

Mathematical Reasoning

Short Answer Type Questions

Q. 1 Which of the following sentences are statements? Justify

- (i) A triangle has three sides.
- (ii) 0 is a complex number.
- (iii) Sky is red.
- (iv) Every set is an infinite set.
- (v) $15 + 8 > 23$.
- (vi) $y + 9 = 7$
- (vii) Where is your bag?
- (viii) Every square is a rectangle.
- (ix) Sum of opposite angles of a cyclic quadrilateral is 180° .
- (x) $\sin^2 x + \cos^2 x = 0$

Sol. As we know, a statement is a sentence which is either true or false but not both simultaneously.

- (i) It is true statement.
- (ii) It is true statement.
- (iii) It is false statement.
- (iv) It is false statement.
- (v) It is false statement.
- (vi) $y + 9 = 7$
It is not considered as a statement, since the value of y is not given.
- (vii) It is a question, so it is not a statement.
- (viii) It is a true statement.
- (ix) It is true statement.
- (x) It is false statement.

Q. 2 Find the component statements of the following compound statements.

- (i) Number 7 is prime and odd.
- (ii) Chennai is in India and is the capital of Tamil Nadu.
- (iii) The number 100 is divisible by 3, 11 and 5.
- (iv) Chandigarh is the capital of Haryana and UP.
- (v) $\sqrt{7}$ is a rational number or an irrational number.
- (vi) 0 is less than every positive integer and every negative integer.
- (vii) Plants use sunlight, water and carbon dioxide for photosynthesis.
- (viii) Two lines in a plane either intersect at one point or they are parallel.
- (ix) A rectangle is a quadrilateral or a 5 sided polygon.

Sol. (i) p : Number 7 is prime.

q : Number 7 is odd.

(ii) P : Chennai is in India.

q : Chennai is capital of Tamil Nadu.

(iii) p : 100 is divisible by 3.

q : 100 is divisible by 11.

r : 100 is divisible by 5.

(iv) p : Chandigarh is capital of Haryana.

q : Chandigarh is capital of UP.

(v) p : $\sqrt{7}$ is a rational number.

q : $\sqrt{7}$ is an irrational number.

(vi) p : 0 is less than every positive integer.

q : 0 is less than every negative integer.

(vii) p : Plants use sunlight for photosynthesis.

q : Plants use water for photosynthesis.

r : Plants use carbon dioxide for photosynthesis.

(viii) p : Two lines in a plane intersect at one point.

q : Two lines in a plane are parallel.

(ix) p : A rectangle, is a quadrilateral.

q : A rectangle is a 5-sided polygon.

Q. 3 Write the component statements of the following compound statements and check whether the compound statement is true or false.

- (i) 57 is divisible by 2 or 3.
- (ii) 24 is a multiple of 4 and 6.
- (iii) All living things have two eyes and two legs.
- (iv) 2 is an even number and a prime number.

Sol. (i) Given compound statement is of the form ' $p \vee q$ '. Since, the statement ' $p \vee q$ ' has the truth value T whenever either p or q or both have the truth value T .

So, it is true statement.

Its component statements are

p : 57 is divisible by 2. [false]

q : 57 is divisible by 3. [true]

- (ii) Given compound statement is of the form ' $p \wedge q$ '. Since, the statement ' $p \wedge q$ ' have the truth value T whenever both p and q have the truth value T .

So, it is a true statement.

Its component statements are

p : 24 is multiple of 4 [true]

q : 24 is multiple of 6. [true]

- (iii) It is a false statement. Since ' $p \wedge q$ ' has truth value F whenever either p or q or both have the truth value F .

Its component statements are

p : All living things have two eyes. [false]

q : All living things have two legs. [false]

- (iv) It is a true statement.

Its component statements are

p : 2 is an even number. [true]

q : 2 is a prime number. [true]

Q. 4 Write the negative on the following simple statements.

- (i) The number 17 is prime.
- (ii) $2 + 7 = 6$.
- (iii) Violets are blue.
- (iv) $\sqrt{5}$ is a rational number.
- (v) 2 is not a prime number.
- (vi) Every real number is an irrational number.
- (vii) Cow has four legs.
- (viii) A leap year has 366 days.
- (ix) All similar triangles are congruent.
- (x) Area of a circle is same as the perimeter of the circle.

Sol. (i) The number 17 is not prime.

(ii) $2 + 7 \neq 6$.

(iii) Violets are not blue.

(iv) $\sqrt{5}$ is not a rational number.

(v) 2 is a prime number.

(vi) Every real number is not an irrational number.

(vii) Cow has not four legs.

(viii) A leap year has not 366 days.

(ix) There exist similar triangles which are not congruent.

(x) Area of a circle is not same as the perimeter of the circle.

Q. 5 Translate the following statements into symbolic form

- (i) Rahul passed in Hindi and English.
- (ii) x and y are even integers.
- (iii) 2, 3 and 6 are factors of 12.
- (iv) Either x or $x + 1$ is an odd integer.
- (v) A number is either divisible by 2 or 3.
- (vi) Either $x = 2$ or $x = 3$ is a root of $3x^2 - x - 10 = 0$.
- (vii) Students can take Hindi or English as an optional paper.

Sol. (i) p : Rahul passed in Hindi.

q : Rahul passed in English.

$p \wedge q$: Rahul passed in Hindi and English.

(ii) p : x is even integers.

q : y is even integers.

$p \cap q$: x and y are even integers.

(iii) p : 2 is factor of 12.

q : 3 is factor of 12.

r : 6 is factor of 12.

$p \wedge q \wedge r$: 2, 3 and 6 are factor of 12.

(iv) p : x is an odd integer.

q : $(x + 1)$ is an odd integer.

$p \vee q$: Either x or $(x + 1)$ is an odd integer.

(v) p : A number is divisible by 2.

q : A number is divisible by 3.

$p \vee q$: A number is either divisible by 2 or 3.

(vi) p : $x = 2$ is a root of $3x^2 - x - 10 = 0$.

q : $x = 3$ is a root of $3x^2 - x - 10 = 0$.

$p \vee q$: Either $x = 2$ or $x = 3$ is a root of $3x^2 - x - 10 = 0$.

(vii) p : Students can take Hindi as an optional paper.

q : Students can take English as an optional subject.

$p \vee q$: Students can take Hindi or English as an optional paper.

Q. 6 Write down the negation of following compound statements.

- (i) All rational numbers are real and complex.
- (ii) All real numbers are rationals or irrationals.
- (iii) $x = 2$ and $x = 3$ are roots of the quadratic equation $x^2 - 5x + 6 = 0$.
- (iv) A triangle has either 3-sides or 4-sides.
- (v) 35 is a prime number or a composite number.
- (vi) All prime integers are either even or odd.
- (vii) $|x|$ is equal to either x or $-x$.
- (viii) 6 is divisible by 2 and 3.

Thinking Process

Use (i) $\sim(p \wedge q) = \sim p \vee \sim q$

(ii) $\sim(p \vee q) = \sim p \wedge \sim q$

Sol. (i) Let p : All rational numbers are real.

q : All rational numbers are complex.

$\sim p$: All rational number are not real.

$\sim q$: All rational numbers are not complex.

$\sim(p \wedge q)$: All rational numbers are not real or not complex. [$\because \sim(p \wedge q) = \sim p \vee \sim q$]

(ii) Let p : All real numbers are rationals.

q : All real numbers are irrational.

Then, the negation of the above statement is given by

$\sim(p \vee q)$: All real numbers are not rational and all real numbers are not irrational.

[$\because \sim(p \vee q) = \sim p \wedge \sim q$]

(iii) Let p : $x = 2$ is root of quadratic equation $x^2 - 5x + 6 = 0$.

q : $x = 3$ is root of quadratic equation $x^2 - 5x + 6 = 0$.

Then, the negation of conjunction of above statement is given by

$\sim(p \wedge q)$: $x = 2$ is not a root of quadratic equation $x^2 - 5x + 6 = 0$ or $x = 3$ is not a root of the quadratic equation $x^2 - 5x + 6 = 0$.

(iv) Let p : A triangle has 3-sides.

q : A triangle has 4-sides.

Then, negation of disjunction of the above statement is given by

$\sim(p \vee q)$: A triangle has neither 3-sides nor 4-sides.

(v) Let p : 35 is a prime number.

q : 35 is a composite number.

Then, negation of disjunction of the above statement is given by

$\sim(p \vee q)$: 35 is not a prime number and it is not a composite number.

(vi) Let p : All prime integers are even.

q : All prime integers are odd.

Then negation of disjunction of the above statement is given by

$\sim(p \vee q)$: All prime integers are not even and all prime integers are not odd.

(vii) Let p : $|x|$ is equal to x .

q : $|x|$ is equal to $-x$.

Then negation of disjunction of the above statement is given by

$\sim(p \vee q)$: $|x|$ is not equal to x and it is not equal to $-x$.

(viii) Let p : 6 is divisible by 2.

q : 6 is divisible by 3.

Then, negation of conjunction of above statement is given by

$\sim(p \wedge q)$: 6 is not divisible by 2 or it is not divisible by 3

Q. 7 Rewrite each of the following statements in the form of conditional statements.

- (i) The square of an odd number is odd.
- (ii) You will get a sweet dish after the dinner.
- (iii) You will fail, if you will not study.
- (iv) The unit digit of an integer is 0 or 5, if it is divisible by 5.
- (v) The square of a prime number is not prime.
- (vi) $2b = a + c$, if a , b and c are in AP.

Sol. We know that, some of the common expressions of conditional statement $p \rightarrow q$ are

- (i) if p , then q
- (ii) q if p
- (iii) p only if q
- (iv) p is sufficient for q
- (v) q is necessary for p
- (vi) $\sim q$ implies $\sim p$

So, use above information to get the answer

- (i) If the number is odd number, then its square is odd number.
- (ii) If you take the dinner, then you will get sweet dish.
- (iii) If you will not study, then you will fail.
- (iv) If an integer is divisible by 5, then its unit digits are 0 or 5.
- (v) If the number is prime, then its square is not prime.
- (vi) If a , b and c are in AP, then $2b = a + c$.

Q. 8 Form the biconditional statement $p \leftrightarrow q$, where

- (i) p : The unit digits of an integer is zero.
 q : It is divisible by 5.
- (ii) p : A natural number n is odd.
 q : Natural number n is not divisible by 2.
- (iii) p : A triangle is an equilateral triangle.
 q : All three sides of a triangle are equal.

Sol. (i) $p \leftrightarrow q$: The unit digit of an integer is zero, if and only if it is divisible by 5.

(ii) $p \leftrightarrow q$: A natural number n is odd if and only if it is not divisible by 2.

(iii) $p \leftrightarrow q$: A triangle is an equilateral triangle if and only if all three sides of triangle are equal.

Q. 9 Write down the contrapositive of the following statements.

- (i) If $x = y$ and $y = 3$, then $x = 3$.
- (ii) If n is a natural number, then n is an integer.
- (iii) If all three sides of a triangle are equal, then the triangle is equilateral.
- (iv) If x and y are negative integers, then xy is positive.
- (v) If natural number n is divisible by 6, then n is divisible by 2 and 3.
- (vi) If it snows, then the weather will be cold.
- (vii) If x is a real number such that $0 < x < 1$, then $x^2 < 1$.

💡 Thinking Process

We know that, the statement $(\sim q) \rightarrow (\sim p)$ is called contrapositive of the statement $p \rightarrow q$.

- Sol.**
- (i) If $x \neq 3$, then $x \neq y$ or $y \neq 3$.
 - (ii) If n is not an integer, then it is not a natural number.
 - (iii) If the triangle is not equilateral, then all three sides of the triangle are not equal.
 - (iv) If xy is not positive integer, then either x or y is not negative integer.
 - (v) If natural number n is not divisible by 2 or 3, then n is not divisible by 6.
 - (vi) The weather will not be cold, if it does not snow.
 - (vii) If $x^2 \not< 1$, then x is not a real number such that $0 < x < 1$.

Q. 10 Write down the converse of following statements.

- (i) If a rectangle ' R ' is a square, then R is a rhombus.
- (ii) If today is Monday, then tomorrow is Tuesday.
- (iii) If you go to Agra, then you must visit Taj Mahal.
- (iv) If sum of squares of two sides of a triangle is equal to the square of third side of a triangle, then the triangle is right angled.
- (v) If all three angles of a triangle are equal, then the triangle is equilateral.
- (vi) If $x : y = 3 : 2$, then $2x = 3y$.
- (vii) If S is a cyclic quadrilateral, then the opposite angles of S are supplementary.
- (viii) If x is zero, then x is neither positive nor negative.
- (ix) If two triangles are similar, then the ratio of their corresponding sides are equal.

💡 Thinking Process

We know that, the converse of the statement " $p \rightarrow q$ " is " $(q) \rightarrow (p)$ ".

- Sol.**
- (i) If the rectangle ' R ' is rhombus, then it is square.
 - (ii) If tomorrow is Tuesday, then today is Monday.
 - (iii) If you must visit Taj Mahal, you go to Agra.
 - (iv) If the triangle is right angle, then sum of squares of two sides of a triangle is equal to the square of third side.
 - (v) If the triangle is equilateral, then all three angles of triangle are equal.

- (vi) If $2x = 3y$, then $x:y = 3:2$
 (vii) If the opposite angles of a quadrilateral are supplementary, then S is cyclic.
 (viii) If x is neither positive nor negative, then x is 0.
 (ix) If the ratio of corresponding sides of two triangles are equal, then triangles are similar.

Q. 11 Identify the quantifiers in the following statements.

- (i) There exists a triangle which is not equilateral.
 (ii) For all real numbers x and y , $xy = yx$.
 (iii) There exists a real number which is not a rational number.
 (iv) For every natural number x , $x + 1$ is also a natural number.
 (v) For all real numbers x with $x > 3$, x^2 is greater than 9.
 (vi) There exists a triangle which is not an isosceles triangle.
 (vii) For all negative integers x , x^3 is also a negative integers.
 (viii) There exists a statement in above statements which is not true.
 (ix) There exists an even prime number other than 2.
 (x) There exists a real number x such that $x^2 + 1 = 0$.

Sol. Quantifier are the phrases like 'There exist' and 'For every', 'For all' etc.

- | | |
|--------------------|---------------------|
| (i) There exists | (ii) For all |
| (iii) There exists | (iv) For every |
| (v) For all | (vi) There exists |
| (vii) For all | (viii) There exists |
| (ix) There exists | (x) There exists |

Q. 12 Prove by direct method that for any integer ' n ', $n^3 - n$ is always even.

Thinking Process

We know that, in direct method to show a statement, if p then q is true, we assume p is true and show q is true i.e., $p \rightarrow q$.

Sol. Here, two cases arise

Case I When n is even,

$$\begin{aligned} \text{Let } n &= 2K, K \in N \\ \Rightarrow n^3 - n &= (2K)^3 - (2K) = 2K(4K^2 - 1) \\ &= 2\lambda, \text{ where } \lambda = K(4K^2 - 1) \end{aligned}$$

Thus, $(n^3 - n)$ is even when n is even.

Case II When n is odd,

$$\begin{aligned} \text{Let } n &= 2K + 1, K \in N \\ \Rightarrow n^3 - n &= (2K + 1)^3 - (2K + 1) \\ &= (2K + 1)[(2K + 1)^2 - 1] \\ &= (2K + 1)[4K^2 + 1 + 4K - 1] \\ &= (2K + 1)(4K^2 + 4K) \\ &= 4K(2K + 1)(K + 1) \\ &= 2\mu, \text{ when } \mu = 2K(K + 1)(2K + 1) \end{aligned}$$

Then, $n^3 - n$ is even when n is odd.

So, $n^3 - n$ is always even.

Q. 13 Check validity of the following statement.

- (i) p : 125 is divisible by 5 and 7.
- (ii) q : 131 is a multiple of 3 or 11.

Sol. (i) p : 125 is divisible by 5 and 7.

Let q : 125 is divisible by 5.

r : 125 is divisible by 7.

q is true, r is false.

$\Rightarrow q \wedge r$ is false.

[since, $p \wedge q$ has the truth value F (false) whenever either p or q or both have the truth value F.]

Hence, p is not valid.

(ii) p : 131 is a multiple of 3 or 11.

Let q : 131 is multiple of 3.

r : 131 is a multiple of 11.

p is true, r is false.

$\Rightarrow p \vee r$ is true.

[since, $p \vee q$ has the truth value T (true) whenever either p or q or both have the truth value T]

Hence, q is valid.

Q.14 Prove the following statement by contradiction method

p : The sum of an irrational number and a rational number is irrational.

Sol. Let p is false i.e., sum of an irrational and a rational number is rational.

Let \sqrt{m} is irrational and n is rational number.

$\Rightarrow \sqrt{m} + n = r$ [rational]

$\Rightarrow \sqrt{m} = r - n$

\sqrt{m} is irrational, where as $(r - n)$ is rational. This is contradiction.

Then, our supposition is wrong.

Hence, p is true.

Q. 15 Prove by direct method that for any real number x, y if $x = y$, then

$$x^2 = y^2.$$

Thinking Process

In direct method assume p is true and show q is true i.e., $p \Rightarrow q$.

Sol. Let $p : x = y, \quad x, y \in R$

On squaring both sides,

$$x^2 = y^2 : q \quad \text{[say]}$$

$$p \Rightarrow q$$

Hence, we have the result.

Q. 16 Using contrapositive method prove that, if n^2 is an even integer, then n is also an even integer.

Thinking Process

In contrapositive method assume $\sim q$ is true and show $\sim p$ is true i.e., $\sim q \Rightarrow \sim p$.

Sol. Let $p : n^2$ is an even integer.
 $q : n$ is also an even integer.
 Let $\sim p$ is true i.e., n is not an even integer.
 $\Rightarrow n^2$ is not an even integer. [since, square of an odd integer is odd]
 $\Rightarrow \sim p$ is true.
 Therefore, $\sim q$ is true $\Rightarrow \sim p$ is true. **Hence proved.**

Objective Type Questions

Q. 17 Which of the following is a statement?

- (a) x is a real number
- (b) Switch off the fan
- (c) 6 is a natural number
- (d) Let me go

Sol. (c) As we know a statement is a sentence which is either true or false.
 So, 6 is a natural number, which is true.
 Hence, it is a statement.

Q. 18 Which of the following is not a statement.

- (a) Smoking is injurious to health
- (b) $2 + 2 = 4$
- (c) 2 is the only even prime number
- (d) Come here

Sol. (d) 'Come here' is not a statement. Since, no sentence can be called a statement, if it is an order.

Q. 19 The connective in the statement ' $2 + 7 > 9$ or $2 + 7 < 9$ ' is

- (a) and
- (b) or
- (c) $>$
- (d) $<$

Sol. (b) In ' $2 + 7 > 9$ or $2 + 7 < 9$ ', or is the connective.

Q. 20 The connective in the statement "Earth revolves round the Sun and Moon is a satellite of earth" is

- (a) or
- (b) Earth
- (c) Sun
- (d) and

Sol. (d) Connective word is 'and'.

- Q. 21** The negation of the statement "A circle is an ellipse" is
 (a) An ellipse is a circle (b) An ellipse is not a circle
 (c) A circle is not an ellipse (d) A circle is an ellipse

Sol. (c) Let p : A circle is an ellipse.
 $\sim p$: A circle is not an ellipse.

- Q. 22** The negation of the statement "7 is greater than 8" is
 (a) 7 is equal to 8 (b) 7 is not greater than 8
 (c) 8 is less than 7 (d) None of these

Sol. (b) Let p : 7 is greater than 8.
 $\sim p$: 7 is not greater than 8.

- Q. 23** The negation of the statement "72 is divisible by 2 and 3" is
 (a) 72 is not divisible by 2 or 72 is not divisible by 3
 (b) 72 is not divisible by 2 and 72 is not divisible by 3
 (c) 72 is divisible by 2 and 72 is not divisible by 3
 (d) 72 is not divisible by 2 and 72 is divisible by 3

Sol. (b) Let p : 72 is divisible by 2 and 3.
 Let q : 72 is divisible by 2.
 r : 72 is divisible by 3.
 $\sim q$: 72 is not divisible by 2.
 $\sim r$: 72 is not divisible by 3.
 $\sim(q \wedge r) : \sim q \vee \sim r$
 \Rightarrow 72 is not divisible by 2 or 72 is not divisible by 3.

- Q. 24** The negation of the statement "Plants take in CO_2 and give out O_2 " is
 (a) Plants do not take in CO_2 and do not given out O_2
 (b) Plants do not take in CO_2 or do not give out O_2
 (c) Plants take is CO_2 and do not give out O_2
 (d) Plants take in CO_2 or do not give out O_2

Sol. (b) Let p : Plants take in CO_2 and give out O_2 .
 Let q : Plants take in CO_2 .
 r : Plants give out O_2 .
 $\sim q$: Plants do not take in CO_2 .
 $\sim r$: Plants do not give out O_2 .
 $\sim(q \wedge r) : \sim q \vee \sim r$
 \Rightarrow Plants do not take in CO_2 or do not give out O_2 .

- Q. 25** The negative of the statement "Rajesh or Rajni lived in Bangaluru" is
 (a) Rajesh did not live in Bengaluru or Rajni lives in Bengaluru
 (b) Rajesh lives in Bengaluru and Rajni did not live in Bengaluru
 (c) Rajesh did not live in Bengaluru and Rajni did not live in Bengaluru
 (d) Rajesh did not live in Bengaluru or Rajni did not live in Bengaluru