6

Linear Inequalities

Short Answer Type Questions

Solve for x, the inequalities in following questions.

Q. 1 $\frac{4}{x-1} \le 3 \le \frac{6}{x+1} (x > 0)$

Thinking Process

First solve the first two inequalities, then solve the last two inequality to get range of x.

Sol. Consider first two inequalities,

$$\frac{4}{x+1} \le 3$$

$\Rightarrow \quad 4 \le 3(x+1)$

$\Rightarrow \quad 4 \le 3x + 3$

$\Rightarrow \quad 4 - 3 \le 3x$ [subtracting 3 on both sides]

$\Rightarrow \quad 1 \le 3x$

$\therefore \quad x \ge \frac{1}{3}$ ...(i)

and consider last two inequalities,

$$3 \le \frac{6}{x+1}$$

$\Rightarrow \quad 3(x+1) \le 6$

$\Rightarrow \quad 3x + 3 \le 6$

$\Rightarrow \quad 3x \le 6 - 3$ [subtracting 3 to both sides]

$\Rightarrow \quad 3x \le 3$ [dividing by 3]

$\therefore \quad x \le 1$ ...(ii)

From Eqs. (i) and (ii),

$$x \in \left[\frac{1}{3}, 1\right]$$

$$\frac{1}{3} \le x \le 1$$

Q. 2 $\dfrac{|x-2|-1}{|x-2|-2} \le 0$

Thinking Process

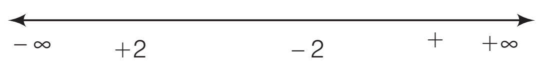
First, let $y=|x-2|$ and then for the obtained values of y use the property $|x-a| \ge k \Leftrightarrow x \le a-k$ or $x \ge a+k$ to get the range of x.

Sol. Let $|x-2| = y$

$$\frac{y-1}{y-2} \le 0$$

$\Rightarrow$ $y-1=0$ and $y-2=0$

$\Rightarrow$ $y=1$ and $y=2$



$\Rightarrow$ $1 \le y < 2$

$\Rightarrow$ $1 \le |x-2| < 2$

$\Rightarrow$ $1 \le |x-2|$ and $|x-2| < 2$

$\Rightarrow$ $x-2 \le -1$

$\Rightarrow$ $x-2 \ge 1$

and $-2 < x-2 < 2$

$\Rightarrow$ $x \le 1$ or $x \ge 3$ and $0 < x < 4$

$\Rightarrow$ $x \in (0, 1] \cup [3, 4)$

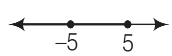
Q. 3 $\dfrac{1}{|x|-3} \le \dfrac{1}{2}$

Sol. Given, $\dfrac{1}{|x|-3} \le \dfrac{1}{2}$

$\Rightarrow$ $|x|-3 \ge 2$ $\left[\because \dfrac{1}{a} < \dfrac{1}{b} \Rightarrow a > b\right]$

$\Rightarrow$ $|x| \ge 5$ [adding 3 to both sides]

$\Rightarrow$ $x \le -5$ or $x \ge 5$ $[\because |x| \ge a \Rightarrow |x| \le -a \Rightarrow |x| \ge a]$



$\Rightarrow$ $x \in (-\infty, -5] \cup [5, \infty)$ ...(i)

But $|x|-3 \ne 0$

Either $|x|-3 < 0$ or $|x|-3 > 0$

$\Rightarrow$ $|x| < 3$ or $|x| > 3$

$\Rightarrow$ $-3 < x < 3$ or $x < -3$ or $x > 3$ ...(ii)

$[\because |x| < a \Rightarrow -a < x < a$ and $|x| > a \Rightarrow x < -a\ x > a]$

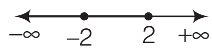
On combining results of Eqs. (i) and (ii), we get

$$x \in (-\infty, -5] \cup (-3, 3) \cup [5, \infty)$$

Q. 4 $|x-1| \le 5, |x| \ge 2$

Sol.
$$|x-1| \le 5$$
$$-5 \le x-1 \le 5$$
$$\Rightarrow \quad -4 \le x \le 6$$
$$\Rightarrow \quad x \in [-4, 6] \quad \ldots(i)$$

and
$$|x| \ge 2$$
$$\Rightarrow \quad x \le -2 \text{ or } x \ge 2$$
$$\Rightarrow \quad x \in (-\infty, -2] \cup [2, \infty) \quad \ldots(ii)$$



On combining Eqs. (i) and (ii), we get
$$x \in (-4, -2] \cup [2, 6]$$

Q. 5 $-5 \le \frac{2-3x}{4} \le 9$

Sol. We have,
$$-5 \le \frac{2-3x}{4}$$
$$\Rightarrow \quad -20 \le 2-3x \quad \text{[multiplying by 4 on both sides]}$$
$$\Rightarrow \quad 3x \le 2+20$$
$$\Rightarrow \quad 3x \le 22$$
$$\Rightarrow \quad x \le \frac{22}{3}$$

and
$$\frac{2-3x}{4} \le 9$$
$$\Rightarrow \quad 2-3x \le 36$$
$$\Rightarrow \quad -3x \le 36-2$$
$$\Rightarrow \quad -3x \le 34$$
$$\Rightarrow \quad 3x \ge -34$$
$$\Rightarrow \quad x \ge -\frac{34}{3}$$
$$\Rightarrow \quad -\frac{34}{3} \le x \le \frac{22}{3}$$
$$\Rightarrow \quad x \in \left[\frac{-34}{3}, \frac{22}{3}\right]$$

Q. 6 $4x+3 \ge 2x+17, 3x-5 < -2$

Sol. We have,
$$4x+3 \ge 2x+17$$
$$\Rightarrow \quad 4x-2x \ge 17-3 \Rightarrow 2x \ge 14$$
$$\Rightarrow \quad x \ge \frac{14}{2}$$
$$\Rightarrow \quad x \ge 7 \quad \ldots(i)$$

Also, we have
$$3x-5 < -2$$
$$\Rightarrow \quad 3x < -2+5 \Rightarrow 3x < 3$$
$$\Rightarrow \quad x < 1 \quad \ldots(ii)$$

On combining Eqs. (i) and (ii), we see that solution is not possible because nothing is common between these two solutions. (*i.e.*, $x < 1, x \ge 7$).

Q. 7 A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$, respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit?

Sol. Cost function, $C(x) = 26000 + 30x$

and revenue function, $R(x) = 43x$

For profit, $R(x) > C(x)$

$\Rightarrow \quad 26000 + 30x < 43x$

$\Rightarrow \quad 30x - 43x < -26000$

$\Rightarrow \quad -13x < -26000$

$\Rightarrow \quad 13x > 26000$

$\Rightarrow \quad x > \dfrac{26000}{13}$

$\therefore \quad x > 2000$

Hence, more than 2000 cassettes must be produced to get profit.

Q. 8 The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, then find the range of pH value for the third reading that will result in the acidity level being normal.

Sol. Given, first pH value = 8.48

and second pH value = 8.35

Let third pH value be x.

Since, it is given that average pH value lies between 8.2 and 8.5.

$\therefore \quad 8.2 < \dfrac{8.48 + 8.35 + x}{3} < 8.5$

$\Rightarrow \quad 8.2 < \dfrac{16.83 + x}{3} < 8.5$

$\Rightarrow \quad 3 \times 8.2 < 16.83 + x < 8.5 \times 3$

$\Rightarrow \quad 24.6 < 16.83 + x < 25.5$

$\Rightarrow \quad 24.6 - 16.83 < x < 25.5 - 16.83$

$\Rightarrow \quad 7.77 < x < 8.67$

Thus, third pH value lies between 7.77 and 8.67.

Q. 9 A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 L of the 9% solution, how many litres of 3% solution will have to be added?

Sol. Let x L of 3% solution be added to 460 L of 9% solution of acid.

Then, total quantity of mixture = $(460 + x)$ L

Total acid content in the $(460 + x)$ L of mixture

$$= \left(460 \times \frac{9}{100} + x \times \frac{3}{100}\right)$$

It is given that acid content in the resulting mixture must be more than 5% but less than 7% acid.

Therefore, $5\% \text{ of } (460 + x) < 460 \times \frac{9}{100} + \frac{3x}{100} < 7\% \text{ of } (460 + x)$

$\Rightarrow \frac{5}{100} \times (460 + x) < 460 \times \frac{9}{100} + \frac{3}{100}x < \frac{7}{100} \times (460 + x)$

$\Rightarrow 5 \times (460 + x) < 460 \times 9 + 3x < 7 \times (460 + x)$ [multiplying by 100]

$\Rightarrow 2300 + 5x < 4140 + 3x < 3220 + 7x$

Taking first two inequalities, $2300 + 5x < 4140 + 3x$

$\Rightarrow 5x - 3x < 4140 - 2300$

$\Rightarrow 2x < 1840$

$\Rightarrow x < \frac{1840}{2}$

$\Rightarrow x < 920$ ...(i)

Taking last two inequalities, $4140 + 3x < 3220 + 7x$

$\Rightarrow 3x - 7x < 3220 - 4140$

$\Rightarrow -4x < -920$

$\Rightarrow 4x > 920$

$\Rightarrow x > \frac{920}{4}$

$\Rightarrow x > 230$ ...(ii)

Hence, the number of litres of the 3% solution of acid must be more than 230 L and less than 920 L.

Q. 10 A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree fahrenheit, if the conversion formula is $F = \frac{9}{5}C + 32$?

Sol. Let the required temperature be x°F.

Given that, $F = \frac{9}{5}C + 32$

$\Rightarrow 5F = 9C + 32 \times 5$

$\Rightarrow 9C = 5F - 32 \times 5$

$\therefore C = \frac{5F - 160}{9}$

Since, temperature in degree calcius lies between 40°C to 45°C.

Therefore, $40 < \frac{5F - 160}{9} < 45$

$\Rightarrow 40 < \frac{5x - 160}{9} < 45$

$\Rightarrow 40 \times 9 < 5x - 160 < 45 \times 9$ [multiplying throughout by 9]

$\Rightarrow 360 < 5x - 160 < 405$ [adding 160 throughout]

$\Rightarrow 360 + 160 < 5x < 405 + 160$

$\Rightarrow 520 < 5x < 565$

$\Rightarrow \frac{520}{5} < x < \frac{565}{5}$ [divide throughout by 5]

$\Rightarrow 104 < x < 113$

Hence, the range of temperature in degree fahrenheit is 104 °F to 113°F.

Q. 11 **The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm, then find the minimum length of the shortest side.**

Sol. Let the length of shortest side be x cm.

According to the given information,

Longest side $= 2 \times$ Shortest side

$= 2x$ cm

and third side $= 2 +$ Shortest side

$= (2 + x)$ cm

Perimeter of triangle $= x + 2x + (x + 2) = 4x + 2$

According to the question,

Perimeter > 166 cm

$\Rightarrow \quad 4x + 2 > 166$

$\Rightarrow \quad 4x > 166 - 2$

$\Rightarrow \quad 4x > 164$

$\therefore \quad x > \frac{164}{4} = 41$ cm

Hence, the minimum length of shortest side be 41cm.

Q. 12 **In drilling world's deepest hole it was found that the temperature T in degree celcius, x km below the earth's surface was given by $T = 30 + 25(x - 3)$, $3 \le x \le 15$. At what depth will the temperature be between 155°C and 205°C?**

Sol. Given that, $T = 30 + 25(x - 3)$, $3 \le x \le 15$

According to the question,

$155 < T < 205$

$\Rightarrow \quad 155 < 30 + 25(x - 3) < 205$

$\Rightarrow \quad 155 - 30 < 25(x - 3) < 205 - 30$ [subtracting 30 in whole]

$\Rightarrow \quad 125 < 25(x - 3) < 175$

$\Rightarrow \quad \frac{125}{25} < x - 3 < \frac{175}{25}$ [dividing by 25 in whole]

$\Rightarrow \quad 5 < x - 3 < 7$

$\Rightarrow \quad 5 + 3 < x < 7 + 3$ [adding 3 in whole]

$\Rightarrow \quad 8 < x < 10$

Hence, at the depth 8 to10 km temperature lies between 155° to 205°C.

Long Answer Type Questions

Q.13 Solve the following system of inequalities $\frac{2x+1}{7x-1} > 5, \frac{x+7}{x-8} > 2$.

Sol. The given system of inequations is

$$\frac{2x+1}{7x-1} > 5 \quad \ldots\text{(i)}$$

and $$\frac{x+7}{x-8} > 2 \quad \ldots\text{(ii)}$$

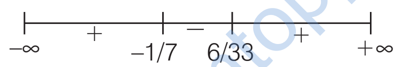
Now, $$\frac{2x+1}{7x-1} - 5 > 0$$

$$\Rightarrow \frac{(2x+1) - 5(7x-1)}{7x-1} > 0$$

$$\Rightarrow \frac{2x+1-35x+5}{7x-1} > 0$$

$$\Rightarrow \frac{-33x+6}{7x-1} > 0 \Rightarrow \frac{33x-6}{7x-1} < 0$$

$$\Rightarrow x \in \left(\frac{1}{7}, \frac{6}{33}\right) \quad \ldots\text{(iii)}$$



and $$\frac{x+7}{x-8} > 2 \Rightarrow \frac{x+7}{x-8} - 2 > 0$$

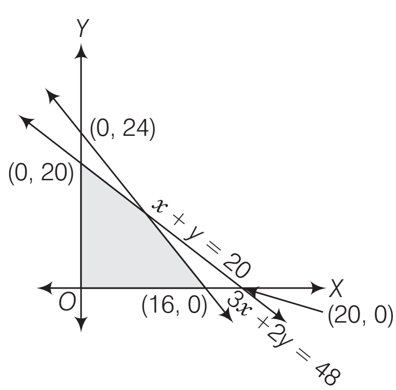
$$\Rightarrow \frac{x+7-2(x-8)}{x-8} > 0 \Rightarrow \frac{x+7-2x+16}{x-8} > 0$$

$$\Rightarrow \frac{-x+23}{x-8} > 0 \Rightarrow \frac{x-23}{x-8} < 0$$

$$\Rightarrow x \in (8, 23) \quad \ldots\text{(iv)}$$

Since, the intersection of Eqs. (iii) and (iv) is the null set. Hence, the given system of equation has no solution.

Q.14 Find the linear inequalities for which the shaded region in the given figure is the solution set.



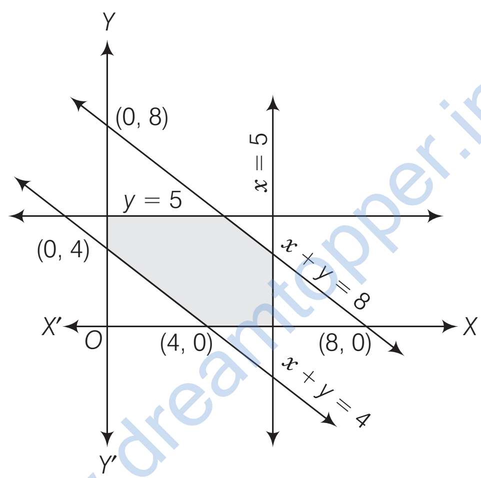
Sol. Consider the line $3x + 2y = 48$, we observe that the shaded region and the origin are on the same side of the line $3x + 2y = 48$ and (0, 0) satisfy the linear constraint $3x + 2y \leq 48$. So, we must have one inequation as $3x + 2y \leq 48$.

Now, consider the line $x + y = 20$. We find that the shaded region and the origin are on the same side of the line $x + y = 20$ and (0, 0) satisfy the constraints $x + y \leq 20$. So, the second inequation is $x + y \leq 20$.

We also notice that the shaded region is above X-axis and is on the right side of Y-axis, so we must have $x \geq 0$, $y \geq 0$.

Thus, the linear inequations corresponding to the given solution set are $3x + 2y \leq 48$, $x + y \leq 20$ and $x \geq 0$, $y \geq 0$.

Q. 15 Find the linear inequalities for which the shaded region in the given figure is the solution set.



Sol. Consider the line $x + y = 4$.

We observe that the shaded region and the origin lie on the opposite side of this line and (0, 0) satisfies $x + y \leq 4$. Therefore, we must have $x + y \geq 4$ as the linear inequation corresponding to the line $x + y = 4$.

Consider the line $x + y = 8$, clearly the shaded region and origin lie on the same side of this line and (0, 0) satisfies the constraints $x + y \leq 8$. Therefore, we must have $x + y \leq 8$, as the linear inequation corresponding to the line $x + y = 8$.

Consider the line $x = 5$. It is clear from the graph that the shaded region and origin are on the left of this line and (0, 0) satisfy the constraint $x \leq 5$.

Hence, $x \leq 5$ is the linear inequation corresponding to $x = 5$.

Consider the line $y = 5$, clearly the shaded region and origin are on the same side (below) of the line and (0, 0) satisfy the constrain $y \leq 5$.

Therefore, $y \leq 5$ is an inequation corresponding to the line $y = 5$.

We also notice that the shaded region is above the X-axis and on the right of the Y-axis *i.e.*, shaded region is in first quadrant. So, we must have $x \geq 0$, $y \geq 0$.

Thus, the linear inequations comprising the given solution set are

$$x + y \geq 4;\ x + y \leq 8;\ x \leq 5;\ y \leq 5;\ x \geq 0 \text{ and } y \geq 0.$$

Q.16 Show that the following system of linear inequalities has no solution $x+2y\le 3, 3x+4y\ge 12, x\ge 0, y\ge 1$.

Sol. Consider the inequation $x+2y\le 3$ as an equation, we have

$$x+2y=3$$

$\Rightarrow$ $$x=3-2y$$

$\Rightarrow$ $$2y=3-x$$

x	3	1	0
y	0	1	1.5

Now, (0, 0) satisfy the inequation $x+2y\le 3$.

So, half plane contains (0, 0) as the solution and the line $x+2y=3$ intersect the coordinate axis at (3, 0) and (0, 3/2).

Consider the inequation $3x+4y\ge 12$ as an equation, we have $3x+4y=12$

$\Rightarrow$ $$4y=12-3x$$

x	0	4	2
y	3	0	3/2

Thus, coordinate axis intersected by the line $3x+4y=12$ at points (4, 0) and (0, 3).

Now, (0, 0) does not satisfy the inequation $3x+4y=12$.

Therefore, half plane of the solution does not contained (0, 0).

Consider the inequation $y\ge 1$ as an equation, we have

$$y=1.$$

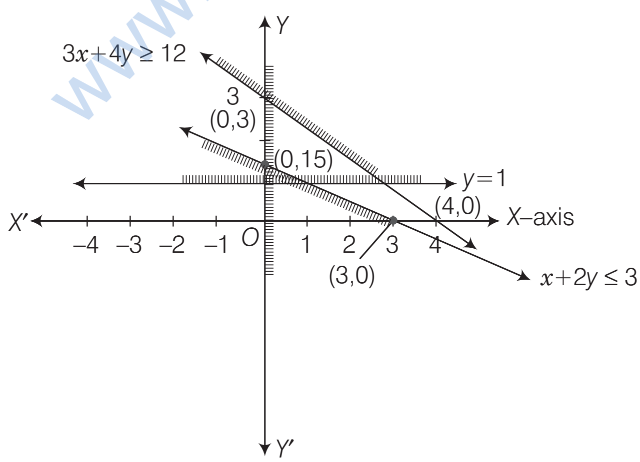
It represents a straight line parallel to X-axis passing through point (0, 1).

Now, (0, 0) does not satisfy the inequation $y\ge 1$.

Therefore, half plane of the solution does not contains (0, 0).

Clearly $x\ge 0$ represents the region lying on the right side of Y-axis.

The solution set of the given linear constraints will be the intersection of the above region.



It is clear from the graph the shaded portions do not have common region.

So, solution set is null set.

Q.17 Solve the following system of linear inequalities
$3x + 2y \geq 24,\ 3x + y \leq 15,\ x \geq 4.$

Sol. Consider the inequation $3x + 2y \geq 24$ as an equation, we have $3x + 2y = 24$.

$\Rightarrow$ $$2y = 24 - 3x$$

x	0	8	4
y	12	0	6

Hence, line $3x + y = 24$ intersect coordinate axes at points (8 , 0) and (0,12).
Now, (0,0) does not satisfy the inequation $3x + 2y \geq 24$.
Therefore, half plane of the solution set does not contains (0,0).
Consider the inequation $3x + y \leq 15$ as an equation, we have

$$3x + y = 15$$

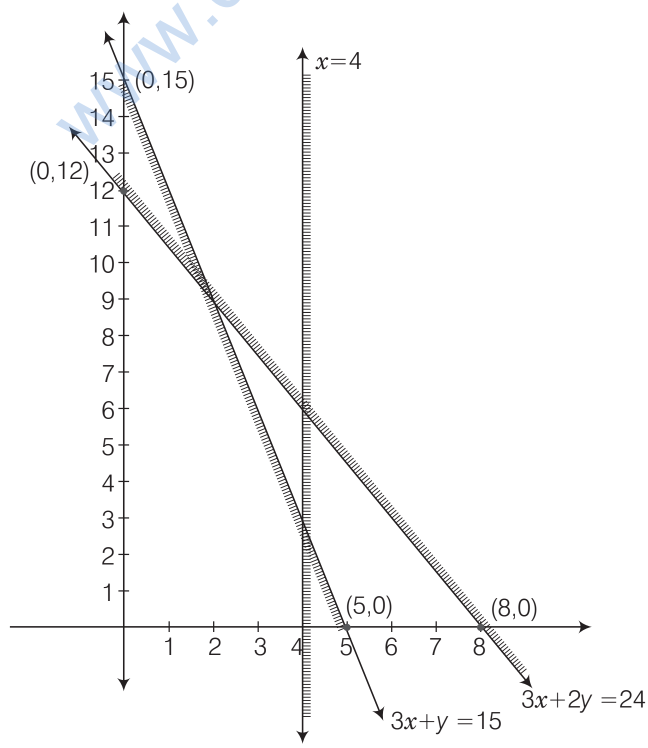
$\Rightarrow$ $$y = 15 - 3x$$

x	0	5	3
y	15	0	6

Line $3x + y = 15$ intersects coordinate axes at points (5,0) and (0,15).
Now, point (0,0) satisfy the inequation $3x + y \leq 15$.
Therefore, the half plane of the solution contain origin.
Consider the inequality $x \geq 4$ as an equation, we have

$$x = 4$$

It represents a straight line parallel to Y-axis passing through (4,0). Now, point (0,0) does not satisfy the inequation $x \geq 4$.
Therefore, half plane does not contains (0,0),
The graph of the above inequations is given below.



It is clear from the graph that there is no common region corresponding to these inequality.
Hence, the given system of inequalities have no solution.

Q.18 Show that the solution set of the following system of linear inequalities is an unbounded region $2x + y \geq 8$, $x + 2y \geq 10$, $x \geq 0$, $y \geq 0$.

Sol. Consider the inequation $2x + y \geq 8$ as an equation, we have

$$2x + y = 8$$

$$\Rightarrow \quad y = 8 - 2x$$

x	0	4	3
y	8	0	2

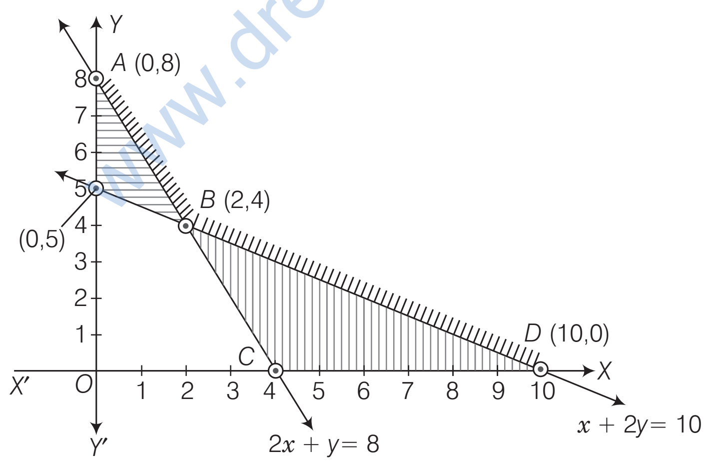
The line $2x + y = 8$ intersects coordinate axes at (4, 0) and (0, 8).
Now, point (0, 0) does not satisfy the inequation $2x + y \geq 8$.
Therefore, half plane does not contain origin.
Consider the inequation $x + 2y \geq 10$, as an equation, we have

$$x + 2y = 10$$

$$\Rightarrow \quad 2y = 10 - x$$

x	10	0	8
y	0	5	1

The line $2x + y = 8$ intersects the coordinate axes at (10, 0) and (0, 5).
Now, point (0, 0) does not satisfy the inequation $x + 2y \geq 10$.
Therefore, half plane does not contain (0, 0).
Consider the inequation $x \geq 0$ and $y \geq 0$ clearly, it represents the region in first quadrant.
The graph of the above inequations is given below



It is clear from the graph that common shaded portion is unbounded.

Objective Type Questions

Q. 19 If $x < 5$, then

(a) $-x < -5$ (b) $-x \le -5$ (c) $-x > -5$ (d) $-x \ge -5$

Sol. *(c)* If $x < 5$, then $-x > -5$

[if we multiply by negative numbers, then inequality get reversed]

Q. 20 If x, y and b are real numbers and $x < y$, $b < 0$, then

(a) $\frac{x}{b} < \frac{y}{b}$ (b) $\frac{x}{b} \le \frac{y}{b}$ (c) $\frac{x}{b} > \frac{y}{b}$ (d) $\frac{x}{b} \ge \frac{y}{b}$

Sol. *(c)* It is given that, $x < y$, $b < 0$

$\Rightarrow \frac{x}{b} > \frac{y}{b}$ [$\because b < 0$]

Q. 21 If $-3x + 17 < -13$, then

(a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$ (c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10)$

Sol. *(a)* Given that, $-3x + 17 < -13$

$\Rightarrow 3x - 17 > 13$ [multiplying by -1 on both sides]

$\Rightarrow 3x > 13 + 17$ [adding 17 on both sides]

$\Rightarrow 3x > 30$

$\therefore x > 10$

Q. 22 If x is a real number and $|x| < 3$, then

(a) $x \ge 3$ (b) $-3 < x < 3$ (c) $x \le -3$ (d) $-3 \le x \le 3$

Sol. *(b)* Given, $|x| < 3$

$\Rightarrow -3 < x < 3$ [$\because |x| < a \Rightarrow -a < x < a$]

Q. 23 Let x and b are real numbers. If $b > 0$ and $|x| > b$, then

(a) $x \in (-b, \infty)$ (b) $x \in [-\infty, b)$

(c) $x \in (-b, b)$ (d) $x \in (-\infty, -b) \cup (b, \infty)$

Sol. *(d)* Given, $|x| > b$ and $b > 0$

$\Rightarrow x < -b$ or $x > b$

$\Rightarrow x \in (-\infty, -b) \cup (b, \infty)$

Q. 24 If $|x - 1| > 5$, then

(a) $x \in (-4, 6)$ (b) $x \in [-4, 6]$

(c) $x \in (-\infty, -4) \cup (6, \infty)$ (d) $x \in [-\infty, -4) \cup [6, \infty)$

Sol. *(c)* Given, $|x - 1| > 5$

$\Rightarrow (x - 1) < -5$ or $(x - 1) > 5$ [$\because |x| > a \Rightarrow x < a$ or $x > a$]

$\Rightarrow x < -4$ or $x > 6$

$\Rightarrow x \in (-\infty, -4) \cup (6, \infty)$

Q. 25 If $|x+2| \leq 9$, then

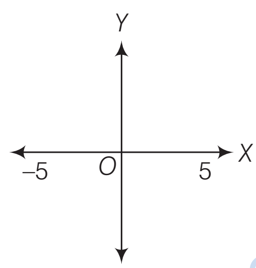
(a) $x \in (-7, 11)$ (b) $x \in [-11, 7]$
(c) $x \in (-\infty, -7) \cup (11, \infty)$ (d) $x \in (-\infty, -7) \cup [11, \infty)$

Sol. ***(b)*** Given, $|x+2| \leq 9$,

$\Rightarrow$ $-9 \leq x + 2 \leq 9$ $[\because |x| \leq a \Rightarrow -a \leq x \leq a]$
$\Rightarrow$ $-9 - 2 \leq x \leq 9 - 2$ [subtracting 2 througout]
$\Rightarrow$ $-11 \leq x \leq 7$
$\Rightarrow$ $x \in [-11, 7]$

The inequality representing the following graphs is

Q. 26



(a) $|x| < 5$ (b) $|x| \leq 5$
(c) $|x| > 5$ (d) $|x| \geq 5$

Sol. ***(a)*** The given graph represent $x > -5$ and $x < 5$.

On combining these two result, we get
$|x| < 5$.

Solution of a linear inequality in variable x is represented on number line in following questions.

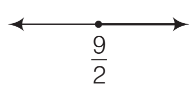
Q. 27

(a) $x \in (-\infty, 5)$ (b) $x \in (-\infty, 5]$
(c) $x \in [5, \infty)$ (d) $x \in (5, \infty)$

Sol. ***(d)*** The given graph represents all the values greater than 5 except $x = 5$ on the real line

So, $x \in (5, \infty)$.

Q. 28

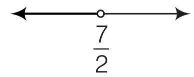


(a) $x \in \left(\frac{9}{2}, \infty\right)$ (b) $x \in \left[\frac{9}{2}, \infty\right)$
(c) $x \in -\left[\infty, \frac{9}{2}\right)$ (d) $x \in \left(-\infty, \frac{9}{2}\right]$

Sol. ***(b)*** The given graph represents all the values greater than $\frac{9}{2}$ including $\frac{9}{2}$ as the real line.

$$x \in \left[\frac{9}{2}, \infty\right)$$

Q. 29



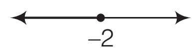
(a) $x \in \left(-\infty, \frac{7}{2}\right)$ (b) $x \in \left(-\infty, \frac{7}{2}\right]$

(c) $x \in \left[\frac{7}{2}, -\infty\right)$ (d) $x \in \left(\frac{7}{2}, \infty\right)$

Sol. ***(a)*** The given graph represents all the values less than $\frac{7}{2}$ on the real line.

$$\Rightarrow \qquad x \in \left(-\infty, \frac{7}{2}\right)$$

Q. 30



(a) $x \in (-\infty, -2)$ (b) $x \in (-\infty, -2]$

(c) $x \in (-2, \infty]$ (d) $x \in [-2, \infty)$

Sol. ***(b)*** The given graph represents all values less than –2 including –2.

$$\Rightarrow \qquad x \in (-\infty, -2]$$

True/False

Q. 31 State which of the following statements is true of false.

(i) If $x < y$ and $b < 0$, then $\frac{x}{b} < \frac{y}{b}$.

(ii) If $xy > 0$, then $x > 0$ and $y < 0$

(iii) If $xy > 0$, then $x < 0$ and $y < 0$

(iv) If $xy < 0$, then $x < 0$ and $y < 0$

(v) If $x < -5$ and $x < -2$, then $x \in (-\infty, -5)$

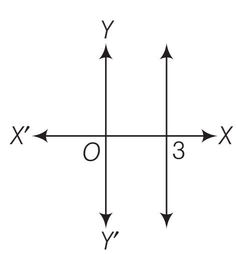
(vi) If $x < -5$ and $x > 2$, then $x \in (-5, 2)$

(vii) If $x > -2$ and $x < 9$, then $x \in (-2, 9)$

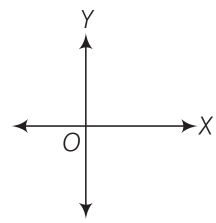
(viii) If $|x| > 5$, then $x \in (-\infty, -5) \cup [5, \infty)$

(ix) If $|x| \le 4$, then $x \in [-4, 4]$

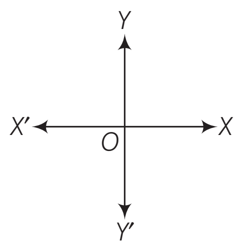
(x) Graph of $x < 3$ is



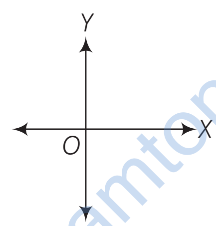
(xi) Graph of $x \geq 0$ is



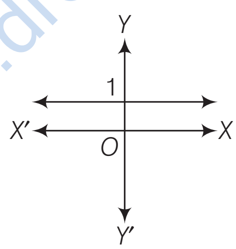
(xii) Graph of $y \leq 0$ is



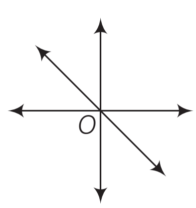
(xiii) Solution set of $x \geq 0$ and and $y \leq 0$ is



(xiv) Solution set of $x \geq 0$ and $y \leq 1$ is



(xv) Solution set of $x + y \geq 0$ is



Sol. **(i)** If $x < y$ and $b < 0$

$\Rightarrow \qquad \dfrac{x}{b} > \dfrac{y}{b}$

Hence, statement (i) is false.

(ii) If $x\,y > 0$, then, $x > 0$, $y > 0$ or $x < 0$, $y < 0$.
Hence, statement (ii) is true.

(iii) If $x\,y > 0$, then $x < 0$ and $y < 0$.
Hence, statement (iii) is true.

(iv) If $xy<0 \Rightarrow x<0$, $y>0$ or $x>0$, $y<0$.
Hence, statement (iv) is false.

(v) If $x<-5$ and $x<-2$, then

$$x\in(-\infty, -5)$$

Hence, statement (v) is true.

(vi) If $x<-5$ and $x>2$, then x have no value.
Hence, statement (vi) is false.

(vii) If $x>-2$ and $x<9$, then $x\in(-2, 9)$.
Hence, statement (vii)is true.

(viii) If $|x|>5$, then either $x<-5$ or $x>5$.

$$\Rightarrow \quad x\in(-\infty, -5)\cup(5, \infty)$$

Hence, statement (viii) is false.

(ix) If $|x|\leq 4$, then

$$-4\leq x\leq 4$$

$$\Rightarrow \quad x\in[-4, 4]$$

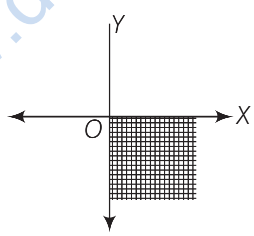
Hence, statement (ix) is true.

(x) The given graph represents $x\leq 3$.
Hence, statement (x) is false.

(xi) The given graph represents $x\geq 0$.
Hence, statement (xi) is true.

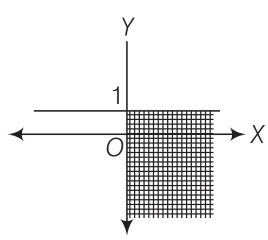
(xii) The given graph represent $y\geq 0$.
Hence, statement (xii) is false.

(xiii) Solution set of $x\geq 0$ and $y\leq 0$ is



Hence, statement (xiii) is false.

(xiv) Solution set of $x\geq 0$ and $y\leq 1$ is



Hence, statement (xiv) is false.

(xv) The given graph represents $x+y\geq 0$.
Hence, statement (xv) is correct.

Fillers

Q. 32 Fill in the blanks of the following

(i) If $-4x \geq 12$, then $x \ldots -3$.

(ii) If $\frac{-3}{4}x \leq -3$, then $x \ldots 4$.

(iii) If $\frac{2}{x+2} > 0$, then $x \ldots -2$.

(iv) If $x > -5$, then $4x \ldots -20$.

(v) If $x > y$ and $z < 0$, then $-xz \ldots -yz$.

(vi) If $p > 0$ and $q < 0$, then $p - q \ldots p$.

(vii) If $|x+2| > 5$, then $x \ldots -7$ or $x \ldots 3$.

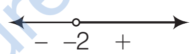
(viii) If $-2x + 1 \geq 9$, then $x \ldots -4$.

Sol. **(i)** If $-4x \geq 12 \Rightarrow x \leq -3$

(ii) If $\frac{-3}{4}x \leq -3$

$\Rightarrow \quad x \geq (-3) \times \frac{4}{-3} \Rightarrow x \geq 4$

(iii) If $\frac{2}{x+2} > 0$



$\Rightarrow \quad x > -2$

(iv) If $x > -5 \Rightarrow 4x > -20$

(v) If $x > y$ and $z < 0$, then

$xz < yz$ [since, $z < 0$]

$\Rightarrow \quad -xz > -yz$

(vi) If $p > 0$ and $q < 0$,

then $p - q > p$

e.g., consider $2 > 0$ and $-3 < 0$.

$\Rightarrow \quad 2 - (-3) = 2 + 3 = 5 > 2$

(vii) If $|x+2| > 5$, then

$x + 2 < -5$ or $x + 2 > 5$

$\Rightarrow \quad x < -5 - 2$ or $x > 5 - 2$

$\Rightarrow \quad x < -7$ or $x > 3$

$\Rightarrow \quad x \in (-\infty, -7) \cup (3, \infty)$

(viii) If $-2x + 1 \geq 9$, then

$-2x \geq 9 - 1 \Rightarrow -2x \geq 8$

$\Rightarrow \quad 2x \leq -8 \Rightarrow x \leq -4$