix}$$

Let

$$P = \frac{1}{2}(A + A')$$

$$= \frac{1}{2}\begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{pmatrix}$$

Now,

$$P' = \begin{pmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{pmatrix}$$

$$= P$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now,

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$$(A-A')=\begin{pmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{pmatrix}-\begin{pmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{pmatrix}$$
$$=\begin{pmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{pmatrix}$$

Let

$$Q=\frac{1}{2}(A-A')$$
$$=\frac{1}{2}\begin{pmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{pmatrix}$$
$$=\begin{pmatrix}0 & \frac{5}{2} & \frac{3}{2}\\ \frac{-5}{2} & 0 & 3\\ \frac{-3}{2} & -3 & 0\end{pmatrix}$$

Now,

$$Q'=\begin{pmatrix}0 & \frac{-5}{2} & \frac{-3}{2}\\ \frac{5}{2} & 0 & -3\\ \frac{3}{2} & 3 & 0\end{pmatrix}$$
$$=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing $A$ as the sum of $P$ and $Q$:

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$$P+Q=\begin{pmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{pmatrix}+\begin{pmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$$

$$=A$$

(iv) Let $A=\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}$

Hence,

$$A'=\begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix}$$

Now,

$$(A+A')=\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}+\begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix}$$

$$=\begin{pmatrix} 2 & 4 \\ 4 & 4 \end{pmatrix}$$

Let

$$P=\frac{1}{2}(A+A')$$

$$=\frac{1}{2}\begin{pmatrix} 2 & 4 \\ 4 & 4 \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$$

Now,

$$P'=\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$$

$$=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

Now,

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$$\begin{aligned}(A-A') &= \begin{pmatrix}1 & 5\\ -1 & 2\end{pmatrix}-\begin{pmatrix}1 & -1\\ 5 & 2\end{pmatrix}\\ &= \begin{pmatrix}0 & 6\\ -6 & 0\end{pmatrix}\end{aligned}$$

Let

$$\begin{aligned}Q &= \frac{1}{2}(A-A')\\ &= \frac{1}{2}\begin{pmatrix}0 & 6\\ -6 & 0\end{pmatrix}\\ &= \begin{pmatrix}0 & 3\\ -3 & 0\end{pmatrix}\end{aligned}$$

Now,

$$\begin{aligned}Q' &= \begin{pmatrix}0 & -3\\ 3 & 0\end{pmatrix}\\ &= -Q\end{aligned}$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing $A$ as the sum of $P$ and $Q$:

$$\begin{aligned}P+Q &= \begin{pmatrix}1 & 2\\ 2 & 2\end{pmatrix}+\begin{pmatrix}0 & 3\\ -3 & 0\end{pmatrix}\\ &= \begin{pmatrix}1 & 5\\ -1 & 2\end{pmatrix}\\ &= A\end{aligned}$$

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## Question 11:

If $A, B$ are symmetric matrices of the same order, then $AB-BA$ is a

(A) Skew symmetric matrix (B) Symmetric matrix

(C) Zero matrix (D) Identity matrix

## Solution:

If $A$ and $B$ are symmetric matrices of the same order, then

$$A' = A \text{ and } B' = B \qquad \ldots(1)$$

Now consider,

$$(AB-BA)' = (AB)' - (BA)' \qquad \left[\because (A-B)' = A'-B'\right]$$
$$= B'A' - A'B \qquad \left[\because (AB)' = B'A'\right]$$
$$= BA - AB \qquad \left[from\,(1)\right]$$
$$= -(AB-BA)$$

Therefore,

$$(AB-BA)' = -(AB-BA)$$

Thus, $AB-BA$ is a skew symmetric matrix.
The Correct option is A.

**Question 12:**

If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then $A+A' = I$, if the value of $\alpha$ is:

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\pi$ (D) $\frac{3\pi}{2}$

**Solution:**

It is given that $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

Hence,

$$A' = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$

Now,

$$A+A' = I$$

Therefore,

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} + \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

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$$\Rightarrow 2\cos\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{2}$$

$$\Rightarrow \alpha = \cos^{-1}\frac{1}{2}$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

Thus, the correct option is B.

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# EXERCISE 3.4

**Question 1:**

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$, if exists.

**Solution:**

Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix} A \qquad \left(R_2 \to \frac{1}{5} R_2\right)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix} A \qquad (R_1 \to R_1 + R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix}$$

**Question 2:**

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, if exists.

**Solution:**

Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

We know that $A = IA$

Therefore,

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$$\Rightarrow \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} A \qquad (R_2 \to R_2 - R_1)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

## Question 3:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$, if exists.

**Solution:**

Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - 3R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix}$$

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## Question 4:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$, if exists.

**Solution:**

Let $A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to \frac{1}{2} R_1\right)$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & \frac{-1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{-5}{2} & 1 \end{pmatrix} A \qquad (R_2 \to R_2 - 5R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{-1}{2} \end{pmatrix} = \begin{pmatrix} -7 & 3 \\ \frac{-5}{2} & 1 \end{pmatrix} A \qquad (R_1 \to R_1 + 3R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & 3 \\ 5 & -2 \end{pmatrix} A \qquad (R_2 \to -2R_1)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} -7 & 3 \\ 5 & -2 \end{pmatrix}$$

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## Question 5:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$, if exists.

## Solution:

Let $A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to \frac{1}{2} R_1\right)$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{-7}{2} & 1 \end{pmatrix} A \qquad (R_2 \to R_2 - 7R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ \frac{-7}{2} & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix} A \qquad (R_2 \to 2R_1)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$$

## Question 6:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$, if exists.

## Solution:

Let $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$

We know that $A = IA$

Therefore,

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$$\Rightarrow \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to \frac{1}{2} R_1\right)$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{-1}{2} & 1 \end{pmatrix} A \qquad (R_2 \to R_2 - R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ \frac{-1}{2} & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - 5R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} A \qquad (R_2 \to 2R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

**Question 7:**

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$, if exists.

**Solution:**

Let $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

We know that $A = IA$

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Therefore,

$$\Rightarrow \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = A \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = A \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \qquad (C_1 \to C_1 - 2C_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = A \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} \qquad (C_2 \to C_2 - C_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \qquad (C_1 \to C_1 - C_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$$

## Question 8:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$, if exists.

## Solution:

Let $A = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -3 & 4 \end{pmatrix} A \qquad (R_2 \to R_2 - 3R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix}$$

## Question 9:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$, if exists.

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**Solution:**

Let $A=\begin{pmatrix}3 & 10\\ 2 & 7\end{pmatrix}$

We know that $A=IA$

Therefore,

$$\Rightarrow \begin{pmatrix}3 & 10\\ 2 & 7\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}A$$

$$\Rightarrow \begin{pmatrix}1 & 3\\ 2 & 7\end{pmatrix}=\begin{pmatrix}1 & -1\\ 0 & 1\end{pmatrix}A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & -1\\ -2 & 3\end{pmatrix}A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}7 & -10\\ -2 & 3\end{pmatrix}A \qquad (R_1 \to R_1 - 3R_2)$$

$$\Rightarrow A^{-1}=\begin{pmatrix}7 & -10\\ -2 & 3\end{pmatrix}$$

**Question 10:**

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix}3 & -1\\ -4 & 2\end{pmatrix}$, if exists.

**Solution:**

Let $A=\begin{pmatrix}3 & -1\\ -4 & 2\end{pmatrix}$

We know that $A=IA$

Therefore,

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$$\Rightarrow \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} = A\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = A\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \qquad (C_1 \to C_1 + 2C_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = A\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \qquad (C_2 \to C_2 + C_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix} \qquad \left(C_2 \to \frac{1}{2}C_2\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix}$$

## Question 11:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$, if exists.

## Solution:

Let $A = \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix} = A\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} = A\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \qquad (C_2 \to C_2 + 3C_1)$$

$$\Rightarrow \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = A\begin{pmatrix} -2 & 3 \\ -1 & 1 \end{pmatrix} \qquad (C_1 \to C_1 - C_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A\begin{pmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{pmatrix} \qquad \left(C_1 \to \frac{1}{2}C_1\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{pmatrix}$$

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## Question 12:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$, if exists.

## Solution:

Let $A = \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{-1}{2} \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to \frac{1}{6} R_1\right)$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{-1}{2} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{pmatrix} A \qquad \left(R_2 \to R_2 + 2R_1\right)$$

In the above equation, we can see all the zeros in the second row of the matrix on the L.H.S.

Thus, $A^{-1}$ does not exist.

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## Question 13:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$, if exists.

## Solution:

Let $A = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} A \qquad (R_1 \to R_1 + R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} A \qquad (R_2 \to R_2 + R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} A \qquad (R_1 \to R_1 + R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$$

## Question 14:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$, if exists.

**Solution:**

Let $A = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 0 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & \frac{-1}{2} \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to R_1 - \frac{1}{2} R_2\right)$$

In the above equation, we can see all the zeros in the first row of the matrix on the L.H.S.

Thus, $A^{-1}$ does not exist.

## Question 15:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{pmatrix}$, if exists.

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**Solution:**

Let $A=\begin{pmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{pmatrix}$

We know that $A=IA$

Therefore,

$$\Rightarrow\begin{pmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A$$

$$\Rightarrow\begin{pmatrix}2 & -3 & 3\\ 0 & 5 & 0\\ 3 & -2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ -1 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A \qquad (R_2\to R_2-R_1)$$

$$\Rightarrow\begin{pmatrix}2 & -3 & 3\\ 0 & 1 & 0\\ 3 & -2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ \frac{-1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1\end{pmatrix}A \qquad \left(R_2\to\frac{1}{5}R_2\right)$$

$$\Rightarrow\begin{pmatrix}-1 & -1 & 1\\ 0 & 1 & 0\\ 3 & -2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & -1\\ \frac{-1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1\end{pmatrix}A \qquad (R_1\to R_1-R_3)$$

$$\Rightarrow\begin{pmatrix}-1 & 0 & 1\\ 0 & 1 & 0\\ 3 & 0 & 2\end{pmatrix}=\begin{pmatrix}\frac{4}{5} & \frac{1}{5} & -1\\ \frac{-1}{5} & \frac{1}{5} & 0\\ \frac{-2}{5} & \frac{2}{5} & 1\end{pmatrix}A \qquad (R_1\to R_1+R_2 \text{ and } R_3\to R_3+2R_2)$$

$$\Rightarrow\begin{pmatrix}-1 & 0 & 1\\ 0 & 1 & 0\\ 0 & 0 & 5\end{pmatrix}=\begin{pmatrix}\frac{4}{5} & \frac{1}{5} & -1\\ \frac{-1}{5} & \frac{1}{5} & 0\\ 2 & 1 & -2\end{pmatrix}A \qquad (R_3\to R_3+3R_1)$$

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$$\Rightarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix} A \qquad \left( R_3 \to \frac{1}{5} R_3 \right)$$

$$\Rightarrow \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & 0 & \frac{-3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix} A \qquad \left( R_1 \to R_1 - R_3 \right)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix} A \qquad \left( R_1 \to (-1) R_1 \right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix}$$

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## Question 16:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$, if exists.

## Solution:

Let $A = \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} A \qquad (R_2 \to R_2 + 3R_1 \text{ and } R_3 \to R_3 - 2R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{pmatrix} A \qquad (R_1 \to R_1 + 3R_3 \text{ and } R_2 \to R_2 + 8R_3)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{pmatrix} A \qquad (R_3 \to R_3 + R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{pmatrix} A \qquad \left(R_3 \to \frac{1}{25} R_3\right)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{pmatrix} A \qquad (R_1 \to R_1 - 10R_3 \text{ and } R_2 \to R_2 - 21R_3)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{pmatrix}$$

## Question 17:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$, if exists.

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**Solution:**

Let $A=\begin{pmatrix}2&0&-1\\5&1&0\\0&1&3\end{pmatrix}$

We know that $A=IA$

Therefore,

$$\Rightarrow\begin{pmatrix}2&0&-1\\5&1&0\\0&1&3\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}A$$

$$\Rightarrow\begin{pmatrix}1&0&\frac{-1}{2}\\5&1&0\\0&1&3\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&0&0\\0&1&0\\0&0&1\end{pmatrix}A \qquad \left(R_1\to\frac{1}{2}R_1\right)$$

$$\Rightarrow\begin{pmatrix}1&0&\frac{-1}{2}\\0&1&\frac{5}{2}\\0&1&3\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&0&0\\\frac{-5}{2}&1&0\\0&0&1\end{pmatrix}A \qquad \left(R_2\to R_2-5R_1\right)$$

$$\Rightarrow\begin{pmatrix}1&0&\frac{-1}{2}\\0&1&\frac{5}{2}\\0&0&\frac{1}{2}\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&0&0\\\frac{-5}{2}&1&0\\\frac{5}{2}&-1&1\end{pmatrix}A \qquad \left(R_3\to R_3-R_2\right)$$

$$\Rightarrow\begin{pmatrix}1&0&\frac{-1}{2}\\0&1&\frac{5}{2}\\0&0&1\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&0&0\\\frac{-5}{2}&1&0\\5&-2&2\end{pmatrix}A \qquad \left(R_3\to 2R_3\right)$$

$$\Rightarrow\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}3&-1&1\\-15&6&-5\\5&-2&2\end{pmatrix}A \qquad \left(R_1\to R_1+\frac{1}{2}R_3\ and\ R_2\to R_2-\frac{5}{2}R_3\right)$$

$$\Rightarrow A^{-1}=\begin{pmatrix}3&-1&1\\-15&6&-5\\5&-2&2\end{pmatrix}$$

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**Question 18:**

Matrices $A$ and $B$ will be the inverse of each other only if:

(A) $AB = BA$

(B) $AB = BA = 0$

(C) $AB = 0, BA = I$

(D) $AB = BA = I$

**Solution:**

We know that if $A$ is a square matrix of order $m$, and
if there exists another square matrix $B$ of the same order $m$,
such that $AB = BA = I$, then $B$ is said to be the inverse of $A$.

In this case, it is clear that $A$ is the inverse of $B$.

Thus, matrices $A$ and $B$ will be inverses of each other only if $AB = BA = I$.

The correct option is D.

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# MISCELLANEOUS EXERCISE

**Question 1:**

Let $A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$, show that $(aI+bA)^n=a^nI+na^{n-1}bA$, where $I$ is the identity matrix of order 2 and $n\in N$.

**Solution:**

It is given that $A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$

We shall prove the result by using the principle of mathematical induction.

For $n=1$, we have:

$$P(1):(aI+bA)=aI+ba^0A=aI+bA$$

Therefore, the result is true for $n=1$.

Let the result be true for $n=k$

That is, $P(k):(aI+bA)^k=a^kI+ka^{k-1}bA$

Now, we have to prove that the result is true for $n=k+1$.

Consider,

$$\begin{aligned}(aI+bA)^{k+1}&=(aI+bA)^k(aI+bA)\\&=(a^kI+ka^{k-1}bA)(aI+bA)\\&=a^{k+1}I+ka^kbAI+a^kbIA+ka^{k-1}b^2A^2\\&=a^{k+1}I+(k+1)a^kbA+ka^{k-1}b^2A^2 \qquad \ldots(1)\end{aligned}$$

Now,

$$A^2=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}=0$$

From $(1)$, we have

$$\begin{aligned}(aI+bA)^{k+1}&=a^{k+1}I+(k+1)a^kbA+0\\&=a^{k+1}I+(k+1)a^kbA\end{aligned}$$

Therefore, the result is true for $n=k+1$.

Thus, by the principle of mathematical induction, we have:

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$$(aI+bA)^n = a^n I + na^{n-1}bA$$ where $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, n \in N$

**Question 2:**

If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, prove that $A^n = \begin{pmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{pmatrix}, n \in N$.

**Solution:**

It is given that $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

We shall prove the result by using the principle of mathematical induction.
For $n = 1$, we have:

$$P(1): \begin{pmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{pmatrix} = \begin{pmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k): A^k = \begin{pmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{pmatrix}$$

Now, we have to prove that the result is true for $n = k+1$.

Since,

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$$A^{k+1} = A.A^k$$
$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{pmatrix}$$
$$= \begin{pmatrix} 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \end{pmatrix}$$
$$= \begin{pmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{pmatrix}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{pmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{pmatrix}, n \in N$$

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## Question 3:

If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$, prove that $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$, where $n$ is any positive integer.

## Solution:

It is given that $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): A^1 = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$$
$$= \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$$
$$= A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k): A^k = \begin{pmatrix} 1+2k & -4k \\ k & 1-2k \end{pmatrix}, n \in N$$

Now, we have to prove that the result is true for $n = k+1$.

Since,

$$\begin{aligned}
A^{k+1} &= A.A^k \\
&= \begin{pmatrix} 1+2k & -4k \\ k & 1-2k \end{pmatrix} \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \\
&= \begin{pmatrix} 3(1+2k)-4k & -4(1+2k)+4k \\ 3k+1-2k & -4k-1(1-2k) \end{pmatrix} \\
&= \begin{pmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{pmatrix} \\
&= \begin{pmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{pmatrix} \\
&= \begin{pmatrix} 1+2(k+1) & -4(k+1) \\ 1+k & 1-2(k+1) \end{pmatrix}
\end{aligned}$$

Therefore, the result is true for $n = k+1$.

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}; n \in N$$

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**Question 4:**

If $A$ and $B$ are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix.

**Solution:**

It is given that $A$ and $B$ are symmetric matrices.
Therefore, we have:

$$A' = A \text{ and } B' = B \qquad \ldots(1)$$

Now,

$$(AB-BA)' = (AB)' - (BA)' \qquad \left[(A-B)' = A' - B'\right]$$
$$= B'A' - A'B' \qquad \left[(AB)' = B'A'\right]$$
$$= BA - AB \qquad [\text{Using } (1)]$$
$$= -(AB-BA)$$

Hence,

$$(AB-BA)' = -(AB-BA)$$

Thus, $AB-BA$ is a skew symmetric matrix.

**Question 5:**

Show that the matrix $B'AB$ is symmetric or skew symmetric according as $A$ is symmetric or skew symmetric.

**Solution:**

We suppose that $A$ is a symmetric matrix, then

$$A' = A \qquad \ldots(1)$$

Consider,

$$(B'AB)' = \{B'(AB)\}'$$
$$= (AB)'(B')' \qquad \left[\because (AB)' = B'A'\right]$$
$$= B'A'(B) \qquad \left[\because (B')' = B\right]$$
$$= B'(A'B)$$
$$= B'(AB) \qquad [\text{Using } (1)]$$

Therefore,

$$(B'AB)' = B'AB$$

Thus, if $A$ is symmetric matrix, then $B'AB$ is a symmetric matrix.

Now, we suppose that $A$ is a skew symmetric matrix, then

$$A' = -A \qquad \ldots(2)$$

Consider,

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$$
\begin{aligned}
(B'AB)' &= \{B'(AB)\}' \\
&= (AB)'(B')' \\
&= (B'A')B \\
&= B'(-A)B && [\text{Using } (2)] \\
&= -B'AB
\end{aligned}
$$

Therefore,

$$(B'AB)' = -B'AB$$

Thus, if $A$ is a skew symmetric matrix, then $B'AB$ is a skew symmetric matrix.

Hence, if $A$ is symmetric or skew symmetric matrix, then $B'AB$ is symmetric or skew symmetric accordingly.

**Question 6:**

Find the values of $x, y, z$ if the matrix $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$ satisfy the equation $A'A = I$.

**Solution:**

It is given that $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$

Therefore,

$$A' = \begin{pmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{pmatrix}$$

Now, $A'A = I$

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Hence,

$$\Rightarrow \begin{pmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{pmatrix}\begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-xz+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

On comparing the corresponding elements, we have:

$$2x^2 = 1$$
$$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

$$6y^2 = 1$$
$$\Rightarrow y = \pm\frac{1}{\sqrt{6}}$$

$$3z^2 = 1$$
$$\Rightarrow z = \pm\frac{1}{\sqrt{3}}$$

Thus, $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$

**Question 7:**

For what values of $x : \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$ ?

**Solution:**

We have:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

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Hence,

$$\Rightarrow \begin{bmatrix}1+4+1 & 2+0+0 & 0+2+2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix}=0$$

$$\Rightarrow \left[6(0)+2(2)+4(x)\right]=0$$

$$\Rightarrow \left[4+4x\right]=0$$

$$\Rightarrow 4x=-4$$

$$\Rightarrow x=-1$$

Thus, the required value of $x=-1$.

## Question 8:

If $A=\begin{pmatrix}3 & 1\\-1 & 2\end{pmatrix}$, show that $A^2-5A+7I=0$

## Solution:

It is given that $A=\begin{pmatrix}3 & 1\\-1 & 2\end{pmatrix}$

Therefore,

$$A^2 = A.A$$

$$=\begin{pmatrix}3 & 1\\-1 & 2\end{pmatrix}\begin{pmatrix}3 & 1\\-1 & 2\end{pmatrix}$$

$$=\begin{pmatrix}3(3)+1(-1) & 3(1)+1(2)\\-1(3)+2(-1) & -1(1)+2(2)\end{pmatrix}$$

$$=\begin{pmatrix}8 & 5\\-5 & 3\end{pmatrix}$$

Now,

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$$\begin{aligned}
LHS &= A^2 - 5A + 7I \\
&= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - 5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} \\
&= \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} \\
&= 0 \\
&= RHS
\end{aligned}$$

Thus, $A^2 - 5A + 7I = 0$

## Question 9:

Find $x$, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

## Solution:

We have

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

Hence,

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \left[x(x-2) - 40 + 2x - 8\right] = 0$$

$$\Rightarrow \left[x^2 - 2x - 40 + 2x - 8\right] = 0$$

$$\Rightarrow \left[x^2 - 48\right] = 0$$

$$\Rightarrow x^2 - 48 = 0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm 4\sqrt{3}$$

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Thus, $x = \pm 4\sqrt{3}$.

## Question 10:

A manufacturer produces three products $x, y, z$ which he sells in two markets. Annual sales are indicated below:

| **Market** | **Products** | | |
|---|---|---|---|
| I | 10000 | 2000 | 18000 |
| II | 6000 | 20000 | 8000 |

(a) If unit sale prices of $x, y$ and $z$ are ₹2.50, ₹1.50 and ₹1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively. Find the gross profit.

## Solution:

(a) The unit sale prices of $x, y$ and $z$ are ₹2.50, ₹1.50 and ₹1.00 respectively.

Consequently, the total revenue in market I can be represented in the form of a matrix as:

$$\begin{aligned}\begin{bmatrix}10000 & 2000 & 18000\end{bmatrix}\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix} &= 10000\times 2.50 + 2000\times 1.50 + 18000\times 1.00\\ &= 25000 + 3000 + 18000\\ &= 46000\end{aligned}$$

The total revenue in market II can be represented in the form of a matrix as:

$$\begin{aligned}\begin{bmatrix}6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix} &= 6000\times 2.50 + 20000\times 1.50 + 8000\times 1.00\\ &= 15000 + 30000 + 8000\\ &= 53000\end{aligned}$$

Thus, the total revenue in market I is ₹46000 and the total revenue in market II is ₹ 53000.

(b) The unit costs of $x, y$ and $z$ are ₹2.00, ₹1.00 and 50 paise respectively.

Consequently, the total cost prices of all the products in market I can be represented in the form of a matrix as:

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$$\begin{bmatrix}10000 & 2000 & 18000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix} = 10000\times 2.00 + 2000\times 1.00 + 18000\times 0.50$$
$$= 20000 + 2000 + 9000$$
$$= 31000$$

Since the total revenue in market I is ₹ $46000$, the gross profit in this market in ₹ is

$$46000 - 31000 = 15000$$

The total cost prices of all the products in market II can be represented in the form of a matrix as:

$$\begin{bmatrix}6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix} = 6000\times 2.00 + 20000\times 1.00 + 8000\times 0.50$$
$$= 12000 + 20000 + 4000$$
$$= 36000$$

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Since the total revenue in market I is ₹$53000$, the gross profit in this market in ₹ is

$$53000 - 36000 = 17000$$

Thus, the gross profit in market I is ₹$15000$ and in market II is ₹$17000$.

**Question 11:**

Find the matrix $X$ so that $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

**Solution:**

It is given that $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

The matrix given on the R.H.S. of the equation is a $2\times 3$ matrix and the one given on the L.H.S. of the equation is a $2\times 3$ matrix.

Therefore, X has to be a $2\times 2$ matrix.

Now, let $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Therefore,

$$\Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$$a+4c=7 \quad 2a+5c=-8 \quad 3a+6c=-9a$$
$$b+4d=2 \quad 2b+5d=4 \quad 3b+6d=6$$

Now,

$$a+4c=-7$$
$$\Rightarrow a=-7-4c$$

Therefore,

$$2a+5c=-8$$
$$\Rightarrow 2(-7-4c)+5c=-8$$
$$\Rightarrow -14-8c+5c=-8$$
$$\Rightarrow -3c=6$$
$$\Rightarrow c=-2$$

Hence,

$$\Rightarrow a=-7-4(-2)$$
$$\Rightarrow a=-7+8$$
$$\Rightarrow a=1$$

Now,

$$b+4d=2$$
$$\Rightarrow b=2-4d$$

Therefore,

$$2b+5d=4$$
$$\Rightarrow 2(2-4d)+5d=4$$
$$\Rightarrow 4-8d+5d=4$$
$$\Rightarrow -3d=0$$
$$\Rightarrow d=0$$

Hence,

$$b=2-4d$$
$$\Rightarrow b=2$$

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Thus, $a=1, b=2, c=-2$ and $d=0$

Hence, the required matrix $X=\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$

**Question 12:**

If $A$ and $B$ are square matrices of the same order such that $AB=BA$, then prove by induction that $AB^n=B^nA$. Further, prove that $(AB)^n=A^nB^n$ for all $n\in N$.

**Solution:**

**Given:** $A$ and $B$ are square matrices of the same order such that $AB=BA$.

**To prove:** $P(n): AB^n=B^nA, n\in N$

For n = 1, we have:

$$P(1): AB=BA \qquad [\text{Given}]$$
$$\Rightarrow AB^1=B^1A$$

Therefore, the result is true for $n=1$.

Let the result be true for $n=k$.

$$P(k)=AB^k=B^kA \qquad \ldots(1)$$

Now, we prove that the result is true for $n=k+1$.

$$\begin{aligned}AB^{k+1} &= AB^k.B\\ &=(B^kA)B && [By\ (1)]\\ &=B^k(AB) && [\text{Associative law}]\\ &=B^k(BA) && [AB=BA\ (\text{Given})]\\ &=(B^kB)A && [\text{Associative law}]\\ &=B^{k+1}A\end{aligned}$$

Therefore, the result is true for $n=k+1$.

Thus, by the principle of mathematical induction, we have $AB^n=B^nA, n\in N$

Now, we have to prove that $(AB)^n=A^nB^n$ for all $n\in N$

For $n=1$, we have:

$$(AB)^1=A^1B^1=AB$$

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Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$(AB)^k = A^k B^k \qquad \ldots(2)$$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned} AB^{k+1} &= (AB)^k . (AB) \\ &= (A^k B^k).(AB) && [\text{By } (2)] \\ &= A^k (B^k A) B && [\text{Associative law}] \\ &= A^k (AB^k) B && [AB^n = B^n A, n \in N] \\ &= (A^k A).(B^k B) && [\text{Associative law}] \\ &= A^{k+1} B^{k+1} \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $(AB)^n = A^n B^n, n \in N$

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**Question 13:**

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then,

(A) $1 + \alpha^2 + \beta\gamma = 0$

(B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$

(D) $1 + \alpha^2 - \beta\gamma = 0$

**Solution:**

It is given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

Therefore,

$$A^2 = A.A$$
$$= \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \alpha^2+\beta\gamma & \alpha\beta-\alpha\beta \\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2 \end{pmatrix}$$
$$= \begin{pmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{pmatrix}$$

Now, $A^2 = I$

Hence,

$$\begin{pmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

On comparing the corresponding elements, we have:

$$\alpha^2+\beta\gamma = 1$$
$$\Rightarrow \alpha^2+\beta\gamma-1=0$$
$$\Rightarrow 1-\alpha^2-\beta\gamma=0$$

Thus, the correct option is C.

## Question 14:

If the matrix $A$ is both symmetric and skew symmetric, then

(A) $A$ is a diagonal matrix
(B) $A$ is a zero matrix
(C) $A$ is a square matrix
(D) None of these

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## Solution:

If the matrix $A$ is both symmetric and skew symmetric, then

$$A' = A \text{ and } A' = -A$$

Hence,

$$\Rightarrow A = -A$$
$$\Rightarrow A + A = 0$$
$$\Rightarrow 2A = 0$$
$$\Rightarrow A = 0$$

Therefore, $A$ is a zero matrix.

Thus, the correct option is B.

## Question 15:

If $A$ is a square matrix such that $A^2 = A$, then $(I+A)^3 - 7A$ is equal to

(A) $A$ (B) $I - A$ (C) $I$ (D) $3A$

## Solution:

It is given that $A$ is a square matrix such that $A^2 = A$.

Now,

$$\begin{aligned}(I+A)^3 - 7A &= I^3 + A^3 + 3I^2A + 3A^2I - 7A \\ &= I + A^2.A + 3A + 3A^2 - 7A \\ &= I + A.A + 3A + 3A - 7A \qquad [\because A^2 = A] \\ &= I + A^2 - A \\ &= I + A - A \qquad [\because A^2 = A] \\ &= I\end{aligned}$$

Hence,

$$(I+A)^3 - 7A = I$$

Thus, the correct option is C.

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