Using a Simple Pendulum plot $\mathrm{L}-\mathrm{T}$ and $\mathrm{L}-\mathrm{T}^{2}$ graphs, hence find the effective length of second's pendulum using appropriate graph.

## Apparatus and material required

Clamp stand; a split cork; a heavy metallic (brass/iron) spherical bob with a hook; a long, fine, strong cotton thread/string (about 2.0 m ); stop-watch; metre scale, graph paper, pencil, eraser.

## DeSCRIPTION OF TIME MEASURING DEVICESIN, A SCHOOL LABORATORY

The most common device used for measuring time in a school laboratory is a stop-watch or a stop-clock (analog). As the names suggest, these have the provision to start or stop their working as desired by the experimenter.

## (a) Stop-Watch

## Analog

A stop-watch is a special kind of watch. It has a multipurpose knob or button (B) for start/stop/back to zero position [Fig. E 6.1(b)]. It has two circular dials, the bigger one for a longer second's hand and the other smaller one for a shorter minute's hand. The


Fig.E 6.1(a): Stop - Watch second's dial has 30 equal divisions, each division representing 0.1 second. Before using a stop-watch you should find its least count. In one rotation, the seconds hand covers 30 seconds (marked by black colour) then in the second rotation another 30 seconds are covered (marked by red colour), therefore, the least count is 0.1 second.

## (b) Stop-Clock

The least count of a stop-watch is generally about 0.1 s [Fig. E 6.1(b)] while that of a stop-clock is 1 s , so for more accurate measurement of time intervals in a school laboratory, a stop-watch is preferred. Digital stop-watches are also available now. These watches may be started by pressing the button and can be stopped by pressing the same button
once again. The lapsed time interval is directly displayed by the watch.

## TERMS AND DEFINITIONS

1. Second's pendulum: It is a pendulum which takes precisely one second to move from one extreme position to other. Thus, its times period is precisely 2 seconds.
2. Simple pendulum: A point mass suspended by an inextensible, mass less string from a rigid point support. In practice a small heavy


Fig.E 6.1(b): Stop - Clock spherical bob of high density material of radius $r$, much smaller than the length of the suspension, is suspended by a light, flexible and strong string/thread supported at the other end firmly with a clamp stand. Fig. E 6.2 is a good approximation to an ideal simple pendulum.
3. Effective length of the pendulum: The distance $L$ between the point of suspension and the centre of spherical bob (centre of gravity), $L=l+r+e$, is also called the effective length where $l$ is the length of the string from the top of the bob to the hook, $e$, the length of the hook and $r$ the radius of the bob.


The simple pendulum executes Simple Harmonic Motion (SHM) as the acceleration of the pendulum bob is directly proportional to its displacement from the mean position and is always directed towards it.
The time period (T) of a simple pendulum for oscillations of small amplitude, is given by the relation

$$
T=2 \pi \sqrt{L / g}
$$

(E 6.1)
where $L$ is the length of the pendulum, and $g$ is the acceleration due to gravity at the place of experiment.
Eq. (6.1) may be rewritten as

$$
T^{2}=\frac{4 \pi^{2} L}{g}
$$

1. Place the clamp stand on the table. Tie the hook, attached to the pendulum bob, to one end of the string of about 150 cm in length. Pass the other end of the string through two half-pieces of a split cork.
2. Clamp the split cork firmly in the clamp stand such that the line of separation of the two pieces of the split cork is at right angles to the line OA along which the pendulum oscillates [Fig. E 6.2(a)]. Mark, with a piece of chalk or ink, on the edge of the table a vertical line parallel to and just behind the vertical thread OA, the position of the bob at rest. Take care that the bob hangs vertically (about 2 cm above the floor) beyond the edge of the table so that it is free to oscillate.
3. Measure the effective length of simple pendulum as shown in Fig. E 6.2(b).


Fig.E 6.2 (a): A simple pendulum; $B$ and $C$ show the extreme positions

Fig.E 6.2 (b): Effective length of a simple pendulum
4. Displace the bob to one side, not more than 15 degrees angular displacement, from the vertical position OA and then release it gently. In case you find that the stand is shaky, put some heavy object on its base. Make sure that the bob starts oscillating in a vertical plane about its rest (or mean) position OA and does not (i) spin about its own axis, or (ii) move up and down while oscillating, or (iii) revolve in an elliptic path around its mean position.
5. Keep the pendulum oscillating for some time. After completion of a few oscillations, start the stop-watch/clock as the thread attached to the pendulum bob just crosses its mean position (say, from left to right). Count it as zero oscillation.
6. Keep on counting oscillations $1,2,3, \ldots, n$, everytime the bob crosses the mean position OA in the same direction (from left to right).

Stop the stop-watch/clock, at the count $n$ (say, 20 or 25 ) of oscillations, i.e., just when $n$ oscillations are complete. For better results, $n$ should be chosen such that the time taken for $n$ oscillations is 50 s or more. Read, the total time ( $t$ ) taken by the bob for $n$ oscillations. Repeat this observation a few times by noting the time for same number ( $n$ ) of oscillations. Take the mean of these readings. Compute the time for one oscillation, i.e., the time period $T(=t / n)$ of the pendulum.
7. Change the length of the pendulum, by about 10 cm . Repeat the step 6 again for finding the time (t) for about 20 oscillations or more for the new length and find the mean time period. Take 5 or 6 more observations for different lengths of penduLum and find mean time period in each case.
8. Record observations in the tabular form with proper units and significant figures.
9. Take effective length $L$ along x -axis and $T^{2}$ (or $T$ ) along y -axis, using the observed values from Table E 6.1. Choose suitable scales on these axes to represent $L$ and $T^{2}$ (or $T$ ). Plot a graph between $L$ and $T^{2}$ (as shown in Fig. E 6.4) and also between $L$ and $T$ (as shown in Fig. E 6.3). What are the shapes of $L-T^{2}$ graph and $L-T$ graph? Identify these shapes.

## ObSERVATIONS

(i) Radius ( $r$ ) of the pendulum bob (given)

Length of the hook (given) (e)
Least count of the metre scale
Least count of the stop-watch/clock
$=. . \mathrm{cm}$
= ... cm
$=\ldots \mathrm{mm}=\ldots \mathrm{cm}$
= ... s

Table E 6.1: Measuring the time period $T$ and effective length $L$ of the simple pendulum

| S. No. | Length of the string from the top of the bob to the point of suspension $l$ | $\begin{gathered} \text { Effective } \\ \text { length, } L= \\ (l+r+e) \end{gathered}$ |  | Number of oscillations counted, $n$ | Time |  | $\begin{gathered} n \mathrm{osc} \\ t(\mathrm{~s}) \end{gathered}$ | ations | $\begin{aligned} & \text { Time } \\ & \text { period } T \\ & (=t / n) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | cm | (cm) | m |  | (i)... | (ii) | (iii) | $\begin{aligned} & \text { Mean } \\ & t(\mathrm{~s}) \end{aligned}$ | s |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

## (i) $L$ vs $T$ graphs

Plot a graph between $L$ versus $T$ from observations recorded in Table E 6.1, taking $L$ along x -axis and $T$ along y -axis. You will find that this graph is a curve, which is part of a parabola as shown in Fig. E 6.3.

## (ii) $L$ vs $T^{2}$ graph

Plot a graph between $L$ versus $T^{2}$ from observations recorded in Table E 6.1, taking $L$ along x -axis and $T^{2}$ along y -axis. You will find that the graph is a straight line passing through origin as shown in Fig. E. 6.4.
(iii) From the $T^{2}$ versus $L$ graph locate the effective length of second's pendulum for $T^{2}=4 \mathrm{~s}^{2}$.


Fig. E 6.3: Graph of $L$ vs $T$


Fig. E 6.4: Graph $L$ vs $T^{2}$

1. The graph $L$ versus $T$ is curved, convex upwards.
2. The graph $L$ versus $T^{2}$ is a straight line.
3. The effective length of second's pendulum from $L$ versus $T^{2}$ graph is ... cm.

Note : The radius of bob may be found from its measured diameter with the help of callipers by placing the pendulum bob between the two jaws of (a) ordinary callipers, or (b) Vernier Callipers, as described in Experiment E 1.1 (a). It can also be found by placing the spherical bob between two parallel card boards and measuring the spacing (diameter) or distance between them with a metre scale.

1. The accuracy of the result for the length of second's pendulum depends mainly on the accuracy in measurement of effective length (using metre scale) and the time period $T$ of the pendulum (using stop-watch). As the time period appears as $T^{2}$ in Eq. E 6.2, a small uncertainty in the measurement of $T$ would result in appreciable error in $T^{2}$, thereby significantly affecting the result. A stop-watch with accuracy of 0.1 s may be preferred over a less accurate stop-watch/clock.
2. Some personal error is always likely to be involved due to stop-watch not being started or stopped exactly at the instant the bob crosses the mean position. Take special care that you start and stop the stop-watch at the instant when pendulum bob just crosses the mean position in the same direction.
3. Sometimes air currents may not be completely eliminated. This may result in conical motion of the bob, instead of its motion in vertical plane. The spin or conical motion of the bob may cause a twist in the thread, thereby affecting the time period. Take special care that the bob, when it is taken to one side of the rest position, is released very gently.
4. To suspend the bob from the rigid support, use a thin, light, strong, unspun cotton thread instead of nylon string. Elasticity of the string is likely to cause some error in the effective length of the pendulum.
5. The simple pendulum swings to and fro in SHM about the mean, equilibrium position. Eq. (E 6.1) that expresses the relation between $T$ and $L$ as $T=2 \pi \sqrt{L / g}$, holds strictly true for small amplitude or swing $\theta$ of the pendulum.

Remember that this relation is based on the assumption that sin $\theta \approx \theta$, (expressed in radian) holds only for small angular displacement $\theta$.
6. Buoyancy of air and viscous drag due to air slightly increase the time period of the pendulum. The effect can be greatly reduced to a large extent by taking a small, heavy bob of high density material (such as iron/ steel/brass).

## Self Assessment

1. Interpret the graphs between $L$ and $T^{2}$, and also between $L$ and $T$ that you have drawn for a simple pendulum.
2. Examine, using Table E 6.1, how the time period Tchanges as the
effective length $L$ of a simple pendulum; becomes 2 -fold, 4-fold, and so on.
3. How can you determine the value of ' $g$ ', acceleration due to gravity, from the $T^{2}$ vs $L$ graph?

## SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. To determine ' $g$ ', the acceleration due to gravity, at a given place, from the $L-T^{2}$ graph, for a simple pendulum.
2. Studying the effect of size of the bob on the time period of the simple pendulum.
[Hint: With the same experimental set-up, take a few spherical bobs of same material (density) but of different sizes (diameters). Keep the length of the pendulum the same for each case. Clamp the bobs one by one, and starting from a small angular displacement of about $10^{\circ}$, each time measure the time for 50 oscillations. Find out the time period of the pendulum using bobs of different sizes. Compensate for difference in diameter of the bob by adjusting the length of the thread.

Does the time period depend on the size of the pendulum bob? If yes, see the order in which the change occurs.]
3. Studying the effect of material (density) of the bob on the time period of the simple pendulum.
[Hint: With the same experimental set-up, take a few spherical bobs (balls) of different materials, but of same size. Keep the length of the pendulum the same for each case. Find out, in each case starting from a small angular displacement of about $10^{\circ}$, the time period of the pendulum using bobs of different materials,
Does the time period depend on the material (density) of the pendulum bob? If yes, see the order in which the change occurs. If not, then do you see an additional reason to use the pendulum for time measurement.]
4. Studying the effect of mass of the bob on the time period of the simple pendulum.
[Hint: With the same experimental set-up, take a few bobs of different materials (different masses) but of same size. Keep the length of the pendulum same for each case. Starting from a small angular displacement of about $10^{\circ}$ find out, in each case, the time period of the pendulum, using bobs of different masses.

Does the time period depend on the mass of the pendulum bob? If yes, then see the order in which the change occurs. If not, then do you see an additional reason to use the pendulum as a time measuring device.]
5. Studying the effect of amplitude of oscillation on the time period of the simple pendulum.
[Hint: With the same experimental set-up, keep the mass of the bob and length of the pendulum fixed. For measuring the angular amplitude, make a large protractor on the cardboard and have a scale marked on an arc from $0^{\circ}$ to $90^{\circ}$ in units of $5^{\circ}$. Fix it on the edge of a table by two drawing pins such that its $0^{\circ}$ - line coincides
with the suspension thread of the pendulum at rest. Start the pendulum oscillating with a very large angular amplitude (say $70^{\circ}$ ) and find the time period $T$ of the pendulum. Change the amplitude of oscillation of the bob in small steps of $5^{\circ}$ or $10^{\circ}$ and determine the time period in each case till the amplitude becomes small (say $5^{\circ}$ ). Draw a graph between angular amplitude and $T$. How does the time period of the pendulum change with the amplitude of oscillation?

How much does the value of $T$ for $A=10^{\circ}$ differ from that for $A=$ $50^{\circ}$ from the graph you have drawn?

Find at what amplitude of oscillation , the time period begins to vary?
Determine the limit for the pendulum when it ceases to be a simple pendulum.]
6. Studying the effect on time period of a pendulum having a bob of varying mass (e.g. by filling the hollow bob with sand, sand being drained out in steps)
[Hint: The change in $T$, if any, in this experiment will be so small that it will not be possible to measure it due to the following reasons:

The centre of gravity (CG) of a hollow sphere is at the centre of the sphere. The length of this simple pendulum will be same as that of a solid sphere (same size) or that of the hollow sphere filled completely with sand (solid sphere).

Drain out some sand from the sphere. The situation is as shown in Fig. E. 6.5. The CG of bob now goes down to point say A. The effective length of the pendulum increases and therefore the $T_{A}$ increases ( $T_{A}>T_{O}$ ), some more sand is drained out, the CG goes down further to a point $B$. The effective length further increases, increasing $T$.

The process continues and $L$ and $T$ change in the same direction (increasing), until finally the entire sand is drained out. The bob is now a hollow sphere with CG shifting back to centre C . The time period will now become $T_{0}$ again.]


Fig. E 6.5: Variation of centre of gravity of sand filled hollow bob on time period of the pendulum; sand being drained out of the bob in steps.

## EXPERIMENT

To study the relation between force of limiting friction and normal reaction and to find the coefficient of friction between surface of a moving block and that of a horizontal surface.

## Apparatus and material required

A wooden block with a hook, a horizontal plane with a glass or laminated table top (the table top itself may be used as a horizontal plane), a frictionless pulley which can be fixed at the edge of the horizontal table/plane, spirit level, a scale, pan, thread or string, spring balance, weight box and five masses of 100 g each.

## $\lceil$ ERMS AND DEFINITIONS

Friction: The tendency to oppose the relative motion between two surfaces in contact is called friction.

Static Friction: It is the frictional force acting between two solid surfaces in contact at rest but having a tendency to move (slide) with respect to each other.
Limiting Friction: It is the maximum value of force of static friction when one body is at the verge of sliding with respect to the other body in contact.

Kinetic (or Dynamic) Friction: It is the frictional force acting between two solid surfaces in contact when they are in relative motion.

## $)_{\text {RINCIPLE }}$

The maximum force of static friction, i.e., limiting friction, $\mathrm{F}_{\mathrm{L}}$ between two dry, clean and unlubricated solid surfaces is found to obey the following empirical laws:
(i) The limiting friction is


Fig. E 7.1: The body is at rest due to static friction
directly proportional to the normal reaction, $R$, which is given by the total weight $W$ of the body (Fig. E 7.1). The line of action is same for both $W$ and $R$ for horizontal surface,

$$
\begin{aligned}
& F_{L} \propto R \Rightarrow F_{L}=\mu_{L} R \\
& \text { i.e. } \mu_{L}=\frac{F_{L}}{R}
\end{aligned}
$$

Thus, the ratio of the magnitude of the limiting friction, $F_{L}$, to the magnitude of the normal force, $R$, is a constant known as the coefficient of limiting friction $\left(\mu_{L}\right)$ for the given pair of surfaces in contact.
(ii) The limiting friction depends upon the nature of surfaces in contact and is nearly independent of the surface area of contact over wide limits so long as normal reaction remains constant.

Note that $F_{L}=\mu_{L} R$ is an equation of a straight line passing through the origin. Thus, the slope of the straight-line graph between $F_{l}$ (along Yaxis) and $R$ (along Xaxis) will give the value of coefficient of limiting friction $\mu_{L}$.
In this experiment, the relationship between the limiting


Fig. E 7.2: Experimental set up to study limiting friction friction and normal reaction is studied for a wooden block. The wooden block is made to slide over a horizontal surface (say glass or a laminated surface) (Fig. E 7.2).

## PROCEDURE

1. Find the range and least count of the spring balance.
2. Measure the mass ( $M$ ) of the given wooden block with hooks on its sides and the scale pan ( m ) with the help of the spring balance.
3. Place the glass (or a laminated sheet) on a table and make it horizontal, if required, by inserting a few sheets of paper or cardboard below it. To ensure that the table-top surface is horizontal use a spirit level. Take care that the top surface must be clean and dry.
4. Fix a frictionless pulley on one edge of table-top as shown in Fig. E 7.2. Lubricate the pulley if need be.
5. Tie one end of a string of suitable length (in accordance with the size and the height of the table) to a scale pan and tie its other end to the hook of the wooden block.
6. Place the wooden block on the horizontal plane and pass the string over the pulley (Fig. E7.2). Ensure that the portion of the string between pulley and the wooden block is horizontal. This can be done by adjusting the height of the pulley to the level of hook of block.
7. Put some mass $(q)$ on the scale pan. Tap the table-top gently with your finger. Check whether the wooden block starts moving.
8. Keep on increasing the mass $(q)$ on the scale pan till the wooden block just starts moving on gently tapping the glass top. Record the total mass kept on the scale pan in Table E 7.1.
9. Place some known mass (say p) on the top of wooden block and adjust the mass $(q)$ on the scale pan so that the wooden block alongwith mass $p$ just begins to slide on gently tapping the table top. Record the values of $p^{\prime}$ and $q^{\prime}$ in Table E 7.1.
10. Repeat step 9 for three or four more values of $p$ and record the corresponding values of $q$ in Table E 7.1. A minimum of five observations may be required for plotting a graph between $F_{L}$ and $R$.
$\mathrm{O}_{\text {bSERVATIONS }}$
11. Range of spring balance
$=$... to ... g
12. Least count of spring balance
$=. . . \mathrm{g}$
13. Mass of the scale pan, $(m)$
$=\ldots \mathrm{g}$
14. Mass of the wooden block ( $M$ )
$=. . . \mathrm{g}$
15. Acceleration due to gravity $(g)$ at the place of experiment $=\ldots \mathrm{m} / \mathrm{s}^{2}$

Table E 7.1: Variation of Limiting Friction with Normal Reaction

| S. No. | Mass on the wooden block (p) (g) |  | Normal force $R$ due to mass $(M+p)$ |  | Mass on the pan (q) g | Force of limiting friction $F_{L}$ | Coefficient of friction $\mu_{L}=\frac{F_{L}}{R}$ | Mean $\mu_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (g) | (kg) | N | (g) | (kg) | (N) |  |  |
| $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \end{aligned}$ |  |  |  |  |  |  |  |  |

Plot a graph between the limiting friction $\left(F_{L}\right)$ and normal force $(R)$ between the wooden block and the horizontal surface, taking the limiting friction $F_{L}$ along the $y$-axis and normal force $R$ along the x -axis. Draw a line to join all the points marked on it (Fig. E 7.3). Some points may not lie on the straight-line graph and may be on either side of it. Extend the straight line backwards to check whether the graph passes through the origin. The slope of this straight-line graph gives the coefficient of limiting friction $\left(\mu_{L}\right)$ between the wooden block and the horizontal surface. To find the slope of straight line, choose two points A and B that are far apart from each other on the straight line as shown in Fig. E 7.3. Draw a line parallel to x-axis through point A and another line parallel to $y$-axis through point $B$. Let point $Z$ be the point of intersection of these two lines. Then, the slope $\mu_{\mathrm{L}}$ of


Fig. E 7.3: Graph between force of limiting friction $F_{L}$ and normal reaction, $R$ straight line graph AB would be

$$
\mu_{L}=\frac{F_{L}}{R}=\frac{B Z}{A Z}
$$

## $R_{\text {ESULT }}$

The value of coefficient of limiting friction $\mu_{\mathrm{L}}$ between surface of wooden block and the table-top (laminated sheet/glass) is:
(i) From calculation (Table E 7.1) = ...
(ii) From graph = .. PRECAUTIONS

1. Surface of the table should be horizontal and dust free.
2. Thread connecting wooden block and pulley should be horizontal.
3. Friction of the pulley should be reduced by proper oiling.
4. Table top should always be tapped gently.

## Sources of error

1. Always put the mass at the centre of wooden block.
2. Surface must be dust free and dry.
3. The thread must be unstretchable and unspun.
4. The friction depends on the roughness of the surfaces in contact. If the surfaces in contact are ideally (perfactly) smooth, there would be no friction between the two surfaces. However, there cannot be an ideally smooth surface as the distribution of atoms or molecules on solid surface results in an inherent roughness.
5. In this experimental set up and calculations, friction at the pulley has been neglected, therefore, as far as possible, the pulley, should have minimum friction as it cannot be frictionless.
6. The presence of dust particles between the wooden block and horizontal plane surface may affect friction and therefore lead to errors in observations. Therefore, the surface of the horizontal plane and wooden block in contact must be clean and dust free.
7. The presence of water or moisture between the wooden block and the plane horizontal surface would change the nature of the surface. Thus, while studying the friction between the surface of the moving body and horizontal plane these must be kept dry.
8. Elasticity of the string may cause some error in the observation. Therefore, a thin, light, strong and unspun cotton thread must be used as a string to join the scale pan and the moving block.
9. The portion of string between the pulley and wooden block must be horizontal otherwise only a component of tension in the string would act as the force to move the block.
10. It is important to make a judicious choice of the size of the block and set of masses for this experiment. If the block is too light, its force of limiting friction may be even less than the weight of empty pan and in this situation, the observation cannot be taken with the block alone. Similarly, the maximum mass on the block, which can be obtained by putting separate masses on it, should not be very large otherwise it would require a large force to make the block move.
11. The additional mass, $p$, should always be put at the centre of wooden block.
12. The permissible error in measurements of coefficient of friction

$$
=\frac{\Delta F_{L}}{F_{L}}+\frac{\Delta R}{R}=\ldots
$$

1. On the basis of your observations, find the relation between limiting friction and the mass of sliding body.
2. Why do we not choose a spherical body to study the limiting friction between the two surfaces?
3. Why should the horizontal surfaces be clean and dry?
4. Why should the portion of thread between the moving body and pulley be horizontal?
5. Why is it essential in this experiment to ensure that the surface on which the block moves should be horizontal?
6. Comment on the statement: "The friction between two surfaces can never be zero".
7. In this experiment, usually unpolished surfaces are preferred, why?
8. What do you understand by self-adjusting nature of force of friction?
9. In an experiment to study the relation between force of limiting friction and normal reaction, a body just starts sliding on applying a force of 3 N . What will be the magnitude of force of friction acting on the body when the applied forces on it are $0.5 \mathrm{~N}, 1.0 \mathrm{~N}$, $2.5 \mathrm{~N}, 3.5 \mathrm{~N}$, respectively.

## SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. To study the effect of the nature of sliding surface. [Hint: Repeat the same experiment for different types of surfaces say, plywood, carpet etc. Or repeat the experiment after putting oil or powder on the surface.]
2. To study the effect of changing the area of the surfaces in contact. [Hint: Place the wooden block vertically and repeat the experiment. Discuss whether the readings and result of the experiment are same.]
3. To find the coefficient of limiting friction by sliding the block on an inclined plane.
