

# EXPERIMENT 10

## A<sub>IM</sub>

To find the force constant and effective mass of a helical spring by plotting  $T^2 - m$  graph using method of oscillation.

## A<sub>PPARATUS AND MATERIAL REQUIRED</sub>

Light weight helical spring with a pointer attached at the lower end and a hook/ring for suspending it from a hanger, (diameter of the spring may be about 1-1.5 cm inside or same as that in a spring balance of 100 g); a rigid support, hanger and five slotted weights of 10 g each (in case the spring constant is of high value one may use slotted weight of 20 g), clamp stand, a balance, a measuring scale (15-30 cm) and a stop-watch (with least count of 0.1s).

## P<sub>RINCIPLE</sub>

Spring constant (or force constant) of a spring is given by

$$\text{Spring constant, } K = \frac{\text{Restoring Force}}{\text{Extension}} \quad \text{(E 10.1)}$$

Thus, spring constant is the restoring force per unit extension in the spring. Its value is determined by the elastic properties of the spring. A given object is attached to the free end of a spring which is suspended from a rigid point support (a nail, fixed to a wall). If the object is pulled down and then released, it executes simple harmonic oscillations.

The time period ( $T$ ) of oscillations of a helical spring of spring constant  $K$  is given by the relation  $T$ ,

$T = 2\pi\sqrt{\frac{m}{K}}$  where  $m$  is the load that is the mass of the object. If the spring has a large mass of its own, the expression changes to

$$T = 2\pi\sqrt{\frac{m_o + m}{K}} \quad \text{(E 10.2)}$$

where  $m_0$  and  $m$  define the effective mass of the spring system (the spring along with the pointer and the hanger) and the suspended object (load) respectively. The time period of a stiff spring (having large spring constant) is small.

One can easily eliminate the term  $m_0$  of the spring system appearing in Eq. (E 10.2) by suspending two different objects (loads) of masses  $m_1$  and  $m_2$  and measuring their respective periods of oscillations  $T_1$  and  $T_2$ . Then,

(E 10.3)

$$T_1 = 2\pi\sqrt{\frac{m_0 + m_1}{K}}$$

(E 10.4)

and

$$T_2 = 2\pi\sqrt{\frac{m_0 + m_2}{K}}$$

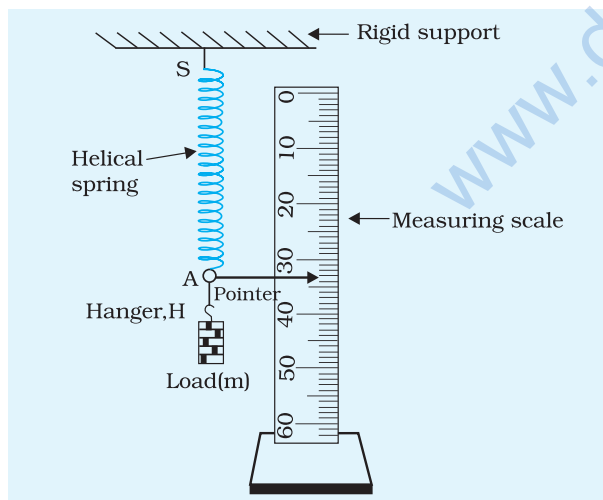
Eliminating  $m_0$  from Eqs. (E 2.3) and (E 2.4), we get

(E 10.5)

$$K = \frac{4\pi^2(m_1 - m_2)}{(T_1^2 - T_2^2)}$$

Using Eq. (E 10.5), and knowing the values of  $m_1$ ,  $m_2$ ,  $T_1$  and  $T_2$ , the spring constant  $K$  of the spring system can be determined.

## PROCEDURE



**Fig. E 10.1:** Experimental arrangement for studying spring constant of a helical spring

1. Suspend the helical spring SA (having pointer P and the hanger H at its free end A), from a rigid support, as shown in Fig. E 10.1.
2. Set the measuring scale, close to the spring vertically. Take care that the pointer P moves freely over the scale without touching it.
3. Find out the least count of the measuring scale (It is usually 1mm or 0.1 cm).
4. Familiarise yourself with the working of the stop-watch and find its least count.
5. Suspend the load or slotted weight with mass  $m_1$  on the hanger gently. Wait till the pointer comes to rest. This is the equilibrium position for the given load. Pull the load slightly downwards and then release it gently so that it is set into oscillations in a vertical plane

about its rest (or equilibrium) position. The rest position ( $x$ ) of the pointer P on the scale is the reference or mean position for the given load. Start the stop-watch as the pointer P just crosses its mean position (say, from upwards to downwards) and simultaneously begin to count the oscillations.

6. Keep on counting the oscillations as the pointer crosses the mean position ( $x$ ) in the same direction. Stop the watch after  $n$  (say, 5 to 10) oscillations are complete. Note the time ( $t$ ) taken by the oscillating load for  $n$  oscillations.
7. Repeat this observation atleast thrice and in each occasion note the time taken for the same number ( $n$ ) of oscillations. Find the mean time ( $t_1$ ), for  $n$  oscillations and compute the time for one oscillation, i.e., the time period  $T_1 (= t_1/n)$  of oscillating helical spring with a load  $m_1$ .
8. Repeat steps 5 and 6 for two more slotted weights.
9. Calculate time period of oscillation  $T = \frac{t}{n}$  for each weight and tabulate your observations.
10. Compute the value of spring constant ( $K_1, K_2, K_3$ ) for each load and find out the mean value of spring constant  $K$  of the given helical spring.
11. The value of  $K$  can also be determined by plotting a graph of  $T^2$  vs  $m$  with  $T^2$  on y-axis and  $m$  on x-axis.

**[Note:** The number of oscillations,  $n$ , should be large enough to keep the error minimum in measurement of time. One convenient method to decide on the number  $n$  is based on the least count of the stop-watch. If the least count of the stop-watch is 0.1 s. Then to have 1% error in measurement, the minimum time measured should be 10.0 s. Hence, the number  $n$  for oscillations should be so chosen that oscillating mass takes more than 10.0 s to complete them.]

## OBSERVATIONS

Least count of the measuring scale = ... mm = ... cm

Least count of the stop-watch = ... s

Mass of load 1,  $m_1 = \dots$  g = ... kg

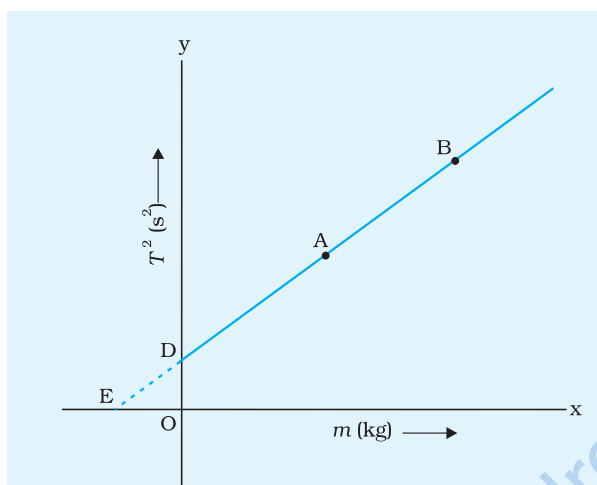
Mass of load 2,  $m_2 = \dots$  g = ... kg

Mass of load 3,  $m_3 = \dots$  g = ... kg

**Table E 10.1: Measuring the time period  $T$  of oscillations of helical spring with load**

S. No.	Mass of the load, $m$ (kg)	Mean position of pointer, $x$ (cm)	No. of oscillations, ( $n$ )	Time for ( $n$ ) oscillations, $t$ (s)				Time period, $T = t/n$ (s)
				1	2	3	Mean $t$ (s)	

## CALCULATION


**Fig. E 10.2:** Expected graph between  $T^2$  and  $m$  for a helical spring

Substitute the values of  $m_1$ ,  $m_2$ ,  $m_3$  and  $T_1$ ,  $T_2$ ,  $T_3$ , in Eq. (E 10.5):

$$K_1 = 4\pi^2 (m_1 - m_2) / (T_1^2 - T_2^2);$$

$$K_2 = 4\pi^2 (m_2 - m_3) / (T_2^2 - T_3^2);$$

$$K_3 = 4\pi^2 (m_1 - m_3) / (T_1^2 - T_3^2)$$

Compute the values of  $K_1$ ,  $K_2$  and  $K_3$  and find the mean value of spring constant  $K$  of the given helical spring. Express the result in proper SI units and significant figures.

Alternately one can also find the spring constant and effective mass of the spring from the graph between  $T^2$  and  $m$ , which is expected to be a straight line as shown in Fig. E 10.2.

The value of spring constant  $K (= 4\pi^2 / m')$  of the helical spring can be calculated from the slope  $m'$  of the straight line graph.

From the knowledge of intercept  $c$  on  $y$ -axis and the slope  $m$ , the value of effective mass  $m_0 (= c/m')$  of the helical spring can be computed. Alternatively, the effective mass  $m_0 (= -c')$  of the helical spring can be directly computed from the knowledge of the intercept  $c'$  made by the straight line on  $x$ -axis.

## RESULT

Spring constant of the given helical spring = ...  $\text{N/m}^{-1}$

Effective mass of helical spring = ...  $\text{g} = \dots \text{kg}$

Error in  $K$ , can be calculated from the error in slope

$$\frac{\Delta K}{K} = \frac{\Delta \text{slope}}{\text{slope}}$$

The error in effective mass  $m_0$  will be equal to the error in intercept and the error in slope. Once the error is calculated the result may be stated indicating the error.

## DISCUSSION

1. The accuracy in determination of the spring constant depends mainly on the accuracy in measurement of the time period  $T$  of oscillation of the spring. As the time period appears as  $T^2$  in Eq. (E 10.5), a small uncertainty in the measurement of  $T$  would result in appreciable error in  $T^2$ , thereby significantly affecting the result. A stop-watch with accuracy of 0.1s may be preferred.
2. Some personal error is always likely to occur in measurement of time due to delay in starting or stopping the watch.
3. Sometimes air currents may affect the oscillations thereby affecting the time period. The time period of oscillation may also get affected if the load is released with a jerk. Take special care that the load while being taken to one side (upwards or downwards) of the rest (or mean) position, is released very gently.
4. The load attached to the spring executes to and fro motion (in SHM) about the mean, equilibrium position. Eqs. (E 10.1) and (E 10.2) hold true for small amplitude of oscillations or small extensions of the spring within the elastic limit (Hook's law). Take care that initially the load is pulled only through a small distance before being released gently to let it oscillate vertically.
5. Oscillations of the helical spring are not likely to be absolutely undamped. Buoyancy of air and viscous drag due to it may slightly increase the time period of the oscillations. The effect can be greatly reduced by taking a small and stiff spring of high density material (such as steel/brass).
6. A rigid support is required for suspending the helical spring. The slotted weights may not have exactly the same mass as engraved on them. Some error in the time period of its oscillation is likely to creep in due to yielding (sometimes) of the support and inaccuracy in the accepted value of mass of load.

## SELF-ASSESSMENT

1. Two springs A (soft) and B (stiff), loaded with the same mass on their hangers, are suspended one by one from the same rigid support. They are set into vertical oscillations at different times, and the time period of their oscillations are noted. In which spring will the oscillations be slower?

2. You are given six known masses ( $m_1, m_2, \dots, m_6$ ), a helical spring and a stop-watch. You are asked to measure time periods ( $T_1, T_2, \dots, T_6$ ) of oscillations corresponding to each mass when it is suspended from the given helical spring.
- What is the shape of the curve you would expect by using Eq. (E 10.2) and plotting a graph between load of mass  $m$  along x-axis and  $T^2$  on y-axis?
  - Interpret the slope, the x-and y-intercepts of the above graph, and hence find (i) spring constant  $K$  of the helical spring, and (ii) its effective mass  $m_0$ .

[Hint: (a) Eq. (E 10.2), rewritten as:  $T^2 = (4\pi^2/K) m + (4\pi^2/K) m_0$ , is similar to the equation of a straight line:  $y = mx + c$ , with  $m$  as the slope of the straight line and  $c$  the intercept on y-axis. The graph between  $m$  and  $T^2$  is expected to be a straight line AB, as shown in Fig. E 10.2. From the above equations given in (a):

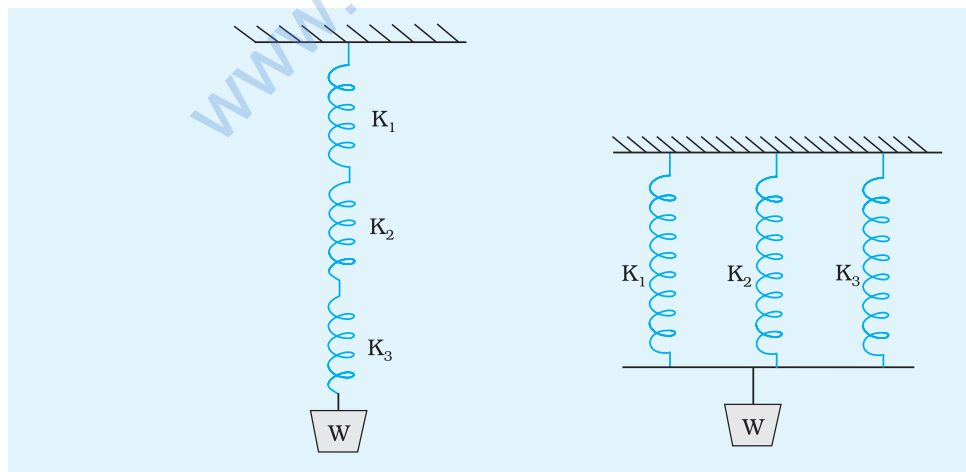
Intercept on y-axis (OD),  $c = (4\pi^2/K) m_0$ ; ( $x = 0, y = c$ )

Intercept on x-axis (OE),  $c' = -c/m' = -m_0$ ; ( $y = 0, x = -c/m'$ )

Slope,  $m' = \tan \theta = OD/OE = c/c' = -c/m_0 = (4\pi^2/K)$

#### SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

- Take three springs with different spring constants  $K_1, K_2, K_3$  and join them in series as shown in Fig. E. 10.3. Determine the time period of oscillation of combined spring and check the relation between individual spring constant and combined system.
- Repeat the above activity with the set up shown in Fig. E. 10.4 and find out whether there is any difference in the time period and spring constant between the two set ups?
- What is the physical significance of spring constant  $20.5 \text{ Nm}^{-1}$ ?
- If possible, measure the mass of the spring. Is this related to the effective mass  $m_0$ ?



**Fig. E 10.3:** Springs joined in series

**Fig. E 10.4:** Springs joined in parallel